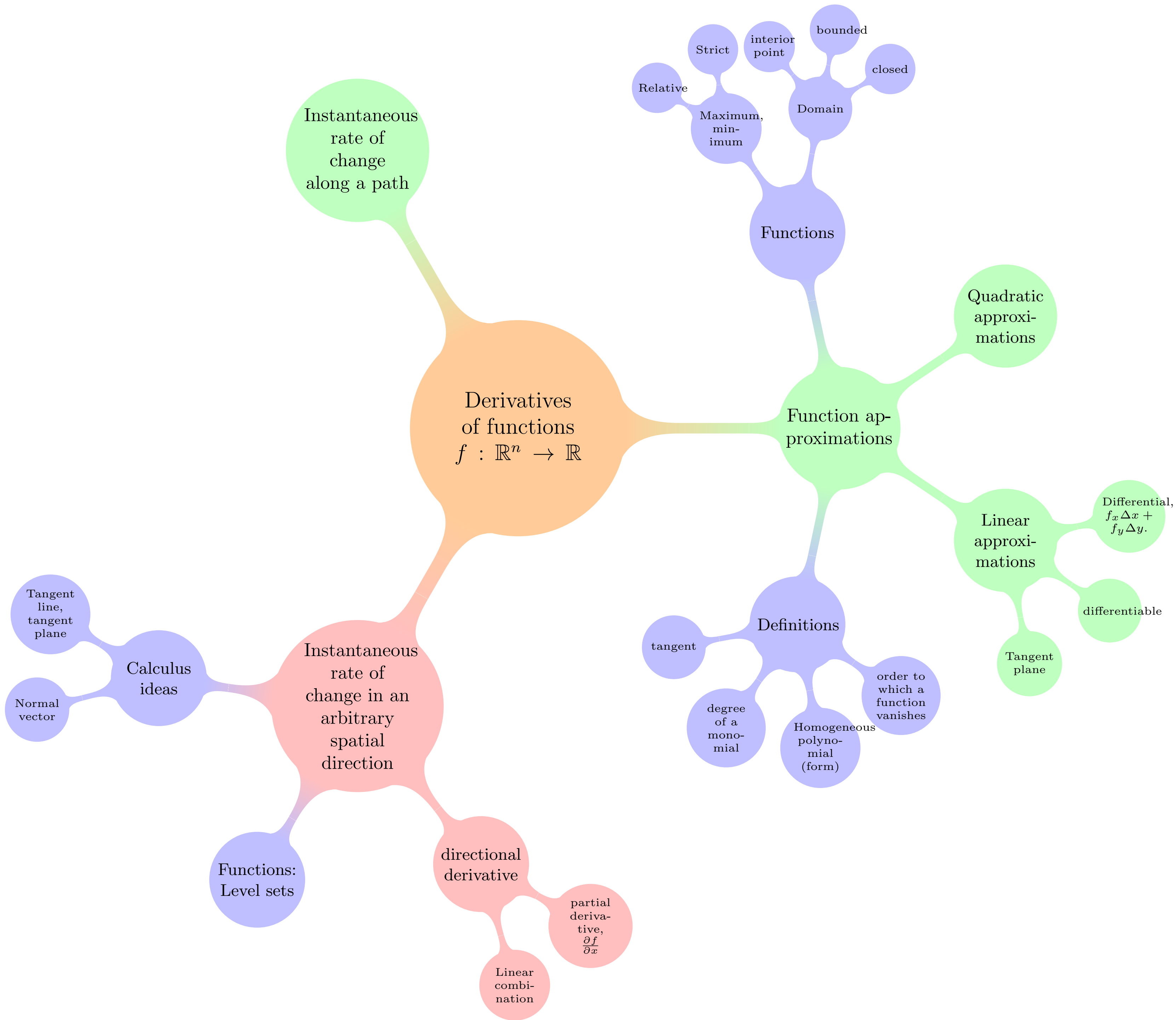
1 Partial differentiation mindmap



Glossary

degree The *degree of a monomial* $a_{mn}x^m y^n$ is the sum $m+n$ of the exponents of the variables appearing in it (given that $a_{mn} \neq 0$). The *degree of a polynomial* written as a linear combination of monomials is the highest degree of the monomial terms (Courant and John, Wikipedia).

differentiable The function $u = f(x,y)$ is called *differentiable* at the point (x,y) if it can be approximated in the neighborhood of this point by a linear function. That is, if $f(x+h, y+k) = Ah + Bk + C + \epsilon\sqrt{h^2+k^2}$ where A, B, C are independent of the variables h and k, and ϵ tends to 0 as h and k do (Courant and John).

directional derivative The *derivative in the direction* $\vec{v}$ is the rate of change of f at the point (x,y) with respect to distance as we leave (x,y) along the ray in the direction of $\vec{v}$. The *directional derivative* is a linear combination of the derivatives f_x and f_y in the x- and y- directions with the coefficients $\cos\alpha$ and $\sin\alpha$ where α is the angle the vector $\vec{v}$ makes with the x-axis (Courant and John).

form A *homogeneous polynomial*, or a *form*, is a polynomial consisting of monomials which all have the same degree N. For example, $x^2 + 4xy$ is a quadratic form (Courant and John).

monomial Term of the form $a_{mn}x^m y^n$ where m, n are non-negative integers and a_{mn} is arbitrary (Courant and John).

order Consider a function $\phi(h,k)$ which tends to 0 as h and k do. Let $\rho = \sqrt{h^2+k^2}$. A function $\phi(h,k)$ vanishes as $\rho \to 0$ to at least the same order as ρ provided there exists a constant C independent of h and k such that $\left|\frac{\phi(h,k)}{\rho}\right| \leq C$ holds for all sufficiently small values of ρ. We write $\phi(h,k) = O(\rho)$. More generally, if a comparison function $\omega(h,k)$ is defined for all nonzero values of (h,k) in a sufficiently small circle about the origin and is not equal to 0 then $\phi(h,k)$ *vanishes to at least the same order as* $\omega(h,k)$ as $\rho \to 0$ if for some suitably chosen constant C the relation $\left|\frac{\phi(h,k)}{\omega(h,k)}\right| \leq C$ holds in a neighborhood of the point $(0,0)$. We write $\phi(h,k) = O(\omega(h,k))$ (Courant and John).

partial derivative The *partial derivative of* $f(x,y)$ *with respect to* x at the point (x_0, y_0) is $\lim\limits_{h\to 0} \frac{f(x_0+h,y_0)-f(x_0,y_0)}{h}$, denoted $\frac{\partial f}{\partial x}$ with the round letter ∂ (Courant and John).

tangent A tangent line to a function $f(x)$ at a point a has equation $y = f(a) + (x-a)f'(a)$. A tangent plane to a function $f(x,y)$ at a point (a,b) has equation $z = f(a,b) + (x-a)f_x(a,b) + (y-a)f_y(a,b)$. The tangent is a linear function with matching position and slope to the original function at the point of tangency (Wolfram MathWorld).