

# CPSC-354 Report

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November 10, 2024

## Abstract

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## 1 Introduction

## 2 Week by Week

### 2.1 Week 1

#### Notes

Learned about some tactics and theorems

rfl: a tactic that proves theorems that take the form of  $X = X$

rw: a tactic that rewrites a proof

one\_eq\_succ\_zero: a theorem that proves  $1 = \text{succ } 0$  (there are also other similar existing theorems like two\_eq\_succ\_one and so on)

add\_zero: a theorem that proves  $a + 0 = a$ .

add\_succ: a theorem that proves  $a + \text{succ } b = \text{succ}(a + b)$   
succ\_eq\_add\_one: a theorem that proves  $\text{succ } a = a + 1$

## Homework

Problem 5:

$a$   $b$   $c$  are in the set of natural numbers.

Prove that both sides are equal to each other.

$$a + (b + 0) + (c + 0) = a + b + c$$

rw [add\_zero] - uses the add\_zero theorem to prove that  $b + 0 = b$

This is rewritten to:

$$a + b + (c + 0) = a + b + c$$

rw [add\_zero] - this is done again to prove that  $c + 0 = c$

This is rewritten to:

$$a + b + c = a + b + c$$

rfl - this proves that both sides that look the same are equal to each other

Problem 6:

This is the same problem as 5 but will be approached in a different manner.

$$a + (b + 0) + (c + 0) = a + b + c$$

rw [add\_zero c] - specifically applies the add\_zero theorem to  $c$ , making  $c + 0$  into  $c$

This is rewritten to:

$$a + (b + 0) + c = a + b + c$$

rw [add\_zero b] - specifically applies the add\_zero theorem to  $b$ , making  $b + 0$  into  $b$

This is rewritten to:

$$a + b + c = a + b + c$$

rfl - this proves that both sides that look the same are equal to each other

Problem 7:

$n$  is in the set of natural numbers.

Prove that both sides are equal to each other.

$\text{succ } n = n + 1$  rw [one\_eq\_succ\_zero] - rewrite 1 into successor 0 This is rewritten to:

$$\text{succ } n = n + \text{succ } 0$$

rw [add\_succ] - uses the add\_succ theorem to change  $n + \text{succ } 0$  into  $\text{succ}(n + 0)$

This is rewritten to:

$$\text{succ } n = \text{succ } (n + 0)$$

rw [add\_zero] - uses the add\_zero theorem to prove that  $n + 0 = n$

This is rewritten to:

$$\text{succ } n = \text{succ } n$$

rfl - this proves that both sides that look the same are equal to each other

Problem 8:

Prove that both sides are equal to each other.

$$2 + 2 = 4$$

rw [two\_eq\_succ\_one] - rewrites 2 into succ 1

This is rewritten to:

$$\text{succ } 1 + \text{succ } 1 = 4$$

rw [one\_eq\_succ\_zero] - rewrites 1 into succ 0

This is rewritten to:

```

succ (succ 0) + succ (succ 0) = 4
rw[four_eq_succ_three] - rewrites 4 into succ 3
This is rewritten to:
succ (succ 0) + succ (succ 0) = succ 3
rw[three_eq_succ_two] - rewrites 3 into succ 2
This is rewritten to:
succ (succ 0) + succ (succ 0) = succ (succ 2)
rw[two_eq_succ_one] - rewrites 2 into succ 1
This is rewritten to:
succ (succ 0) + succ (succ 0) = succ (succ (succ 1))
rw[one_eq_succ_zero] - rewrites 1 into succ 0
This is rewritten to:
succ (succ 0) + succ (succ 0) = succ (succ (succ (succ 0)))
rw[add_succ] - changes succ (succ 0) + succ (succ 0) into succ (succ (succ 0) + succ 0)
This is rewritten to:
succ (succ (succ 0) + succ 0) = succ (succ (succ (succ 0)))
rw[add_succ] - changes succ (succ (succ 0) + succ 0) into succ (succ (succ (succ 0) + 0))
This is rewritten to:
succ (succ (succ (succ 0) + 0)) = succ (succ (succ (succ 0)))
rw[add_zero] - changes succ (succ 0) + 0 into
This is rewritten to:
succ (succ (succ (succ 0))) = succ (succ (succ (succ 0)))
rfl - this proves that both sides that look the same are equal to each other

```

For level 5: `add.zero` is a Lean proof that  $a + 0 = a$  (a representing any number). In mathematics, there are laws for arithmetic. One of them is called the identity which applies to addition and multiplication. For addition, it states that  $m + 0 = m = 0 + m$ . This is the exact same as the Lean proof,  $a + 0 = a$ , which can also be written as  $a = 0 + a$ .

