- There is a skill check in class on Monday for skill C04 and C05. The C05 sample problem is at the end of this handout. Skill checks (and retakes) are done from memory without any aids (notes, calculators, etc).
- The C02 / C03 retake will be available on Monday. It is optional and should be completed for any skill that was not satisfied on the original skill check.
- There will be a pre-class assignment due on Monday.
- PSet 01 is due on Thursday Feb 4th at 6pm.
- PSet 02 will be due on Thursday Feb 11th at 6pm.

Big picture

This week we are working with vectors and vector products, with a focus on moving between the algebraic and geometric definitions of the dot product and the cross product. These vector products will become very important in March: we'll be using them often once we start working with functions that have vectors for their output.

Skill Check Practice

1. Let $\underline{u}$ be the vector  and $\underline{v}$ be the vector .

   Is $\underline{u} \times \underline{v}$ into the page/screen (away from you) or out of the page/screen (towards you)?

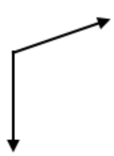
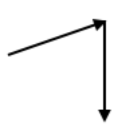
   ◯ into the page ◯ out of the page

2. Compute $(2\underline{i}+\underline{j})\times(2\underline{j})$ **using distributive and scalar multiplication properties** of the cross product along with the cross product relationships between $\underline{i}, \underline{j}, \underline{k}$.

cross product:

Skill check solution

1. Two options for finding the direction using the right hand rule:

   - Draw . Make a flat hand. Leave your thumb and index (pointer) finger in place. Bend your other three fingers towards your palm. Rotate your hand to align the index finger with the first vector ($\underline{u}$) in such a position that you can also align the other fingers with ($\underline{v}$). Which way is your thumb pointing?
   - Draw , with $\underline{u}$ first and $\underline{v}$ drawn from its tip. Place your palm along $\underline{u}$ oriented so that you can bend your fingers to align with $\underline{v}$. Which way is your thumb pointing?

Your thumb should be into the page/screen in both cases.

2. Distribute:

$$\begin{aligned}(2\underline{i}+\underline{j})\times(2\underline{j}) &= (2\underline{i})\times(2\underline{j})+(\underline{j})\times(2\underline{j})\\ &= 4(\underline{i}\times\underline{j})+2(\underline{j}\times\underline{j})\\ &= 4(\underline{i}\times\underline{j})\\ &= 4\underline{k}.\end{aligned}$$

Teams

New teams today; introduce yourself to your group.

1. student names

From C04 Handout:

The **dot product** between two vectors of the same size, $\underline{u}$ and $\underline{v}$, is given by $\underline{u}\cdot\underline{v} = u_1v_1 + u_2v_2 + \ldots + u_nv_n$ where $\underline{u} = (u_1, u_2, \ldots, u_n)^T$ and $\underline{v} = (v_1, v_2, \ldots, v_n)^T$. This is the **algebraic definition** of the dot product.
The **angle** between the vectors $\underline{u}$ and $\underline{v}$ can be found using the relationship $\underline{u}\cdot\underline{v} = \|\underline{u}\|\|\underline{v}\|\cos\theta$. This is the **geometric definition** of the dot product.

Dot product example. Find $\underline{u}\cdot\underline{v}$ where $\underline{u} = 3\underline{i}+\underline{j}-\underline{k}$ and $\underline{v}$ is a vector of length 2 oriented at an angle of $\pi/4$ away from the direction of $\underline{u}$.

Computing the cross product §13.4

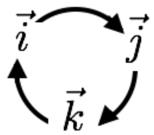
The **cross product** of vectors $\underline{u} = \begin{pmatrix} u_1\\ u_2\\ u_3\end{pmatrix}$ and $\underline{v} = \begin{pmatrix} v_1\\ v_2\\ v_3\end{pmatrix}$, denoted $\underline{u}\times\underline{v}$, is

$$\underline{u}\times\underline{v} = \begin{pmatrix} u_2v_3 - u_3v_2\\ u_3v_1 - u_1v_3\\ u_1v_2 - u_2v_1\end{pmatrix}.$$

The cross product is **anti-commutative**: $\underline{u}\times\underline{v} = -\underline{v}\times\underline{u}$.

