- There is a pre-class assignment in advance of class on Friday Sept 4th. Find the reading and videos via the Check Yourself C02 assignment on Canvas.
- The first problem set will be due on Friday Sept 11th.

Big picture

We will introduce the idea of a dynamical system along with two important types (maps and flows). We will distinguish between linear and nonlinear systems and will introduce a core question that we will be tackling all semester.

After today, the next few classes will focus on analyzing one-dimensional flows to identify (and interpret) the long term behaviors.

Teams

You have been pre-assigned to a breakout room and team. There is extra overhead to meeting a group remotely, so we will stay in the same teams for a few class meetings.

1.

Zoom poll 1: dynamical systems

> **Dynamical systems**. In this course we will study *dynamical systems*: systems that evolve in time, with a rule that specifies their evolution.

Dynamical systems can have deterministic evolution rules (meaning that the current state of the system uniquely determines its future state) or can have evolution rules where randomness (stochasticity) is involved. In this course we will learn about *deterministic dynamical systems*. In subjects such as differential equations, mechanics, chemical kinetics, and biology, dynamical models are used to describe, predict, and inform the control of the behavior of a time-evolving system.

> **Team work**. In the classroom, this course takes a team-based approach to learning.

- Explicating your ideas for your peers while working to advocate for your own understanding will contribute to the development of your communication and collaboration skills within a technical context.
- Your learning will be strengthened by your efforts to explain your ideas to others.
- Your learning will be strengthened by working to understand the perspective of your classmate.
- Approaching mathematical ideas in a team will expose you to a range of problem solving approaches.

Slack poll 1: #classactivities channel

Before Friday's class There is a pre-class video assignment for Friday.

This will be the only time this semester that there is a pre-class assignment for a Friday. See Canvas for the assignment. It is due by 1:20pm ET on Friday September 4th. If possible, please submit it a few hours in advance of the class meeting. Your discussion board posts with questions and comments will shape the information I share during the first part of class.

Goals for today:

- Provide an explanation for the term "dynamical system"
- Work collaboratively with another student
- Develop reasoning to identify the possible long term behaviors that can occur in a particular dynamical system (the cosine map).
- Work to identify the qualitatively different types of long term behavior that occur in a particular dynamical system as we vary a parameter (population model).

Identifying, and interpreting, qualitatively different long term behaviors is the central question we will work on this semester.

1. Once you're in your breakout room, introduce yourself to your teammates.

2. Connect to Jamboard and find the right slide for your group.

   Head to the C01 jamboard `https://jamboard.google.com/d/1N_y-wNYbhnrBVlLcuQjoiYDlithAHa5D` `viewer?f=0`.

   Change slides (the slide changer is at the top) to find the slide (1 - 8) that corresponds to the number of your breakout group.

   To practice writing and to identify your group, write your own name in the corner of your group's jamboard slide.

3. (Map example) Consider the map $x \mapsto \cos x$. ($x \in \mathbb{R}$ defines the *state* of the system). Given an initial value, x_0, we have

$$
\begin{aligned}
x_1 &= \cos(x_0) \\
x_2 &= \cos(x_1) \\
&\vdots
\end{aligned}
$$

   Using your Jamboard as a shared whiteboard, work together on the following questions. If you run out of space on your slide, take a screenshot, clear your work, and keep going on the same slide.

   You can zoom in to a part of the slide to work, so that your work is more compact.

   - Select a starting value for x_0 and try iterating this map. *You may use a calculator to do this exactly or a graph of* $\cos x$ *to do this approximately*. Plot a time series of the iterates (x_k vs k). What happens?

     Use dots for your plot, rather than a connected line or curve. Why?
   - How does your starting value of x matter?
   - Identify the group member whose birthday is closest to today. Have them post on Slack about your group's approach and reasoning. Compare it with the reasoning developed in that group.

4. (Simple population model) In a simple linear model of population, the population at the next timestep, x_{n+1}, is modeled as a constant multiple (use constant a with $a > 1$) of the population at this timestep, x_n.

   - Write down an equation relating x_{n+1} to x_n. This equation is linear in x_n. What does that mean?
   - Let the initial population be $x_0 = b$ with $b \in (0, \infty)$. Find formulas for x_1 and for x_n in terms of b and a.
   - What happens to x_n at long times?
   - Critique this as a population model. Based on your prior knowledge, when could you imagine it might be reasonable and when would it not be?

- Now remove the constraint that $a > 1$ on a. Let $a \in \mathbb{R}$. What different behavior do you see as you change a? Describe all of the possibilities.
- When do you think different values of a might be more or less appropriate for a population model? Justify your answer.

Consider $x_{n+1} - x_n$, the change in the state of the system with a single timestep. In this **linear system** the change is proportional to the value of x_n. As x_n changes, the constant of proportionality is not changing. We will refer to this as not having a **feedback** in the system.

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5. (Ordinary differential equation population model) Now we switch away from maps (where time was discrete) to a (differential equation) population model where time is continuous (a **flow**). We will work with continuous models for much of the semester.

   Instead of using discrete generations, we make the assumption that the population grows continuously at a rate α.
   $$\frac{dN}{dt} = \alpha N$$
   describes the rate of change in population with time. Note: We will often write $\dot{N}$ in place of $\frac{dN}{dt}$.

   The rate of change per person is $\frac{dN/dt}{N} = \alpha$. This is an extremely simple population model: the rate of change in population per member of the population is assumed to be constant, rather than depending on how many individuals are in the population. This means that whether the population is large or small the growth rate per member is fixed.

   - Plot $\frac{dN/dt}{N}$ as a function of population, N. What does $\frac{dN/dt}{N}$ represent in the context of the model?
   - Show that $N(t) = N_0 e^{\alpha t}$ is a solution of this differential equation, and graph a time series of this solution for a few values of N_0 and α. *To show that an expression is a solution to an equation, plug the expression in and show that the equation then holds.* **Don't approach this by solving for the solution of the diff eq.**
   - What is the long term behavior of the population?
   - How does this compare to the behavior of the discrete model above?

Technical terms: dynamical system, deterministic, stochastic, map, system state, long term behavior, initial condition, qualitative, discrete, continuous, differential equation

Only a few kinds of long term behavior of a solution are possible in the linear systems we have encountered today:

- exponential decay towards a particular value
- exponential growth
- starting and staying at a fixed value
- in the map example, oscillation while doing one of the above was also possible

In the reading/videos for Friday's class, you will see examples of nonlinear systems. Many more possible behaviors exist in nonlinear systems. Our analysis of those systems will rely on linear approximation. Thus the intuition that linear systems show exponential growth or exponential decay of solutions will be helpful for understanding nonlinear systems as well.