- The next skill check will be for C19, C20, C21 on Monday Mar 22nd.
- There is a pre-class assignment for Wed Mar 17th.
- There is a discussion board post due Thurs Mar 18th (no problem set).
- Quiz 03 will be posted on Friday Mar 19th (info is on Canvas).
- There are no office hours this Tuesday (wellness day).

Skill Check C19 Practice

1. Assume $x, y > 0$. For $\underline{F} = x\underline{j}$, decide if
   (a) the vectors in the vector field are
   ◯ parallel to the x-axis ◯ parallel to the y-axis ◯ neither
   (b) As x increases the length of the vectors
   ◯ increases ◯ decreases ◯ neither
   (c) As y increases the length of the vectors
   ◯ increases ◯ decreases ◯ neither

Skill Check C19 Practice Solution

1. (a) The vector field is $\langle 0, x \rangle$ so vectors are parallel to the y-axis.
   (b) As x increases $\|\underline{F}\| = |x|$ also increases.
   (c) As y increases $\|\underline{F}\| = |x|$ does not change, so neither increases nor decreases.

Big picture

Today is focused on vector valued functions, today interpreted as vector fields rather than as parameterized curves or surfaces. We will soon return to integration (with these new functions).

Teams

You will work with this team on the in-class problems today.

1. student names

Vector fields §17.3

A function $f : \mathbb{R}^n \to \mathbb{R}^k$ is a **vector-valued function** because there are multiple outputs associated with each input.
The output of the function can be interpreted in two different ways:

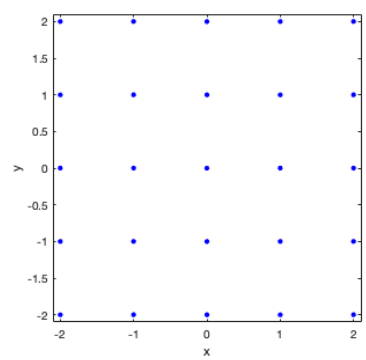
- as a mapping assigning a vector in k-space to each point in $\mathbb{R}^n$. This is a **vector field**
- as a mapping assigning a point in k-space to each point in n-space. This is a **parameterization**.

To create a representation of a vector field,

- we sketch vectors at points in $\mathbb{R}^n$.
- Although we only draw the vectors at select points, the vector field is defined for every point in the domain.

Example (vector field).

Consider $\underline{F}(x, y) = x\underline{i} + 0\underline{j}$. At each point (x, y) in the xy-plane we have a corresponding vector $x\underline{i}$. We can sketch these vectors. This is the vector field $\underline{F}(x, y) = \langle x, 0\rangle$.

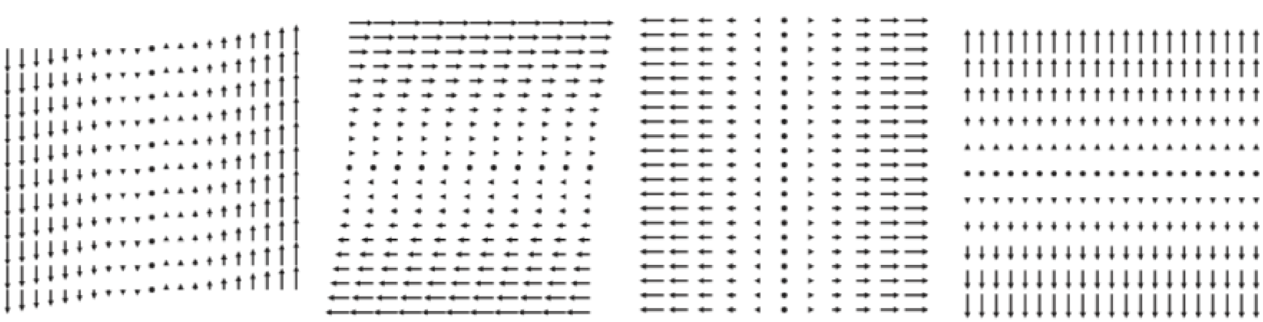


Sketch this vector field using a unit grid ranging from $-2 \le x \le 2$, $2 \le y \le 2$.

