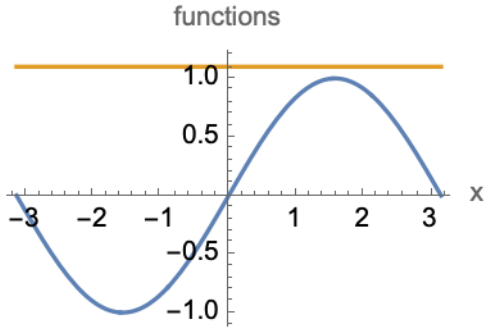
Class 07: flow on a circle

Activity

Teams 5 and 6: Post screenshots of your work to the course Google Drive today (make or use a C07 folder). Include words, labels, and other short notes that might make those solutions useful to you or your classmates. Find the link in Canvas.

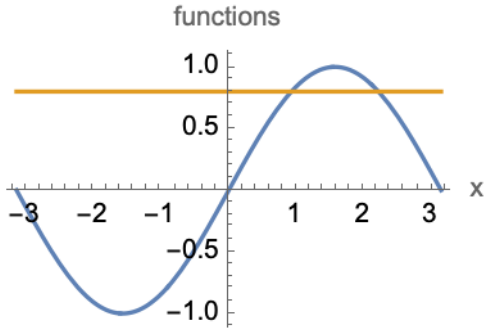
1. (a) Assume the time evolution of the phase difference, ϕ, between an oscillator and a reference signal is given by the system $\dot{\phi} = 1.1 - \sin\phi$.



What is the long term behavior of the phase difference in this system? *If it approaches a fixed value, provide an estimate of that value.*

(b) Assume the time evolution of the phase difference, ϕ, between an oscillator and a reference signal is given by the system $\dot{\phi} = 0.8 - \sin\phi$.

What is the long term behavior of the phase difference in this system? *If it approaches a fixed value, provide an estimate of that value.*



2. (4.1.1)

For which values of a does the equation $\dot{\theta} = \sin a\theta$ give a well-defined vector field on the circle?

For $a = 3$, find and classify all the fixed points and sketch the phase portrait on the circle.

You might plot $\sin 3\theta$ *to help you.*

3. (4.3.3) For $\dot{\phi} = \mu \sin\phi - \sin 2\phi$:

(a) Check that the vector field is well-defined on the circle.

(b) Draw phase portraits for:

- large positive μ

- $\mu = 0$
- large negative μ

(c) Use $\sin 2\phi = 2\cos\phi\sin\phi$ to find mathematical expressions for fixed points.

(d) Classify the bifurcations that occur as μ varies.

(e) Find the bifurcation values of μ.

(f) Think of ϕ as describing the **phase of a single oscillator**. For what values of μ is the system "oscillating"?

(g) Think of ϕ as describing the **phase difference** between an oscillator and a reference. For what values of μ is the oscillator entrained (phase-locked) to the reference? What sets their phase difference?