1 PROPOSAL: Solving the inverse problem for the Elastic Wave Equation to Localize Impact Source

The long term goal of my project to build a distributed network of PZT sensors whose cards can localize an impact in a composite board based on a method called Sparse Reconstruction. Figure 1 describes the physical setup of the PZT sensors.

For the puposes of this course, I will use [2] and [3] to find a finite volume solution to the elastic wave equation and I will also use a book called "Integral Geometry and Inverse Problems for Hyperbolic Equations" in order to solve the inverse problem. I will try to extrapolate the methods applied in [1]. In that paper, the source localization and solution to the inverse problem was done in the diffusion equation, not in a hyperbolic equation.

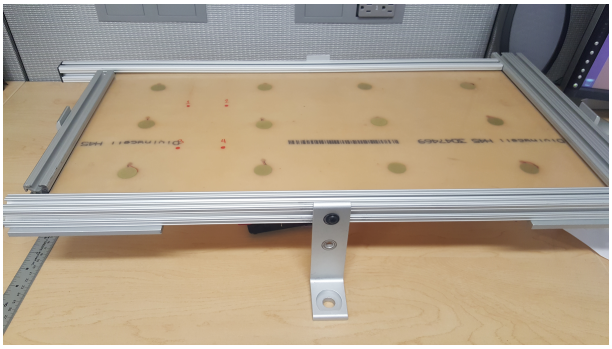

Figure 1: Physical setup of the sensors. It is a composite board with a grid of PZT sensors where each one is connected to a Particle Photon.

It is important to mention that the piezo-sensors can only measure the shear waves that occur during the impact/touch because the voltage comes from having the piezos get "squished." Another consideration to take into account is that solving time is a very important variable since we would like to obtain near-real-time results. Finally, a source position estimate $\hat{\mathbf{r}}^{(j)}(0) = [\hat{x}_{sk}, \hat{y}_{sk}, \hat{z}_{sk}]^T$ is what I intend to determine after the solution of the elastic wave equation. In Figure 1, the dots drawn with sharpies indicates the positions that I will be testing.

Here is my detailed plan of attack:

- Set up the elastic wave equations in rectangular coordinates and solve for the case where there is no external force or impact applied to the board
- Try then to solve for the inverse problem using the methods described in the book.
- Produce a heat map of those solutions.

- If I have enough time left, I will try to solve the same problem in an unstructured grid.

References

[1] G. a. Zou and B. Wang. Numerical methods for solving the direct and inverse problems of the parabolic equation. In *2011 International Conference on Internet Technology and Applications*, pages 1–3, Aug 2011.

[2] Emmanuel Dormy and Albert Tarantola. Numerical simulation of elastic wave propagation using a finite volume method. *Journal of Geophysical Research: Solid Earth*, 100(B2):2123–2133, 1995.

[3] Wensheng Zhang, Yuan Zhuang, and Eric T. Chung. A new spectral finite volume method for elastic wave modelling on unstructured meshes. *Geophysical Journal International*, 206(1):292–307, 2016.