Question (matching). Match the vector fields

$$f(x)\underline{i}, \quad g(x)\underline{j}, \quad h(y)\underline{i}, \quad k(y)\underline{j}$$

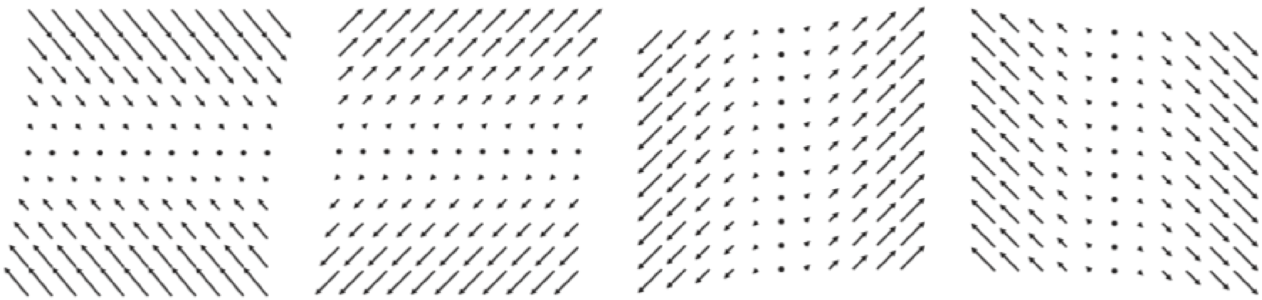
to the images below.



Question (matching). Match the vector fields

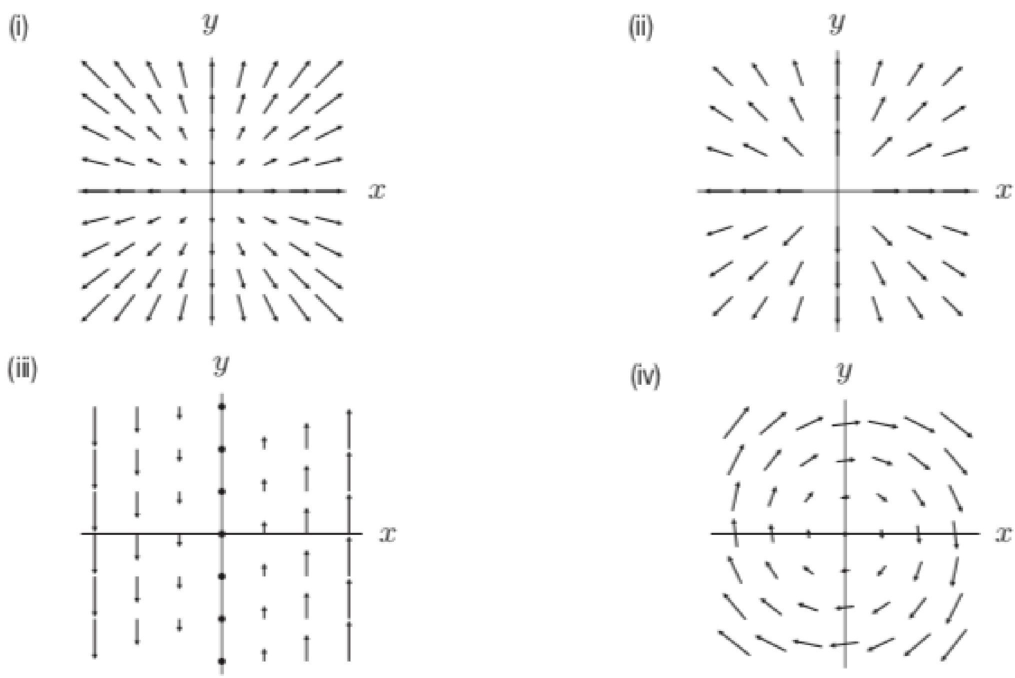
$$f(x)\underline{i} + f(x)\underline{j}, \quad g(x)\underline{i} - g(x)\underline{j}, \quad h(y)\underline{i} + h(y)\underline{j}, \quad k(y)\underline{i} - k(y)\underline{j}$$

to the images below.