## Comments and Questions

Learning the root of mathematics is very eye-opening, and I am confident it will be the same for programming languages. It provides another perspective for elementary functions like  $2 + 2$  equals 4, which is different from just knowing it through memorization. I feel as though this is why people have been able to expand mathematically. This makes me wonder: how can looking through the core of programming help us better current languages (e.g. python, rust)?

## 2.2 Week 2

### Notes

Recursion as a concept using the Towers of Hanoi: It is broken down into: moving a tower of  $n$  disks from  $x$  to  $y$  moving a tower of  $n+1$  disks when it is already known how to move a tower of  $n$  disks The algorithm is made up of a bunch of "pushes" and "pops" The logic overall is a bunch of back and forth movement of the disks

Lean: induction proof with: `induction n with d hd succ.add`: proves that  $\text{succ } a + b = \text{succ } (a + b)$   
`add.comm x y`: proves that  $x + y = y + x$  `add.assoc`: proves that  $a + b + c = a + (b + c)$  `add.right_comm a b c`: proves that  $a + b + c = a + c + b$

## Homework

Problem 1:

$n$  is in the natural number set

Prove  $0 + n = n$ .

induction  $n$  with  $d$  hd - starting a proof by induction

Now our first goal is:

$$0 + 0 = 0$$

`rw[add_zero]` - proves that  $0 + 0 = 0$

This is rewritten to:

$$0 = 0$$

`rfl` - this proves that both sides that look the same are equal to each other

Now, we prove our second goal

$$\text{hd: } 0 + d = d$$

$$0 + \text{succ } d = \text{succ } d$$

`rw[add_succ]` - proves that  $0 + \text{succ } d = \text{succ } (0 + d)$

This is rewritten to:

$$\text{succ } (0 + d) = \text{succ } d$$

`rw[hd]` - this replaces  $0 + d$  with  $d$

This is rewritten to:

$$\text{succ } d = \text{succ } d$$

`rfl` - this proves that both sides that look the same are equal to each other

Problem 2:

$a$   $b$  is in the set of natural numbers

Prove  $\text{succ } a + b = \text{succ } (a + b)$

inductin  $b$  with  $d$  hd - starting a proof by induction

Now our first goal is:

$$\text{succ } a + 0 = \text{succ } (a + 0)$$

`rw[add_zero]` - proves that  $\text{succ } a + 0 = \text{succ } a$

This is rewritten to:

$$\text{succ } a = \text{succ}(a + 0)$$

`rw[add_zero]` - proves that  $\text{succ } (a + 0) = \text{succ } a$

This is rewritten to:

$$\text{succ } a = \text{succ } a$$

`rfl` - this proves that both sides that look the same are equal to each other

Now, we prove our second goal

$$\text{hd: } \text{succ } a + d = \text{succ } (a + d)$$

$$\text{succ } a + \text{succ } d = \text{succ } (a + \text{succ } d)$$

`rw[add_succ]` - proves that  $\text{succ } a + \text{succ } d = \text{succ } (\text{succ } a + d)$

This is rewritten to:

$$\text{succ } (\text{succ } a + d) = \text{succ } (a + \text{succ } d)$$

`rw[hd]` - this replaces  $\text{succ } a + d$  with  $\text{succ } (a + d)$

This is rewritten to:

$$\text{succ } (\text{succ } (a + d)) = \text{succ } (a + \text{succ } d)$$

`rw[add_succ]` - proves that  $\text{succ } (a + \text{succ } d) = \text{succ } (\text{succ } (a + d))$

This is rewritten to:

$$\text{succ } (\text{succ } (a + d)) = \text{succ } (\text{succ } (a + d))$$

`rfl` - this proves that both sides that look the same are equal to each other

Problem 3:

$a$   $b$  is in the set of natural numbers  
 Prove  $a + b = b + a$   
 induction  $b$  with  $hd$  - starting a proof by induction  
 Now our first goal is:  
 $a + 0 = 0 + a$   
 $rw[zero\_add]$  - proves that  $0 + a = a$   
 This is rewritten to:  
 $a + 0 = a$   
 $rw[add\_zero]$  - proves that  $a + 0 = a$   
 This is rewritten to:  
 $a = a$   
 $rfl$  - this proves that both sides that look the same are equal to each other  
 Now, we prove our second goal  
 $n\_ih$ :  $a + hd = hd + a$   
 $a + succ\ hd = succ\ hd + a$   
 $rw[add\_succ]$  - proves that  $a + succ\ hd = succ\ (a + hd)$   
 This is rewritten to:  
 $succ\ (a + hd) = succ\ hd + a$   
 $rw[succ\_add]$  - proves that  $succ\ hd + a = succ\ (hd + a)$   
 This is rewritten to:  
 $succ\ (a + hd) = succ\ (hd + a)$   
 $rw[n\_ih]$  - replaces  $succ\ (a + hd)$  with  $succ\ (hd + a)$   
 This is rewritten to:  
 $succ\ (hd + a) = succ\ (hd + a)$   
 $rfl$  - this proves that both sides that look the same are equal to each other

Problem 4:  
 $a$   $b$   $c$  is in the set of natural numbers  
 Prove  $a + b + c = a + (b + c)$   
 induction  $a$  with  $hd$  - starting a proof by induction  
 Now our first goal is:  
 $0 + b + c = 0 + (b + c)$   
 $rw[zero\_add]$  - proves that  $0 + b = b$   
 This is rewritten to:  
 $b + c = 0 + (b + c)$   
 $rw[zero\_add]$  - proves that  $0 + (b + c) = b + c$   
 This is rewritten to:  
 $b + c = b + c$   
 $rfl$  - this proves that both sides that look the same are equal to each other  
 Now, we prove our second goal  
 $n\_ih$ :  $hd + b + c = hd + (b + c)$   
 $succ\ hd + b + c = succ\ hd + (b + c)$   
 $rw[succ\_add]$  - proves that  $succ\ hd + b + c = succ\ (hd + b) + c$   
 This is rewritten to:  
 $succ\ (hd + b) + c = succ\ hd + (b + c)$   
 $rw[succ\_add]$  - proves that  $succ\ (hd + b) + c = succ\ (hd + b + c)$   
 This is rewritten to:  
 $succ\ (hd + b + c) = succ\ hd + (b + c)$   
 $rw[n\_ih]$  - replaces  $succ\ (hd + b + c)$  with  $succ\ (hd + (b + c))$   
 This is rewritten to:  
 $succ\ (hd + (b + c)) = succ\ hd + (b + c)$

$\text{rw}[\text{succ\_add}]$  - proves  $\text{succ } \text{hd} + (\text{b} + \text{c}) = \text{succ } (\text{hd} + (\text{b} + \text{c}))$   
 This is rewritten to:  
 $\text{succ } (\text{hd} + (\text{b} + \text{c})) = \text{succ } (\text{hd} + (\text{b} + \text{c}))$   
 $\text{rfl}$  - this proves that both sides that look the same are equal to each other

Problem 5:

a b c is in the set of natural numbers

Prove  $a + b + c = a + c + b$

induction c with hd - starting a proof by induction

Now our first goal is:

$a + b + 0 = a + 0 + b$

$\text{rw}[\text{add\_zero}]$  - proves that  $b + 0 = b$

This is rewritten to:

$a + b = a + 0 + b$

$\text{rw}[\text{add\_zero}]$  0 proves that  $a + 0 = a$

This is rewritten to:

$a + b = a + b$

$\text{rfl}$  - this proves that both sides that look the same are equal to each other

Now, we prove our second goal

n\_ih:  $a + b + \text{hd} = a + \text{hd} + b$

$a + b + \text{succ } \text{hd} = a + \text{succ } \text{hd} + b$

$\text{rw}[\text{add\_succ}]$  - proves that  $a + b + \text{succ } \text{hd} = \text{succ } (a + b + \text{hd})$

