Sarah Clapoff
V00886385 CSC421 A#3 11,22,2022

Q1:

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CSC421 A#3
GOOD

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#1 Using propositional resolution, show the following sentence is unsatisfiable:

$$(p \mid q \mid \neg r) \;\&\; (\neg r \mid q \mid p) \rightarrow ((r \mid q) \;\&\; \neg q \;\&\; \neg p))$$

-Convert to clausal form and derive the empty clause. using resolution.

$$(p \mid q \mid \neg r) \;\&\; (\neg r \mid q \mid p) \rightarrow ((r \mid q) \;\&\; \neg q \;\&\; \neg p)) \iff (p \vee q \vee \neg r) \wedge ((\neg r \vee q \vee p) \Rightarrow ((r \vee q) \wedge \neg q \wedge \neg p))$$

CLAUSAL FORM:

I:
$$(p \vee q \vee \neg r) \wedge ((\neg r \vee q \vee p) \Rightarrow ((r \vee q) \wedge \neg q \wedge \neg p)) \iff (p \vee q \vee \neg r) \wedge (\neg(\neg r \vee q \vee p) \vee ((r \vee q) \wedge \neg q \wedge \neg p))$$

N:
$$(p \vee q \vee \neg r) \wedge ((\neg r \wedge \neg q \wedge p) \vee ((r \vee q) \wedge \neg q \wedge \neg p)) \iff (p \vee q \vee \neg r) \wedge ((r \vee q) \wedge \neg q) \vee ((\neg r \wedge p) \wedge \neg q \wedge \neg p))$$

D: let $\varnothing_1 = (r \wedge \neg q \wedge \neg p)$,

$$(p \vee q \vee \neg r) \wedge (\varnothing_1 \vee ((r \vee q) \wedge \neg q \wedge \neg p)) \iff (p \vee q \vee \neg r) \wedge ((\varnothing_1 \vee (r \vee q)) \wedge (\varnothing_1 \vee \neg q) \wedge (\varnothing_1 \vee \neg p))$$

$$\varnothing_1 \vee (r \vee q) \iff (r \wedge \neg q \wedge \neg p) \vee (r \vee q) \iff (r \vee r \vee q) \wedge (\neg q \vee r \vee q) \wedge (\neg p \vee r \vee q) \iff (r \vee q) \vee (\top) \vee (\neg p \vee r \vee q) \iff (r \vee q) \vee (\neg p \vee r \vee q)$$

$$\varnothing_1 \vee (\neg q) \iff (r \wedge \neg q \wedge \neg p) \vee \neg q \iff (r \vee \neg q) \wedge (\neg q \vee \neg q) \wedge (\neg p \vee \neg q) \iff (r \vee \neg q) \wedge \neg q \wedge (\neg p \vee \neg q)$$

$$\varnothing_1 \vee (\neg p) \iff (r \wedge \neg q \wedge \neg p) \vee \neg p \iff (r \vee \neg p) \wedge (\neg q \vee \neg p) \wedge (\neg p \vee \neg p) \iff (r \vee \neg p) \wedge (\neg q \vee \neg p) \wedge \neg p$$

$$(p \vee q \vee \neg r) \wedge [((r \vee q) \wedge (\neg p \vee r \vee q)) \wedge ((r \vee q) \wedge \neg q \wedge (\neg p \vee r \vee q)) \wedge ((r \vee q) \wedge \neg q \wedge \neg p)]$$

$$\iff (p \vee q \vee \neg r) \wedge (r \vee q) \wedge (\neg p \vee r \vee q) \wedge (r \vee q) \wedge \neg q \wedge (\neg p \vee r \vee q) \wedge (r \vee q) \wedge \neg q \wedge \neg p$$

O:

① $\{p, q, \neg r\}$	Premise	⑨ $\{\neg q, r, q\}$ Premise	⑭ $\{p\}$ (10, 13)
② $\{r, r, q\}$	"	⑩ $\{\neg q\}$ (5)	⑮ $\{\}$ (11, 14)
③ $\{\neg p, r, q\}$	"	⑪ $\{\neg p\}$ (8)	
④ $\{r, \neg q\}$	"	⑫ $\{r\}$ (2, 10)	
⑤ $\{\neg q, \neg q\}$	"	⑬ $\{p, q\}$ (1, 12)	
⑥ $\{\neg p, \neg q\}$	"		
⑦ $\{r, \neg p\}$	"		
⑧ $\{\neg p, \neg p\}$	"		

We have derived the empty clause,
∴ The sentence is UNSATISFIABLE.

