

CPSC-354 Report

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Abstract

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1 Introduction

2 Week by Week

2.1 Week 1

Notes

Learned about some tactics and theorems

rfl: a tactic that proves theorems that take the form of $X = X$

rw: a tactic that rewrites a proof

one_eq_succ_zero: a theorem that proves $1 = \text{succ } 0$ (there are also other similar existing theorems like two_eq_succ_one and so on)

add_zero: a theorem that proves $a + 0 = a$.

add_succ: a theorem that proves $a + \text{succ } b = \text{succ}(a + b)$

succ_eq_add_one: a theorem that proves $\text{succ } a = a + 1$

Homework

Problem 5:

a, b, c are in the set of natural numbers.

Prove that both sides are equal to each other.

$$a + (b + 0) + (c + 0) = a + b + c$$

rw [add_zero] - uses the add_zero theorem to prove that $b + 0 = b$

This is rewritten to:

$$a + b + (c + 0) = a + b + c$$

rw [add_zero] - this is done again to prove that $c + 0 = c$

This is rewritten to:

$$a + b + c = a + b + c$$

rfl - this proves that both sides that look the same are equal to each other

Problem 6:

This is the same problem as 5 but will be approached in a different manner.

$$a + (b + 0) + (c + 0) = a + b + c$$

rw [add_zero c] - specifically applies the add_zero theorem to c, making $c + 0$ into c

This is rewritten to:

$$a + (b + 0) + c = a + b + c$$

rw [add_zero b] - specifically applies the add_zero theorem to b, making $b + 0$ into b

This is rewritten to:

$$a + b + c = a + b + c$$

rfl - this proves that both sides that look the same are equal to each other

Problem 7:

n is in the set of natural numbers.

Prove that both sides are equal to each other.

succ n = n + 1 rw[one_eq_succ_zero] - rewrite 1 into successor 0 This is rewritten to:

$$\text{succ } n = n + \text{succ } 0$$

rw[add_succ] - uses the add_succ theorem to change $n + \text{succ } 0$ into $\text{succ}(n + 0)$

This is rewritten to:

$$\text{succ } n = \text{succ } (n + 0)$$

rw[add_zero] - uses the add_zero theorem to prove that $n + 0 = n$

This is rewritten to:

$$\text{succ } n = \text{succ } n$$

rfl - this proves that both sides that look the same are equal to each other

Problem 8:

Prove that both sides are equal to each other.

$$2 + 2 = 4$$

rw[two_eq_succ_one] - rewrites 2 into succ 1

This is rewritten to:

$$\text{succ } 1 + \text{succ } 1 = 4$$

rw[one_eq_succ_zero] - rewrites 1 into succ 0

This is rewritten to:

$$\text{succ } (\text{succ } 0) + \text{succ } (\text{succ } 0) = 4$$

rw[four_eq_succ_three] - rewrites 4 into succ 3

This is rewritten to:

$$\text{succ } (\text{succ } 0) + \text{succ } (\text{succ } 0) = \text{succ } 3$$

rw[three_eq_succ_two] - rewrites 3 into succ 2

This is rewritten to:

$$\text{succ } (\text{succ } 0) + \text{succ } (\text{succ } 0) = \text{succ } (\text{succ } 2)$$

rw[two_eq_succ_one] - rewrites 2 into succ 1

This is rewritten to:

$\text{succ} (\text{succ } 0) + \text{succ} (\text{succ } 0) = \text{succ} (\text{succ} (\text{succ } 1))$

`rw[one_eq_succ.zero]` - rewrites 1 into `succ 0`

This is rewritten to:

$\text{succ} (\text{succ } 0) + \text{succ} (\text{succ } 0) = \text{succ} (\text{succ} (\text{succ} (\text{succ } 0)))$

`rw[add_succ]` - changes $\text{succ} (\text{succ } 0) + \text{succ} (\text{succ } 0)$ into $\text{succ} (\text{succ} (\text{succ } 0) + \text{succ } 0)$

This is rewritten to:

$\text{succ} (\text{succ} (\text{succ } 0) + \text{succ } 0) = \text{succ} (\text{succ} (\text{succ} (\text{succ } 0)))$

`rw[add_succ]` - changes $\text{succ} (\text{succ} (\text{succ } 0) + \text{succ } 0)$ into $\text{succ} (\text{succ} (\text{succ} (\text{succ } 0) + 0))$

This is rewritten to:

$\text{succ} (\text{succ} (\text{succ} (\text{succ } 0) + 0)) = \text{succ} (\text{succ} (\text{succ} (\text{succ } 0)))$

`rw[add_zero]` - changes $\text{succ} (\text{succ } 0) + 0$ into

This is rewritten to:

$\text{succ} (\text{succ} (\text{succ} (\text{succ } 0))) = \text{succ} (\text{succ} (\text{succ} (\text{succ } 0)))$

`rfl` - this proves that both sides that look the same are equal to each other

For level 5: `add.zero` is a Lean proof that $a + 0 = a$ (a representing any number). In mathematics, there are laws for arithmetic. One of them is called the identity which applies to addition and multiplication. For addition, it states that $m + 0 = m = 0 + m$. This is the exact same as the Lean proof, $a + 0 = a$, which can also be written as $a = 0 + a$.