This is rewritten to:

$\text{succ } (a + b + \text{hd}) = a + \text{succ } \text{hd} + b$

$\text{rw}[\text{add\_succ}]$  - proves that  $a + \text{succ } \text{hd} + b = \text{succ } (a + \text{hd}) + b$

This is rewritten to:

$\text{succ } (a + b + \text{hd}) = \text{succ } (a + \text{hd}) + b$

$\text{rw}[\text{succ\_add}]$  - proves that  $\text{succ } (a + \text{hd}) + b = \text{succ } (a + \text{hd} + b)$

This is rewritten to:

$\text{succ } (a + b + \text{hd}) = \text{succ } (a + \text{hd} + b)$

$\text{rw}[\text{n\_ih}]$  - replaces  $a + b + \text{hd}$  with  $a + \text{hd} + b$

This is rewritten to

$\text{succ } (a + \text{hd} + b) = \text{succ } (a + \text{hd} + b)$

$\text{rfl}$  - this proves that both sides that look the same are equal to each other

Problem 5 Proof in Mathematics:

$a + b + c = a + (b + c)$

$0 + b + c = 0 + (b + c)$  - Basis

$b + c = 0 + (b + c)$  - Addition Identity

$b + c = b + c$  - Addition Identity

Inductive Step:

$k + b + c = k + (b + c)$

The goal is to prove that  $Sk + b + c = Sk + (b + c)$

$S(k + b + c) = Sk + (b + c)$  - Definition of Addition

$S(k + b + c) = S(k + (b + c))$  - Definition of Addition

$S(k + (b + c)) = S(k + (b + c))$  - Inductive Hypothesis

Therefore, by the Axiom of induction  $a + b + c = a + (b + c)$  for all a in the natural numbers set

Math to Lean

Basis: induction a with hd

Addition Identity: `zero_add`  
Definition of Addition: `succ_add`  
Inductive Hypothesis: `n_ih`

### Comments and Questions

The Towers of Hanoi reminded me of solving certain problems by simply using recursion. I also remember applying this method to the Fibonacci sequence. This makes me wonder how it transfers to math. How does recursion appear in mathematics or, specifically, in Lean?

## 2.3 Week 3

### Homework

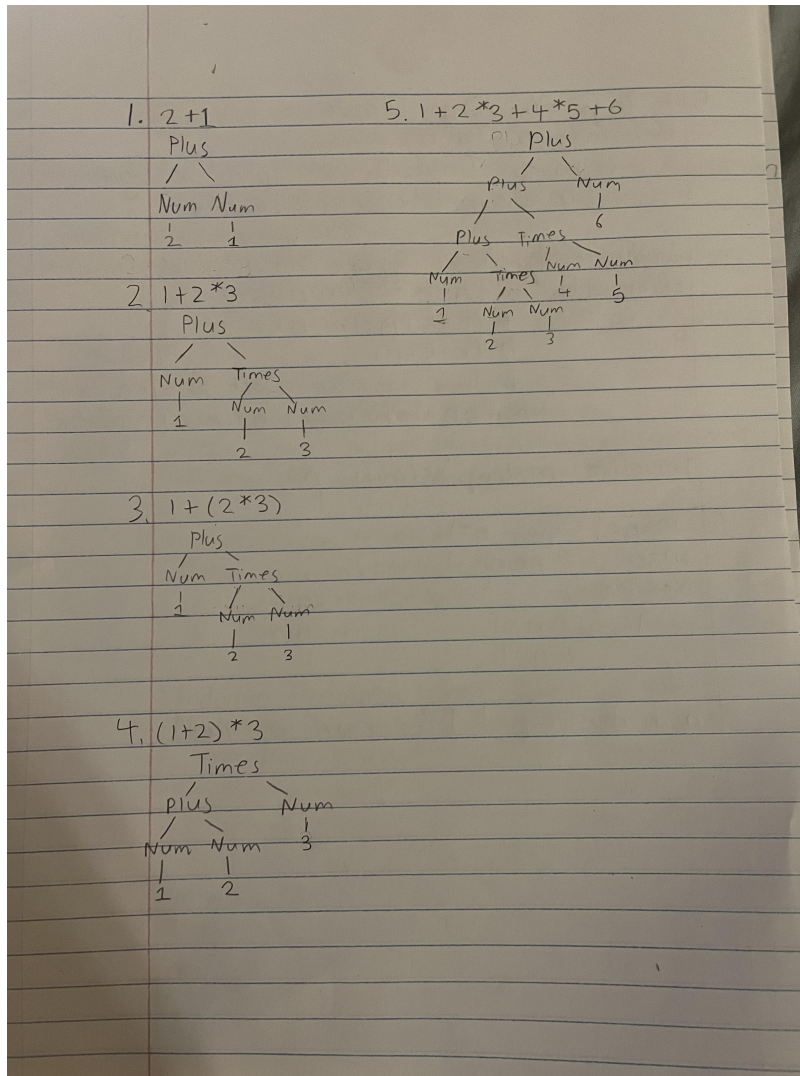
[Week 3 Assignment](#)