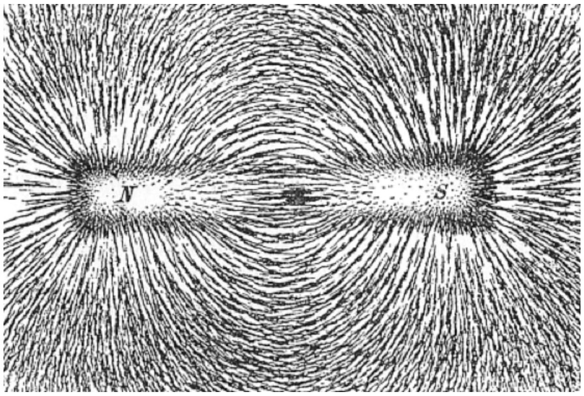
Examples (vector fields to know).

$\underline{F}_i = \underline{r}$, $\underline{F}_{ii} = \frac{r}{\|\underline{r}\|}$. $\underline{F}_{iii} = x\underline{j}$. $\underline{F}_{iv} = y\underline{i} - x\underline{j}$.



> **Force fields** are a common type of vector field. Because forces have a direction as well as a magnitude, and force fields act at each point in space, a vector field is an appropriate representation.

Below is a visualization of a magnetic field. The vectors themselves are not quite visible. Each iron filing is aligned with the magnetic field. However the length of filings does not vary with the strength of the field, as it would it we were drawing in vectors:



http://www.txessrevolution.org/MagneticSun_Info

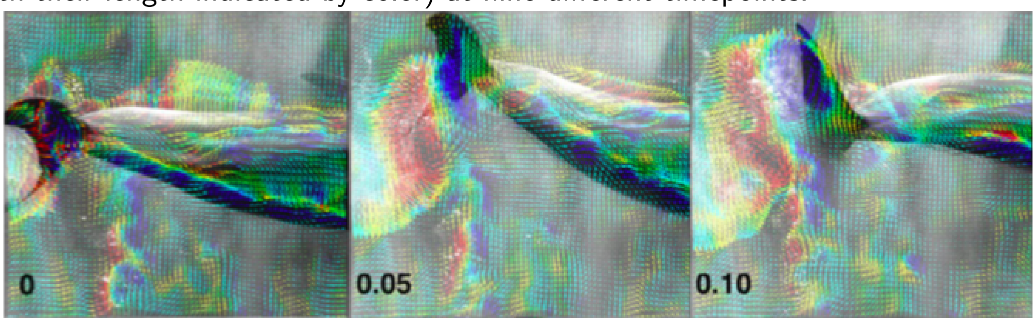
Example: gravitational force

$\underline{F}(\underline{r}) = -\frac{GMm\underline{r}}{\|\underline{r}\|^3}$. $\underline{r} = \langle x, y, z\rangle$, so this is a compact notation for $\underline{F}(\underline{r}) = -GMm(\frac{x}{(x^2+y^2+z^2)^{3/2}}\underline{i} + \frac{y}{(x^2+y^2+z^2)^{3/2}}\underline{j} + \frac{z}{(x^2+y^2+z^2)^{3/2}}\underline{k}$

$\underline{F}(\underline{r})$ is a radial vector field where all vectors point towards the origin.

