Tópico 06:
Filtragem na frequência – (Processamento na frequência)

Prof. Dr. Matheus Cardoso Moraes

Contexto

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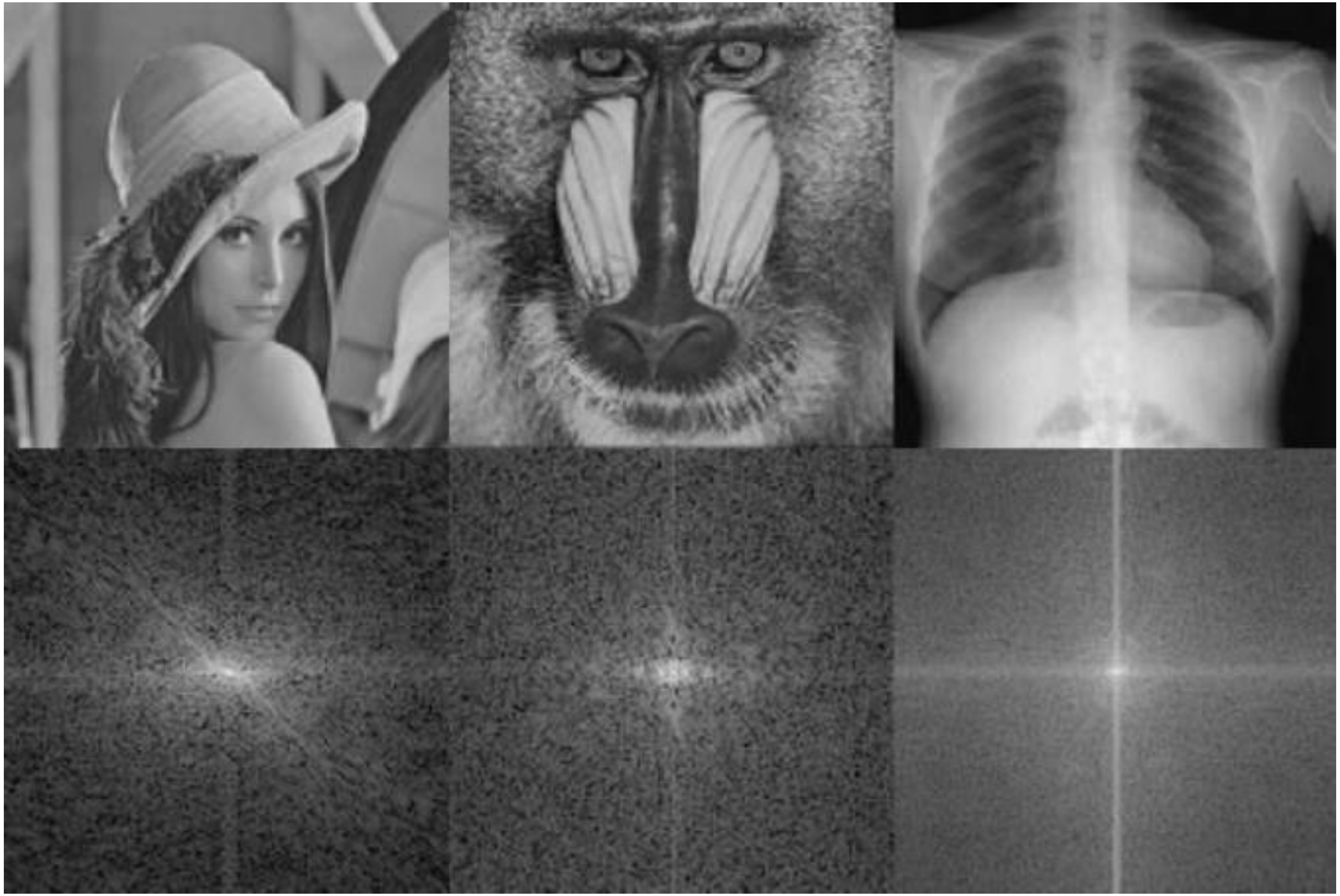
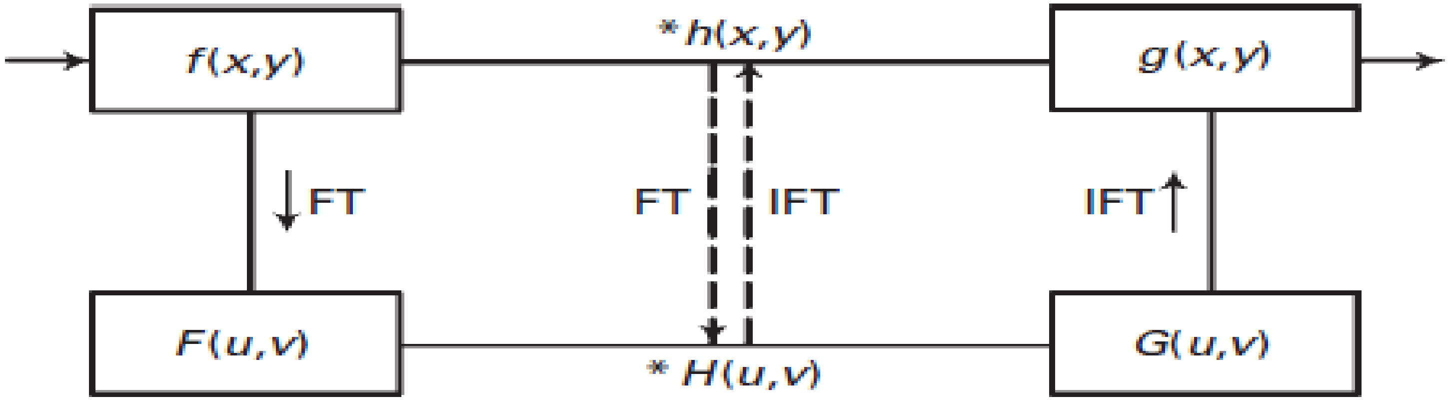


Filtragem no domínio da Frequência

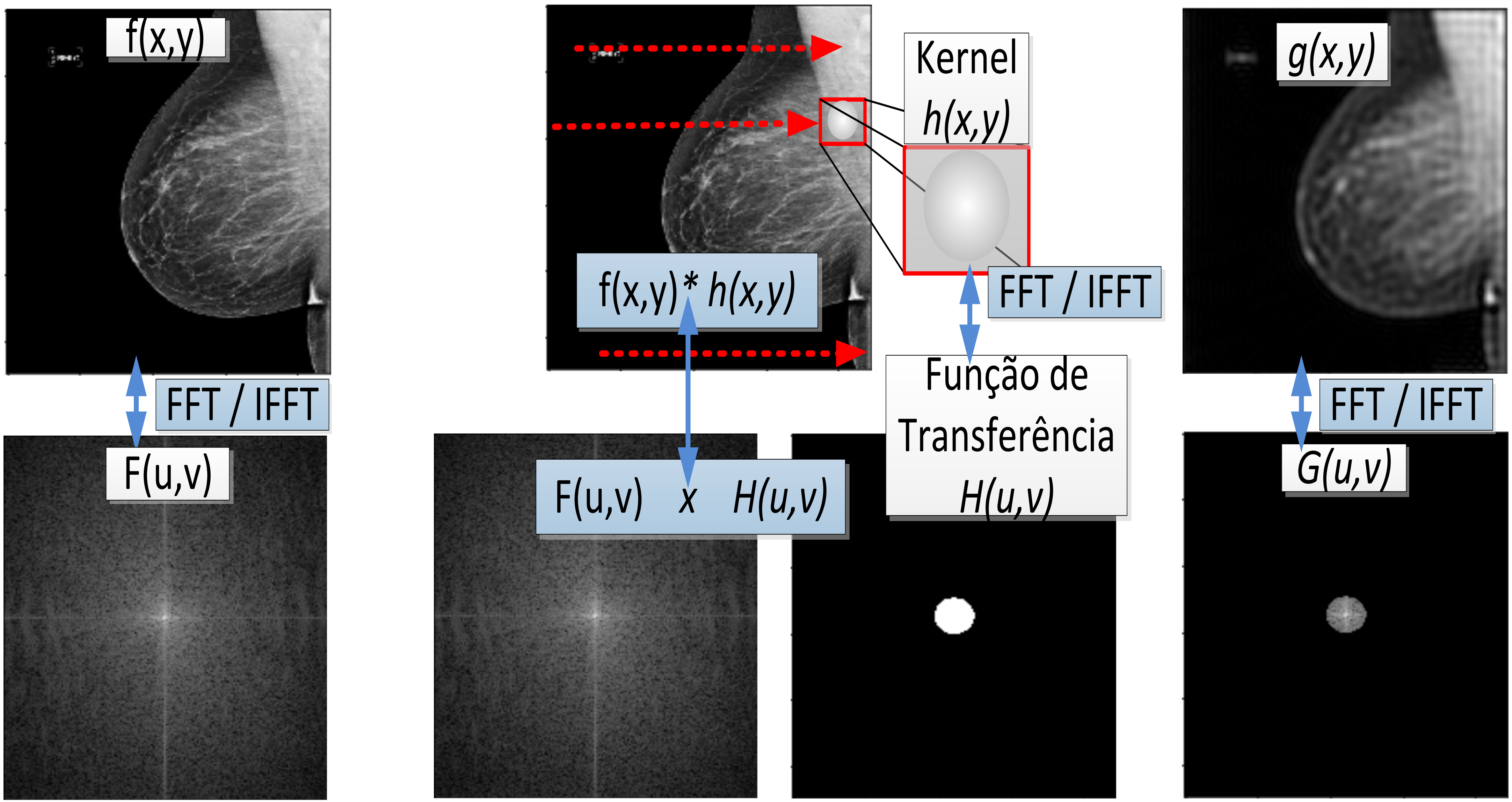
Correspondência tempo/espaço com frequência



