Q1 Rayleigh waves decay exponentially w/ depth into the earth, with the P component decaying more rapidly than the S component since $K_p > K_s$. Starting w/ Eq ②, consider a depth such that the $\exp(K_p z)$ terms are negligable compared to the $\exp(K_s z)$ terms, and show that Rayleigh wave particle motion is prograde elliptic at this depth: Consider x and z components of displacement and particle velocity at this depth and plot the motion over one period of the wave (as done in class for $z=0$).

Eq ②: THE PHYSICAL DISPLACEMENT OF RAYLEIGH WAVE PARTICLE:

$$Re\left\{\frac{u_x}{A}\right\} = \left(Ke^{+K_p z} - \frac{2KK_pK_s}{K^2+K_s^2}e^{+K_s z}\right)\cos(\omega t - Kx)$$

$$Re\left\{\frac{u_z}{A}\right\} = -\left(K_p e^{+K_p z} - \frac{2K^2K_p}{K^2+K_s^2}e^{+K_s z}\right)\sin(\omega t - Kx)$$

→ CONSIDERING DEPTH S.T $\exp(K_p z)$ are negligible compared to $\exp(K_s z)$: (ie. $\exp(K_s z) >> \exp(K_p z)$)

$$Re\left\{\frac{u_x}{A}\right\} = \overbrace{\left(\frac{-2KK_pK_s}{K^2+K_s^2}e^{K_s z}\right)}^{=a/A}\cos(\omega t - Kx)$$

$$Re\left\{\frac{u_z}{A}\right\} = \underbrace{\left(\frac{2K^2K_p}{K^2+K_s^2}e^{K_s z}\right)}_{=b/A}\sin(\omega t - Kx)$$

⇒ MOTION AT $z<0$ ⇒ $e^{K_s z} < 1$

$K^2 = \frac{\omega^2}{C_R^2}$

$K_s^2 = K^2 - \frac{\omega^2}{\beta^2} = \omega^2\left(\frac{1}{C_R^2} - \frac{1}{\beta^2}\right)$

$K_p^2 = K^2 - \frac{\omega^2}{\alpha^2} = \omega^2\left(\frac{1}{C_R^2} - \frac{1}{\alpha^2}\right)$

→ $\frac{a}{A} < 0$ since $K, K_p, K_s, e^{K_s z}$ all > 0

→ $\frac{b}{A} > 0$

DISPLACEMENT FOR $z<0$:

$u_x = a\cos(\omega t - Kx)$, $u_z = b\sin(\omega t - Kx)$, for $\frac{a}{A} = \frac{-2KK_pK_s}{K^2+K_s^2}e^{K_s z} < 0, b>0$

OR:

$u_x = -a\cos(\omega t - Kx)$, $u_z = b\sin(\omega t - Kx)$, for $\frac{a}{A} = \frac{2KK_pK_s}{K^2+K_s^2}e^{K_s z}$

⇒ $a, b > 0$

Sarah Clapoff
V00886385

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#1 CONT'D:

The second set of displacement eq. also satisfies equation of ellipse:

$$\frac{u_x^2}{a^2} + \frac{u_z^2}{b^2} = 1$$

$$\frac{(-a\cos(\omega t - kx))^2}{a^2} + \frac{(b\sin(\omega t - kx))^2}{b^2} = \frac{\cancel{a^2}\cos^2(\omega t - kx)}{\cancel{a^2}} + \frac{\cancel{b^2}\sin^2(\omega t - kx)}{\cancel{b^2}} = 1$$

PARTICLE VELOCITY FOR $z<0$:

$$\dot{u}_x = \frac{du_x}{dt} = -a(-\omega\sin(\omega t - kx)) = a\omega\sin(\omega t - kx)$$

$$\dot{u}_z = \frac{du_z}{dt} = b\omega\cos(\omega t - kx)$$

PLOT MOTION @ $x=0$ OVER 1 PERIOD:

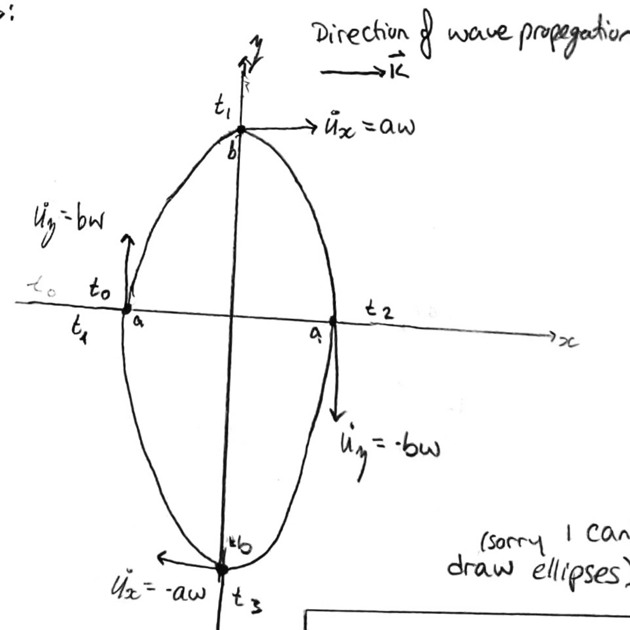
@ $t_0 = 0$: $u_x = -a\cos(0) = -a$
$u_z = b\sin(0) = 0$
$\dot{u}_x = a\omega\sin(0) = 0$
$\dot{u}_z = b\omega\cos(0) = b\omega$

@ $t_1 = \frac{\pi}{2\omega}$: $u_x = -a\cos(\frac{\pi}{2}) = 0$
$u_z = b\sin(\frac{\pi}{2}) = b$
$\dot{u}_x = a\omega$
$\dot{u}_z = 0$

@ $t_2 = \frac{\pi}{\omega}$: $u_x = -a\cos(\pi) = a$
$u_z = 0$
$\dot{u}_x = 0$
$\dot{u}_z = b\omega\cos(\pi) = -b\omega$

@ $t_3 = \frac{3\pi}{2\omega}$: $u_x = -a\cos(\frac{3\pi}{2}) = 0$
$u_z = b\sin(\frac{3\pi}{2}) = -b$
$\dot{u}_x = a\omega\sin(\frac{3\pi}{2}) = -a\omega$
$\dot{u}_z = 0$

@ $t_4 = \frac{2\pi}{\omega}$: $u_x = -a\cos(2\pi) = -a$
$u_z = b\sin(2\pi) = 0$
$\dot{u}_x = a\omega\sin(2\pi) = 0$
$\dot{u}_z = b\omega\cos(2\pi) = b\omega$



(sorry I can't draw ellipses) :)

⇒ THE PARTICLE MOTION @ THE TOP OF THE ELLIPSE IS IN THE SAME DIRECTION AS THE WAVE PROPEGATION

∴ FOR DEPTH AT WHICH $e^{k_s z} \gg e^{k_p z}$, $z<0$, THE RAYLEIGH-WAVE PARTICLE MOTION IS PROGRADE.

2. (a) For a Rayleigh wave propagating at a frequency of 1 Hz in a uniform medium with $\alpha=5$ km/s and $\beta=2.9$ km/s, use matlab or python to plot (together on the same graph) the relative amplitude of the x and z displacements of the wave as a function of depth over one Rayleigh-wave wavelength. For this, set the amplitude factor $A=1$ and neglect the sine/cosine dependences in Eq. (2) in class notes; you can use $c_R=0.92\beta$ for this case. (20 pts)

In [ ]:

```
import numpy as np
import matplotlib.pyplot as plt
```

In [ ]:

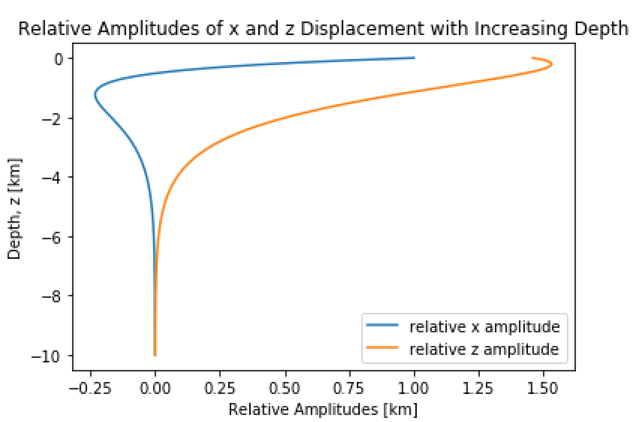
```
alpha = 5 #km/s, p velocity
beta = 2.9 #km/s, s velocity
cR = 0.92*beta #km/s, rayleigh wave velocity
f = 1 #Hz


#want as function of depth (aka z) over one R-wave wavelength
#so what is our max/deepest z?
z = np.linspace(0,-10,500) #km


omega = 2*np.pi*f #s^-1
k = omega/cR #km^-1
kappaS = omega*np.sqrt((1/cR**2) - (1/beta**2)) #km^-1
kappaP = omega*np.sqrt((1/cR**2) - (1/alpha**2)) #km^-1

#neglecting sin and cos dependance (a/A, b/A = a,b)
# --> we want relative amplitudes which is why we're neglecting sin and cos. note also A=1
a = k*np.exp(kappaP*z) - np.exp(kappaS*z)*(2*k*kappaP*kappaS)/(k**2 + kappaS**2) #km^-1
b = -(kappaP*np.exp(kappaP*z) - np.exp(kappaS*z)*(2*k**2 * kappaP)/(k**2 + kappaS**2)) #km^-1

plt.plot(a,z, label='relative x amplitude')
plt.plot(b,z, label='relative z amplitude')
plt.legend()
plt.ylabel('Depth, z [km]')
plt.xlabel('Relative Amplitudes [km]')
plt.title('Relative Amplitudes of x and z Displacement with Increasing Depth')
plt.show()
```



(b) Based on your plot from (a), explain the behaviour of Rayleigh-wave particle motion as a function of depth, making reference to the figure on page 2 of this assignment showing measured Rayleigh-wave particle motion at increasing depths. (10 pts)

As described on page 49 of class notes, the equation for the elliptic particle motion is:

$$\frac{u_x^2}{a^2} + \frac{u_z^2}{b^2} = 1$$

So we can explain the x and z particle displacement from considering the changes in their respective relative amplitudes, a and b.

First, we can note that the relative z amplitude is larger than the relative x amplitude for the entirety of the particle motion, until they converge to zero. This tell us that, in the elliptic particle motion, the semi-major axis will correspond to b. In addition, both amplitudes are approaching zero, which tells us that the motion is becoming smaller as the depth increases, as shown in the figure.

From the equations for Rayleigh-wave particle velocity (pg 49),

$$\dot{u_x} = -a\omega sin(\omega t - kx)$$

$$\dot{u_z} = b\omega sin(\omega t - kx)$$

From a depth of 0km to -1km, a and b are both positive, corresponding to $\dot{u_z} > 0$ and $\dot{u_x} < 0$. This describes retrograde motion, as shown in the figure. As a approaches zero, the semi-minor axis is decreasing faster than the semi-major axis, resulting in a more eccentric ellipse in the z direction, as shown in the figure. At a depth of approximately -1km, $a = 0$, and the motion is linearly polarized. For depths greater than -1km (that is, $z < -1$), $a > 0$ and $b > 0$. This corresponds to positive particle velocities in both the x and z, which is prograde elliptic motion, as seen in the figure.

(c) By what factors are the P and S components attenuated at one Rayleigh-wave wavelength depth? (5 pts)

The P motion is attenuated by a factor of $e^{\kappa_P z} = e^{2\pi\sqrt{(1/c_R^2 - 1/\alpha^2)}} = e^{1.268\pi}$

The S motion is attenuated by a factor of $e^{\kappa_S z} = e^{2\pi\sqrt{(1/c_R^2 - 1/\beta^2)}} = e^{0.294\pi}$

```
In [ ]: 2*np.sqrt((1/cR**2)-(1/alpha**2))
        2*np.sqrt((1/cR**2)-(1/beta**2))
```

```
0.2937918731973826
```