- Review sections §13.1-§13.4 in Hughes-Hallett, the course text.
- The odd numbered problems in the Exercise sections (the first chunk of problems) in each section are worthwhile practice for before a problem set. *Answers are in the back for odd numbered problems, and solutions are in the student solutions manual is available in Cabot.*
- The 'Check your understanding' or 'Strengthen your understanding' questions are a great way to check whether you're building intuition for course concepts.

1. Complete the problems assigned via WeBWorK.

2. The point P in the figure below has position vector $\vec{v}$ obtained by rotating the position vector $\vec{r}$ of the point (x, y) by 90 degrees counterclockwise about the origin.

   Our text uses the term **position vector** *to refer to a vector where the tail is drawn at the origin.*

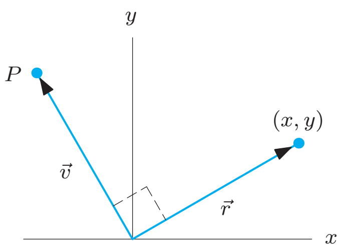


Figure 13.43

   (a) Use the geometric definition of the cross product to explain why $\underline{v} = \underline{k} \times \underline{r}$.

   (b) Find the coordinates of P. Explain your mathematical reasoning.

3. (a) Find a vector $\underline{v}$ with all of the following properties:
      - Magnitude 20,
      - angle of $45°$ with the positive x-axis,
      - angle of $60°$ with the positive y-axis,
      - negative z-component.

   (b) A vector $\underline{v}$ of magnitude v makes an angle α with the positive x-axis, β with the positive y-axis and γ with the positive z-axis. Show that $\underline{v} = v\cos\alpha\underline{i} + v\cos\beta\underline{j} + v\cos\gamma\underline{k}$.

   (c) For angles α, β, γ as in (b), show that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$.

4. We want to relate the area of a parallelogram to the area of its shadow in the xy-plane.

   Consider a parallelogram S lying in the plane $z = mx + ny + c$. It has a projection R into the xy-plane. Let S be determined by the vectors $\underline{u} = u_1\underline{i} + u_2\underline{j} + u_3\underline{k}$ and $\underline{v} = v_1\underline{i} + v_2\underline{j} + v_3\underline{k}$ as in the figure on the left below. The figure on the right shows S for a steeper plane.

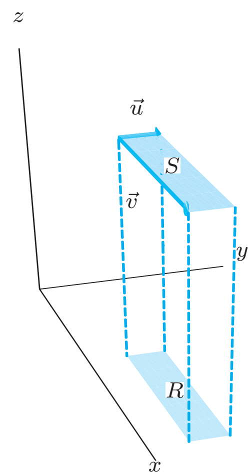
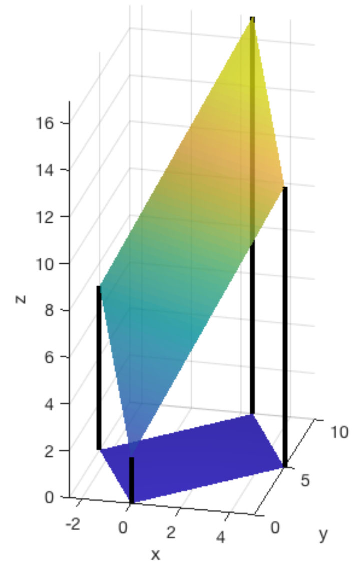


Figure 13.45



(a) Use the matlab template for this problem set to produce three different plots of parallelograms and their projections into the xy-plane. Insert the plots into your solution and submit the m-file with your code as a separate file when you submit your problem set.

Show your mathematical reasoning for the following:

(b) Find the area of R. *Find this in terms of* u_1, u_2, v_1, v_2.

(c) Find the area of S. *Find this in terms of the vector components.*

(d) Show that

$$\text{Area of } S = \sqrt{1 + m^2 + n^2} \cdot \text{ Area of } R.$$

(e) Show that the area of the projection of S into the xy-plane is $|\underline{A} \cdot \underline{k}|$ where $\underline{A}$ is the area vector for S. Write a similar expression that you think would give the area of the projection of S into the xz-plane.

5. (Revisiting land lost to sea level change). Last week you approximated the land area lost to sea level rise in an image that included Massachusetts.

   (a) After discussing the problem with your classmates, choose two different approximation schemes that you'll use to redo the calculation this week. Implement each method yourself. *You may choose the method you used last week, as well as a different method of approximating.*

   (b) Use each of these methods on

   - the cropped image with no smoothing and no downsampling
   - the cropped image with a little smoothing and a little downsampling
   - the cropped image with more smoothing and a little downsampling
   - the cropped image with a little smoothing and more downsampling
   - the cropped image with more smoothing and more downsampling

   to produce ten different estimates of the land area lost (measure the land area in km^2).

(c) Plot your ten estimates. *Insert the plot into your problem set write-up and remember to add labels to the axes.* Given these ten (presumably different) estimates, provide a single estimate for the approximate land lost and explain how you chose that estimate.

6. (cosine similarity) The following time series data is course enrollment data for four courses at Harvard:

course	2009	2010	2011	2012	2013	2014
cs50	337	478	607	715	693	825
stat 110	216	258	272	308	454	321
multi	620	568	649	690	767	777
greek hero	95	68	219	221	236	40

The 'multi' column aggregates all multivariable courses taught at Harvard in that year

These vectors are in 6-dimensional space.

(a) Use the cosine of the angle between the vectors as a measure of their similarity. This is the **cosine similarity**. Edit the Matlab code provided to compute the cosine similarity between each pair of vectors.

According to this measure,

- which two course enrollment time-series are most similar?
- which two are least similar?
- how wide is the variation?

(b) Instead of directly comparing the values in the time series, it is common to compare the variation of each time-series from its own mean. This is called **centering** the data. Use centered data to answer the questions above. *Add code for this to the end of your matlab file.*

(c) The cosine similarity of centered data is also referred to as the **correlation coefficient** of the two data vectors. Describe, in words, the relationship between the data in two vectors when the correlation coefficient is

- near 1
- near 0
- near -1

Late work policy: Because of the unusual circumstances of our semester, all students have access to deadline flexibility when needed. You may assume that extensions of up to two days will be approved without issue. Request those via direct message on Slack. When you request an extension, specify your preferred new deadline for the assignment.

Late WeBWorK is difficult to arrange (it requires a manual override of the course settings), so I suggest planning ahead to complete it on time.