$$f(x, y) * h(x, y) \leftrightarrow F(u, v)H(u, v)$$

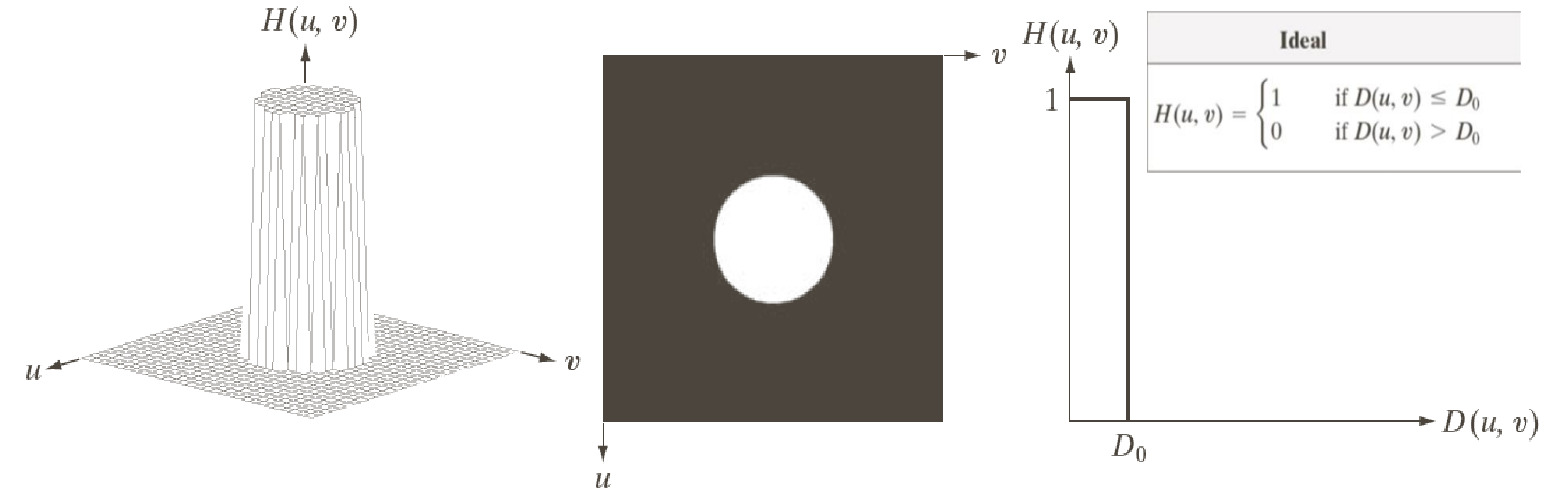
$$f(x, y)h(x, y) \leftrightarrow F(u, v) * H(u, v)$$

Equivalência de Filtragem Espaço e Frequência



Filtro 2D ideal

- Banda passante plana.
- Queda acentuada na frequência de corte *Do*.
- Como calcular *Do*? Explicar, tamanho da imagem e *fc.*
- *D* distância em relação ao centro.



a b c

FIGURE 4.40 (a) Perspective plot of an idea ... ed as an image. (c) Filter radial cross section.

```
function HIdeal = FazerMascaraIdeal2D(M, N, fc)
H_Ideal = zeros(M,N);
Do=fc*(M/2);
    for l = 1:M
        for c=1:N
            distx = c - (M/2);
            disty = l - (N/2);
            D = sqrt(distx.^2 + disty.^2);
            if (D<=Do)
                H_Ideal(l,c)=1;
            end
        end
    end
end
```

```
## Função Mascara Ideal##############
def fazerMascaraIdeal2D(M, N, fc):
    # importar bibliotecas

    import numpy as np
    import matplotlib.pyplot as plt
    import cv2 # OpenCV
    import skimage
    import skimage.exposure
    import scipy.signal

    H_Ideal = np.zeros((M,N), complex)

    Do=fc*(M/2)
    for l in range(M):
        for c in range(N):
            distx = c - (N/2)
            disty = l - (M/2)
            D = np.math.sqrt(distx**2 + disty**2)
            if D<=Do:
                H_Ideal[l,c] = 1 + 0j

    return H_Ideal
```

Filtragem 2D com filtro ideal

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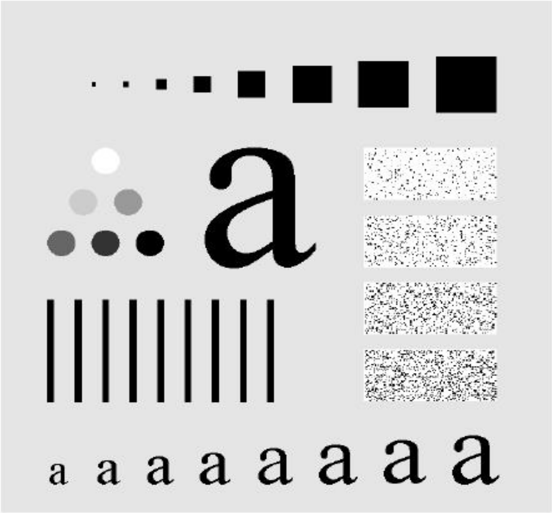
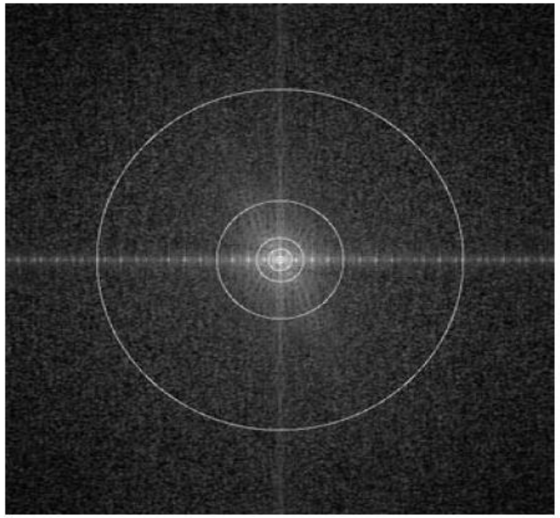
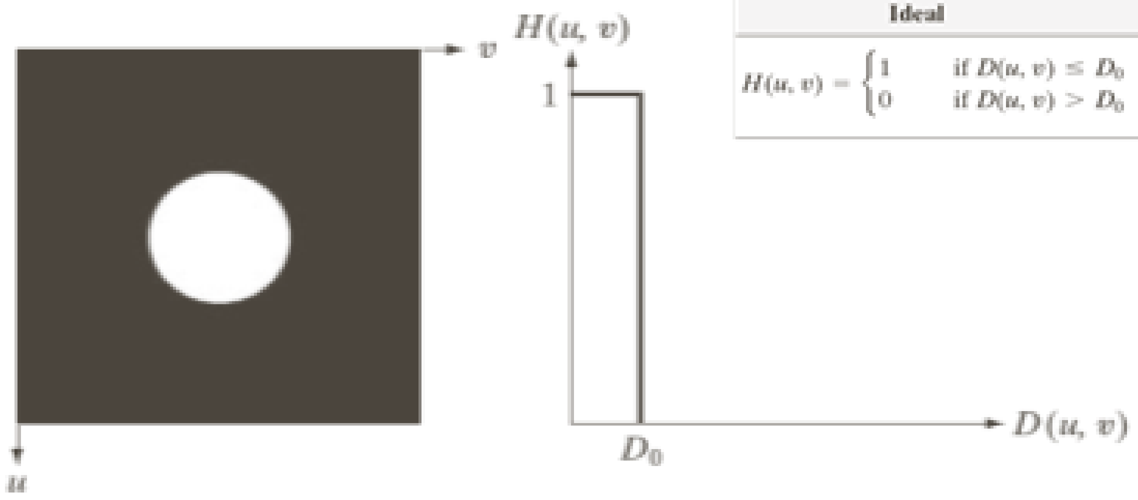
Frequência de Corte (Passa Baixa) é o raio do filtro
Normalizada → fc= 0,1; 0,2; ....0,9 da frequência máxima
Em termos de pixels Do = fc.(raiomax)

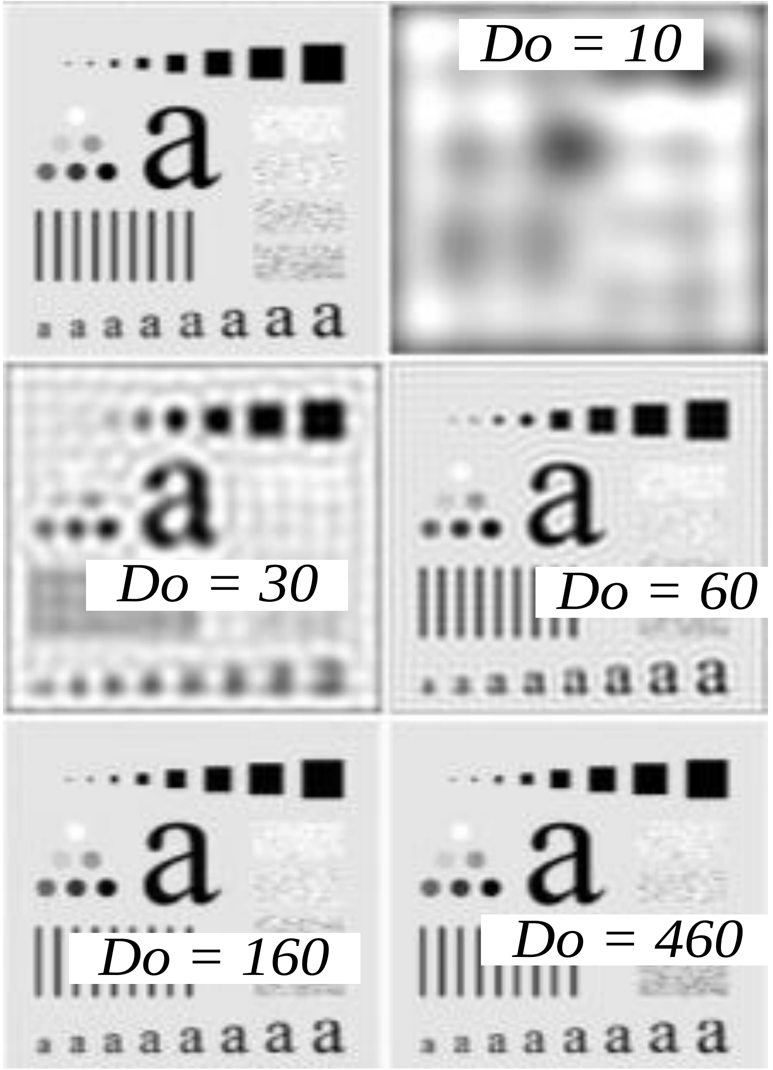






Ideal

$$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \le D_0 \\ 0 & \text{if } D(u,v) > D_0 \end{cases}$$



