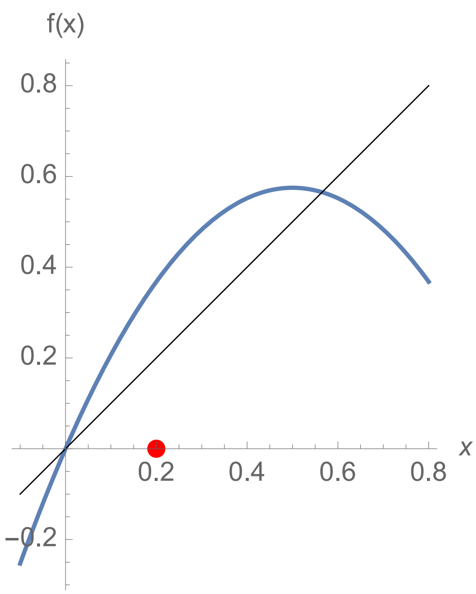
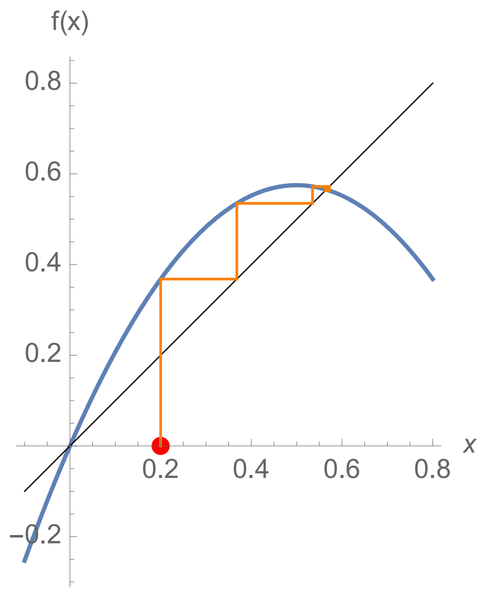
- There will be a skill check in class on Monday. The problem info is below.
- Problem set 08 is due today.
- Problem set 09 is due next Friday.

Skill check practice Let $x_{n+1} = f(x_n)$ where $f(x) = 2.3x(1-x)$. Add a cobweb to the graph to represent the orbit, using the red dot as your starting value of x.



Skill check practice solution



More explanation:

Starting from $x_0 = 0.2$ drawn a vertical line upwards to the point $(x_0, f(x_0))$ where $x_1 = f(x_0)$. Then move horizontally to the point (x_1, x_1). Next use a vertical line to find $(x_1, f(x_1))$ where $x_2 = f(x_1)$. Continue to move horizontally to (x_n, x_n) and then vertically to $(x_n, f(x_n))$ to continue the cobwebbing.

Big picture

We have explored 1d and 2d continuous time dynamical systems, learning about possible long term behaviors and bifurcations.

What additional long term behaviors are possible in 3d continuous time dynamical systems?

We will also begin our study of 1d discrete time dynamical systems: we will use these 1d systems to help us understand chaotic behaviors.

3D systems: Example 1

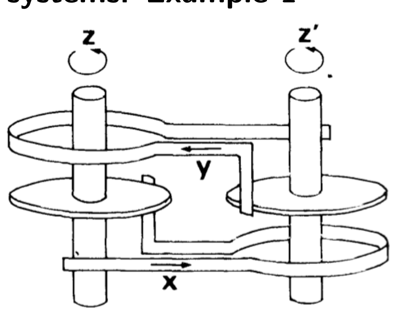
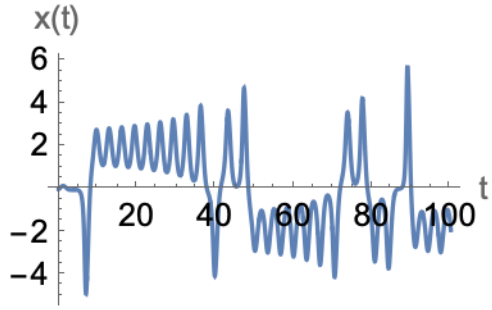


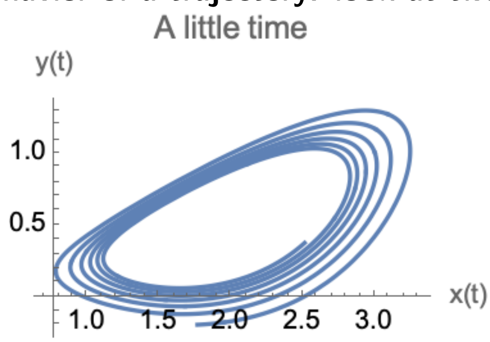
Fig. 1. Rikitake two-disc dynamo system.