Figure 17.19: The gravitational field of the earth

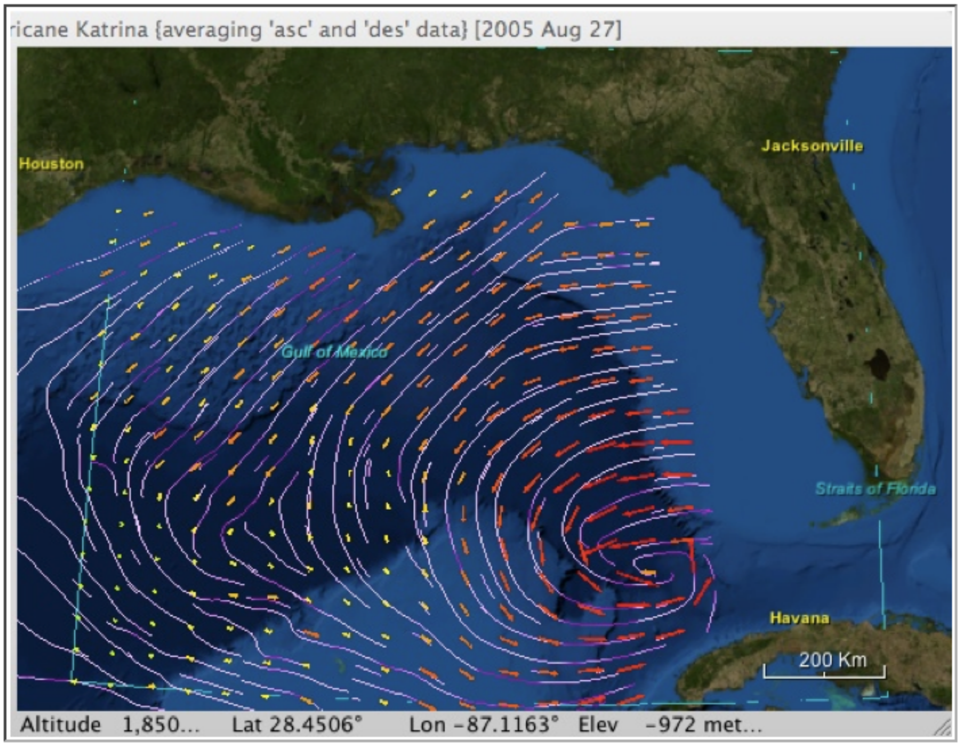
Example (dolphin motion). Velocity vectors of the water motion as a dolphin swims are shown (with their length indicated by color) at nine different timepoints.



[Fish et al.(2014)Fish, Legac, Williams, and Wei]

Example (wind velocity).

Here is an example where vectors are indicating the wind velocity near Hurricane Katrina (measured by a satellite in Aug 2005):



https://people.eecs.ku.edu/~miller/WorldWindProjects/VectorFieldVis/5DayKatrinaNoGrid.html

The length of each vector indicates its relative magnitude. The vectors are color coded by magnitude: red is high speed, and yellow low.