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Q2:

#2 a) FOL:

$\forall x \forall y (Horse(x) \wedge Dog(y)) \Rightarrow Faster(x,y)$ //All horses are faster than all dogs

$\exists y \forall z (Greyhound(y) \wedge Rabbit(z)) \Rightarrow Faster(y,z)$ //Some greyhounds are faster than all rabbits.

$\forall y \; Greyhound(y) \Rightarrow Dog(y)$ //All greyhounds are dogs.

$\forall x \forall y \forall z (Faster(x,y) \wedge Faster(y,z)) \Rightarrow Faster(x,z)$ //If x faster than y, and y faster than z, then x faster than z.

$\neg(\forall x \forall y (Horse(x) \wedge Rabbit(y)) \Rightarrow Faster(x,y))$. //Negated conclusion.

b) All hummingbirds are richly coloured.
No large birds live on honey. $\forall x (Large(x) \wedge Bird(x) \Rightarrow \neg Lives(x, Honey)$
Birds that do not live on honey are dull in colour. $\forall x (Bird(x) \wedge \neg Lives(x, Honey)) \Rightarrow \neg Colourful(x)$
Conclusion: All hummingbirds are small.

FOL:

$\forall x \; Hummingbird(x) \Rightarrow Colourful(x)$.

$\forall x \; Hummingbird(x) \Rightarrow Bird(x)$.

$\forall x (Large(x) \wedge Bird(x)) \Rightarrow \neg Lives(x, Honey)$.

$\forall x (Bird(x) \wedge \neg Lives(x, Honey)) \Rightarrow \neg Colourful(x)$.

$\neg(\forall x \; Humingbird(x) \Rightarrow \neg Large(x))$. //Negated conclusion.

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#2 c)

My gardener is well worth listening to on military subjects.
↳If gardener then worth listening to
No one can remember the battle of Waterloo, unless he is very old.
↳If remember Waterloo, old.
Nobody is really worth listening to on military subjects, unless he can remember the battle of Waterloo. →Worth listening to, then they remember Waterloo.
Conclusion: My gardener is very old. -If gardener, then old.

$\forall x (gardener(x) \Rightarrow listen(x))$. //if x is my gardener, then x is worth listening to.

$\forall x (waterloo(x) \Rightarrow old(x))$.

$\forall x (listen(x) \Rightarrow waterloo(x))$.

$\neg(\forall x (gardener(x) \Rightarrow old(x)))$. //Negated conclusion.

