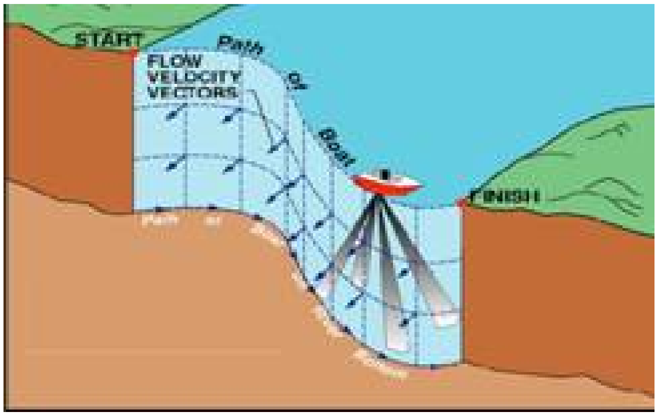
1. Log in to WeBWorK and complete the problems assigned there under pset09.

2. 

   Refer back to problem set 08 for the formula that was derived to compute the flux of the water given boat velocity and water velocity information.

   (a) We have velocity measurements for the water flow in a single vertical column of a river. For this column, the boat is moving 0.318 ft/s east and -0.205 ft/s north for a time period of $\Delta t = 0.56$ s. The column is made up of multiple boxes, and each of the boxes has a depth of 0.2 ft. Find an approximation of the area of one of the boxes in this column.

   (b) Open AM22bPSet09.m to access velocity data at thirteen different depths in the water column. Use the boat velocity and time information from part (a) along with this flow data to find an approximation of the flux of water downstream through the column. Provide a written explanation of how you used the data to approximate the flux, and include a screenshot of your calculation code, along with your estimated value, in your Gradescope submission.

3. Consider the second order differential equation $a\frac{d^2y}{dt^2} + b\frac{dy}{dt} + cy = 0$ where a, b, c are constants. Let $y_1(t) = e^{-2t}\cos 5t$ and $y_2(t) = e^{-2t}\sin 5t$.

   (a) Is the differential equation linear or nonlinear? How do you know?

   (b) Find a, b, c such that $y_1(t)$ and $y_2(t)$ are both solutions to the differential equation.

   (c) Consider a function, $y_3(t)$, that is a linear combination of the two solutions. Let $y_3(t) = k_1 y_1(t) + k_2 y_2(t)$. Either show that $y_3(t)$ is a solution to the differential equation, or show that it is not.

4. (a) Let W be a solid region of volume V surrounded by a closed surface S, oriented outward. Show that $\frac{1}{3}\int_S \vec{r} \cdot d\vec{A} = V$. Here the vector field is $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$.

   This is a remarkable result: it creates a way to approximate the volume of a region when you only have information about points on the surface of the region.

   (b) Suppose div $\underline{F} = x^2 + y^2 + 3$. Find a surface, S such that $\int_S \underline{F} \cdot d\underline{A}$ is negative (for any vector field where div $\underline{F} = x^2 + y^2 + 3$), or explain why no such surface exists.

5. Let S be the portion of the unit sphere centered at the origin that is above the cone $z = \sqrt{x^2 + y^2}$. *Note that the term 'sphere' refers to a surface, not a solid.* Let $\underline{F} = \langle xy + \cos z, -yx + x^2 + z^3, 2z^2 + x\rangle$. Find $\int_S \underline{F} \cdot d\underline{S}$.

Radial symmetry in this problem can help you simplify the integrals. If you use cylindrical or spherical, integrate first with respect to θ, so that cancellations due to radial symmetry happen before you tackle the rest of the integral.

6. Consider the initial value problem $\frac{dy}{dt} = \sqrt{y}$, $y(0) = 1$.
   (a) Working in matlab, use Euler's method to compute three different approximate solutions. Use $\Delta t = 1.0, 0.5, 0.25$ over the interval $0 \leq t \leq 4$. Provide your approximations for $y(2)$ and $y(4)$ for each method.
   (b) Graph your three solutions (all on the same axes, so that they can easily be compared).
   (c) Make a prediction about $y(2)$ and $y(4)$ for the actual solution to the initial value problem.
   (d) Improve your approximation: choose a small enough step size, $h = \Delta t$, so that your approximation for $y(4)$ using h and using $h/2$ differ by less than 0.5%.

```
% sample code for Euler's method
clear('tval','yval')
tval(1) = 1790; % initial time
yval(1) = 3929214; % initial population
k = 1; % index
dt = 2; % time step
const = 0.0223;
endtime = 2050;
while tval(k) < endtime
    dydt(k) = yval(k)*const; % dP/dt = const * P
    tval(k+1) = tval(k) + dt;
    yval(k+1) = yval(k) + dydt(k)*dt; % Pnew approx= Pold + dP/dt*delt t
    k = k+1;
end
plot(tval,yval)
```