P427 A#8 ①

Hi! Sorry this is late, I thought I'd submit it and see if it's possible to get feedback on! Thanks :)

#1

a) Compute the root depth of a 6000m high mountain w/ an average density of 2700 kg/m³ in Airy isostatic equilibrium w/ an underlying substratum of density 3300 kg/m³.

ISOSTASY ≡ Rigid lithosphere "floats" on the deformable asthenosphere

→ Heavy mountains are compensated for by less mass further down.

- AIRY ISOSTASY: usually lithosphere, sometimes crust; usually asthenosphere, sometimes mantel

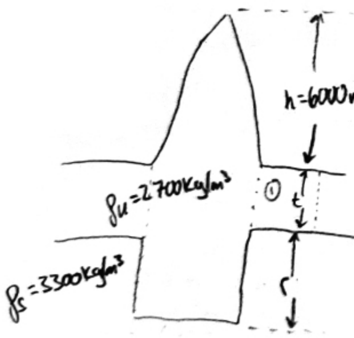
→ Rigid Upper layer sits on deformable substratum, (only 2 densities - ρ_u, ρ_s)

Different columns have different heights (mountains have tall columns ⇒ deep roots)

(vs. PRATT ISOSTASY:

→ All columns have same height, but different densities ⇒ substratum starts @ compensation depth

⇒ Mountains have low density (to make up for more mass))



Take compensation depth some distance d below root.

column ① mass: $t\rho_u + r\rho_s + d\rho_s$

column ② (mountain) mass: $h\rho_u + t\rho_u + r\rho_u + d\rho_s$

- Mass above compensation depth must be equal:

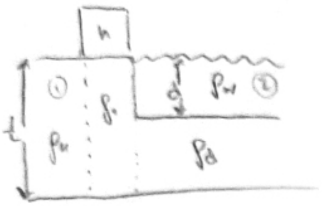
$$t\rho_u + r\rho_s + \cancel{d\rho_s} = (h+t+r)\rho_u + \cancel{d\rho_s}$$

$$r = \frac{(h+t-t)\rho_u}{(\rho_s - \rho_u)} = \frac{h\rho_u}{\rho_s - \rho_u}$$

$$r = \frac{(6000m)(2700\ kg/m^3)}{(3300 - 2700)\ kg/m^3} = 27000\ m$$

$\boxed{\therefore \text{ MOUNTAIN ROOT IS } 27000m = 27 km \text{ DEEP}}$

#1 b) For a compensation depth equal to the base of the continental crust (35 km), continental crust density of 2700 kg/m³, oceanic crust density of 3000 kg/m³, and water density of 1000 kg/m³. Compute the ocean depth assuming Pratt isostatic equilibrium.



ρ_m

t = compensation depth = continental crust thickness

$t = 35\,000\ m$

$\rho_u = 2700\ kg/m^3$

$\rho_w = 1000\ kg/m^3$

ρ_d = oceanic crust density = $3000\ kg/m^3$

ρ_s = substratum density

- BELOW COMPENSATION DEPTH, ALL PRESSURES ARE HYDROSTATIC

⇒ Weight of all columns are equal:

① ②

$$t\rho_u = d\rho_w + \rho_d(t-d)$$

$t\rho_u - t\rho_d = d(\rho_w - \rho_d)$ $\quad d = \frac{t(\rho_u-\rho_d)}{\rho_w-\rho_d}$

$t\rho_u = d\rho_w + t\rho_d - d\rho_d$

$$d = \frac{t(\rho_u-\rho_d)}{\rho_w-\rho_d} = \frac{(35000\,m)(2700-1000)\,kg/m^3}{(1000-3000)\,kg/m^3} = -29750\,m$$

→ (-) because below sea level

$\boxed{\therefore \text{Using Pratt's hypothesis, ocean is } 29750m \approx 30\,Km \text{ deep}}$

P427 A#8 (2) $\qquad 2t - d\left(1 + \left(\frac{\rho_u - \rho_w}{\rho_s - \rho_u}\right)\right)$

K NOTES - WEEK 11 - pg 6, 7

#2 a) Show that the geoid height anomaly for an ocean basin in Airy isostatic equilibrium is given by:

$$\Delta h = \frac{-\pi G d}{g}(\rho_u - \rho_w)\left[2t - d\left(\frac{\rho_s - \rho_w}{\rho_s - \rho_u}\right)\right]$$

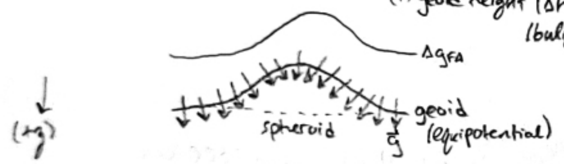
GEOID HEIGHT ANOMALY: Result of lateral density variations in the earth (these also cause gravitational anomalies)

