EOS/PHYS 427 — Assignment 3

Due: 8:30 am Tuesday, February 7, 2023.

1. (a) Show that when shear can be neglected (β_1, $\beta_2 \to 0$), the 4×4 system of Zoepptriz equations given in class notes reduces to the following expressions for P-wave reflection and transmission coefficients:

$$R_p = \frac{A_1}{A_0} = \frac{Z_2^p/\cos\theta_2 - Z_1^p/\cos\theta_1}{Z_2^p/\cos\theta_2 + Z_1^p/\cos\theta_1}, \qquad T_p = \frac{B_1}{A_0} = \frac{2\,Z_1^p/\cos\theta_2}{Z_2^p/\cos\theta_2 + Z_1^p/\cos\theta_1}$$

where $Z_i^p = \rho_i\alpha_i,\ i = 1, 2.$ (10 pts)

(b) In this case (negligible shear), the equation for vertical energy flux simplifies to

$$|R_p|^2 + \left(\frac{\rho_2\alpha_2\cos\theta_2}{\rho_1\alpha_1\cos\theta_1}\right)|T_p|^2 = 1,$$

where the first and second terms represents E_{rp} and E_{tp}, the reflected and transmitted P-wave energy coefficients, respectively. Show that substituting the expressions for R_p and T_p given in (a) into this equation satisfies conservation of energy flux, i.e., that the sum of E_{rp} and E_{tp} really is 1. (10 pts)

2. The angle of intromission θ_I is defined to be the incidence angle that produces a zero reflection coefficient (given certain conditions for wave velocities and densities). Show that this angle can be expressed: (10 pts)

$$\theta_I = \sin^{-1}\sqrt{\frac{(\rho_1\,\alpha_1)^2 - (\rho_2\,\alpha_2)^2}{\alpha_2^2\,(\rho_1^2 - \rho_2^2)}}\ .$$

3. Using MATLAB, phython or any programming language you like, solve for and plot the normalized energy coefficients as a function of incident P-wave angle from 0–90° for the media values given below (assume incident amplitude $A_0 = 1$). This involves solving the full Zoeppritz equations (4×4 system of linear equations) for each angle, which can be done with matrix inversion or a linear system solver.

(a) $\alpha_1 = 2000$ m/s, $\beta_1 = 1070$ m/s, $\rho_1 = 2000$ kg/m^3, $\alpha_2 = 4000$ m/s, $\beta_2 = 2310$ m/s, $\rho_2 = 2500$ kg/m^3 (this is the case considered in the online figures, and can serve as a check for your code). (15 pts)

(b) $\alpha_1 = 1480$ m/s, $\beta_1 = 1$ m/s, $\rho_1 = 1030$ kg/m^3, $\alpha_2 = 3300$ m/s, $\beta_2 = 1300$ m/s, $\rho_2 = 900$ kg/m^3. This case corresponds to ocean acoustic waves incident from below on sea ice (β_1 should be 0 for seawater, but this could cause numerical problems in your code). Briefly discuss/explain the characteristics of this plot. (20 pts)

Note that these calculations involve complex quantities for some angles. MATLAB handles this automatically, but in phython you will have to use cmath for potentially complex calculations, e.g., cmath.arcsin instead of np.arcsin. For either programming language, take absolute values of the reflection and transmission coefficients, and you might need to take the real part of final energy coefficients if small imaginary parts persist through the calculations.

4. Using the plot below of measured reflection coefficients for an ocean acoustic wave reflecting off a sandy seabed in the Mediterranean Sea, and neglecting shear, estimate the seabed P-wave velocity and density given that the sound velocity and density of seawater are 1510 m/s and 1030 kg/m^3. (Hint: consider expressions for the critical angle, which is $\sim$64° here, and for the reflection coefficient at normal incidence, $\theta_1 = 0$.) (15 pts)