### Comments and Questions

My literature review is about the cause for many different programming languages, the abstraction of them in the future, and what needs future ones would need to fulfill. I found that languages evolved depending on the different needs and users over time. For instance, Domain-specific languages were created to meet specific needs. SQL, being one of them, was created to interact with databases. Within the topic of abstraction, PLs are bound to become higher level to not worry about lower-level implementation. Examples of the current progress towards abstraction would be AI-assisted programming and declarative programming. However, abstraction will not change the usage of current languages like Java, Python, and C++ in the foreseeable future. These languages have made a huge impact and it is shown through their extensive ecosystems and sheer amount of existing codebases. On the other hand, some specific areas use newer languages like Rust and Kotlin. They fulfill more modern needs like memory safety and developer productivity. Future programming languages would need to deal with challenges like security, AI, and concurrency.

## 2.4 Week 4

### Homework



### Comments and Questions

Parsing seemed to work really great with expressions. In general, is parsing a good strategy when it comes to breaking things down?

## 2.5 Week 5

### Notes

exact -conclusion of a proof

Operator:  $\wedge$  -Logical And

Example:  $A \wedge B$  -A and B

and.into -takes two pieces of evidence and combines it into one

have -adds new assumptions to the proof



## Homework

Problem 1:  
exact todo\_list

Problem 2:  
exact and\_intro p s

Problem 3:  
exact  $\langle \langle a, i \rangle, \langle o, u \rangle \rangle$

Problem 4:  
have p := and\_left vmS  
exact p

Problem 5:  
have q := and\_right h  
exact q

Problem 6:  
have a := and\_left h1  
have u := and\_right h2  
exact  $\langle a, u \rangle$

Problem 7:  
have h1 := h.left  
have h2 := h1.right  
have h3 := h2.left  
have h4 := h3.left  
have h5 := h4.right  
exact h5

Problem 8:  
have h1 := h.left  
have a := h1.right  
have h2 := h1.left  
have p := h2.left  
have s := h2.right  
have h3 := h.right  
have h4 := h3.right  
have h5 := h4.left  
have c := h5.left

In mathematical proof:  
If  $((P \wedge S) \wedge A) \wedge I \wedge (C \wedge O) \wedge U$  then  $A \wedge C \wedge P \wedge S$ .  
Proof:  
(1)  
 $((P \wedge S) \wedge A) \wedge I \wedge (C \wedge O) \wedge U$

assumption

(2)  
 $(P \wedge S) \wedge A$   
and\_left (1)

(3)  
 $A$   
and\_right (2)

(4)  
 $(P \wedge S)$   
and\_left (2)

(5)  
 $P$   
and\_left (4)

(6)  
 $S$   
and\_right (4)

(7)  
 $I \wedge (C \wedge O) \wedge U$   
and\_right (1)

(8)  
 $(C \wedge O) \wedge U$   
and\_right (7)

(9)  
 $C \wedge O$   
and\_left (8)

(10)  
 $C$   
and\_left (9)

(11)  
 $A \wedge C \wedge P \wedge S$   
and\_intro (3)(10)(5)(6)

### Comments and Questions

How does lean help with problem solving when it comes to programming?

