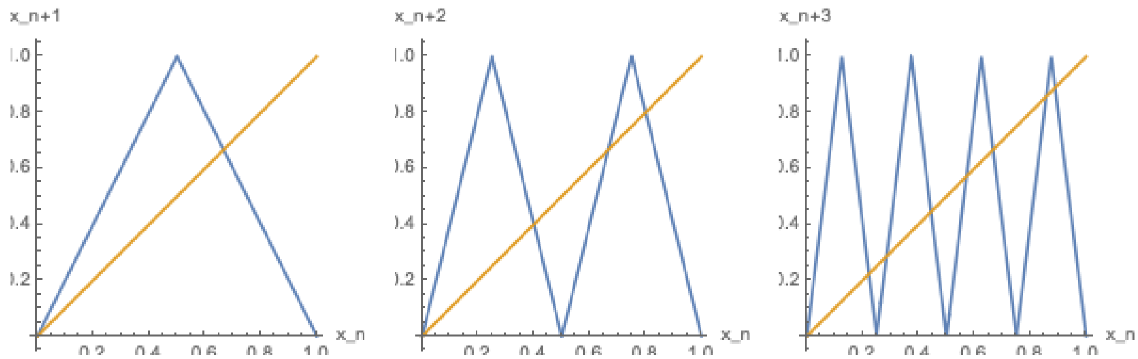
- There will be a skill check in class on Monday. The problem info is below.
- The 2D system analysis assignment will replace a problem set next week. This is an **individual** assignment. You are expected not to discuss your work on it with classmates.
- There is a project log due next Friday (submit to Gradescope, separately from the 2D system analysis).
- There are not pre-class assignments next week.

Skill check practice

For the map shown below, how many period-3 orbits exist?



Skill check practice solution

answer: Two.

more explanation: There are two period-1 points that will show up in the x_{n+3} vs x_n plot. There is a period-2 orbit, but it won't show up in the x_{n+3} vs x_n plot.

There are eight fixed points in the x_{n+3} vs x_n plot. Two of those are the period-1 points. So six of those are period-3 points. Six period-3 points corresponds to two period-3 orbits.

Big picture

We have observed sensitive dependence on initial conditions in 3D flows and 1D maps. We have also seen a 1D map that exhibits chaos used as a model for the behavior in a 3D flow.

We still need to learn more about the geometric structure of chaotic attractors, which will involve learning about fractals. We will also examine one way chaos can arise in a system as a parameter changes: that will require learning a little about bifurcations in maps.

Teams

1.

Teams 1 and 2: Post screenshots of your work to the course Google Drive today. Include words, labels, and other short notes that might make those solutions useful to you or your classmates. Find the link in Canvas.

Questions

1. The last two days of our course will be progress presentations. On May 3rd from 9am-12pm we will have final presentations.

   Goals of the progress presentation:

- Introduce your project topic to your classmates in a way that they can understand
- Connect your project to the mathematical content of the course
- Use slides to provide illustrations of your ideas
- The purpose of the illustrations is to make the ideas more understandable to your classmates
- Slides should not serve as presenter notes, or as lists of information that will be read
- Present your project goals
- Share your progress
- Include references

Goals of the final presentation:

- Provide a reminder of project question/goal
- Refresh background your classmates will need to understand your work (in response to feedback on the progress presentation)
- Present the approach/model that you were replicating.
- Share the results of your replication work and your conclusions
- Provide recommendations for next steps (where you would take this project if you had more time)
- Include references

Examples of presentation slides

(a) What does their project seem to be about?

(b) Identify techniques that this team used to illustrate their ideas.

(c) How much text did they use on their slides? What did they use it for?

(d) If you were to suggest revising a slide, which one would you revise?

(e) Identify a slide you thought was particularly well done.

2. (9.4.2) The tent map is a simple analytical model that has some properties in common with the Lorenz map. Let

$$x_{n+1} = \begin{cases} 2x_n, & 0 \leq x_n \leq \frac{1}{2} \\ 2 - 2x_n, & \frac{1}{2} \leq x_n \leq 1. \end{cases}$$

(a) Sketch the graph of $f(f(x))$. How many times does it intersect the curve $y = x$?

(b) Show the map has a period-2 orbit. This means that there is an x such that $f(f(x)) = x$ (but $f(x) \neq x$).

(c) Let $g(x) = f(f(x))$. Period-2 points are fixed points of g. Apply the derivative condition to $g(x)$ and use the chain rule to classify the stability of any period-2 orbits.
Period-1 points are fixed points of g as well - why?.

(d) Look for a period-3 or period-4 point. If you find one, are such orbits stable or unstable?

(e) If you want, you can think about whether there is a period-k orbit...

3. (An interesting geometric structure. Example 4.9, Alligood et al) Consider the tent map $x_{n+1} = T_3(x_n)$, defined as

$$x_{n+1} = \begin{cases} 3x_n, & x_n \leq \frac{1}{2} \\ 3(1 - x_n), & \frac{1}{2} \leq x_n. \end{cases}$$

This is a slope-3 tent map. Consider it to be defined on the whole real line. Let C be the set of points on the real line whose orbits do not diverge to $-\infty$. This is the set of initial conditions where points in the orbit never become negative for T_3.

The set C has an interesting (fractal) structure.

(a) Sketch the map.

(b)
- Identify the fixed points of the map.
- Identify points that map to zero, so $T_3(x) = 0$.
- Identify points whose second iterate, $T_3^2(x)$, is zero (these are points that map to points that map to zero).

These are all points that are part of C.

(c) Convince your team that initial conditions outside of $[0, 1]$ will all have orbits that eventually diverge. Also convince yourselves that the interval $(1/3, 2/3)$ are the only points that leave the interval $[0, 1]$ under a single iteration of T_3 (note that $1/3$ and $2/3$ both stay in the interval). Sketch the line segments that are still in consideration for potentially being in C. Call this set C_1.

(d) What points will leave $[0, 1]$ under two iterations of T_3? Again sketch the line segments that are still in consideration for staying in the interval. Call this set C_2. Is the set of points closed or open? (i.e. do the intervals contain their endpoints or not?)

(e) Can you generalize this to sketch C_3 (three iterations)? What do you think will happen with k iterations?

(f) What points seem to be in C?