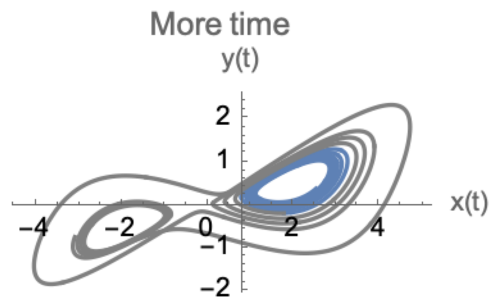
Figure from (Ito 1980).

Behavior of a trajectory: look at one variable



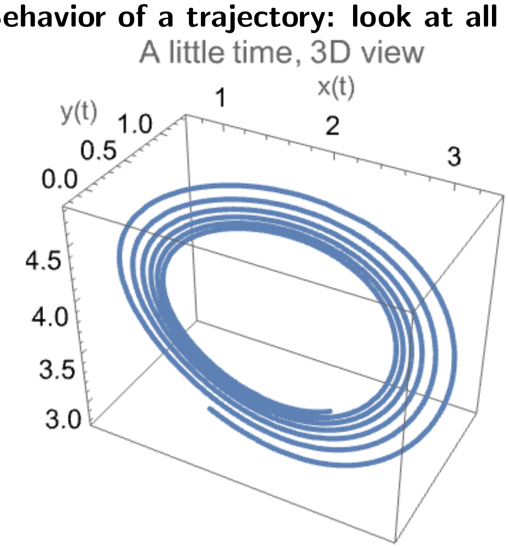
Behavior of a trajectory: look at two variables

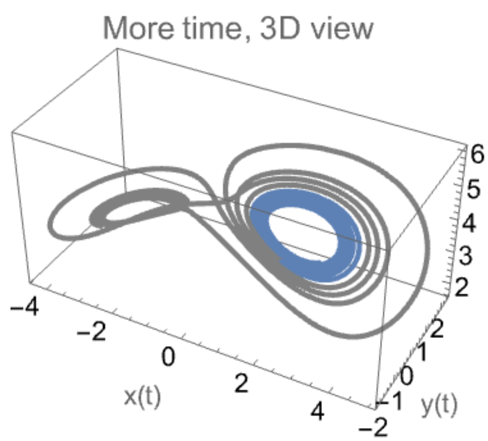




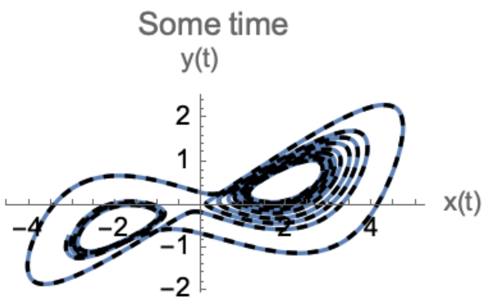
- For a 3D system, can trajectories "cross" themselves in this xy-view?
- What appears to be the long term behavior of this trajectory?

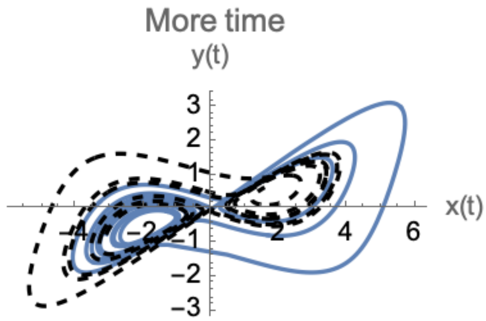
Behavior of a trajectory: look at all three variables

