

1. Find the equation of a tangent plane to $x^3 + 2xy - z = 1$ at the point $(1, 2, 4)$.

2. (a) Compute the directional derivative of $f(x, y) = x^2y - y^2$ at $(1, 2)$ in the direction of $\underline{u} = \langle 2, 2 \rangle$.

- (b) Find the maximum possible directional derivative at $(1, 2)$ (choosing from any direction).

3. Let $z = f(x, y) = xy + y^2$ with $x = u^2 + v^2$ and $y = u^2 - v^2$. Let $\underline{x} = \begin{pmatrix} x \\ y \end{pmatrix}$ and $\underline{u} = \begin{pmatrix} u \\ v \end{pmatrix}$.
Find $\frac{\partial z}{\partial \underline{x}}$ and $\frac{\partial \underline{x}}{\partial \underline{u}}$. Use the chain rule to find $\frac{\partial z}{\partial \underline{u}}$. Evaluate it at $u = 2, v = 1$.