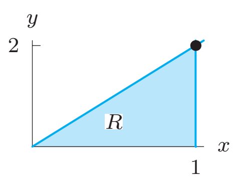
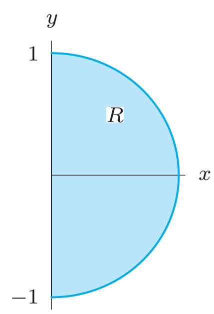
1. Log in to WeBWorK and complete the problems assigned there under pset04.
2. For the following, identify whether the statement is true or false. Include a justification of your choice.
   (a) If $f(x, y) = k$ for all points (x, y) in a region R then $\int_R f\, dA = k \cdot \text{Area}(R)$.
   (b) If R is the rectangle $0 \leq x \leq 1, 0 \leq y \leq 2$ and S is the square $0 \leq x \leq 1, 0 \leq y \leq 1$, then $\int_R f\, dA = 2 \int_S f\, dA$.
3. Evaluating an integral example video: `https://www.youtube.com/watch?v=DgsTXYpbcu4`
   (a) Sketch the region of integration and evaluate the integral for
   $$\int_1^4 \int_{\sqrt{y}}^{y} x^2 y^3\, dx\, dy.$$
   (b) For the two regions shown below, compute the integral of $f(x, y) = xy$ over the region shown.

28.



29.



4. Convert the following from polar coordinates to Cartesian coordinates or from Cartesian coordinates to polar coordinates. Sketch each curve in the xy-plane.
   (a) $r = \frac{2}{\sin\theta}$.
   (b) $y = \sqrt{1 - x^2}$.
   (c) $r = \frac{1}{\cos\theta}$.
   (d) $\theta = 3\pi/4$.
5. Electric charge is distributed throughout 3-space, with density proportional to the distance from the xz-plane. Show that the total charge inside of a cylinder of radius R and height h that sits on the xz-plane and is centered along the y-axis is proportional to $R^2 h^2$. *You may want to think about this using a "cylindrical" coordinate system centered about a different axis than usual.*
6. Suppose W is the region outside the cylinder $x^2 + y^2 = 1$ and inside the sphere $x^2 + y^2 + z^2 = 2$. Calculate
   $$\int_W (x^2 + y^2)\, dV.$$
   Include a sketch of a cross-section of the region in the rz-half plane.

7. (Center of mass of a *Tyrannosaurus rex*)

   This data is from the authors of [1]. We have permission to use it for the purposes of this activity only. It is based on a lidar scan of a *T. rex* skeleton.

   The matlab data in `TRex.mat` is in pixels. There are two sets of pixels: a lower resolution and a higher resolution set. For all of your work on this problem, assume the top of the head of the *T. rex* is at 4.7m for each dataset.

   (a) Make scatter plots of both data sets. Rescale your data so that it is in meters, rather than pixels. Submit the plots (include the code in your Canvas submission, but it does not need to go onto Gradescope). Remember axis labels. Using `axis equal` will help the dinosaur skeleton be in the correct proportions.

   (b) (center of mass) Look up center of mass in the index of our course text (p. 890 in the 6th edition). After reading about center of mass, develop a procedure for estimating the center of mass of the dinosaur skeleton using the data provided. Make an assumption of uniform density for the bones.

   Describe your plan in words (you may use pseudocode or complete sentences).

   (c) Estimate the center of mass for the dinosaur skeleton based on the data that was provided. Do this for each resolution of data. Submit a screenshot of your code on Gradescope, as well as your final estimates.

   (d) (moment of inertia) Modify your algorithm to estimate the moment of inertia instead of the center of mass. Submit analogous deliverables to part (c).

   (e) We have data about the skeleton of this dinosaur. Consider how the center of mass of the actual dinosaur might differ from that of its skeleton. Given the skeleton, how might you estimate the center of mass of the dinosaur? Describe your ideas. Identify at least three assumptions that you would be making.

   There are many possible approaches and choices of assumptions.

   (f) (Optional) If you'd like, you can attempt to estimate the location of the center of mass of the dinosaur.

References

[1] John R Hutchinson, Karl T Bates, Julia Molnar, Vivian Allen, and Peter J Makovicky. A computational analysis of limb and body dimensions in tyrannosaurus rex with implications for locomotion, ontogeny, and growth. *PLoS One*, 6(10):e26037, 2011.