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P427 A#5 GOOD

#1 CASE ①: Layer thickness h, S-wave velocity β_1 and density ρ_1, over a half-space of S-wave velocity β_2 + density ρ_2.

CASE ②: Layer thickness h, w/ rigid lower surface.

Show CASE ① → case ② as $\beta_2, \rho_2 \to \infty$

DISPERSION RELATION CASE ①:

$$\frac{\omega h}{c_L}\sqrt{\frac{c_L^2}{\beta_1^2}-1} - n\pi = \tan^{-1}\left(\frac{\rho_2\sqrt{\frac{\beta_2^2}{c_L^2}-1}}{\rho_1\sqrt{1-\frac{\beta_1^2}{c_L^2}}}\right)$$

CASE ②:

$$c_L(\omega) = \left[\frac{1}{\beta^2} - \frac{(n+\frac{1}{2})^2\pi^2}{h^2\omega^2}\right]^{-\frac{1}{2}}$$

$\lim_{\beta_2, \rho_2 \to \infty}$ ①:

$$LHS = \frac{\omega h}{c_L}\sqrt{\frac{c_L^2}{\beta_1^2}-1} - n\pi$$

$$RHS = \tan^{-1}\left(\frac{\infty}{\rho_1\sqrt{1-\frac{\beta_1^2}{c_L^2}}}\right) \to \frac{\pi}{2}$$

$$\therefore \frac{\omega h}{c_L}\sqrt{\frac{c_L^2}{\beta_1^2}-1} - n\pi = \frac{\pi}{2}$$

$$\frac{c_L^2}{\beta_1^2} = \frac{\pi^2(n+\frac{1}{2})^2 c_L^2}{\omega^2 h^2} + 1$$

$$\frac{1}{\beta_1^2} = \frac{\pi^2(n+\frac{1}{2})^2 \cancel{c_L^2}}{\omega^2 h^2 \cancel{c_L^2}} + \frac{1}{c_L^2}$$

Let $\beta = \beta_1$, as $\beta_2 \to \infty$, as in case ②.

$$\boxed{\Rightarrow c_L = \left[\frac{1}{\beta^2} - \frac{(n+\frac{1}{2})^2\pi^2}{h^2\omega^2}\right]^{-\frac{1}{2}}}$$

↳ THIS IS CASE ② DISPERSION RELATION!

∴ ⇒ DISPERSION RELATION FOR ① BECOMES DISPERSION RELATION FOR ② AS $\beta_2, \rho_2 \to \infty$