Filtragem 2D com filtro ideal -frequência de corte

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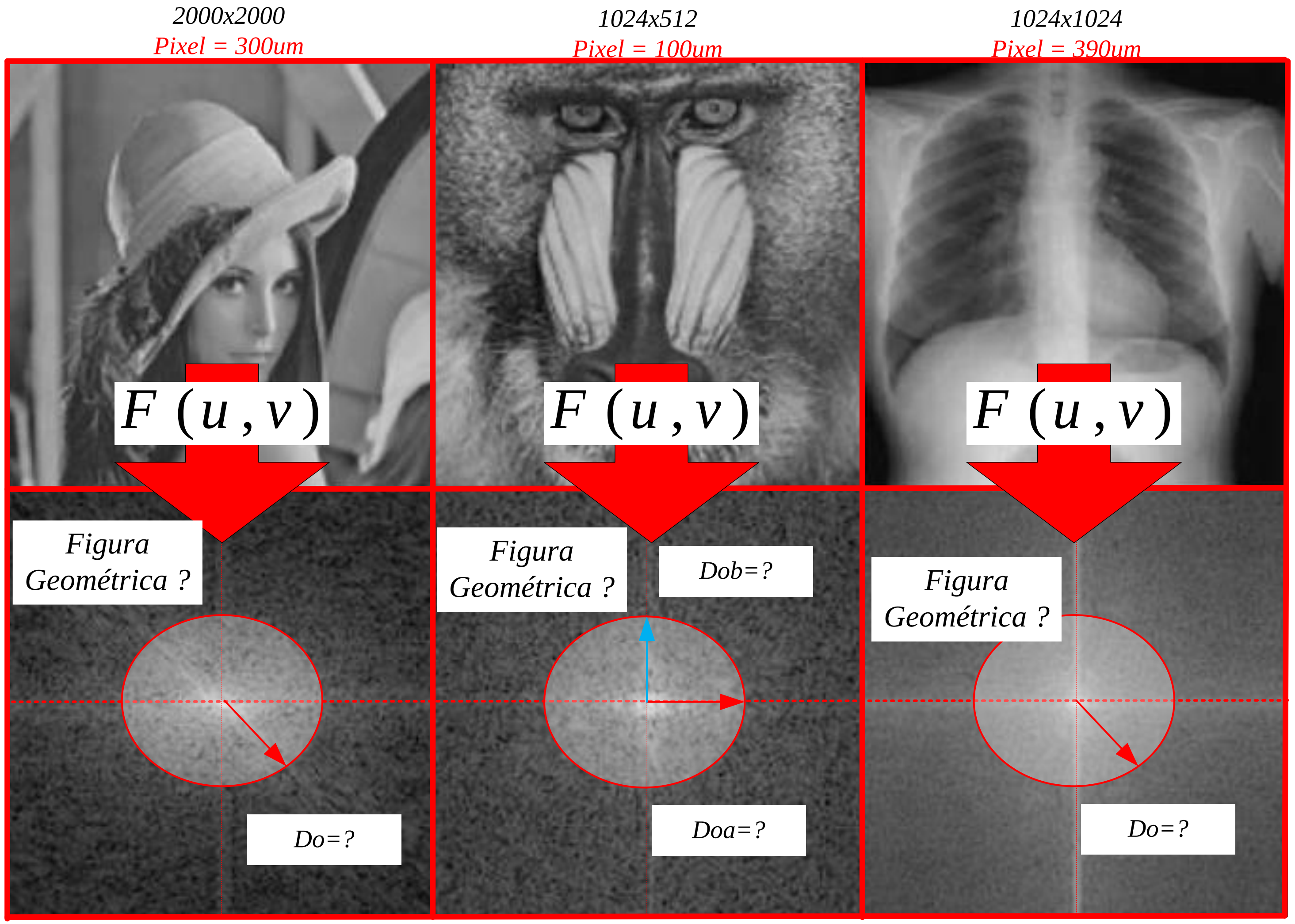
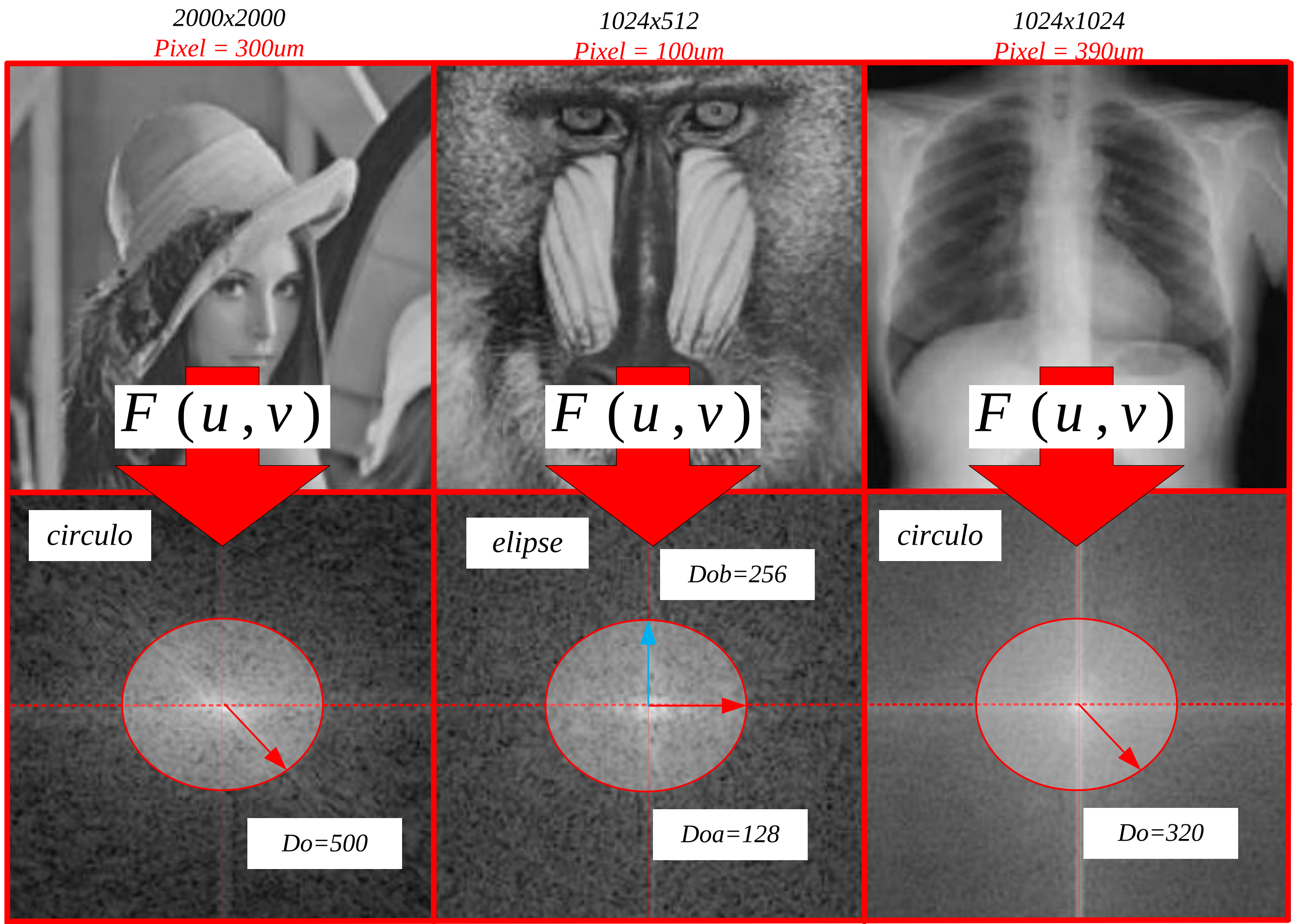
Para as imagens abaixo, calcule o formato e o Do para que seja filtrado todas as frequências acima de 50% da máxima. Passa baixa com fc = 0,5fmáx