Comments and Questions

Learning the root of mathematics is very eye-opening, and I am confident it will be the same for programming languages. It provides another perspective for elementary functions like $2 + 2$ equals 4, which is different from just knowing it through memorization. I feel as though this is why people have been able to expand mathematically. This makes me wonder: how can looking through the core of programming help us better current languages (e.g. python, rust)?

2.2 Week 2

Notes

Recursion as a concept using the Towers of Hanoi: It is broken down into: moving a tower of n disks from x to y moving a tower of $n+1$ disks when it is already known how to move a tower of n disks The algorithm is made up of a bunch of "pushes" and "pops" The logic overall is a bunch of back and forth movement of the disks

Lean: induction proof with: induction n with `hd succ.add`: proves that $\text{succ } a + b = \text{succ} (a + b)$
`add.comm x y`: proves that $x + y = y + x$ `add.assoc`: proves that $a + b + c = a + (b + c)$ `add.right_comm a b c`: proves that $a + b + c = a + c + b$

Homework

Problem 1:

n is in the natural number set

Prove $0 + n = n$.

induction n with `hd` - starting a proof by induction

Now our first goal is:

$0 + 0 = 0$

`rw[add_zero]` - proves that $0 + 0 = 0$

This is rewritten to:

$0 = 0$

rfl - this proves that both sides that look the same are equal to each other
 Now, we prove our second goal
 hd: $0 + d = d$
 $0 + \text{succ } d = \text{succ } d$
 rw[add_succ] - proves that $0 + \text{succ } d = \text{succ } (0 + d)$
 This is rewritten to:
 $\text{succ } (0 + d) = \text{succ } d$
 rw[hd] - this replaces $0 + d$ with d
 This is rewritten to:
 $\text{succ } d = \text{succ } d$
 rfl - this proves that both sides that look the same are equal to each other

Problem 2:

a b is in the set of natural numbers
 Prove $\text{succ } a + b = \text{succ } (a + b)$
 inductin b with d hd - starting a proof by induction
 Now our first goal is:
 $\text{succ } a + 0 = \text{succ } (a + 0)$
 rw[add_zero] - proves that $\text{succ } a + 0 = \text{succ } a$
 This is rewritten to:
 $\text{succ } a = \text{succ}(a + 0)$
 rw[add_zero] - proves that $\text{succ } (a + 0) = \text{succ } a$
 This is rewritten to:
 $\text{succ } a = \text{succ } a$
 rfl - this proves that both sides that look the same are equal to each other
 Now, we prove our second goal
 hd: $\text{succ } a + d = \text{succ } (a + d)$
 $\text{succ } a + \text{succ } d = \text{succ } (a + \text{succ } d)$
 rw[add_succ] - proves that $\text{succ } a + \text{succ } d = \text{succ } (\text{succ } a + d)$
 This is rewritten to:
 $\text{succ } (\text{succ } a + d) = \text{succ } (a + \text{succ } d)$
 rw[hd] - this replaces $\text{succ } a + d$ with $\text{succ } (a + d)$
 This is rewritten to:
 $\text{succ } (\text{succ } (a + d)) = \text{succ } (a + \text{succ } d)$
 rw[add_succ] - proves that $\text{succ } (a + \text{succ } d) = \text{succ } (\text{succ } (a + d))$
 This is rewritten to:
 $\text{succ } (\text{succ } (a + d)) = \text{succ } (\text{succ } (a + d))$
 rfl - this proves that both sides that look the same are equal to each other

Problem 3:

a b is in the set of natural numbers
 Prove $a + b = b + a$
 induction b with hd - starting a proof by induction
 Now our first goal is:
 $a + 0 = 0 + a$
 rw[zero_add] - proves that $0 + a = a$
 This is rewritten to:
 $a + 0 = a$
 rw[add_zero] - proves that $a + 0 = a$
 This is rewritten to:

$a = a$
 rfl - this proves that both sides that look the same are equal to each other
 Now, we prove our second goal
 n_ih: $a + \text{hd} = \text{hd} + a$
 $a + \text{succ } \text{hd} = \text{succ } \text{hd} + a$
 rw[add_succ] - proves that $a + \text{succ } \text{hd} = \text{succ } (a + \text{hd})$
 This is rewritten to:
 $\text{succ } (a + \text{hd}) = \text{succ } \text{hd} + a$
 rw[succ_add] - proves that $\text{succ } \text{hd} + a = \text{succ } (\text{hd} + a)$
 This is rewritten to:
 $\text{succ } (a + \text{hd}) = \text{succ } (\text{hd} + a)$
 rw[n_ih] - replaces $\text{succ } (a + \text{hd})$ with $\text{succ } (\text{hd} + a)$
 This is rewritten to:
 $\text{succ } (\text{hd} + a) = \text{succ } (\text{hd} + a)$
 rfl - this proves that both sides that look the same are equal to each other

Problem 4:
 a b c is in the set of natural numbers
 Prove $a + b + c = a + (b + c)$
 induction a with hd - starting a proof by induction
 Now our first goal is:
 $0 + b + c = 0 + (b + c)$
 rw[zero_add] - proves that $0 + b = b$
 This is rewritten to:
 $b + c = 0 + (b + c)$
 rw[zero_add] - proves that $0 + (b + c) = b + c$
 This is rewritten to:
 $b + c = b + c$
 rfl - this proves that both sides that look the same are equal to each other
 Now, we prove our second goal
 n_ih: $\text{hd} + b + c = \text{hd} + (b + c)$
 $\text{succ } \text{hd} + b + c = \text{succ } \text{hd} + (b + c)$
 rw[succ_add] - proves that $\text{succ } \text{hd} + b + c = \text{succ } (\text{hd} + b) + c$
 This is rewritten to:
 $\text{succ } (\text{hd} + b) + c = \text{succ } \text{hd} + (b + c)$
 rw[succ_add] - proves that $\text{succ } (\text{hd} + b) + c = \text{succ } (\text{hd} + b + c)$
 This is rewritten to:
 $\text{succ } (\text{hd} + b + c) = \text{succ } \text{hd} + (b + c)$
 rw[n_ih] - replaces $\text{succ } (\text{hd} + b + c)$ with $\text{succ } (\text{hd} + (b + c))$
 This is rewritten to:
 $\text{succ } (\text{hd} + (b + c)) = \text{succ } \text{hd} + (b + c)$
 rw[succ_add] - proves $\text{succ } \text{hd} + (b + c) = \text{succ } (\text{hd} + (b + c))$
 This is rewritten to:
 $\text{succ } (\text{hd} + (b + c)) = \text{succ } (\text{hd} + (b + c))$
 rfl - this proves that both sides that look the same are equal to each other

Problem 5:
 a b c is in the set of natural numbers
 Prove $a + b + c = a + c + b$
 induction c with hd - starting a proof by induction

Now our first goal is:

$$a + b + 0 = a + 0 + b$$

rw[add_zero] - proves that $b + 0 = b$

This is rewritten to:

$$a + b = a + 0 + b$$

rw[add_zero] 0 proves that $a + 0 = a$

This is rewritten to:

$$a + b = a + b$$

rfl - this proves that both sides that look the same are equal to each other

Now, we prove our second goal

$$\text{n_ih: } a + b + \text{hd} = a + \text{hd} + b$$

$$a + b + \text{succ hd} = a + \text{succ hd} + b$$

rw[add_succ] - proves that $a + b + \text{succ hd} = \text{succ}(a + b + \text{hd})$

This is rewritten to:

$$\text{succ}(a + b + \text{hd}) = a + \text{succ hd} + b$$

rw[add_succ] - proves that $a + \text{succ hd} + b = \text{succ}(a + \text{hd}) + b$

This is rewritten to:

$$\text{succ}(a + b + \text{hd}) = \text{succ}(a + \text{hd}) + b$$

rw[succ_add] - proves that $\text{succ}(a + \text{hd}) + b = \text{succ}(a + \text{hd} + b)$

This is rewritten to:

$$\text{succ}(a + b + \text{hd}) = \text{succ}(a + \text{hd} + b)$$

rw[n_ih] - replaces $a + b + \text{hd}$ with $a + \text{hd} + b$

This is rewritten to

$$\text{succ}(a + \text{hd} + b) = \text{succ}(a + \text{hd} + b)$$

rfl - this proves that both sides that look the same are equal to each other

Problem 5 Proof in Mathematics:

$$a + b + c = a + (b + c)$$

$0 + b + c = 0 + (b + c)$ - Basis

$b + c = 0 + (b + c)$ - Addition Identity

$b + c = b + c$ - Addition Identity

Inductive Step:

$$k + b + c = k + (b + c)$$

The goal is to prove that $Sk + b + c = Sk + (b + c)$

$S(k + b + c) = Sk + (b + c)$ - Definition of Addition

$S(k + b + c) = S(k + (b + c))$ - Definition of Addition

$S(k + (b + c)) = S(k + (b + c))$ - Inductive Hypothesis

Therefore, by the Axiom of induction $a + b + c = a + (b + c)$ for all a in the natural numbers set

Math to Lean

Basis: induction a with hd

Addition Identity: zero_add

Definition of Addition: succ_add

Inductive Hypothesis: n_ih

Comments and Questions

The Towers of Hanoi reminded me of solving certain problems by simply using recursion. I also remember applying this method to the Fibonacci sequence. This makes me wonder how it transfers to math. How does recursion appear in mathematics or, specifically, in Lean?