## 2.6 Week 6

### Homework

Problem 1:

```
have b := bakery_service p
exact b
```

Problem 2:

```
exact fun h : C => h
```

Problem 3:

```
exact fun h => and_intro h.right h.left
```

Problem 4:

```
exact fun h : C => have a : A := h1 h; h2 a
```

Problem 5:

```
have q : Q := h1 p
have t : T := h3 q
exact h5 t
```

Problem 6:

```
exact fun c : C =>
fun d : D =>
have cd : C ∧ D := ⟨c, d⟩;
h cd
```

Couldn't finish...

### Comments and Questions

Solving lean logic game is a simple view into proving theorems and the foundations of programming languages. When it comes to the lambda problems specifically, how do real-world challenges complicate the applications of lambda?

## 2.7 Week 7

### Homework

1.

$$((\lambda m. \lambda n. m\ n) (\lambda f. \lambda x. f\ (f\ x))) (\lambda f. \lambda x. f\ (f\ (f\ x)))$$

$$((\lambda m. \lambda n. m\ n) (\lambda f. \lambda x. f\ (f\ x))) \rightarrow \lambda n. (\lambda f. \lambda x. f\ (f\ x))\ n$$

$$(\lambda f. \lambda x. f\ (f\ x))\ n \rightarrow \lambda x. n\ (n\ x)$$

$$(\lambda n. \lambda x. n (n x)) (\lambda f. \lambda x. f (f (f x)))$$

$$(\lambda n. \lambda x. n (n x)) (\lambda f. \lambda x. f (f (f x))) \rightarrow \lambda x. (\lambda f. \lambda x. f (f (f x))) ((\lambda f. \lambda x. f (f (f x))) x)$$

$$(\lambda f. \lambda x. f (f (f x))) x \rightarrow \lambda x. x (x (x x))$$

$$\lambda x. (\lambda f. \lambda x. f (f (f x))) (\lambda x. x (x (x x)))$$

$$(\lambda f. \lambda x. f (f (f x))) (\lambda x. x (x (x x))) \rightarrow \lambda x. (\lambda x. x (x (x x))) ((\lambda x. x (x (x x))) x)$$

$$(\lambda x. x (x (x x))) x \rightarrow x (x (x x))$$

$$\lambda x. (\lambda x. x (x (x x))) (x (x (x x)))$$

$$(\lambda x. x (x (x x))) (x (x (x x))) \rightarrow (x (x (x x))) ((x (x (x x))) ((x (x (x x))) (x (x (x x)))))$$

2.

The lambda term implements addition on natural numbers. It is done by combining two Church numerals. It carries out addition by counting the total number of times the function is being applied.

### Comments and Questions

Can church numerals be used to represent recursive functions/processes?

## 2.8 Week 8

### Homework

Answers on week 9

### Comments and Questions

When it comes to evaluation strategies in lambda calculus, what are the trade-offs? How do these strategies affect performance and accuracy in a practical implementation?

## 2.9 Week 9

### Homework

2. a is applied to b, which leads to (ab). Then, (ab) is applied to c, resulting in ((ab)c). Finally, ((ab)c) is applied to d, which leads to (((ab)c)d). (a) reduces to a it's already in its simplest form.

3. When substituting a variable, all instances of that variable are replaced. However, if the variable being substituted for is also bound, the meaning of the expression is changed. This is implemented by traversing the AST of the expression, checking for bound variables, and renaming them.

4. No, not everything is the expected result.

5.

$$Y = \lambda f.(\lambda x.f(xx))(\lambda x.f(xx))$$

### Comments and Questions

How could different evaluation strategies impact the trace output produced by an interpreter?

## 2.10 Week 10

### Homework

1. The challenge of working through Homework 8/9 and Assignment 3 was figuring out why the interpreter wouldn't completely evaluate everything.
2. The key insight for Assignment 3 was just looking through the evaluation function and looking at where exactly it stopped evaluating the function and just returned whatever it found.
3. The most interesting take away was from the homework and assignment was using the debugger to see how far the program would evaluate a lambda function. I never really use the debugger, so it was interesting to break things apart.