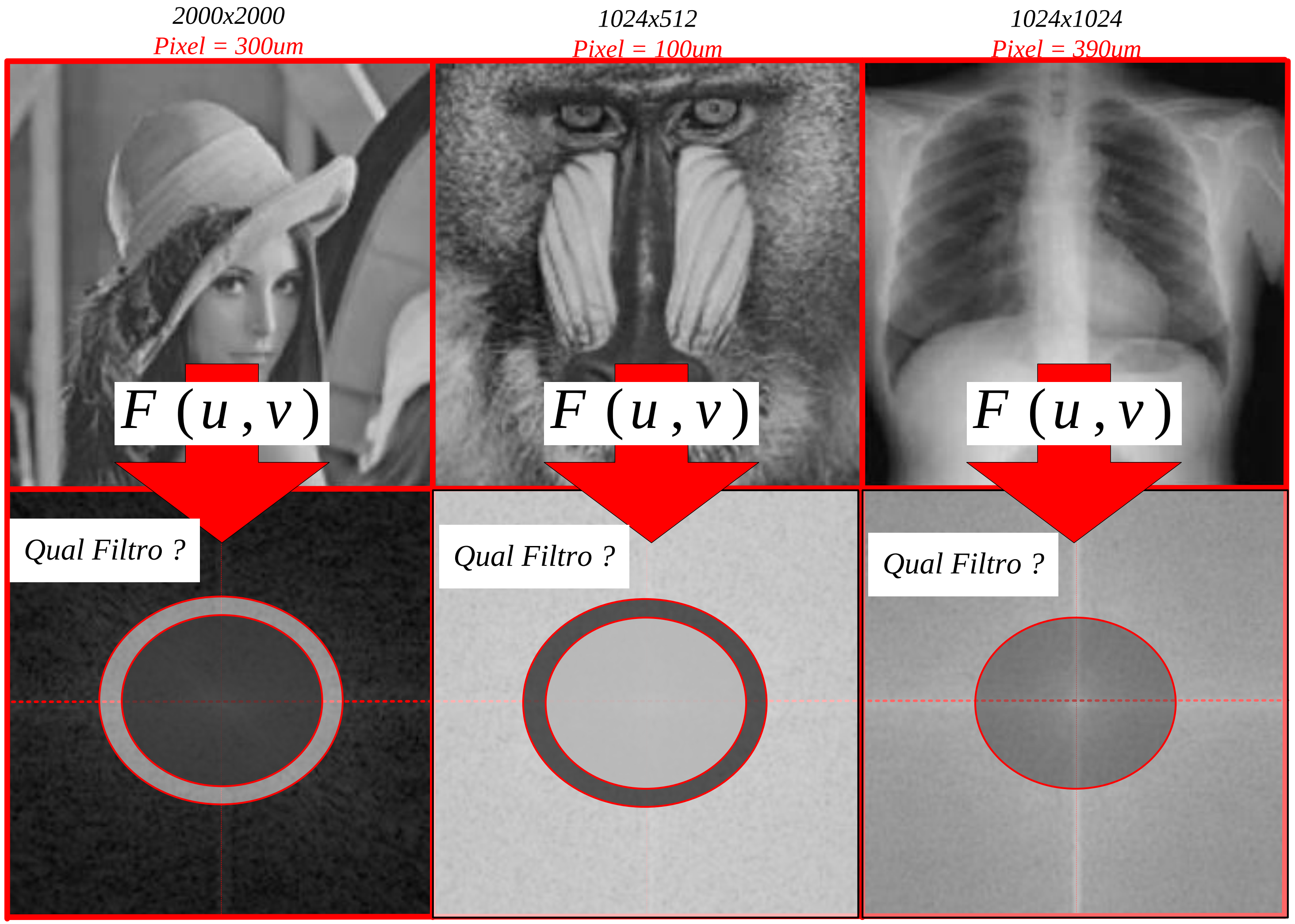
Filtragem 2D com filtro ideal - frequência de corte

Respostas



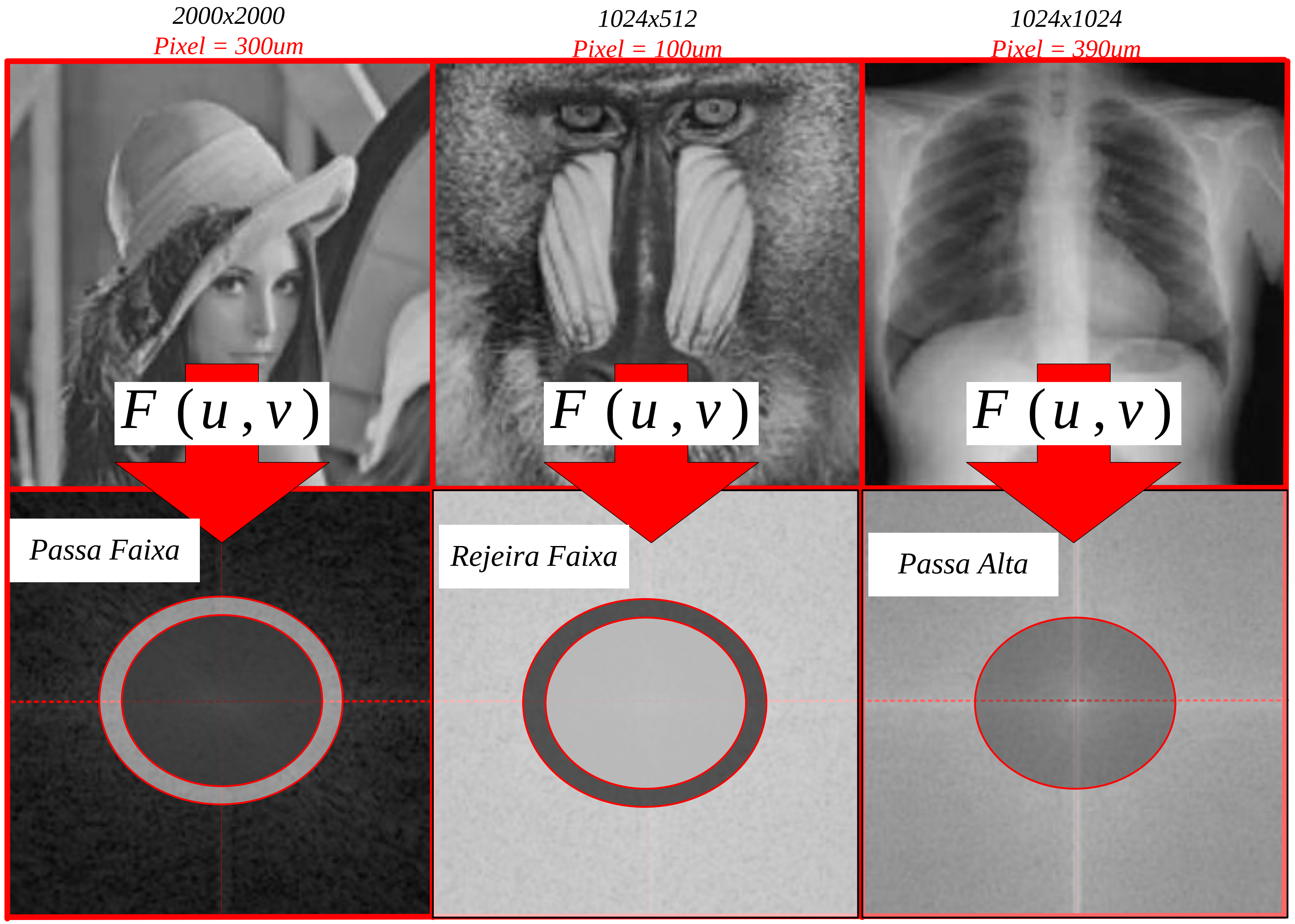
Filtragem 2D com filtro ideal - Tipos de Filtragens

Quais são os tipos de filtros abaixo e quais suas possíveis consequências nas imagens filtradas?



Filtragem 2D com filtro ideal - Tipos de Filtragens

Respostas

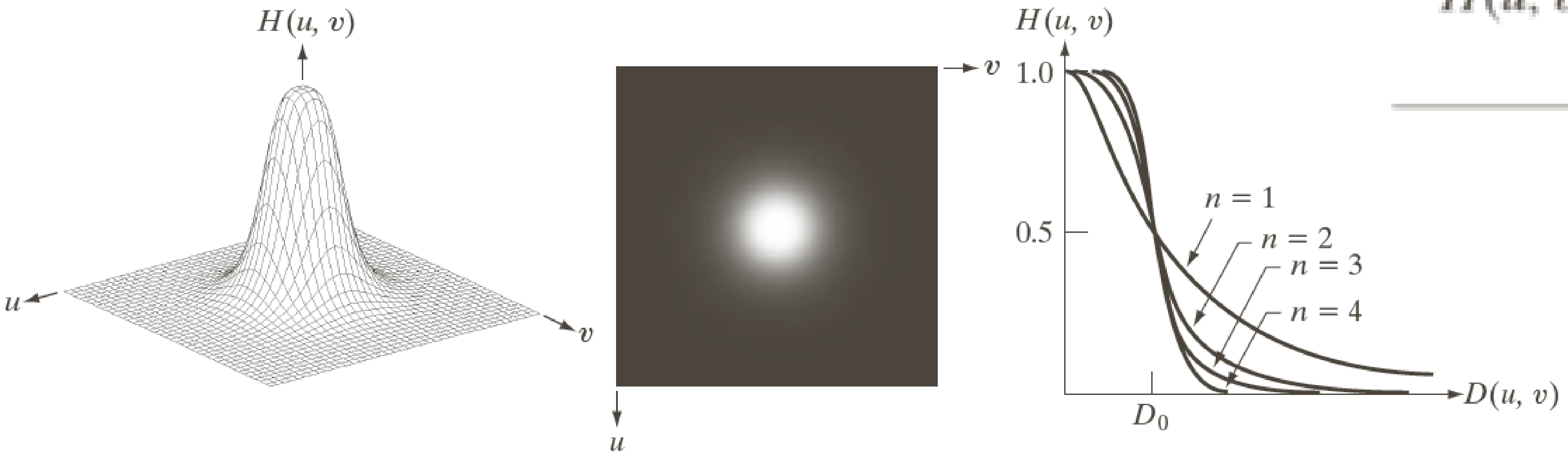


Filtro 2D Butterworth – Passa Baixa

- Banda passante plana.
- Queda proporcional ao numero de polos, *n*
- Energia na frequência de corte Do, igual a metade da máxima

