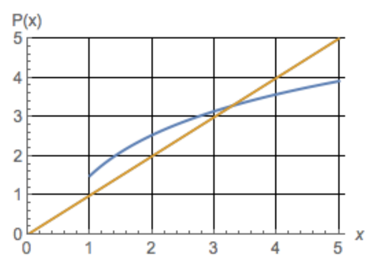
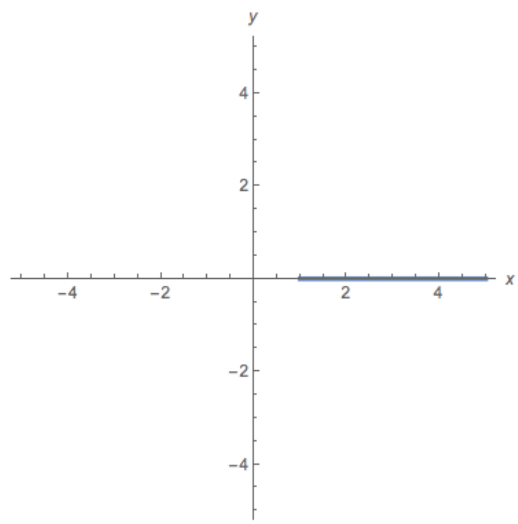
- Problem set 07 is due today.
- Quiz 02 is Monday.
- There will be a skill check on Wednesday. The practice problem is below.
- Problem set 08 is due next Friday. An initial project proposal is part of the problem set.

Skill Check Practice

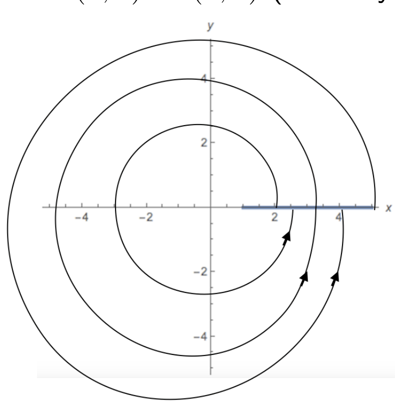
Let the line segment Σ be the section of the x-axis given by $1 \le x \le 5$. A Poincaré map taken along a line segment Σ is shown on the left (blue curve). Sketch two trajectories that are consistent with the Poincaré map on the axes to the right.





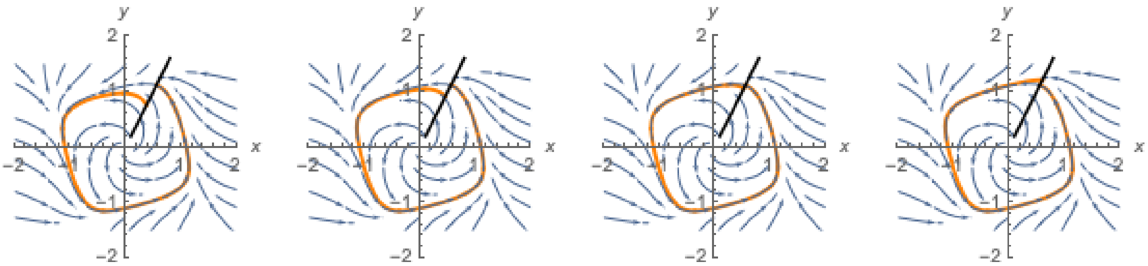
Skill check practice solution

There's a fixed point of $P(x)$ at about 3.4, so a closed orbit through $(3.4, 0)$ (and avoiding the line segment Σ) is one trajectory consistent with the map. $x = 5$ has $P(x) \approx 4$ so a trajectory that connects $(5, 0)$ to $(4, 0)$ (and stays outside the closed orbit) is also consistent with the map.

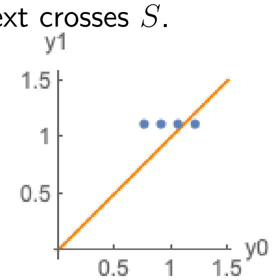


Big picture

Today we are looking at a method for identifying the stability of a closed trajectory and approximating its location.



To approximate a Poincaré map, I start on the line segment S (shown in black in the phase portraits) and integrate forward in time until the trajectory (shown in orange in the phase portraits) next crosses S.

