EOS/PHYS 427 — Assignment 2

Due: Tuesday, January 31, 2023.

1. Given that Poisson's ratio can be related to the bulk and shear moduli as $\nu = \dfrac{3K - 2\mu}{6K + 2\mu}$ fill out the following table: (10 pts)

Material	K (10^9 N/m^2)	μ (10^9 N/m^2)	ρ (kg/m^3)	α (m/s)	β (m/s)	ν
Air	0.00010	0	1.0			
Water	2.2	0	1000			
Ice	8.0	3.9	920			
Sandstone	24	17	2500			
Limestone	38	22	2700			
Granite	88	22	2600			
Peridotite	140	58	3300			

2. Derive an expression for the ratio of S- to P-wave velocity β/α in terms only of Poisson's ratio σ. Using this expression, what are the minimum and maximum values of β relative to α for normal materials? (10 pts)

3. (a) Starting with the scalar displacement potential for a plane P wave, fill in the missing steps in the class notes (compute the gradient) to explicitly show that the particle motion associated with a plane P-wave is given by

$$\mathbf{u_p} = iA\mathbf{k_p}\, e^{i(\mathbf{k_p}\cdot\mathbf{r} - \omega t)},$$

which indicates particle motion parallel to the propagation direction. (5 pts)

(b) Starting with the vector displacement potential for a plane S wave, fill in the missing steps in the class notes (compute the curl) to explicitly show that the particle motion associated with a plane S-wave is given by

$$\mathbf{u_s} = i\mathbf{k_s} \times \mathbf{B}\, e^{i(\mathbf{k_s}\cdot\mathbf{r} - \omega t)},$$

which indicates particle motion perpendicular to the propagation direction. (10 pts)