Butterworth

$$H(u,v) = \frac{1}{1 + [D(u,v)/D_0]^{2n}}$$



a b c

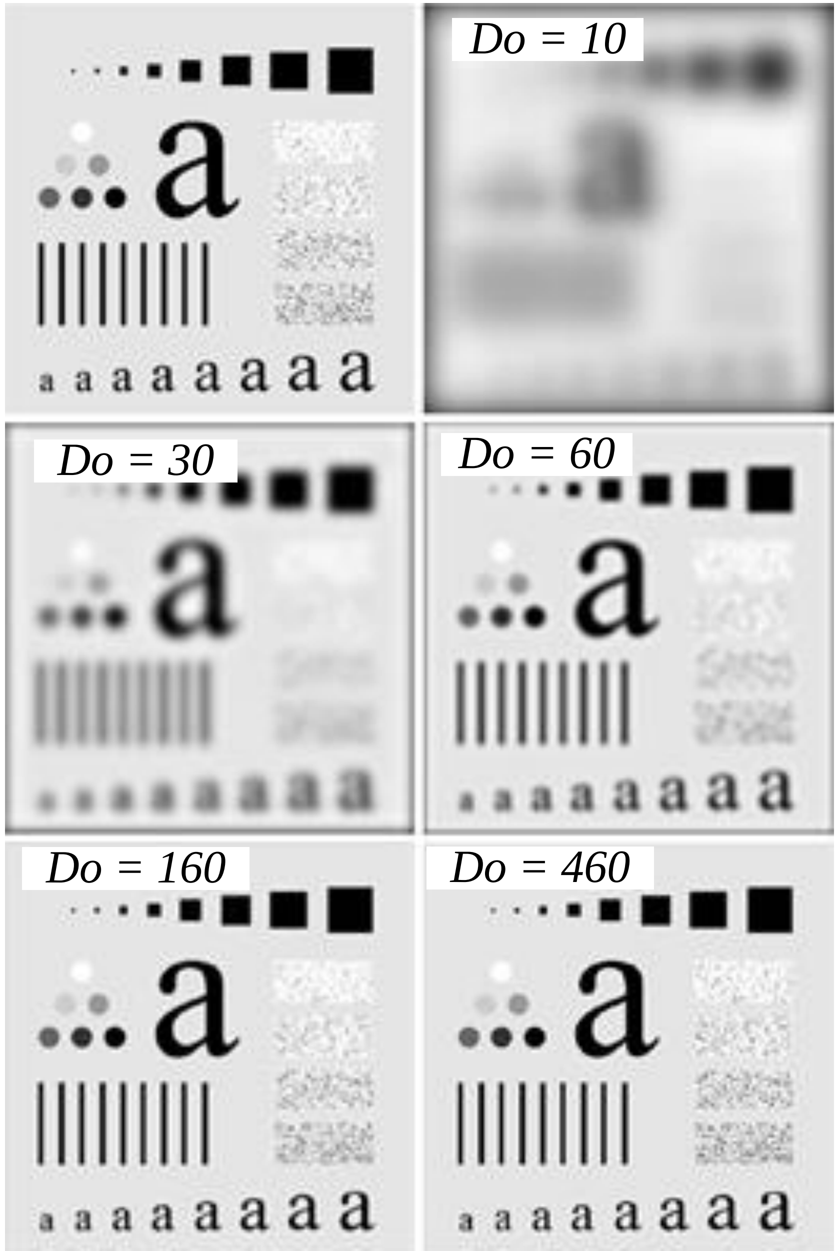
FIGURE 4.44 (a) Perspective plot of a Butterworth lowpass-filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.

Filtro 2D Butterworth – Passa Baixa

- Resultados

Butterworth
$$H(u,v) = \frac{1}{1 + [D(u,v)/D_0]^{2n}}$$

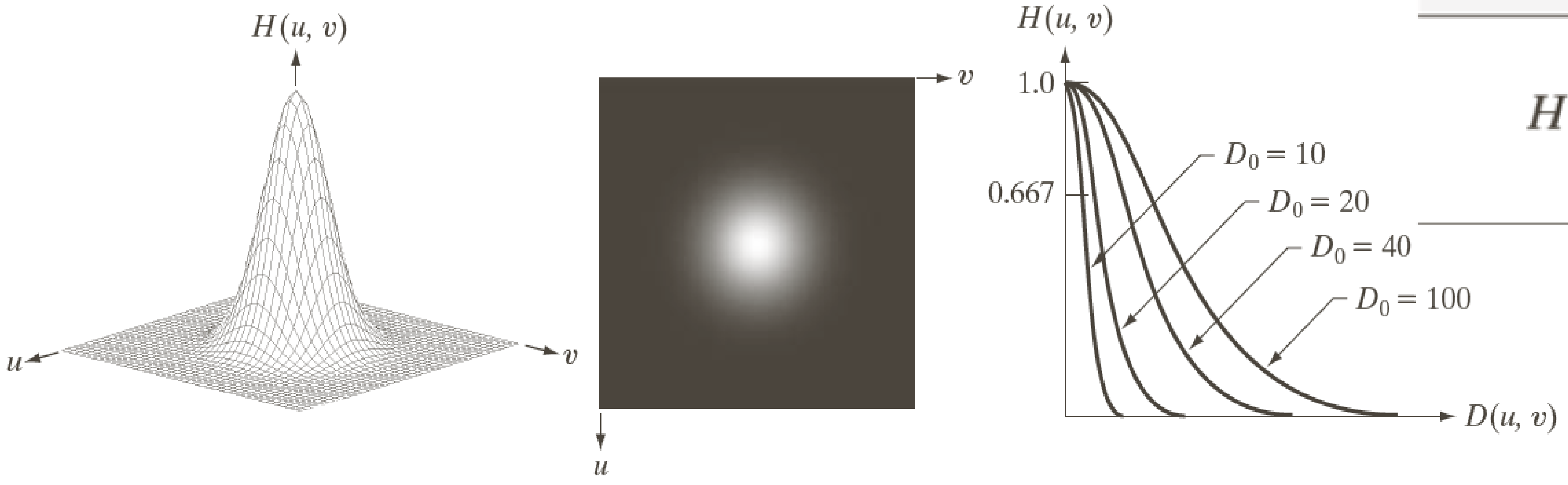
n = 2



Filtro 2D Gaussiano – Passa Baixa

- Banda passante curva.
- Queda proporcional a frequência de corte Do, (desvio padrão)
- Energia na frequência de corte Do, igual a 0.667 da máxima (energia de 1 desvio padrão)

Gaussian
$H(u, v) = e^{-D^2(u,v)/2D_0^2}$



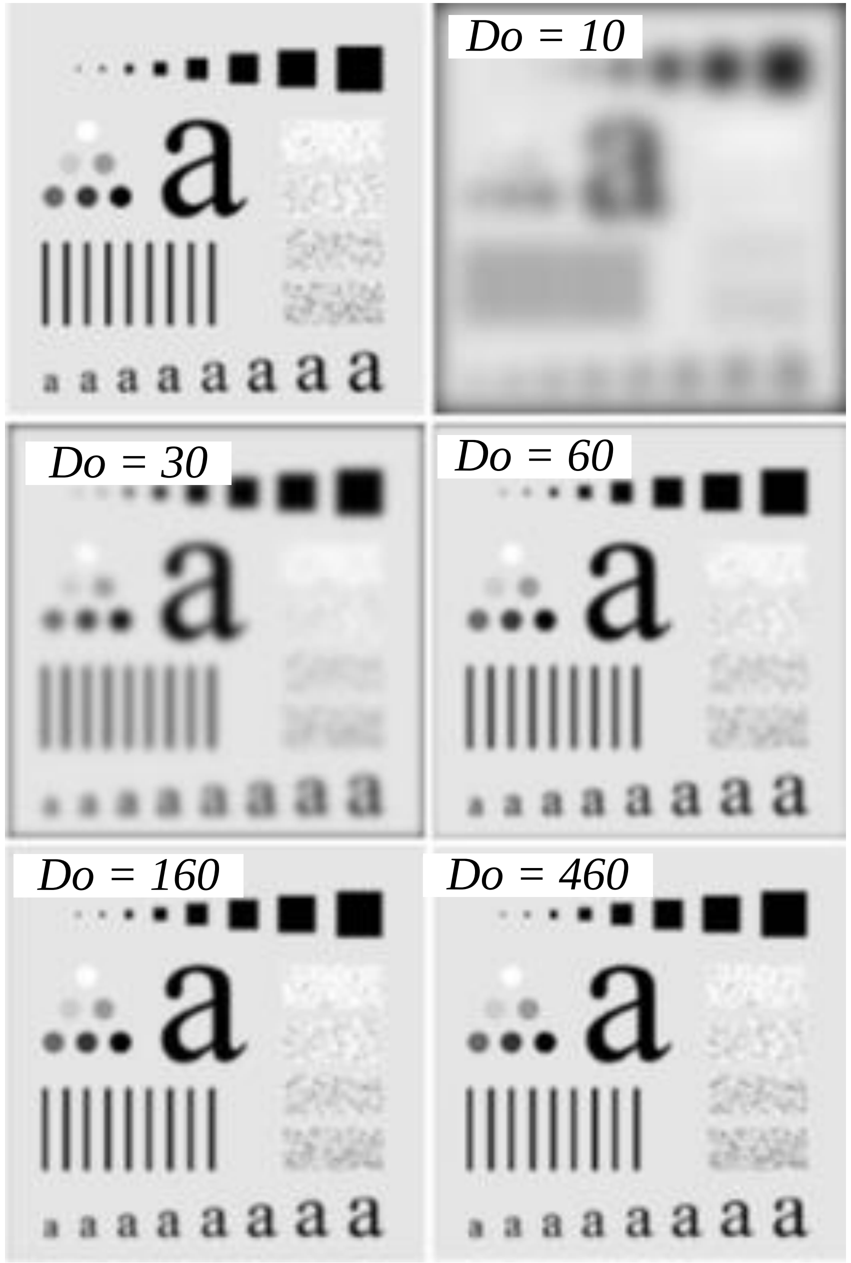
a b c

FIGURE 4.47 (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of D_0.

