- Review sections §14.1-§14.6, §16.1-2 in Hughes-Hallett, the course text.
- The odd numbered problems in the Exercise sections (the first chunk of problems) in each section are worthwhile practice for before a problem set. *Answers are in the back for odd numbered problems, and solutions are in the student solutions manual is available in Cabot.*
- The 'Check your understanding' or 'Strengthen your understanding' questions are a great way to check whether you're building intuition for course concepts.

1. Log in to WeBWorK and complete the problems assigned there under pset03.
2. The cardiac output, c, is the volume of blood flowing through a person's heart per unit time. The systemic vascular resistance (SVR), s, is the resistance to blood flow in the veins and arteries. Let p be a person's blood pressure.
$$p = f(c, s).$$
   (a) What does the instantaneous rate of change $\dfrac{\partial p}{\partial c}$ represent?
   (b) Suppose that p is proportional to c and to s, so $p = kcs$. Create a contour plot in Matlab for the case $k = 1$.
   Use the `'LevelList'` option to set the levels of the contours. For your plot, add labels, thicken your contour lines so that they are visible, add a title, and adjust the font size on the axes so that it is at least 14.
   What do these level curves represent in the context of this problem?
   (c) For a person with a weak heart, it is desirable to have the heart pump against less resistance while maintaining the same blood pressure. Such a person may be given the drug nitroglycerine to decrease the systemic vascular resistance, and the drug dopamine to increase the cardiac output. Add this scenario to your contour plot (within Matlab) by marking a point A that represents the person's state before drugs are administered and a point B for their state after.
   The command `plot(3,2,'k.','MarkerSize',10)` will place a black dot at the point $(3, 2)$. The command `text(3,2.2,'A','fontsize',14)` will place the letter "A" at the point $(3, 2.2)$ (to act as a label).
   (d) After a heart attack a patient's cardiac output drops, causing blood pressure to drop. The text says "a common mistake made by medical residents is to get the patient's blood pressure back to normal by using drugs to increase the SVR, rather than by increasing the cardiac output".
   Using Matlab, add points D, E, and F to your contour diagram to show this scenario. Choose D for the patient before the heart attack, E for the patient immediately after the heart attack, and F after the patient has been given drugs to increase the SVR.
   Make sure to choose a point D where increasing SVR is the wrong thing to do.
3. Two surfaces can be said to be *tangential* at a point (a, b, c) if they have the same tangent plane at that point. In this problem, you'll find all points in 3-space where the two surfaces $z = \sqrt{2x^2 + 2y^2 - 16}$ and $z = \frac{1}{4}(x^2 + y^2)$ are tangential.

(a) Graph the two surfaces in matlab. Use transparency (the `'FaceAlpha'` option in `fsurf`) to make the surfaces more visible.

Label your axes, adjust the font size, and give your plot a title. Use the rotation tool to explore the surfaces. I found the tangency was very visible in my plot.

Submit the graph as part of your problem set. You do not need to submit this code on Gradescope.

(b) Find the set of points where the surfaces are tangential.
It may be helpful to think of $z = \sqrt{2x^2 + 2y^2 - 16}$ *as the surface* $2x^2 + 2y^2 - z^2 = 16$.

(c) Check your work by showing (algebraically) that the each point you've identified satisfies the equation for each surface.

(d) In addition, confirm that the surfaces have the same tangent plane at each point in your set. *Explicitly showing this mathematically is great. However, it is sufficient to explain how you know that you've found the tangent planes, and that they are identical.*

4. Let $x = r\cos\theta$, $y = r\sin\theta$, so $r = \sqrt{x^2 + y^2}$ and for $x > 0, y > 0, \theta = \arctan(y/x)$. This is the polar coordinate system. It is common to need to move back and forth between Cartesian coordinates and polar coordinates depending on which coordinate system is more convenient to use for a particular problem. The chain rule allows us to relate partial derivatives in one coordinate system to partial derivatives in the other.

Let $z = f(x, y)$. Show your mathematical steps for the work below.

(a) Let $\underline{u} = \begin{pmatrix} r \\ \theta \end{pmatrix}$. Let $\underline{w} = \begin{pmatrix} x \\ y \end{pmatrix}$. Find the Jacobian matrix $\dfrac{\partial \underline{w}}{\partial \underline{u}}$.

(b) We have $\dfrac{\partial z}{\partial \underline{u}} = [\partial z/\partial r, \partial z/\partial \theta]$ and $\dfrac{\partial z}{\partial \underline{w}} = [\partial z/\partial x, \partial z/\partial y]$. Use the chain rule and $\dfrac{\partial \underline{w}}{\partial \underline{u}}$ to write $\partial z/\partial r$ and $\partial z/\partial \theta$ in terms of $\partial z/\partial x$ and $\partial z/\partial y$, r, and θ.

(c) Find z_x and z_y in terms of z_r, z_θ, r, and θ. There are two ways to do this:

- Find $\partial \underline{u}/\partial \underline{w}$ and use the chain rule.
- Multiply your z_r equation from (b) by $\cos\theta$ and your z_θ equation by $-\frac{1}{r}\sin\theta$. Then sum the two equations and simplify to find an expression for z_x in terms of z_r and z_θ. Do something similar to isolate z_y.

(d) Show that

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2.$$

5. For each of the following iterated integrals, translate the bounds of each integral into equations, and sketch the region of integration.

(a)

$$\int_0^1 \int_{x-2}^{\cos \pi x} y \; dy \; dx.$$

(b)

$$\int_0^1 \int_{-1}^1 \int_0^{\sqrt{1-z^2}} 3z \; dy \; dz \; dx$$

(c)

$$\int_0^1 \int_{-\sqrt{1-z^2}}^{\sqrt{1-z^2}} \int_0^{\sqrt{1-x^2-z^2}} \sin x \; dy \; dx \; dz.$$

6. (Approximating the volume of the island of Kauai)

   In `imagesKauai.mat` you'll find two elevation data sets:

   - `imALOS` holds the ALOS elevation data. The elevation is in meters and each element of the matrix corresponds to a 30 by 30 meter box on the ground. `infoALOS` holds the latitude and longitude extent for the image.
   - `imUSGS` holds a USGS elevation dataset. The elevation is in meters and each element of the matrix corresponds to a 10 by 10 meter box on the ground. `infoUSGS` holds the latitude and longitude extent for the image.

   (a) On a single set of axes (with the horizontal axis given by longitude and the vertical axis by latitude), plot contours of both datasets. Include axis labels and submit your plot as part of your problem set.

   (b) Use the ALOS data to approximate the volume (above sea level at zero meters) of Kauai. Measure this in cubic meters.

   (c) Use the USGS data to approximate the volume (above sea level at zero meters) of Kauai. Measure this in cubic meters.

   (d) Compute the average height of Kauai.

   (e) If you were to flood the state of Massachusetts with 1 meter of water, would the volume of that water be larger or smaller than the volume of Kauai? *Provide your reasoning.*

   (f) Submit the Matlab code that you used for (a), (b), (c), (d) as part of your problem set submission.