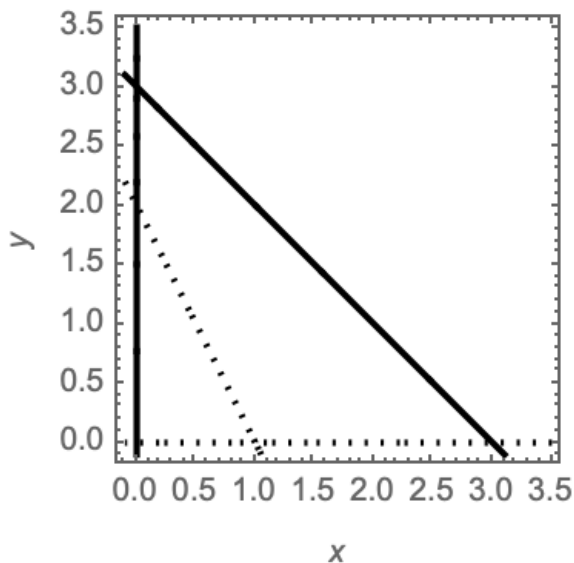# Class 11: 2d nonlinear

Activity

Teams 5 and 6: Post photos of your work to the course Google Drive today. Include words, labels, and other short notes that might make those solutions useful to you or your classmates. Find the link to the drive in Canvas (and add a folder for this class meeting if it doesn't exist yet).

Team problems

1. For the following nonlinear dynamical system: $\dot{x} = x(3 - x - y), \dot{y} = y(2 - 2x - y)$, $x, y \geq 0$ the nullclines are shown below.

   Add a representative vector to each section of the phase space showing direction of the vector field in that region (up and left; up and right; down and left; down and right)

2. (6.4.2) Consider the system $\dot{x} = x(3 - 2x - y), \dot{y} = y(2 - x - y)$, $x, y \geq 0$.

   (a) Find the fixed points.

   (b) Draw the nullclines on the xy-plane.

   (c) Add a representative vector in each region of phase space.

   (d) Compute the Jacobian matrix.

   (e) Classify the fixed points. *Compare your classification to the behavior of the vector field near each fixed point in your diagram from part (c).*

3. Analyzing a 2D System (problem from Dr Alice Nadeau):

   Consider a 2D red fox-coyote system (let "1" denote red foxes and "2" denote coyotes):

   $$\frac{dN_1}{dt} = r_1 N_1 \left(1 - \frac{N_1}{K_1}\right) - \alpha_1 N_1 N_2$$

   $$\frac{dN_2}{dt} = r_2 N_2 \left(1 - \frac{N_2}{K_2}\right) - \alpha_2 N_1 N_2$$

(a) Use $N_1 = A_1 x$, $N_2 = A_2 y$ and $t = T_0 \tau$, where A_1, A_2, T_0 are constants that can be chosen. Substitute, simplify, and identify non-dimensional groups.

(b) The system can be nondimensionalized to give:

$$\frac{dx}{d\tau} = x\,(1 - x) - \beta_1 x y$$

$$\frac{dy}{d\tau} = \rho y\,(1 - y) - \beta_2 x y$$

where $\beta_1 = \alpha_1 K_2 / r_1$, and $\beta_2 = \alpha_2 K_1 / r_1$.

Which non-dimensional groups were set to 1 to create this non-dimensionalization? Find an expression for ρ in terms of parameters of the system.

(c) Show that the nullclines of this nondimensional system are

$$x \text{ nullclines: } \quad x = 0, \;\; y = \frac{1}{\beta_1}(1 - x)$$

$$y \text{ nullclines: } \quad y = 0, \;\; y = 1 - \frac{\beta_2}{\rho}x$$

(d) There are three equilibria whose locations are unaffected by the parameters. What are they? Interpret these equilibria in terms of foxes and coyotes.

(e) Find conditions to ensure that there is exactly one crossing of the nullclines in the first quadrant (not on the axes). How would you interpret these conditions in terms of foxes and coyotes?

I found it helpful to reason about the locations of the x- and y- intercepts.

4. (time permitting)

Continuing to analyse this system:

(a) The Jacobian for our nondimensional system is

$$J = \begin{bmatrix} 1 - 2x - \beta_1 y & -\beta_1 x \\ -\beta_2 y & \rho(1 - 2y) - \beta_2 x \end{bmatrix}$$

   i. Determine the linear stability of equilibrium $e_1 = (1, 0)$ and sketch the phase portraits nearby. Are there any cases where the linear stability cannot tell us about the nonlinear stability of this equilibrium? Relate your analysis back to foxes and coyotes.

   ii. Determine the linear stability of equilibrium $e_2 = (0, 1)$ and sketch the phase portraits nearby. Are there any cases where the linear stability cannot tell us about the nonlinear stability of this equilibrium? Relate your analysis back to foxes and coyotes.

   iii. Determining the linear stability of equilibrium e_4 (the crossing point) is messy without a computer. Instead use the nullclines around this point to determine the stability when $\beta_1 < 1$ and $\beta_2 < \rho$ and when $\beta_1 > 1$ and $\beta_2 > \rho$.

(b) Conceptual questions about the model:

   i. What are some assumptions that are explicit or implicit in this model?

    ii. Briefly describe each of the terms in the model. Which terms give the intraspecies competition? Which give the interspecies competition?

    iii. Which parameters are likely to be hard to measure/constrain? Which would be easy to measure/constrain?

(c) Conceptual questions about results/analysis:

    i. In New York's Adirondack region, coyotes prey on red foxes and the NYS Department of Evironmental Conservation reports that red foxes "avoid coyote territories completely or reside on the periphery of established coyote territories." Is there any inconsistency with this fact and the model we just analyzed? If no, explain why. If yes, can you adjust the model to fix it?

    ii. What in the above analysis would you change if we replaced the red foxes with bobcats. The Adirondack Ecological Center site on bobcats will be helpful: `https://www.esf.edu/aec/adks/mammals/bobcat.htm`

    iii. What in the above analysis would you change if we replaced the coyotes with gray foxes. The Adirondack Ecological Center site on gray foxes will be helpful: `https://www.esf.edu/aec/adks/mammals/gray_fox.htm`