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Prover9 format:

```
sarahclapoff@linux204:~/csc421/a3/q2$ cat 2a.txt
assign(max_seconds, 30).

formulas(assumptions).
    all x all y (Horse(x) & Dog(y) -> Faster(x,y)).
    exists y (Greyhound(y) & all z (Rabbit(z) -> Faster(y,z))).
    all y (Greyhound(y) -> Dog(y)).
    all x all y all z (Faster(x,y) & Faster(y,z) -> Faster(x,z)).
end_of_list.

formulas(goals).
    all x all y (Horse(x) & Rabbit(y) -> Faster(x,y)).
end_of_list.sarahclapoff@linux204:~/csc421/a3/q2$ cat 2b.txt
assign(max_seconds, 30).

formulas(assumptions).
    all x (hummingbird(x) -> colourful(x)).
    all x (hummingbird(x) -> bird(x)).
    all x ((large(x) & bird(x)) -> -lives(x,honey)).
    all x ((bird(x) & -lives(x,honey)) -> -colourful(x)).
end_of_list.

formulas(goals).
    all x (hummingbird(x) -> -large(x)).
end_of_list.sarahclapoff@linux204:~/csc421/a3/q2$ cat 2c.txt
assign(max_seconds, 30).

formulas(assumptions).
    all x (gardener(x) -> listen(x)).
    all x (waterloo(x) -> old(x)).
    all x (listen(x) -> waterloo(x)).
end_of_list.

formulas(goals).
    all x (gardener(x) -> old(x)).
sarahclapoff@linux204:~/csc421/a3/q2$
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Q3:

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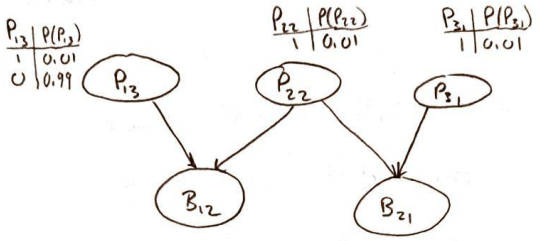
#3 Redo prob. calculations for pits in [1,3], [2,2] assuming that each square contains a pit w/ prob. 0.01, independant of the other squares.



PROB OF PIT IN [1,3]:

$$P(P_{13}|b_{12},b_{21}) = \alpha P(P_{13}) \sum_{P_{22}} \sum_{P_{31}} P(b_{12}|P_{13},P_{22}) P(b_{21}|P_{22},P_{31}) P(P_{22}) P(P_{31})$$

$$= \alpha P(P_{13}) \left[P(b_{12}|P_{13},P_{22}) P(b_{21}|P_{22},P_{31}) P(P_{22}) P(P_{31}) + P(b_{12}|P_{13},\neg P_{22}) P(b_{21}|\neg P_{22},P_{31}) P(\neg P_{22}) P(P_{31}) + P(b_{12}|\neg P_{31},P_{22}) P(b_{21}|P_{22},\neg P_{31}) P(P_{22}) P(\neg P_{31}) + P(b_{12}|\neg P_{31},\neg P_{22}) P(b_{21}|\neg P_{22},\neg P_{31}) P(P_{22}) P(\neg P_{31}) \right]$$

(B_{12}, B_{21} prob. tables taken from slides)

$$= \alpha (0.01)\left[(1*1*0.01*0.01) + (1)(1)(0.99)(0.01) + (1)(1)(0.01)(0.99) + 0\right]$$

