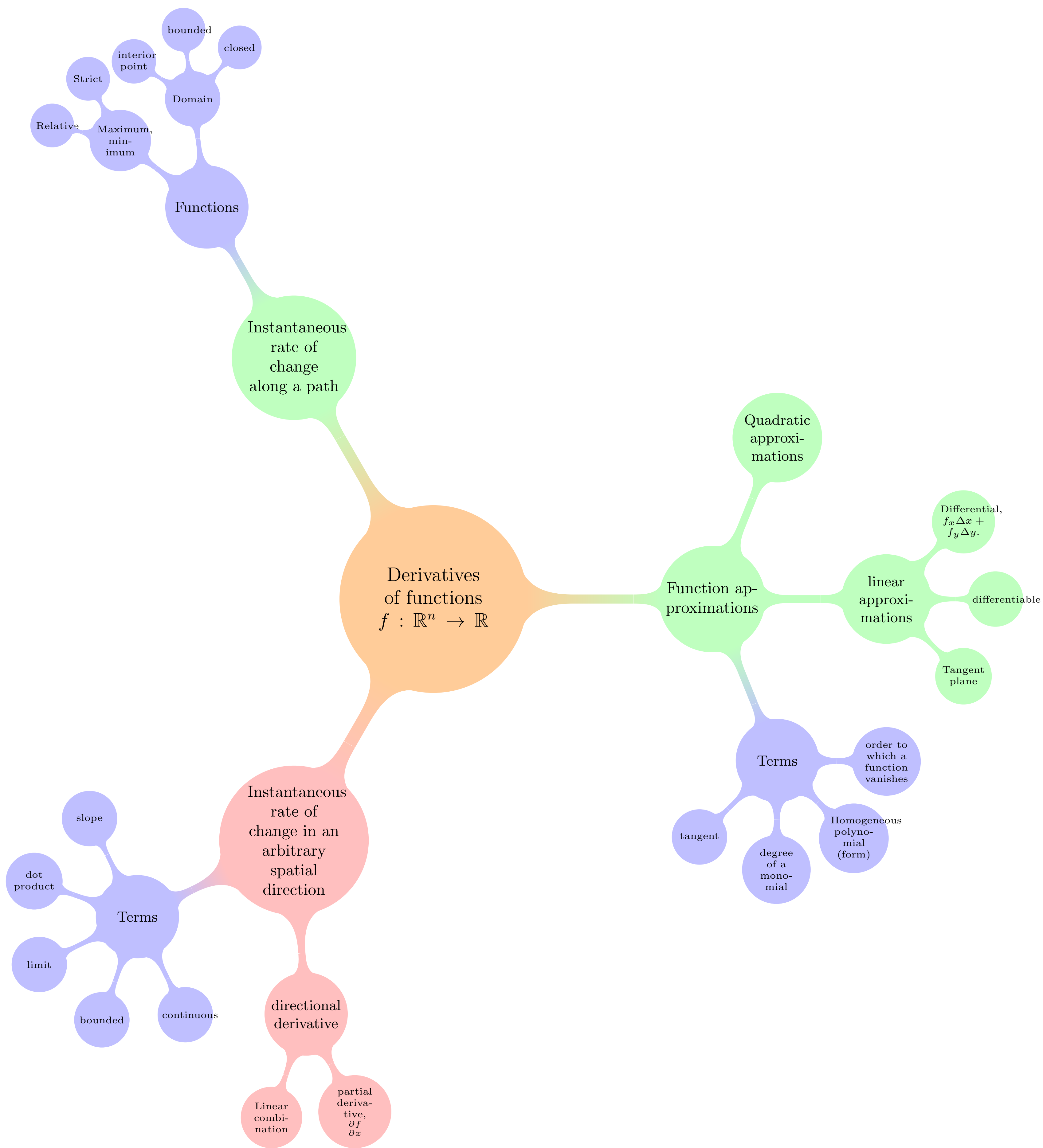
1 Partial differentiation mindmap



Glossary

continuous Roughly speaking: the function $u = f(x, y)$ is continuous at the point (x_0, y_0) when the value of $f(x, y)$ is close to the value of $f(x_0, y_0)$ for (x, y) close to (x_0, y_0). To make this precise: If f has domain R and $P_0 = (x_0, y_0)$ is a point in R then r is continuous at P_0 if for every $\epsilon > 0$ there exists a $\delta > 0$ such that $|f(P) - f(P_0)| = |f(x, y) - f(x_0, y_0)| < \epsilon$ for all $P = (x, y)$ in R within a δ-neighborhood of P_0 (meaning that $d(P, P_0) = \sqrt{(x - x_0)^2 + (y - y_0)^2} < \delta$). Note that the sum, the product, and the difference of two continuous functions are also continuous. The quotient requires more care to avoid zeros of the denominator. (Courant and John).

