1. Log in to WeBWorK and complete the problems assigned there under pset07.

2. (line integrals)

   (a) The vector field $\underline{F}$ has $\|\underline{F}\| \leq 7$ everywhere. Let C be the circle of radius 1 centered at the origin. What is the largest value that $\oint_C \underline{F} \cdot d\underline{r}$ might have? What is the most negative that it could potentially be? What conditions lead to these values?

   (b) Give conditions on the constants a, b, and c to ensure that the line integral $\int_C \underline{F} \cdot \underline{T} ds$ is positive, where $\underline{F} = a\underline{i} + b\underline{j} - \underline{k}$ and C is the line segment from $(1,2,3)$ to $(1,2,c)$

3. (more line integrals).

   (a) Explain why the following statement is true: whenever the circulation of a vector field around every closed curve is zero, the line integral along a curve with particular endpoints has a constant value independent of the path taken between the endpoints.

   Provide a direct explanation, without assuming that circulation free vector fields are gradient fields.

   (b) Explain why the **converse** to the statement is true: whenever the line integral of a vector field depends only on the endpoints and not on the path between them, the circulation around every closed curve is zero.

   Provide a direct explanation, without assuming that circulation free vector fields are gradient fields.

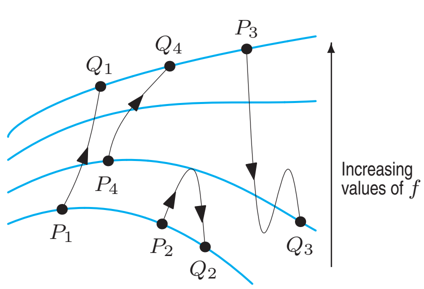
4. Use the Fundamental Theorem (of Line Integrals) to calculate $\int_C \vec{F} \cdot d\vec{r}$:

   (a) Compute $\int_C \left(\cos(xy)e^{\sin(xy)}(y\vec{i} + x\vec{j}) + \vec{k}\right) \cdot d\vec{r}$ where C is the line from $(\pi, 2, 5)$ to $(0.5, \pi, 7)$.

   (b) Let $\vec{F} = 2x\vec{i} + 2y\vec{j} + 2z\vec{k}$. Let C be the line from the origin to $(1,4,9)$. Find $\int_C \vec{F} \cdot d\vec{r}$. Use the result to simplify the computation involved, and find

$$\int_C \left((2x+y)\vec{i} + 2y\vec{j} + 2z\vec{k}\right) \cdot d\vec{r}.$$

5. Consider the line integrals $\int_{C_i} \vec{F} \cdot d\vec{r}$ for $i = 1,2,3,4$ where C_i is the path from P_i to Q_i shown in the figure below and where $\vec{F} = \nabla f$. Level curves of f are shown in the figure. Include your reasoning for each of the following:



(a) Which of the line integrals are zero, if any?

(b) Arrange the line integrals in ascending order from least to greatest.

(c) Two of the line integrals have equal and opposite values. Which are they? Identify which is negative and which is positive.

6. An interesting device called a planimeter (`https://en.wikipedia.org/wiki/Planimeter`) returns an area when the operator traces the boundary of a shape. The action of these devices is often explained via Green's theorem, with the ratchets in the device essentially doing a line integral.

   (a) Show that the line integral of the vector field $\vec{F} = x\vec{j}$ around any closed curve in the xy-plane, oriented as in Green's theorem, measures the area of the region enclosed by the curve.

   (b) Provide a second vector field $\vec{G}$ with the same property (that $\oint_C \vec{G} \cdot d\vec{r}$ returns the area of the region enclosed by the curve).