$$= \alpha (0.01)(0.0199) = 0.000199\,\alpha$$

$$P(\neg P_{13}|b_{12},b_{21}) = \alpha P(\neg P_{13}) \sum_{P_{22}} \sum_{P_{31}} P(b_{12}|\neg P_{13},P_{22}) P(b_{21}|P_{22},P_{31}) P(P_{22}) P(P_{31})$$

$$= \alpha (0.99)\left[P(b_{12}|\neg P_{13},P_{22})\ldots + P(b_{12}|\neg P_{13},\neg P_{12})\ldots \to 0 \right.$$
$$\left. P(b_{12}|\neg P_{13},P_{22}) P(b_{21}|P_{22},\neg P_{31})\ldots + P(b_{12}|\neg P_{13},\neg P_{22})\ldots \to 0 \right]$$

⇒All values are the same except for $P(b_{12}|\neg P_{13},\pm P_{22})$

$$= \alpha (0.99)\left[(1)(1)(0.01)(0.01) + 0 + (1)(1)(0.01)(0.99) + 0\right]$$

$$= \alpha (0.00099)$$

$$\alpha = \frac{1}{P(P_{13}|b_{12},b_{21}) + P(\neg P_{13}|b_{12},b_{21})} = \frac{1}{0.00099 + 0.000199} = 99.0197$$

$$P(P_{13}|b_{12},b_{21}) = \frac{0.000199}{0.00099 + 0.000199} = 0.0197049 \approx \boxed{1.97\% = P(P_{13}|b_{12},b_{21})}$$

#3

PROB OF PIT IN [2,2]:

$$P(p_{22}|b_{12},b_{21}) = \alpha P(p_{22}) \sum_{P_{13}} \sum_{P_{31}} P(b_{12}|p_{13},p_{22}) P(b_{21}|p_{22},p_{31}) P(p_{13}) P(p_{31})$$

$$= \alpha P(p_{22}) \left[P(b_{12}|p_{13},p_{22}) P(b_{21}|p_{22},p_{31}) P(p_{13}) P(p_{31}) + P(b_{12}|\neg p_{13},p_{22}) P(b_{21}|p_{22},p_{31}) P(\neg p_{13}) P(p_{31}) + P(b_{12}|p_{13},p_{22}) P(b_{21}|p_{22},\neg p_{31}) P(p_{13}) P(\neg p_{31}) + P(b_{12}|\neg p_{13},p_{22}) P(b_{21}|p_{22},\neg p_{31}) P(\neg p_{13}) P(\neg p_{31}) \right]$$

$$= \alpha (0.01) \left[(1)(1)(0.01)(0.01) + (1)(1)(0.99)(0.01) + (1)(1)(0.01)(0.99) + (1)(1)(0.99)^2 \right]$$

$$= \alpha (0.01)(1) = 0.01\alpha$$

$$P(\neg p_{22}|b_{12},b_{21}) = \alpha P(\neg p_{22}) \sum_{P_{13}} \sum_{P_{31}} P(b_{12}|p_{13},\neg p_{22}) P(b_{21}|\neg p_{22},p_{31}) P(p_{13}) P(p_{31})$$

$$= \alpha (0.99) \left[(1)(1)(0.01)(0.01) + 0 + 0 + 0 \right]$$

$$= \alpha (0.0000099)$$

$$\alpha = (0.01 + 0.0000099)^{-1} = 99.0197$$

$$P(p_{22}|b_{12},b_{21}) = \frac{0.01}{(0.01 + 0.0000099)} = 0.990197 \approx \boxed{99.02\% = P(p_{22}|b_{12},b_{21})}$$

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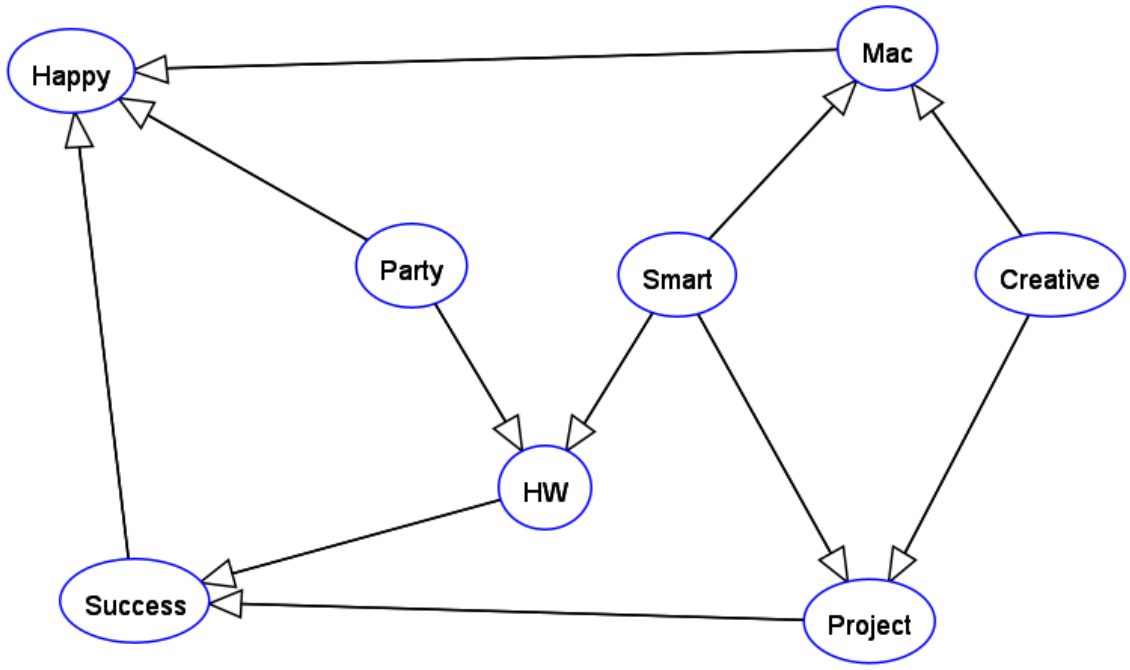
Q4.1:



Q4.2:

Probability Table for HW

