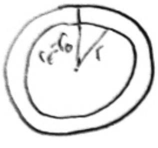
03.20.2023

PHYS 427 A#7

#1 Apply the Adams-Williamson equation in integral form to compute the density $\rho(r)$ at the base of a 30km thick outer-most shell of the Earth (i.e. $r_0 = r_E = 6400$ km and $r = 6370$ km), assuming constant seismic velocities of $\alpha = 7.00$ km/s and $\beta = 4.50$ km/s over the shell, and a density at the top of the shell of $\rho(r_0) = 2800$ kg/m³. Take the mass of the Earth to be $6.00 \times 10^{24}$ kg.

→ Want density at r, i.e. base of shell



A-W Eq: $\dfrac{d\rho}{dr} = \dfrac{-GM_r\,\rho(r)}{r^2\left(\alpha^2(r) - \frac{4}{3}\beta^2(r)\right)} \Rightarrow \ln\left(\dfrac{\rho(r)}{\rho(r_0)}\right) = -GM_r \displaystyle\int_{r_0}^{r} \dfrac{du}{u^2\left(\alpha^2(u) - \frac{4}{3}\beta^2(u)\right)}$

- Since thin shell close to surface, relative to the Earth's radius, can assume mass of shell is negligible relative to Earth's mass, i.e. $M_r = M_E$.

$$\ln(\rho(r)) = -GM_E \int_{r_0}^{r} \frac{du}{u^2\left(\alpha^2 - \frac{4}{3}\beta^2\right)} + \ln(\rho(r_0)) = \frac{-GM_E}{\left(\alpha^2 - \frac{4}{3}\beta^2\right)} \int_{r_0}^{r} \frac{1}{u^2}\,du + \ln(\rho(r_0))$$

$$\ln(\rho(r)) = \frac{GM_E}{\left(\alpha^2 - \frac{4}{3}\beta^2\right)} \left[\frac{1}{r} - \frac{1}{r_0}\right] + \ln(\rho(r_0))$$

$$\ln(\rho(6370)) = \frac{\left(6.67\times10^{-20} \frac{\text{km}^3}{\text{kg s}^2}\right)\left(6.00\times10^{24}\,\text{kg}\right)}{(7.00\,\text{km/s})^2 - \frac{4}{3}(4.50\,\text{km/s})^2} \left[\frac{1}{6370\,\text{km}} - \frac{1}{6400\,\text{km}}\right] + \ln\left(2800\,\text{kg/m}^3\right)$$

$$\rho(r) = e^{\left(0.013386167 + \ln(2800\,\text{kg/m}^3)\right)}$$

$$\rho(r = 6370\,\text{km}) = 2837.733257\ \text{kg/m}^3$$

$$\boxed{\rho(r = 6370\,\text{km}) \approx 2838\ \text{kg/m}^3}$$