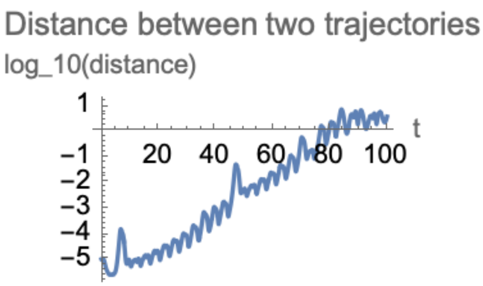




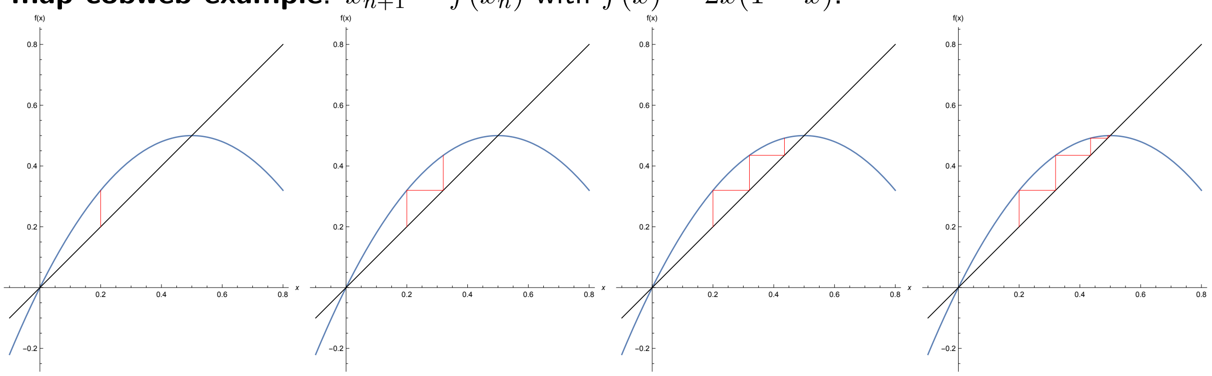
Nearby initial conditions







1D map cobweb example. $x_{n+1} = f(x_n)$ with $f(x) = 2x(1-x)$.



Practice.

Let $x_{n+1} = f(x_n)$ with $f(x) = \cos x$.

- Plot $\cos x$ and x on the same plot.
- Use your plot to identify any fixed points of the map.
- Make cobwebs for at least two qualitatively different cases.

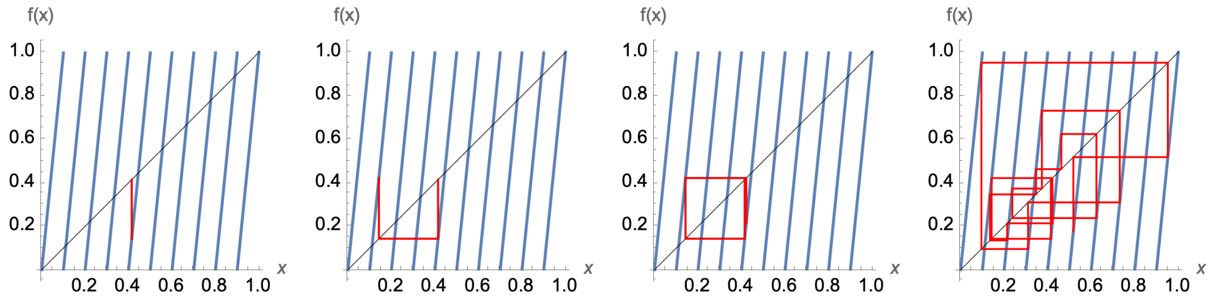
Example of chaos: decimal shift map

Let $x \mapsto 10x \mod 1$.

- This map is not invertible. Knowing $x_1 = 0.75$ does not allow you to identify x_0. ($0.175 \mapsto 0.75$ and $0.375 \mapsto 0.75$).
- Let $x_0 = \sqrt{2} - 1 = 0.414213562...$

  $0.414213562... \mapsto 0.14213562... \mapsto 0.4213562....$
- If we have 9 digits of information about our initial condition, our prediction of the future can only exist for nine iterates (and our predictions are less precise for each further iterate).
- The map is deterministic, but we have difficulty predicting the future due to finite knowledge of the present. That is a hallmark of chaos.

- Two nearby initial conditions: $x_{0a} = 0.123456...$ and $x_{0b} = 0.123439...$ become further apart with each iterate of the map (and have no relationship after just four shifts). This is *sensitive dependence on initial conditions*.



Teams

1. Alexander, Iona, Van, Sophie
2. Joseph, Ada, Noah
3. Mariana, Isaiah, David H
4. Christina, Alice, Dina
5. Hiro, Katheryn, Emily
6. Allison, Margaret, Mallory
7. George, Thea, Michail
8. Shefali, Camilo, David A

Teams 3 and 4: Post screenshots of your work to the course Google Drive today. Include words, labels, and other short notes that might make those solutions useful to you or your classmates. Find the link in Canvas.

Questions

1. (9.4.2) The tent map is a simple map Let

$$x_{n+1} = \begin{cases} 2x_n, & 0 \le x_n \le \frac{1}{2} \\ 2 - 2x_n, & \frac{1}{2} \le x_n \le 1. \end{cases}$$

   (a) Draw $f(x)$ where $x_{n+1} = f(x_n)$. How many times does it intersect the curve $y = x$? Why is this map the "tent map"?

   (b) Find the fixed points of this map.

   (c) Classify the stability of the fixed points. *Use cobwebbing or compare the slope of $f(x)$ at the fixed point to 1.*