Example. Using the definition above, what is $\vec{i}\times\vec{j}$? What is $\vec{j}\times\vec{i}$?

One way people sometimes remember these relationships is with the following diagram:



Example. Find $\underline{v} \times \underline{v}$.

> The cross product **distributes** over addition so $(\underline{u}+\underline{v}) \times \underline{w} = \underline{u} \times \underline{w} + \underline{v} \times \underline{w}$.
> **Scalar multiplication** has the following property: $(c\underline{u}) \times \underline{v} = \underline{u} \times (c\underline{v}) = c(\underline{u} \times \underline{v})$.

Example. Find $\underline{v} \times \underline{w}$ with $\underline{v} = 3\underline{i} - 2\underline{j} + 4\underline{k}$ and $\underline{w} = \underline{i} + 2\underline{j} - \underline{k}$ by using the distributive and scalar multiplication properties above, along with the diagram giving the cross product relationships between $\underline{i}, \underline{j}, \underline{k}$.

Cross product geometry §13.4

> The cross product $\underline{u} \times \underline{v}$ is **orthogonal** to $\underline{u}$ and to $\underline{v}$. *Compute* $\underline{u} \cdot (\underline{u} \times \underline{v})$ *to convince yourself of this.*
> The vector $\underline{u} \times \underline{v}$ points in the direction given by the **right hand rule** (with $\underline{u}$ playing the role of x and $\underline{v}$ playing the role of y in the xyz coordinate system).
> Given three points in a plane that are not co-linear, you can construct two vectors in the plane. Using the cross product to find a normal vector, you can then construct an equation for the plane.

> The **magnitude of the cross product**, $\|\underline{u} \times \underline{v}\|$ is the area of the parallelogram with edges $\underline{u}$ and $\underline{v}$.
> The **area of the parallelogram** is also given by $\|\underline{u}\|\|\underline{v}\| \sin\theta$, where θ is the angle between the vectors $(0 \leq \theta \leq \pi)$, so $\|\underline{u} \times \underline{v}\| = \|\underline{u}\|\|\underline{v}\| \sin\theta$.
> The **area of a triangle** with sides $\underline{u}, \underline{v}, \underline{u} - \underline{v}$ is given by $\frac{1}{2}\|\underline{u} \times \underline{v}\|$.

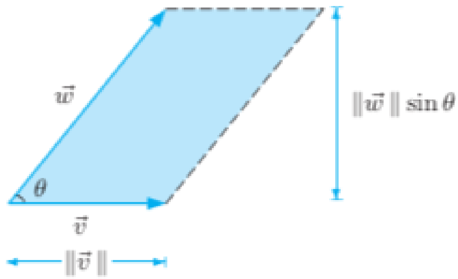


Figure 13.35: Parallelogram formed by $\vec{v}$ and $\vec{w}$ has Area $= \|\vec{v}\|\|\vec{w}\| \sin\theta$

Example. Let points $(0, 1, 2)$, $(2, -1, 3)$ and $(0, 0, 1)$ form a triangle that lies in a plane.

- Find a normal vector to the plane, and construct an equation for the plane

- Find the area of the triangle.

Question.
Figure 13.44 shows the tetrahedron determined by three vectors $\underline{a}, \underline{b}, \underline{c}$.

> The **area vector** of a face is a vector perpendicular to the face, pointing outward, whose magnitude is the area of the face.

We want to show that the sum of the four outward pointing area vectors of the faces equals the zero vector.
Discuss how to approach this problem.

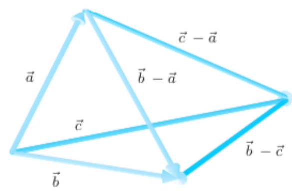


Figure 13.44

Review questions

1. True or False (discuss): There is only one point in the yz-plane that is distance 5 from the point $(3, 0, 0)$.
2. Explain what is wrong with the statement: A contour diagram for $z = f(x, y)$ is a surface in xyz-space.
3. Explain what is wrong with the statement: The functions $f(x, y) = \sqrt{x^2 + y^2}$ and $g(x, y) = x^2 + y^2$ have the same contour diagram.