$\Rightarrow$ $\Delta h \equiv$ geoid radius - spheroid radius, $g\Delta h = -\Delta V$

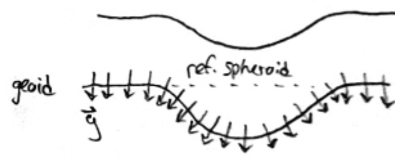
g: ref. gravity for spheroid, $\vec{g} \perp$ geoid (equipotential surface)

ΔV: anomaly in gravitational potential for spheroid

$\Rightarrow$ Mass excess: (+) gravity anomaly, (-) potential (ΔV) anomaly, (+) geoid height (Δh) anomaly (bulge)



$\Rightarrow$ MASS DEFICIT: (-) gravity anomaly, (+) potential anomaly (ΔV), (-) geoid height anomaly (Δh) (TROUGH)



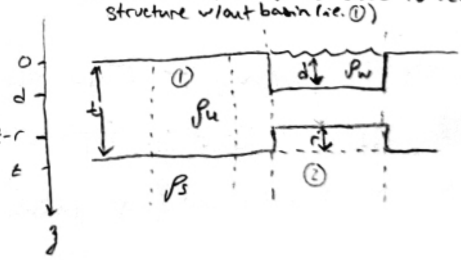
FOR ISOSTATIC DENSITY DIST,

$$\Delta h = \frac{-2\pi G}{g}\int^{D} z\,\Delta\rho(z)\,dz$$

g: ref. gravity; lower limit: Top of column; D: compensation depth; $\Delta\rho(z)$: density anomaly

AIRY ISOSTATIC OCEAN BASIN: Mass deficit

$\rightarrow$ want anomalies relative to ref. structure w/out basin (i.e. (1))



$\rho_u t = \rho_w d + \rho_u(t-d-r) + r\rho_s$

$r = \frac{d(\rho_u - \rho_w)}{(\rho_s - \rho_u)}$ < 0

DENSITY ANOMALY: Relative to ref. structure w/out basin

$$\Delta\rho(z) = \rho_{(2)} - \rho_{(1)} = \begin{cases} \rho_w(z) - \rho_u(z) & 0 \le z \le d \\ \rho_u(z) - \rho_u(z) = 0 & d \le z \le t-r \\ \rho_s(z) - \rho_u(z) & t-r \le z \le t \end{cases}$$

$\Rightarrow$ NOTE: densities are constant in their reigons. $\Rightarrow$ $\rho_u(z) = \rho_u$, etc.

$$\Delta h = \frac{-2\pi G}{g}\left[(\rho_w - \rho_u)\int_0^d z\,dz + 0 + (\rho_s - \rho_u)\int_{t-r}^{t} z\,dz\right] = \frac{-\cancel{2}\pi G}{g}\left[(\rho_w - \rho_u)\left(\frac{d^2}{\cancel{2}}\right) + (\rho_s - \rho_u)\left(\frac{t^2}{\cancel{2}} - \frac{(t-r)^2}{\cancel{2}}\right)\right]$$

$r = \frac{d(\rho_u - \rho_w)}{(\rho_s - \rho_u)}$

$$= \frac{-\pi G}{g}\left[d^2(\rho_w - \rho_u) + (\rho_s - \rho_u)\left(\cancel{t^2} + 2dt\left(\frac{\rho_u - \rho_w}{\cancel{\rho_s - \rho_u}}\right) - d^2\left(\frac{\rho_u - \rho_w}{\cancel{\rho_s - \rho_u}}\right)^2 - \cancel{t^2}\right)\right] = \frac{-\pi G}{g}\left[d^2(\rho_u - \rho_w) + 2dt(\rho_u - \rho_w) - d^2\frac{(\rho_u - \rho_w)^2}{\rho_s - \rho_u}\right]$$

$$= \frac{-\pi G d}{g}(\rho_u - \rho_w)\left[d + 2t - d\left(\frac{\rho_u - \rho_w}{\rho_s - \rho_u}\right)\right] = \frac{-\pi G d}{g}(\rho_u - \rho_w)\left[2t - d\left(1 + \frac{\rho_u - \rho_w}{\rho_s - \rho_u}\right)\right] = \frac{-\pi G d}{g}(\rho_u - \rho_w)\left[2t - d\left(\frac{\rho_s - \rho_u + \rho_u - \rho_w}{\rho_s - \rho_u}\right)\right]$$

$\therefore$ $\boxed{\Delta h = \frac{-\pi G d}{g}(\rho_u - \rho_w)\left[2t - d\left(\frac{\rho_s - \rho_w}{\rho_s - \rho_u}\right)\right]}$ IS THE GEOID HEIGHT ANOMALY FOR AIRY COMPENSATED OCEAN BASIN.