Party	Smart	P(HW=T)	P(HW=F)
T	T	0.8028	0.1972
T	F	0.0935	0.9065
F	T	0.8985	0.1015
F	F	0.3049	0.6951

No observed value for this node.

OK

Probability Table for Mac

Smart	Creative	P(Mac=T)	P(Mac=F)
T	T	0.6858	0.3142
T	F	0.4133	0.5867
F	T	0.8971	0.1029
F	F	0.1216	0.8784

No observed value for this node.

OK

Probability Table for Project

Smart	Creative	P(Project=T)	P(Project=F)
T	T	0.9052	0.0948
T	F	0.7938	0.2062
F	T	0.4029	0.5971
F	F	0.1055	0.8945

No observed value for this node.

OK

Probability Table for Success

HW	Project	P(Success=T)	P(Success=F)
T	T	0.8966	0.1034
T	F	0.3065	0.6935
F	T	0.2067	0.7933
F	F	0.0497	0.9503

No observed value for this node.

OK

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Probability Table for Happy

Party	Success	Mac	P(Happy=T)	P(Happy=F)
T	T	T	0.9594	0.0406
T	T	F	0.7215	0.2785
T	F	T	0.4923	0.5077
T	F	F	0.4201	0.5799
F	T	T	0.358	0.642
F	T	F	0.3068	0.6932
F	F	T	0.205	0.795
F	F	F	0.0939	0.9061

No observed value for this node.

OK

Q4.3 - 4.8:

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#4

3. What is prob of being happy given that you party, are smart, but not creative?

Let h=happy, P=party, S=smart, C=creative.

$$P(h|p,s,\neg c) = \alpha P(h,p,s,\neg c)$$

$$= \alpha \sum_{hw} \sum_{suc} \sum_{pro} \sum_{mac} P(h|p,suc,mac)\,P(p)\,P(\neg c)\,P(s)\,P(hw|p,s)\,P(suc|hw,pro)\,P(pro|s,\neg c)\,P(mac|s,\neg c)$$

$$= \alpha\, P(p)P(s)P(\neg c) \sum_{pro} \sum_{mac} P(pro|s,\neg c)\,P(mac|s,\neg c) \sum_{suc} P(h|p,suc,mac) \sum_{hw} P(hw|p,s)\,P(suc|hw,pro)$$

$$P(\neg h|p,s,\neg c) = \alpha P(\neg h,p,s,\neg c)$$

$$= \alpha\, P(p)P(s)P(\neg c) \sum_{pro} \sum_{mac} P(pro|s,\neg c)\,P(mac|s,\neg c) \sum_{suc} P(\neg h|p,suc,mac) \sum_{hw} P(hw|p,s)\,P(suc|hw,pro)$$

$$\alpha = 1/\left(P(h|p,s,\neg c) + P(\neg h|p,s,\neg c)\right)$$

FROM AITOOL:

$$\boxed{P(h=T|p=T,\ s=T,\ C=F) = 0.69276}$$

(NOTE: I rounded + entered values in AITOOL up to 4 decimal places. This may account for small probability discrepencies)

4. $P(h|s,c) = 0.58155$
5. $P(h|\neg p,hw,pro) = 0.32044$
6. $P(h|mac) = 0.56272$
7. $P(p|s) = 0.6022$
8. $P(p|s,h) = 0.79264$