- This problem set is due by 5pm on Friday Oct 16th. Upload your written work and screenshots of your Mathematica work to Gradescope. Upload your Mathematica file to Canvas.
- Fill out the online cover sheet (on Canvas) for each assignment to name your collaborators, list resources you used, and estimate the time you spent on the assignment.

Academic Integrity and Collaboration on Problem Sets:

Collaborating with classmates in planning and designing solutions to homework problems is encouraged. Collaboration, cooperation, and consultation can all be productive. Work with others by

- discussing the problem,
- brainstorming,
- walking through possible strategies,
- outlining solution methods

For problem sets, you may consult or use:

- Course text (including answers in back)
- Other books
- Internet
- Your notes (taken during class)
- Class notes of other students
- Course handouts
- Piazza or Slack posts from the course staff
- Computational tools such as Mathematica or Desmos
- Calculators

You may **not** consult:

- Solution manuals
- Problem sets from prior years
- Solutions to problem sets from prior years
- Other sources of solutions
- Emails from the course staff

You may:

- Look at communal work while writing up your own solution
- Copy computer code from the source files provided with the problem sets
- Look at a screenshare of another student's computer code

You may **not**

- Look at the individual mathematical work of others
- Post about problems online
- Copy and paste computer code from another student (or otherwise directly use the code of another student)

link to book on Hollis

1. (working with polar, 7.1.5, 7.3.3)
   (a) (7.1.5) Show that the system $\dot{r} = r(1-r^2), \dot{\theta} = 1$ is equivalent to

   $$\dot{x} = x - y - x(x^2+y^2)$$
   $$\dot{y} = x + y - y(x^2+y^2)$$

   where $x = r\cos\theta$ and $y = r\sin\theta$. No need to analyze the system.

   Recall the multivariable chain rule: $\frac{dx}{dt} = \frac{dr}{dt}\cos\theta - r\frac{d\theta}{dt}\sin\theta$.

   You are welcome to use Mathematica for this; in that case explain how you used Mathematica as part of your writeup (and submit screenshots on Gradescope along with your source code on Canvas for completeness).

   (b) (7.3.3) Do this problem as written.

   You are welcome to use Mathematica to help you; in that case explain how you used Mathematica as part of your writeup, include screenshots, etc.

   Remember that trapping regions for the Poincaré-Bendixson theorem need to be closed sets, so they need to include their boundary points.

2. (7.2.9) Do this problem as written. For (c) there's a typo in the text. The system should be $\dot{x} = -2xe^{x^2+y^2}$, $\dot{y} = -2ye^{x^2+y^2}$.

3. (7.2.12) Show that $\begin{cases} \dot{x} = -x + 2y^3 - 2y^4 \\ \dot{y} = -x - y + xy \end{cases}$ has no periodic solutions.

   Try $V(x,y) = x^m + ay^n$, and choose a, m, n so that it is a Liapunov (Lyapunov) function for this system.

   You are welcome to use Mathematica for this; be sure to describe your setup in your writeup and to submit your code on Canvas.

4. (6.8.14) Consider the family of linear systems $\begin{cases} \dot{x} = x\cos\alpha - y\sin\alpha \\ \dot{y} = x\sin\alpha + y\cos\alpha \end{cases}$, where α is a parameter that runs over the range $0 \le \alpha \le \pi$.

   (a) Working analytically, classify the fixed point at the origin as a function of α. Generate phase portraits for a few values and include the computational tool you used along with any specific commands as part of your solution.

   (b)

   $$I_C = \frac{1}{2\pi}\oint_C \frac{f\ dg - g\ df}{f^2+g^2}$$

   is an integral that gives you I_C.

   Use this integral to show that I_C is independent of α.

   You may have questions about notation - please post them to the class on Piazza. df *and* dg *are differentials. They capture the change in* f *or in* g *as either or both* x, y *changes so* $df = f_x dx + f_y dy$.

(c) Let C be a circle centered at the origin. Compute I_C explicitly by evaluating the integral.

You may not have worked with line integrals in some time, and may not have used differential notation for them before. Below is a worked example taken from a calculus textbook:

Example Q: Evaluate $\int_C xydx - y^2dy$ where C is the line segment from $(0,0)$ to $(2,6)$.

Example solution: First, parameterize the curve C. Let $x = t$, so for x to go from 0 to 2 we choose $0 \le t \le 2$. Next, find y in terms of t so that at each time t we are at a point along the curve. Since $y = 3x$ is our curve, let $y = 3t$. Our parameterization is $x(t) = t, y(t) = 3t, 0 \le t \le 2$.

Next, find dx and dy. This is just like what you do in a u-substitution. Here, $dx = 1dt$ and $dy = 3dt$.

Now plug this all in to the integral: $\int_C xydx - y^2dy = \int_0^2 (t)(3t)dt - (3t)^2 3dt$. Next, simplify and evaluate: $\int_0^2 (3t^2 - 27t^2)dt = -24 \int_0^2 t^2 dt$.