Filtro 2D Gaussiano – Passa Baixa

- Resultado

Gaussian
$H(u,v) = e^{-D^2(u,v)/2D_0^2}$



Resultados – Passa Baixa

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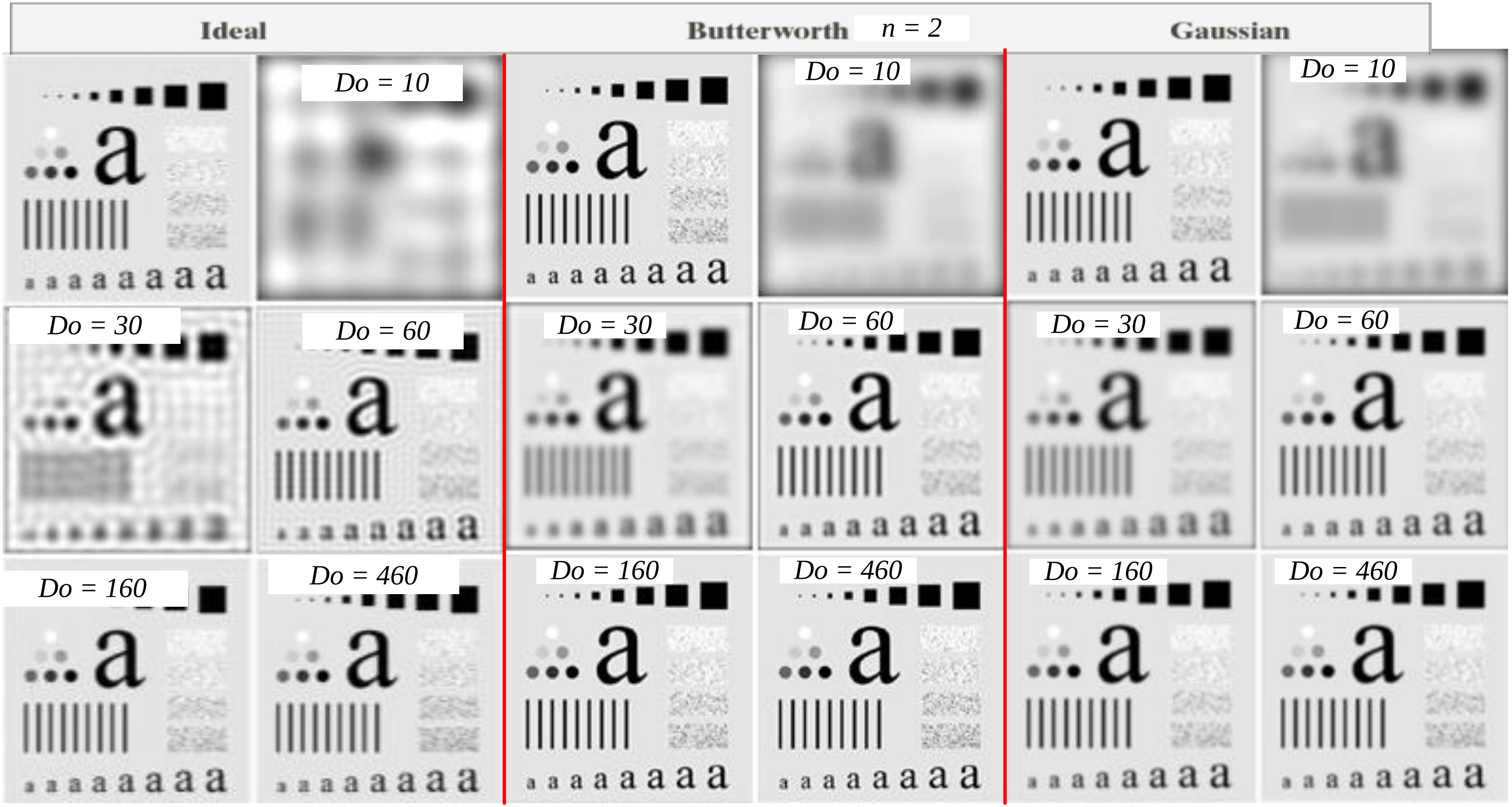
- Comparação entre os 3



Filtros 2D– Passa Alta

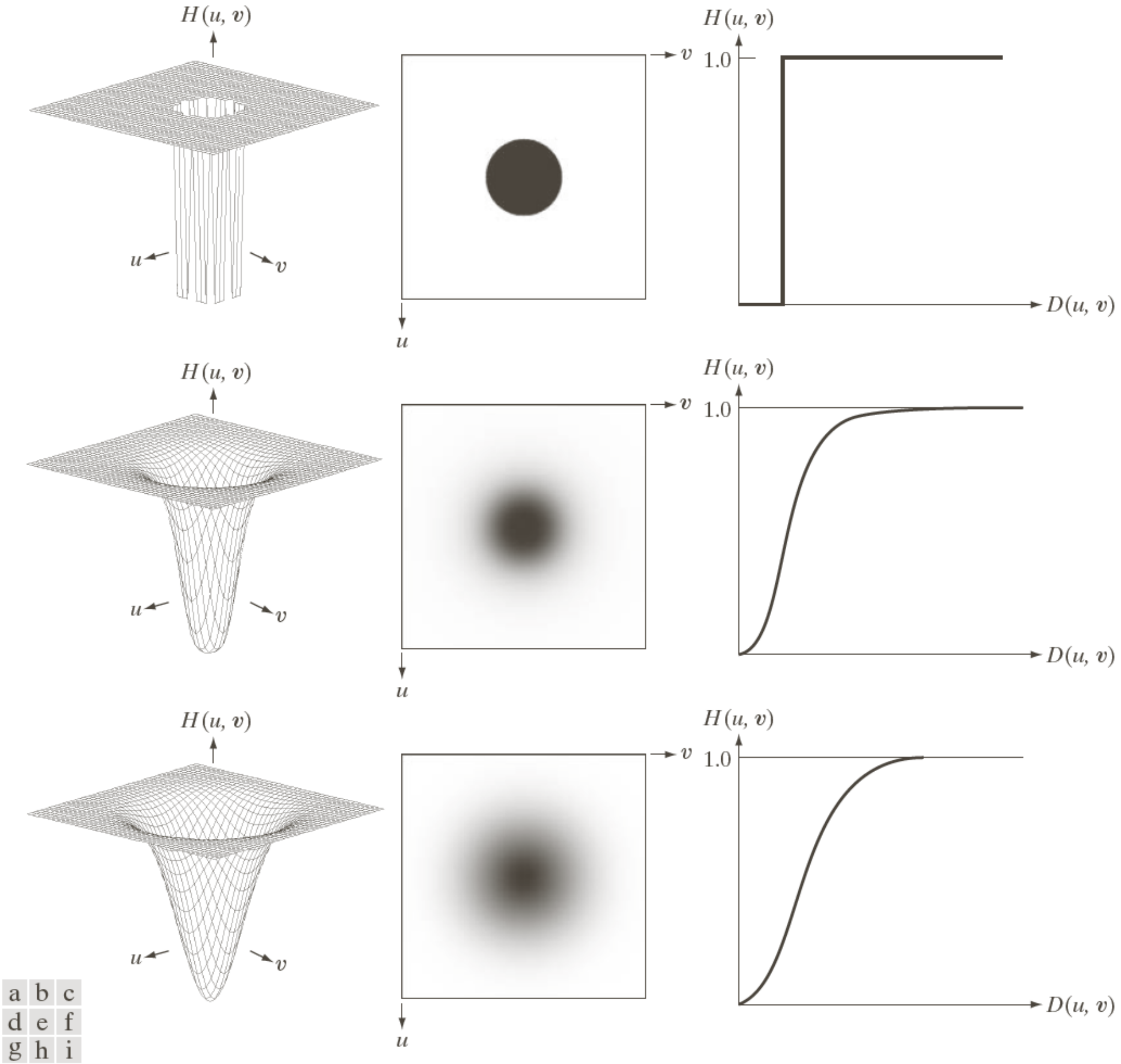


FIGURE 4.52 Top row: Perspective plot, image representation, and cross section of a typical ideal highpass filter. Middle and bottom rows: The same sequence for typical Butterworth and Gaussian highpass filters.

Ideal
$$H(u,v) = \begin{cases} 0 & \text{if } D(u,v) \le D_0 \\ 1 & \text{if } D(u,v) > D_0 \end{cases}$$