#2 b) Evaluate the geoid height anomaly for a 5 km deep ocean given a compensation depth at the base of the crust ($t = 35$ km) w/ $\rho_u = 2800$ kg/m³, $\rho_s = 3300$ kg/m³, $\rho_w = 1000$ kg/m³

$G = 6.67\times10^{-11}\ m^3/kg\,s^2$

$g = 9.81\ m/s^2$ → Reference gravity
→ No latitude given, so use avg. g @ surface of earth. ?

$$\Delta h = \frac{-\pi G d}{g}(\rho_u - \rho_w)\left[2t - d\left(\frac{\rho_s - \rho_w}{\rho_s - \rho_u}\right)\right]$$

$$= \frac{-\pi(6.67\times10^{-11}\ m^3\,kg^{-1}\,s^{-2})(5000m)}{9.81\ m\,s^{-2}}(2800-1000)\,kg\,m^{-3}\left[2(35000m) - (5000m)\left(\frac{3300-1000}{3300-2800}\right)\right]$$

$\Delta h = -9.03539$ m

FROM FOWLER, pg. 216:

$\Delta h \simeq 3.85d(0.7 - 0.09d)$ for d in km, $t = 35$ km, $\rho_u = 2800$ kg/m³, $\rho_s = 3300$ kg/m³

$\simeq 3.85(5)(0.7 - 0.09(5))$

$\simeq 9.625$ m

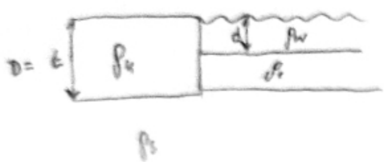
$\boxed{\text{∴ Geoid height anomaly for 5 km deep ocean is } \Delta h \simeq -9.0\text{ m. That is, expect the geoid to have a trough} \sim 9\text{ m deep to compensate for the ocean.}}$

P427 A#8 (3)

#3 a) Show that the geoid height anomally for an ocean basin in Pratt isostatic equilibrium is given by

$$\Delta h = \frac{-\pi G d t}{g}(\rho_u - \rho_w)$$

PRATT:



$$t\rho_u = d\rho_w + (t-d)\rho_1 \Rightarrow \rho_1 = \frac{t\rho_u - d\rho_w}{t-d} \Rightarrow \rho_1 - \rho_u = \frac{t\rho_u - d\rho_w - (t-d)\rho_u}{t-d} = \frac{d(\rho_u - \rho_w)}{t-d}$$

$$\Delta\rho(z) = \begin{cases} \rho_w - \rho_u, & 0 \le z \le d \\ \rho_1 - \rho_u = \frac{d(\rho_u - \rho_w)}{t-d}, & d \le z \le t \end{cases}$$

GEOID HEIGHT ANOMALY
FOR ISOSTATIC DENSITY DIST:

$$\Delta h = \frac{-2\pi G}{g}\int_{\substack{\text{top}\\ \text{of column}}}^{D} \Delta\rho(z) z\,dz = \frac{-2\pi G}{g}\left[(\rho_w - \rho_u)\int_0^d z\,dz + \frac{d(\rho_u - \rho_w)}{t-d}\int_d^t z\,dz\right] = \frac{-2\pi G}{g}(\rho_u - \rho_w)\left[-\left(\frac{d^2}{2}\right) + \frac{d}{t-d}\left(\frac{t^2}{2} - \frac{d^2}{2}\right)\right]$$

$$= \frac{-\pi G}{g}(\rho_u - \rho_w)\left[-d^2 + \frac{dt^2}{t-d} - \frac{d^3}{t-d}\right] = \frac{-\pi G d}{g}(\rho_u - \rho_w)\left[\frac{-dt + d^2 + t^2 - d^2}{t-d}\right] = \frac{-\pi G d t}{g}(\rho_u - \rho_w)\left[\cancel{\frac{-d+t}{t-d}}^{=1}\right]$$

$$\boxed{\therefore \Delta h = \frac{-\pi G d t}{g}(\rho_u - \rho_w) \Rightarrow \text{GEOID HEIGHT ANOMOLY FOR PRATT COMPENSATED OCEAN BASIN}}$$

b) Evaluate the geoid ~~an~~ height anomaly for a 5-km deep ocean assuming a compensation depth at the base of the lithosphere ($t = 100$ km) using densities given in 2b).

$$\Delta h = \frac{-\pi G d t}{g}(\rho_u - \rho_w)$$

$\rho_u = 2800\ kg/m^3$
$\rho_w = 1000\ kg/m^3$

$$= -\pi(6.67\times10^{-11}\ m^3/kgs^2)(5000m)(100\ 000\ m)(2800\ kg/m^3 - 1000\ kg/m^3)\left(\frac{1}{9.81\ m/s^2}\right)$$

$= -19.2\,2424$ m

∴ Geoid height anomaly for Pratt-compensated ocean, w/ compensation depth @ base of lithosphere, is $\Delta h \simeq -19$m. That is, ocean basin is compensated on the geoid by a trough of ~19m deep.