Advanced Algorithms, Homework 6

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due: Friday, 19 November 2021

This homework assignment should be submitted as a single PDF file both to D2L and to Gradescope.

General homework expectations:

- Homework should be typeset using LaTex.
- Answers should be in complete sentences and proofread.
- You will not plagiarize, nor will you share your written solutions with classmates.
- List collaborators at the start of each question using the `collab` command.
- Put your answers where the `todo` command currently is (and remove the `todo`, but not the word `Answer`).
- If you are asked to come up with an algorithm, you are expected to give an algorithm that beats the brute force (and, if possible, of optimal time complexity). With your algorithm, please provide the following:
  - *What*: A prose explanation of the problem and the algorithm, including a description of the input/output.
  - *How*: Describe how the algorithm works, including giving psuedocode for it. Be sure to reference the pseudocode from within the prose explanation.
  - *How Fast*: Runtime, along with justification. (Or, in the extreme, a proof of termination).
  - *Why*: Statement of the loop invariant for each loop, or recursion invariant for each recursive function.
- The lowest HW grade is dropped. However, this HW can only be dropped if the grade is at least 25%. (In other words, please do not choose to drop this homework by not doing any of it).
- This homework is an **individual** assignment.

CSCI 432 Problem 6-1

Collaborators on this problem: *TODO*

Describe an Algorithm Choose one concept or algorithm that you have learned in this class so far. A student who has taken 232 and 246, but not 432, has some questions about this problem:

1. *What* is the problem that this algorithm solves?

   Answer [[**XXX TODO:**]]

2. *How* does it work? (Describe in prose, with or without psuedocode.)

   Answer [[**XXX TODO:**]]

CSCI 432 Problem 6-2

Collaborators on this problem: *Weights on Vertices*

Chapter 8, Question 3, Part (a).

Answer [[**XXX TODO:** answer here]]

CSCI 432 Problem 6-3

Collaborators on this problem: *Weights on Vertices*

MED Consider the randomized minimum enclosing disc (MED) algorithm. In this homework, we investigate the worst-case analysis of it. In class, we will study the randomized analysis.

Algorithm 1 MED(S, Σ)

Input: two finite sets S, $\Sigma \subset \mathbb{R}^2$
Output: B, the smallest ball enclosing S with points of Σ on the boundary.

1: **if** $|S| = 0$ **then**
2: **return** the smallest ball with Σ on boundary
3: **end if**
4: $i \leftarrow$ RANDOM(n)
5: $B =$ MED($S \backslash S[i], \Sigma$)
6: **if** $S[i] \in B$ **then**
7: **return** B
8: **else**
9: **return** MED $((S \backslash S[i], \Sigma \cup \{S[i]\})$
10: **end if**

In the above algorithm, suppose RANDOM(n) returns a random integer between 1 and n (inclusive). Further suppose S is stored as an array with indices 1 though n, and $|\Sigma| \leq 3$. Let's represent the output ball as an ordered pair $B = (b_c, b_r) \in \mathbb{R}^2 \times \mathbb{R}_{\geq 0}$, where $b_c \in \mathbb{R}^2$ is the center of a smallest enclosing ball and b_r is the radius of the smallest enclosing ball.

(a) Suppose $S = \emptyset$ and $\Sigma = \{(a_x, a_y)\}$. What ball is returned on Line 2?

Answer [[**XXX TODO:** answer here]]

(b) Suppose $S = \emptyset$ and $\Sigma = \{a = (a_x, a_y), b = (b_x, b_y)\}$, what ball is returned on Line 2?

Answer [[**XXX TODO:** answer here]]

(c) Suppose $S = \emptyset$ and $\Sigma = \{a = (a_x, a_y), b = (b_x, b_y)\}$, what ball is returned on Line 2?

Answer [[**XXX TODO:** answer here]]

(d) Let $T(n)$ be the time complexity of MED(S, Σ) when $|S| = n$. Give the worst-case recurrence relation.

Answer [[**XXX TODO:** answer here]]

(e) What is the closed form of your answer to (c)?

Answer [[**XXX TODO:** answer here]]

CSCI 432 Problem 6-4

Collaborators on this problem: *None*

Algorithms in the News Find an algorithm discussed in a recent news article (over the past 12 months). Choose ONE of the following:

1. Look up the primary resource for this algorithm (likely to be a research paper). Compare/contrast the similarities and differences between the way the news article describes the problem and algorithm with the way that the primary resource describes it.
2. If the algorithm itself is not given in the article, provide a prose description of the algorithm along with pseudocode. (This might require looking up the primary resource for the algorithm).
3. Analyze the runtime of the algorithm.
4. Prove the correctness of the algorithm.

Answer [[**XXX TODO:** answer here]]

CSCI 432 Problem 6-5

Collaborators on this problem: *TODO*

Decrementing Function Prove that the generic algorithm for SSSP, FordSSSP, terminates using a decrementing function.

Answer [[**XXX TODO:** answer here]]

CSCI 432 Problem 6-6

Collaborators on this problem: *TODO*

All Pairs Shortest Paths Walk through ShimbelAPSP algorithm (Erickson, page 314) for the graph given in class on 11/10/21.

Answer [[**XXX TODO:** answer here]]