Velocity vector fields: flow lines §17.4

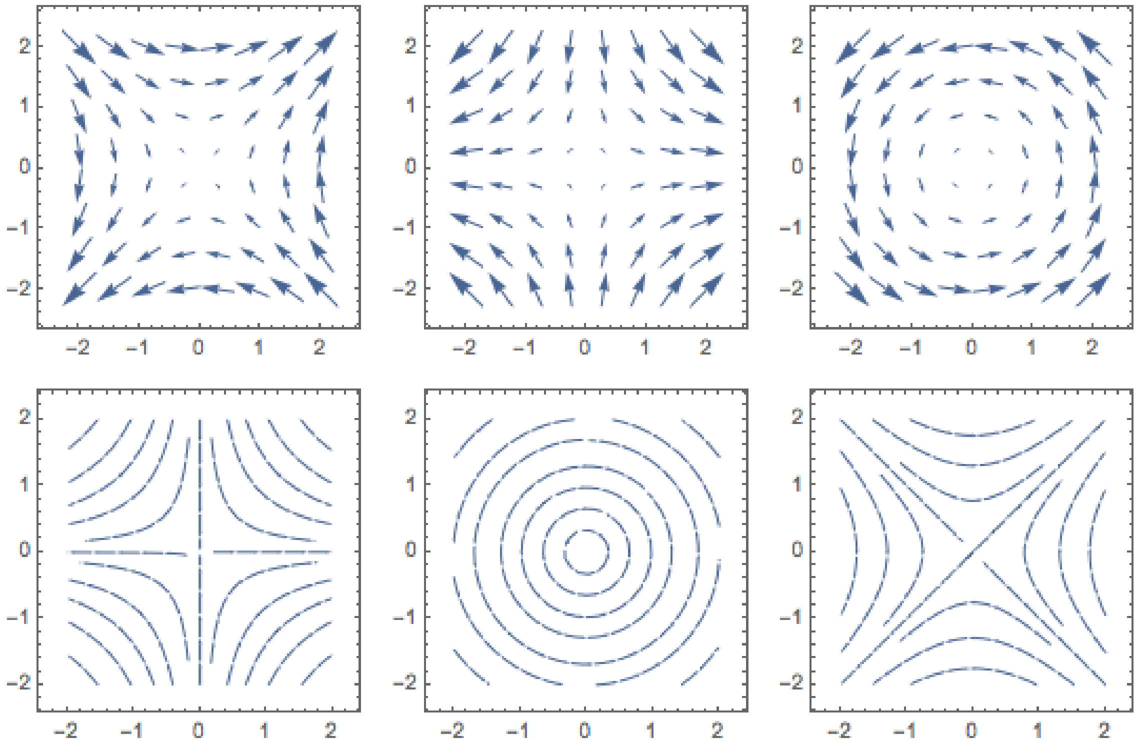
A *flow line* of a vector field $\underline{v} = \underline{F}(\underline{r})$ is a path $\underline{r}(t)$ where the velocity vector at each point on the path is equal to $\underline{v}$. On a flow line $\frac{d\underline{r}}{dt}(t) = \underline{v} = \underline{F}(\underline{r}(t))$.

The *flow* of a vector field is the family of all of its flow lines.

The lines superimposed on the image of wind velocities are flow lines (sometimes called *stream lines*). They indicate the trajectory a massless tracer particle would follow if it were subjected to these wind velocities.

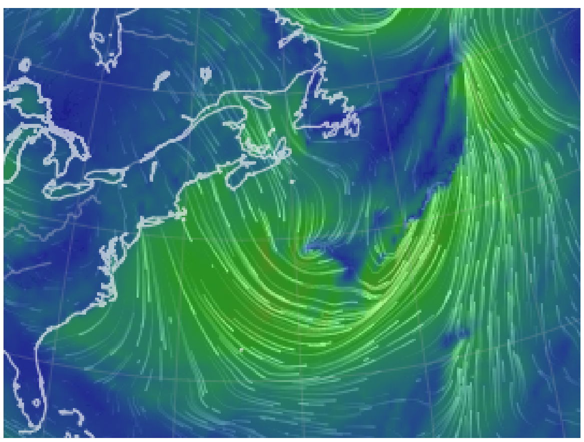
It is common to visualize a vector field using flowlines (streamlines).

Flow lines. Match each of $\underline{F}_1 = -y\underline{i} + x\underline{j}$, $\underline{F}_2 = x\underline{i} - y\underline{j}$ and $\underline{F}_3 = y\underline{i} + x\underline{j}$ to a vector field and to a plot of flow lines below.



Example (today's wind).

Flow lines are found by seeding a region with tracer particles and seeing how they travel under the action of the vector field (where vectors are treated as velocities). Below is a snapshot of a visualization of wind at the surface of the earth in the North Atlantic on the morning of this class.



`https://earth.nullschool.net/#current/wind/surface/level/orthographic=-64.86,47.01,444`

References

[Fish et al.(2014)Fish, Legac, Williams, and Wei] Frank E Fish, Paul Legac, Terrie M Williams, and Timothy Wei. Measurement of hydrodynamic force generation by swimming dolphins using bubble dpiv. *Journal of Experimental Biology*, 217(2):252–260, 2014.