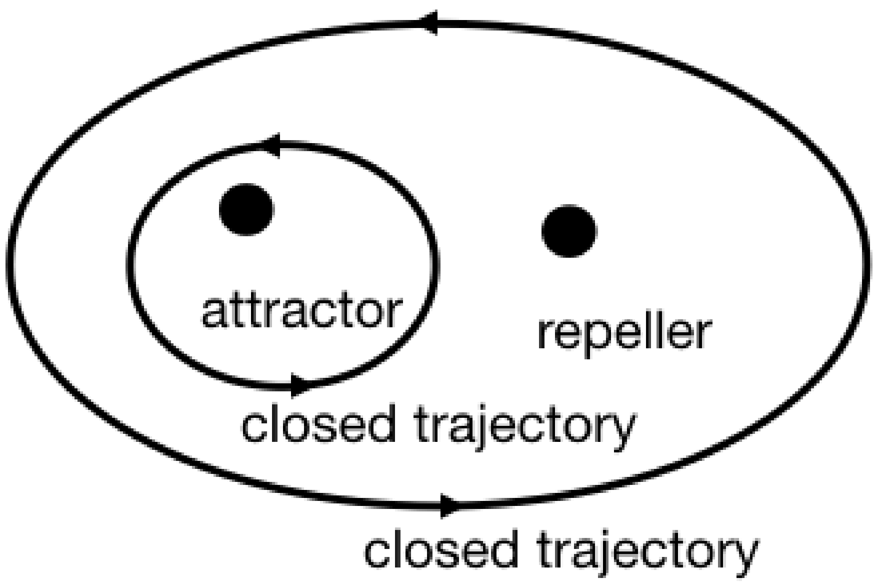
Class 15 index theory

Activity

Teams 3 & 4, post photos of your work to the class Google Drive (see Canvas for link). Make a folder for today's class if one doesn't exist yet.

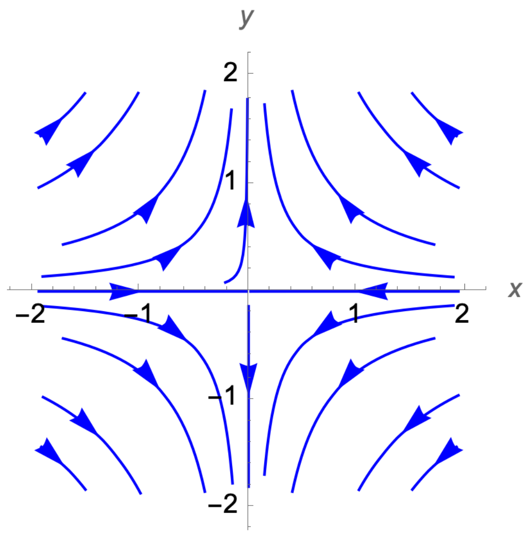
1. According to index theory, is the following phase diagram configuration possible or not? Support your answer by labeling each fixed point with the appropriate index.

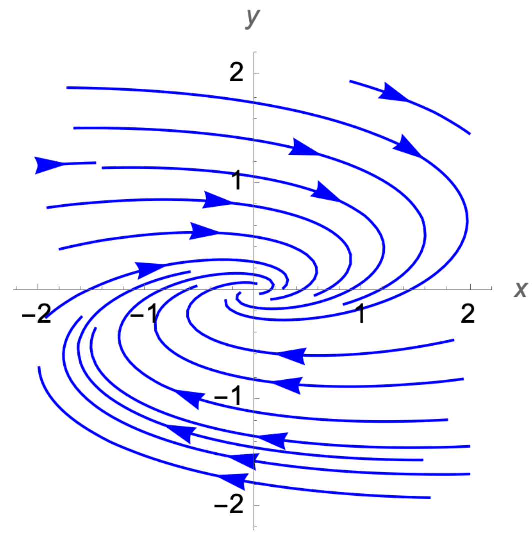


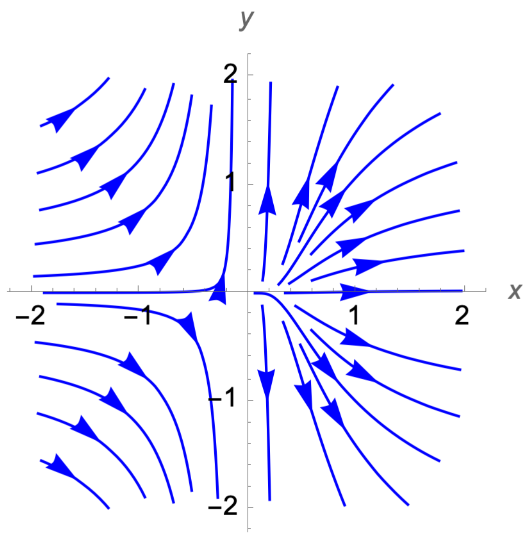
Check one:		
	yes (it is possible):	
	no (it is not possible):	

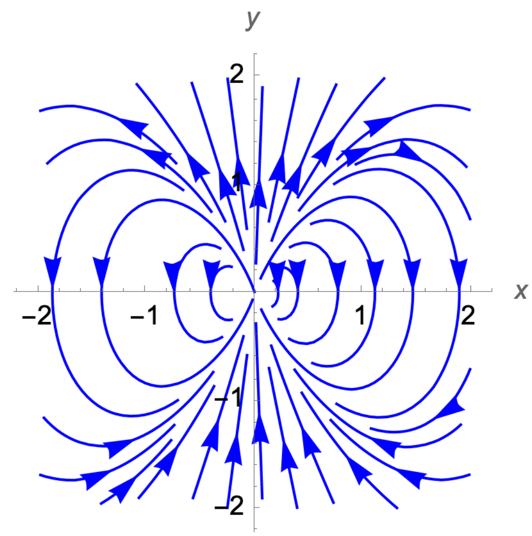
2. (a) Sketch a phase portrait for a system with a saddle point at the origin.
   (b) Choose a closed curve that encloses the saddle point. Find the index on that curve.
   (c) Choose a different closed curve and convince yourself the index is the same.
   (d) When a closed curve is a trajectory of the system it has an index of $+1$. According to index theory, if the fixed point at the origin in part (a) is the sole fixed point in the system, can a closed trajectory exist?
   (e) Now add a stable spiral at $(1, 0)$. No need to connect the two local pictures up.
   (f) Calculate the index about a curve that encircles just the stable spiral.
   (g) Could a closed trajectory exist in this new system? If so, where?

3. Find the index of the fixed points.









4. (Closed orbits) Let

$$\dot{x} = y$$
$$\dot{y} = x - x^3 - \delta y + x^2 y, \quad \delta \geq 0.$$

This system can also be written $\ddot{x} = x - x^3 - \delta \dot{x} + x^2 \dot{x}$.

This is a variation on the unforced Duffing oscillator (Duffing published work on this kind of equation in 1918, and it became a popular model in the 1970s). *Example taken from Wiggins 2003.*

Use index theory to put limits on where closed trajectories might exist.

- Identify the fixed points.
- Use the determinant of the Jacobian matrix to determine the associated index for each fixed point.
- Sketch all of the possible configurations of closed trajectories in the system, based on the index information.