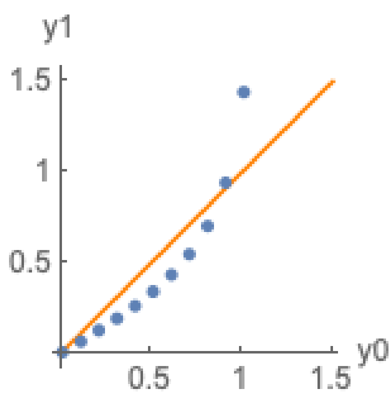


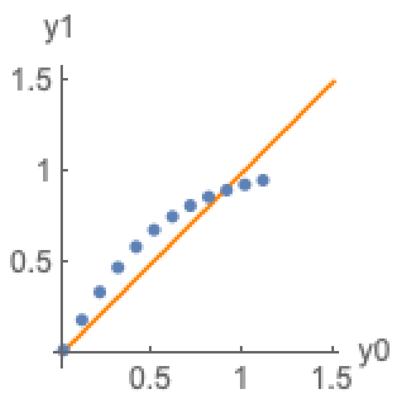
In the graph above, I am plotting the y-value at two successive crossings of S. I start at a point (x_0, y_0) on S and return to S at the point $(x_1, y_1) = P(x_0, y_0)$. I am plotting points (y_0, y_1) in blue. This is a representation of the Poincaré map. In orange, I am plotting the line where $y_1 = y_0$. When $y_1 = y_0$, we have a fixed point.

Stability

Consider the two Poincaré maps represented below.

- For each map, find the approximate y value associated with a closed trajectory.
- If you start close to the closed trajectory, will you approach the closed trajectory or will you move away from it?





A slope between -1 and 1 is associated with moving towards the closed trajectory (stable limit cycle) while a slope steeper than 1 is associated with moving away from it (unstable limit cycle).

Teams

1. Alice, Allison, Margaret
2. George, Ada
3. Christina, Camilo, Katheryn
4. David A, Dina, Shefali
5. Alex, Noah
6. Hiro, Joseph, Van
7. David H, Iona, Michail
8. Mallory, Sophie, Emily
9. Thea, Mariana

Teams1 & 2, post photos of your work to the class Google Drive (see Canvas for link). Make a folder for today's class if one doesn't exist yet.

Project

Meet your project team. Have each team member share their topic interests. Work to identify interests that multiple members of your team have in common.

Questions

1. (8.6.1: "Oscillator death" and bifurcations on a torus) We have worked with models of a single oscillator following a reference oscillator but haven't had the chance to work with a model where each oscillator responds to the other oscillator.

   This model is from Ermentrout and Kopell (1990), where the authors were considering a system of interacting neural oscillators. They developed a simple example with two interacting oscillators that captured many of the interaction properties they wanted for their neural system. Specifically, they wanted to capture that coupling between oscillators can actually suppress oscillation ("oscillator death") and lead to a steady state of the coupled system. Here is their example model:

$$\dot{\theta}_1 = \omega_1 + \sin\theta_1 \cos\theta_2$$
$$\dot{\theta}_2 = \omega_2 + \sin\theta_2 \cos\theta_1.$$

   The oscillators have a natural frequency, but they also are responding to each other.

   There are a number of different behaviors possible in this system. We will work to figure out the possible behaviors by identifying bifurcations and plotting a stability diagram in $\omega_1\omega_2$ space.

   (a) Looking for fixed points of $\phi = \theta_1 - \theta_2$ allows us to identify curves where $\theta_1 = \theta_2 + c$ where c is a constant.
   Here, use both $\phi = \theta_1 - \theta_2$ ("phi") and $\psi = \theta_1 + \theta_2$ ("psi") to aid your analysis.
   If $\dot{\phi} = 0$ and $\dot{\psi} = 0$ (and only if this is true) then the system has a fixed point. Why is that?

   (b) Find $\dot{\phi}$ and $\dot{\psi}$ equations. *Look up trig identities as needed.*

   (c) In what region of the $\omega_1\omega_2$ plane does the system have fixed points?

   (d) In what regions of the $\omega_1\omega_2$ plane does this system have $\dot{\phi} = 0$ or $\dot{\psi} = 0$ but not both? Sketch a phase portrait in the $\theta_1\theta_2$ plane in such a case.