Butterworth
$$H(u,v) = \frac{1}{1 + [D_0/D(u,v)]^{2n}}$$

Gaussian
$$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$$

Filtros 2D– Passa Alta Resultados

Ideal

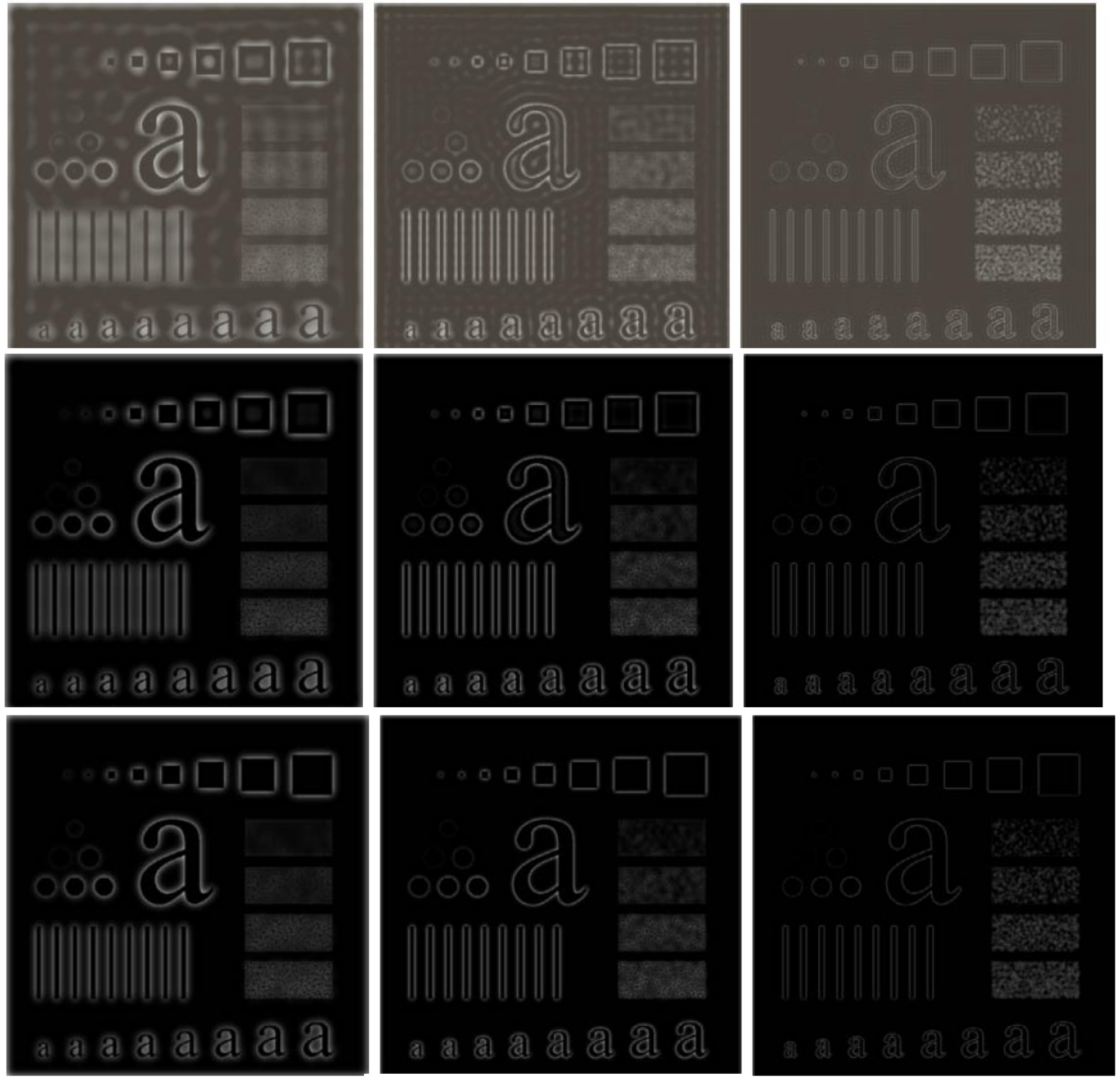
$$H(u,v) = \begin{cases} 0 & \text{if } D(u,v) \leq D_0 \\ 1 & \text{if } D(u,v) > D_0 \end{cases}$$

Butterworth

$$H(u,v) = \frac{1}{1 + [D_0/D(u,v)]^{2n}}$$

Gaussian

$$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$$



Do = 30   *Do = 60*   *Do = 160*

Filtros 2D– Passa Banda e Rejeita Banda

Ideal

Rejeita Banda (H_{BR})

$$H(u,v) = \begin{cases} 0 & \text{if } D_0 - \frac{W}{2} \leq D \leq D_0 + \frac{W}{2} \\ 1 & \text{otherwise} \end{cases}$$

Butterworth

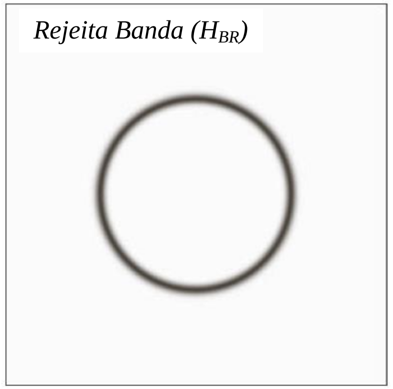
Rejeita Banda (H_{BR})

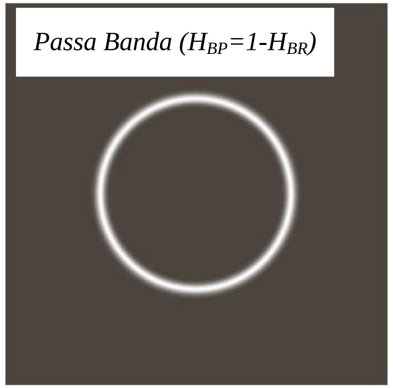
$$H(u,v) = \frac{1}{1 + \left[\frac{DW}{D^2 - D_0^2}\right]^{2n}}$$

Gaussian

Rejeita Banda (H_{BR})

$$H(u,v) = 1 - e^{-\left[\frac{D^2 - D_0^2}{DW}\right]^2}$$





Bandreject filters. W is the width of the band, D is the distance $D(u, v)$ from the center of the filter, D_0 is the cutoff frequency, and n is the order of the Butterworth filter. We show D instead of $D(u, v)$ to simplify the notation in the table.

Filtros 2D– Filtro Notch

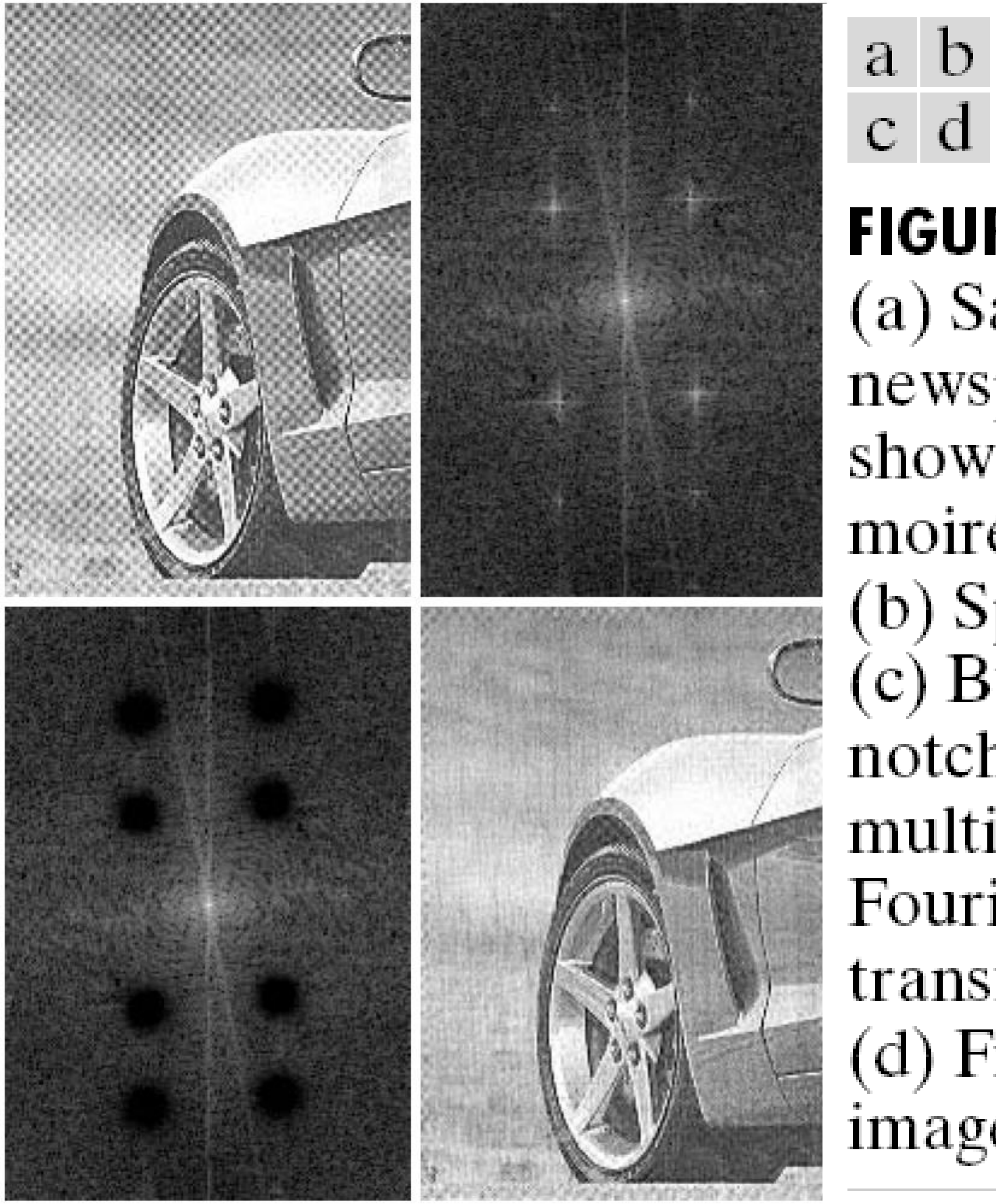


FIGURE 4.64 (a) Sampled newspaper image showing a moiré pattern. (b) Spectrum. (c) Butterworth notch reject filter multiplied by the Fourier transform. (d) Filtered image.