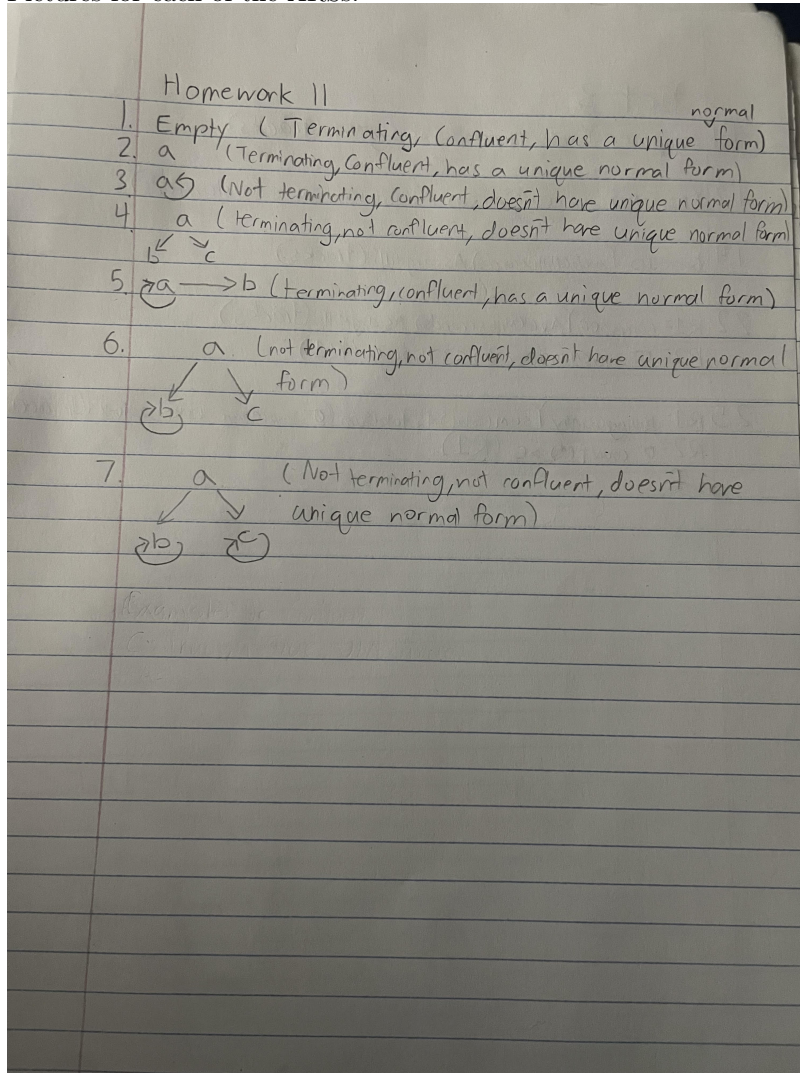
### Comments and Questions

How can the interpreter be expanded upon to handle more complex structures?

## 2.11 Week 10

### Homework

Pictures for each of the ARSs:



Example of ARS for each of the possible 8 combinations

Confluent = True, Terminating: True, Has A Unique Normal Form: True

$$A = a, b, R = (a, b)$$

Confluent = True, Terminating: True, Has A Unique Normal Form: False

$$A = a, b, c, R = (a, b), (a, c)$$

Confluent = True, Terminating: False, Has A Unique Normal Form: True

$$A = a, R = (a, a)$$

Confluent = True, Terminating: False, Has A Unique Normal Form: False

$$A = a, b, R = (a, a), (a, b)$$

Confluent = False, Terminating: True, Has A Unique Normal Form: True

$$A = a, b, c, R = (a, b), (b, b)$$

Confluent = False, Terminating: True, Has A Unique Normal Form: True

$$A = a, b, c, R = (a, b), (a, c)$$

Confluent = False, Terminating: False, Has A Unique Normal Form: True

$$A = a, b, R = (a, b), (b, a)$$

Confluent = False, Terminating: False, Has A Unique Normal Form: False

$$A = a, b, c, R = (a, b), (b, b), (a, c), (c, c)$$

### Comments and Questions

What characteristics of an ARS (confluency, termination, etc) is important when it comes to programming languages and why?

## 3 Lessons from the Assignments

## 4 Conclusion

## References

[BLA] Author, [Title](#), Publisher, Year.