PHYS/EOS 427 — Assignment 1

Due: 8:30 am Tuesday, Jan. 23, 2023. Please submit as a PDF file to Brightspace.

1. Show that the differential displacement can be written

$$\delta \boldsymbol{u} = [\boldsymbol{e} + \boldsymbol{\theta}]\ \delta \boldsymbol{x},$$

where $\delta \mathbf{u} = [\delta u, \delta v, \delta w]^T$, $\delta \boldsymbol{x} = [\delta x, \delta y, \delta z]^T$, $\boldsymbol{e}$ is a symmetric matrix representing volume change (dilatation), and $\boldsymbol{\theta}$ is an anti-symmetric matrix representing rotation. (10 pts)

2. (a) Consider a body under normal stresses σ_{xx}, σ_{yy} and σ_{zz}, and no shear stresses. Show that the total strain e_{xx} from these 3 stresses is given by (5 pts)

$$e_{xx} = \frac{\sigma_{xx}}{E} - \nu\,\frac{\sigma_{yy}}{E} - \nu\,\frac{\sigma_{zz}}{E}.$$

(b) The equation in 2(a) can be re-written as

$$E\,e_{xx} = (1+\nu)\,\sigma_{xx} - \nu\,(\sigma_{xx} + \sigma_{yy} + \sigma_{zz}).$$

Similar equations exist for e_{yy} and e_{zz}, giving three equations in three unknowns σ_{xx}, σ_{yy} and σ_{zz}. Solve these equations to obtain the stress-strain relationship (for normal stresses) in the form:

$$\sigma_{ii} = \lambda\Delta + 2\mu\,e_{ii} \qquad (i = x, y, z)$$

with

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}, \qquad \mu = \frac{E}{2(1+\nu)}.$$

(Hint: Add the three equations and substitute the result for $(\sigma_{xx} + \sigma_{yy} + \sigma_{zz})$ back into each of the three equations in turn.) (10 pts)

3. Starting from Hooke's Law in 3-D for a small cylinder aligned along the x axis with tensional stress σ_{xx} and no other stresses, show for an isotropic elastic solid that (10 pts)

$$\nu = \frac{\lambda}{2(\lambda + \mu)}.$$