- Equação no Livro pg 193

Filtros 2D– Filtro Notch

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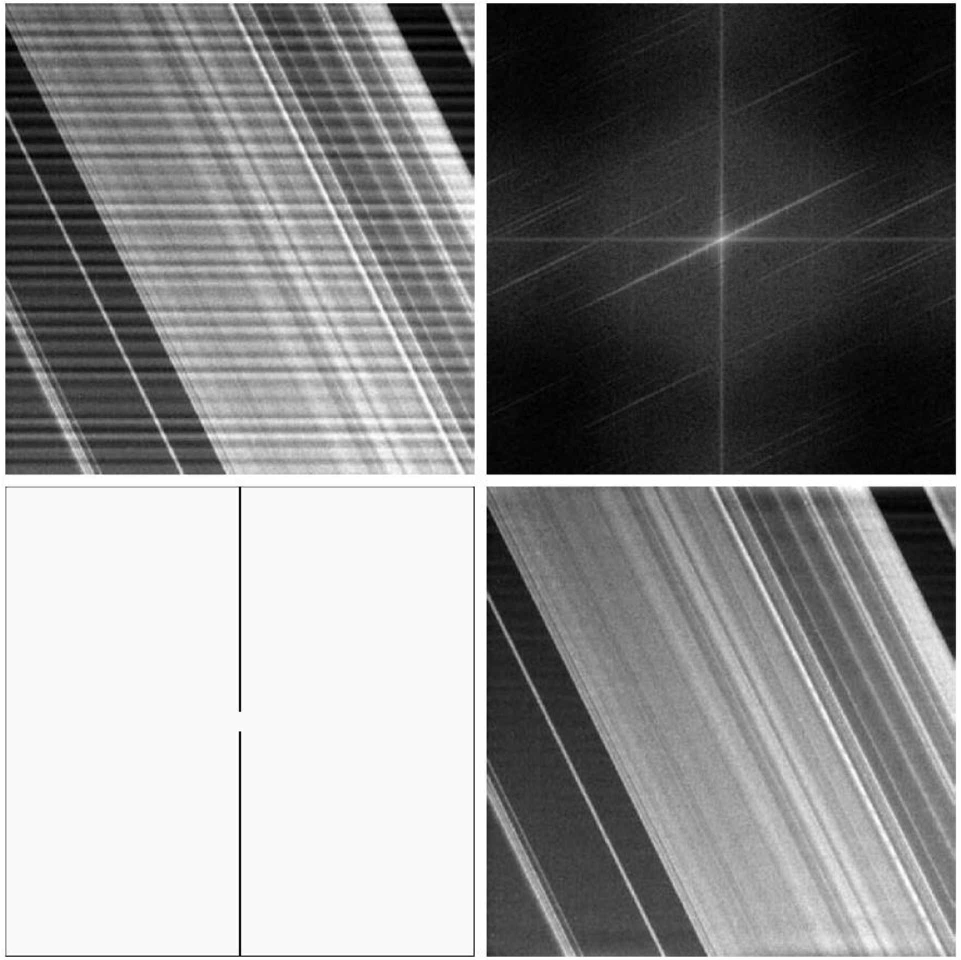
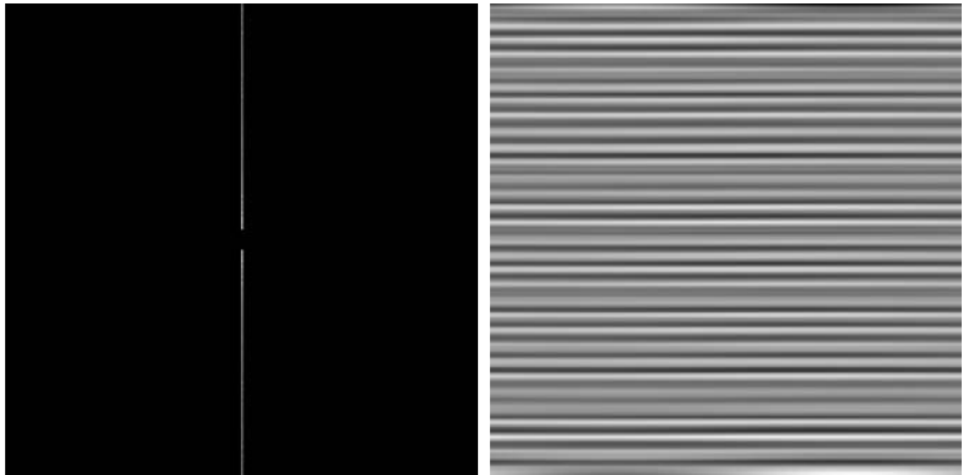


a b
c d

FIGURE 4.65 (a) 674 × 674 image of the Saturn rings showing nearly periodic interference. (b) Spectrum: The bursts of energy in the vertical axis near the origin correspond to the interference pattern. (c) A vertical notch reject filter. (d) Result of filtering. The thin black border in (c) was added for clarity; it is not part of the data. (Original image courtesy of Dr. Robert A. West, NASA/JPL.)

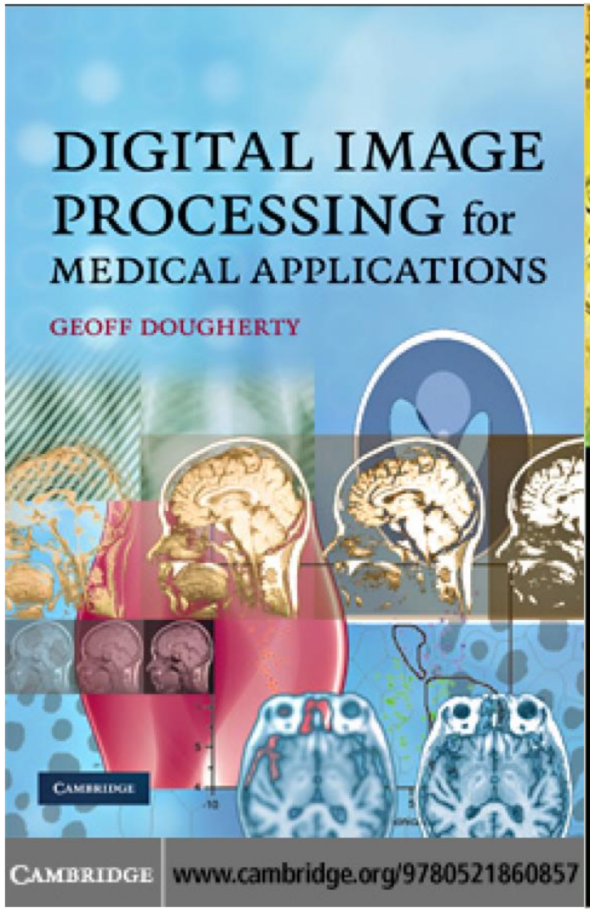


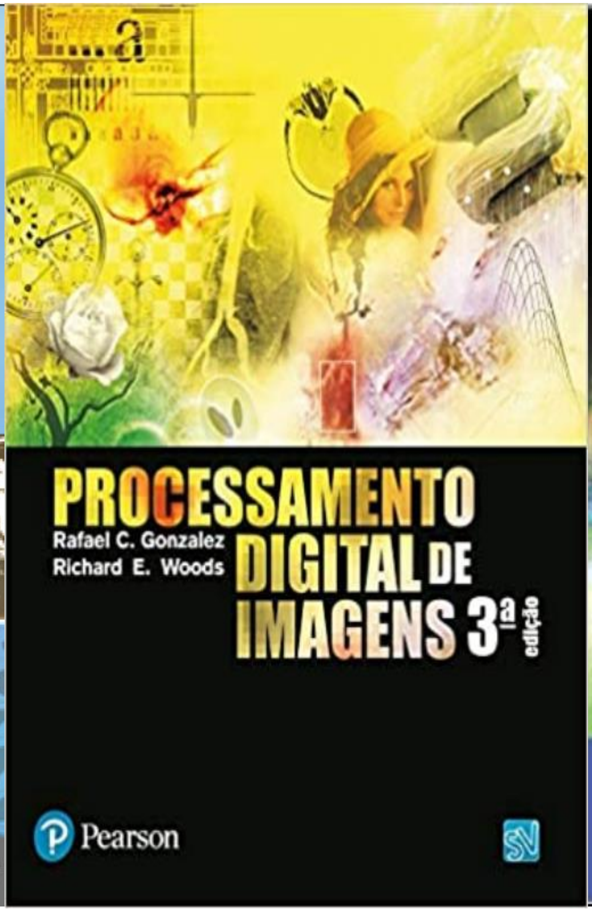
a b

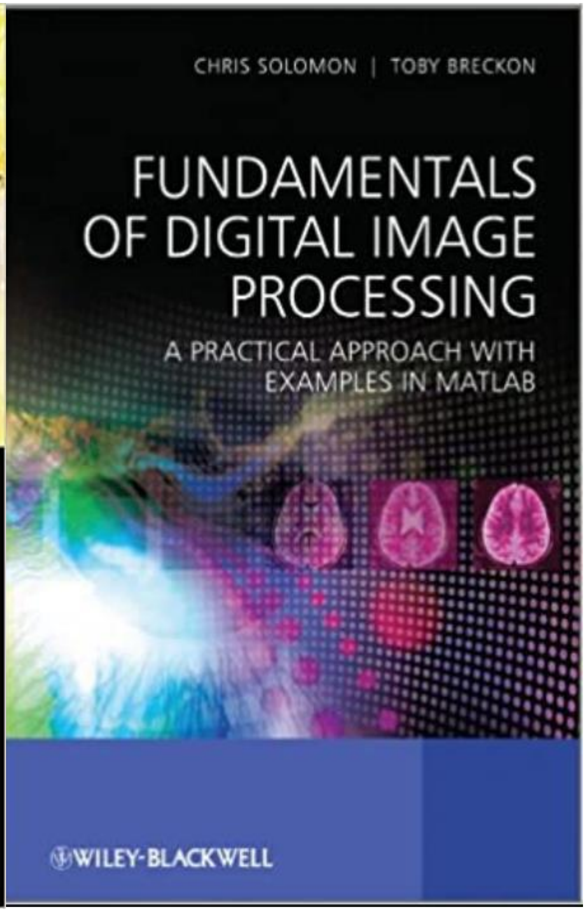
FIGURE 4.66 (a) Result (spectrum) of applying a notch pass filter to the DFT of Fig. 4.65(a). (b) Spatial pattern obtained by computing the IDFT of (a).

- Equação no Livro Gonzales pg 193

Referências







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