2.3 Week 3

Homework

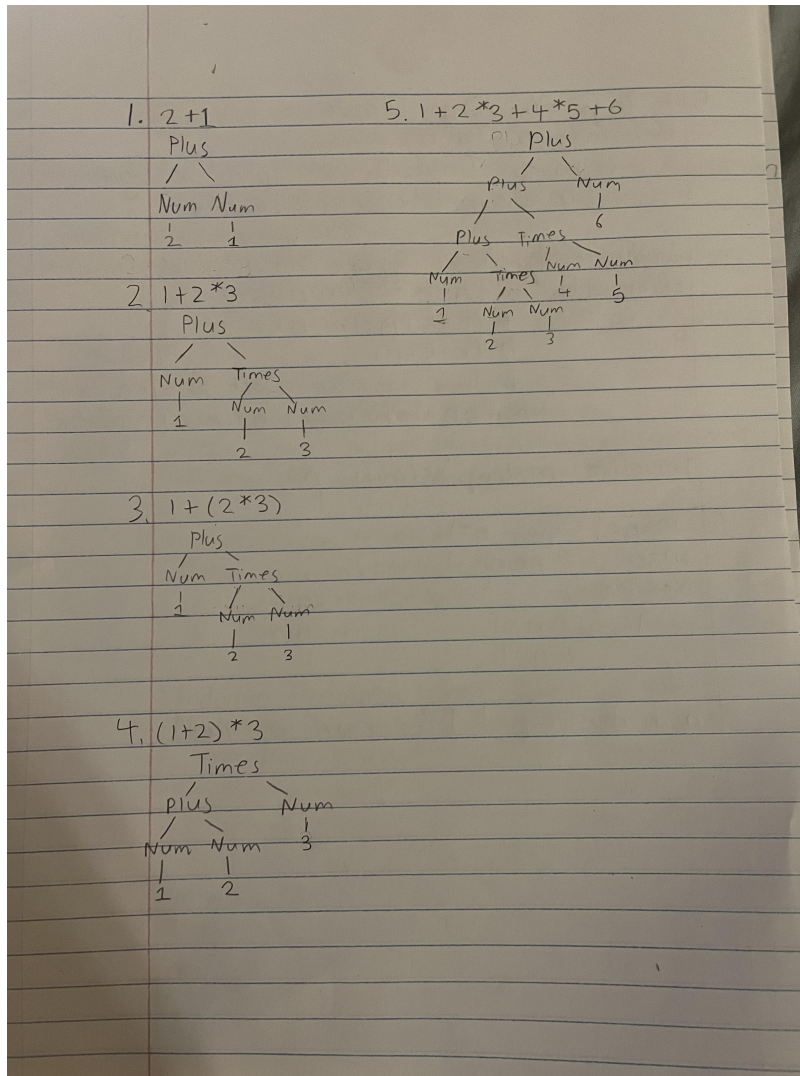
[Week 3 Assignment](#)

Comments and Questions

My literature review is about the cause for many different programming languages, the abstraction of them in the future, and what needs future ones would need to fulfill. I found that languages evolved depending on the different needs and users over time. For instance, Domain-specific languages were created to meet specific needs. SQL, being one of them, was created to interact with databases. Within the topic of abstraction, PLs are bound to become higher level to not worry about lower-level implementation. Examples of the current progress towards abstraction would be AI-assisted programming and declarative programming. However, abstraction will not change the usage of current languages like Java, Python, and C++ in the foreseeable future. These languages have made a huge impact and it is shown through their extensive ecosystems and sheer amount of existing codebases. On the other hand, some specific areas use newer languages like Rust and Kotlin. They fulfill more modern needs like memory safety and developer productivity. Future programming languages would need to deal with challenges like security, AI, and concurrency.

2.4 Week 4

Homework



Comments and Questions

Parsing seemed to work really great with expressions. In general, is parsing a good strategy when it comes to breaking things down?

3 Lessons from the Assignments

4 Conclusion

References

[BLA] Author, [Title](#), Publisher, Year.