degree The *degree of a monomial* $a_{mn}x^m y^n$ is the sum $m + n$ of the exponents of the variables appearing in it (given that $a_{mn} \neq 0$). The *degree of a polynomial* written as a linear combination of monomials is the highest degree of the monomial terms (Courant and John, Wikipedia).

differentiable The function $u = f(x, y)$ is called *differentiable* at the point (x, y) if it can be approximated in the neighborhood of this point by a linear function. That is, if $f(x + h, y + k) = Ah + Bk + C + \epsilon\sqrt{h^2 + k^2}$ where A, B, C are independent of the variables h and k, and ϵ tends to 0 as h and k do (Courant and John).

directional derivative The *derivative in the direction* $\vec{v}$ is the rate of change of f at the point (x, y) with respect to distance as we leave (x, y) along the ray in the direction of $\vec{v}$. The *directional derivative* is a linear combination of the derivatives f_x and f_y in the x- and y- directions with the coefficients $\cos\alpha$ and $\sin\alpha$ where α is the angle the vector $\vec{v}$ makes with the x-axis (Courant and John).

form A *homogeneous polynomial*, or a *form*, is a polynomial consisting of monomials which all have the same degree N. For example, $x^2 + 4xy$ is a quadratic form (Courant and John).

limit Suppose $f(x, y)$ is a function with domain R and $P_0 = (x_0, y_0)$ is a point in the closure or R. We say f *has the limit* L *for* (x, y) *tending to* (x_0, y_0), written $\lim_{(x,y)\to(x_0,y_0)} f(x, y) = L$ or $\lim_{P\to P_0} f(P) = L$ if for every $\epsilon > 0$ we can find a δ-neighborhood of P_0 (meaning points P where

$d(P, P_0) = \sqrt{(x-x_0)^2 + (y-y_0)^2} < \delta$) such that $|f(P) - L| = |f(x,y) - L| < \epsilon$ for all $P = (x,y)$ in that neighborhood and belonging to R. (Courant and John).

linear A linear function is a polynomial of degree one ($u = ax + by + c$ where a, b, c are constants). (Courant and John).

monomial Term of the form $a_{mn}x^m y^n$ where m, n are non-negative integers and a_{mn} is arbitrary (Courant and John).

order Consider a function $\phi(h,k)$ which tends to 0 as h and k do. Let $\rho = \sqrt{h^2 + k^2}$. A function $\phi(h,k)$ vanishes as $\rho \to 0$ to at least the same order as ρ provided there exists a constant C independent of h and k such that $\left|\frac{\phi(h,k)}{\rho}\right| \le C$ holds for all sufficiently small values of ρ. We write $\phi(h,k) = O(\rho)$. More generally, if a comparison function $\omega(h,k)$ is defined for all nonzero values of (h,k) in a sufficiently small circle about the origin and is not equal to 0 then $\phi(h,k)$ *vanishes to at least the same order as* $\omega(h,k)$ as $\rho \to 0$ if for some suitably chosen constant C the relation $\left|\frac{\phi(h,k)}{\omega(h,k)}\right| \le C$ holds in a neighborhood of the point $(0,0)$. We write $\phi(h,k) = O(\omega(h,k))$ (Courant and John).

partial derivative The *partial derivative of* $f(x,y)$ *with respect to* x at the point (x_0, y_0) is $\lim_{h \to 0} \frac{f(x_0+h,y_0) - f(x_0,y_0)}{h}$, denoted $\frac{\partial f}{\partial x}$ with the round letter ∂ (Courant and John).

tangent A tangent line to a function $f(x)$ at a point a has equation $y = f(a) + (x-a)f'(a)$. A tangent plane to a function $f(x,y)$ at a point (a,b) has equation $z = f(a,b) + (x-a)f_x(a,b) + (y-a)f_y(a,b)$. The tangent is a linear function with matching position and slope to the original function at the point of tangency (Wolfram MathWorld).