Interferometry

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Abstract

The purpose of this experiment was to become familiar with the operation of a Michelson interferometer, and to use this interferometer to measure the refractive index of air. The first part determined the path difference d of a He-Ne laser for a count of 100 fringes. This was done through measuring the change in position of the graduated dial on the interferometer as it was turned for 100 fringes. Through this method, the path length was found to be $(3.11 \pm 0.10) \times 10^{-5}\,\text{m}$. The path difference was calculated separately, using the wavelength of the laser, to be $3.164 \times 10^{-5}\,\text{m}$. The second method was thought to be more accurate due to the accepted value of the wavelength and lack of human error. The second part of this experiment calculated the refractive index of air through considering the path difference in an air cell as the pressure changed from a vacuum to room pressure. This was achieved through evacuating the air cell, and counting the number of fringes as air was admitted in. On average, 43 fringes were counted, and the refractive index of air was calculated to be 1.0003 ± 0.0010. This value was consistent with the accepted value.

I. Introduction

The purpose of an interferometer is to produce interference patterns through dividing and recombining light. The Michelson-Morley interferometer can be used to verify the Fresnel-Arago Laws, as well as make accurate measurements of length, since the movement of each fringe corresponds to the mirror being displaced by $\frac{1}{2}$wavelength of the source. [2]

In most amplitude-splitting interferometers (such as the Michelson-Morley interferometer), the light from source S is divided into two paths at the beam splitter by the partial surface mirror; one beam passes through the beam splitter and the other is reflected away. Both beams are then reflected off of fully reflecting mirrors, M_1 and M_2, to the beam splitter which recombines them into constructive and destructive interference patterns which can be displayed on a screen. A beam expander can be inserted in front of the screen to magnify the combined beam such that the fringe pattern is visible. Then, the path length of one beam is altered, either through tilting or displacing one of the mirrors. The result is a change of fringe, where each fringe is displaced 1/2 the source's wavelength, or a phase shift of π. [1]

The Michelson-Morley interferometer is the most well known and historically important. [2] It uses one beam splitter placed at a 45° angle to S, so that the reflected beam reflects 90° to M_1 and the refracted beam refracts within the glass and moves parallel to S towards M_2. The mirrors reflect both beams directly back to the beam splitter, where they recombine, producing the interference pattern shown on the screen (See Figure 1). [1] This makes it easier to align, when compared to a Mach-Zehnder interferometer which has two beam splitters that can create a difference in light paths if not properly aligned. [2] However, since the refracted beam passes through the beam splitter more than the reflected beam, a compensator plate may be inserted in the path of the reflected beam also at 45°, so that the path length is the same as the refracted beam. The compensator plate is the same as the beam splitter, except there is no reflective coating. The mirror M_1 and compensator plate are placed on a steel carriage that moves parallel to the reflected beam. By using the graduated dial to move the steel carriage, the path length of the reflected beam is changed.[1]

For a change of one fringe (phase shift of π), the mirror must be displaced by one wavelength. That is, the path difference, d is equal to the wavelength, λ. In general, for n fringes:

$$n\lambda = 2d \tag{1}$$

The Michelson-Morley interferometer can also be used to determine the refractive index μ of any gas. This is done by evacuating the air cell with a vacuum pump, and slowly letting in the gas; in this experiment we used air. In equation (1), the path difference becomes $(\mu - 1)l$, where l is the length of the air cell. The index of refraction can be found through:

$$n\lambda = 2(\mu - 1)l \tag{2}$$

II. Apparatus

- Michelson-Morley Interferometer
- Vacuum Pump
- Air Cell

See Figure 1. in the Appendix

III. Procedure

The interferometer was set up according to Figure 1. The laser beam was aligned by rotating knobs x_1 and x_2 so that the two spots produced from the beam splitter align and the straight line fringes are turned into a circular fringe pattern. The carriage was moved to both ends of the stops to check that there was no shift to the fringe pattern; if a shift was observed, that meant parallax was present and was corrected by re-positioning the laser.

Warning: Do not look directly into the laser, or into the laser light reflected from the mirrors.

In the first part of the experiment, the interferometer was calibrated through determining the path difference, d, for a count of 100 fringes. This was done by turning the graduated dial until fringes were observed (as there is initial slack in the dial) and recording the difference of dial increments between the start and end positions. This was repeated five times for accuracy. Then, the path difference d was calculated by multiplying the average number of increments by 5.09×10^{-7} m. This value was compared to the value of d given by Equation (1).

In the second part of this experiment, the refractive index of air was determined. The air cell was placed on the interferometer bridge perpendicular to mirror M_2, was evacuated by the vacuum pump, and the roughing valve was closed. Then, the number of fringes was counted as the cell returned to room pressure, through slowly opening the needle valve. This was repeated 6 times for accuracy. The refractive index of air was calculated from Equation (2).

IV. Data

TABLE I. Number of Increments for 100 Fringes

	Start Position	**End Position**	**Number of Dial Increments**
1	0.0	58.8	58.8
2	0.0	58.9	58.9
3	0.0	62.6	62.6
4	5.0	68.4	63.4
5	0.0	61.9	61.9

Average number of dial increments, inc:
$inc = 61.1 \pm 3.1\%$

TABLE II. Number of Fringes Observed as Air Cell Fills

	Number of Fringes, n
1	42
2	42
3	43
4	44
5	43
6	43

Average Number of Fringes Observed, $n = 43$
Length of Air Cell, $l = (48.260 \pm 0.005)$ mm

V. Analysis

See the Appendix for sample calculations.

Part 1:
Path distance d_{obs} from the gear ratio:
$d_{obs} = (3.11 \pm 0.10) \times 10^{-5}$ m

Path distance d_{calc} from Equation (1):
$d_{calc} = 3.164 \times 10^{-5}$ m

A consistency check between d_{obs} and d_{calc} shows that these values are consistent.

Part 2:
Refractive index of air from Equation (2):
$\mu = 1.0003 \pm 0.0010$

Accepted value of refractive index of air at STP:
$\mu_{accept} = 1.00029$ [2]

A consistency check between μ and μ_{accept} show them to be consistent.

VI. Discussion

In part 1, the path difference d for a count of 100 fringes was determined two ways. When determined using the gear ratio, the average number of dial increments was $inc = 61.1 \pm 3.1\%$ and d was found to be $d_{obs} = (3.11 \pm 0.10) \times 10^{-5}\,\mathrm{m}$. When determined using Equation (1), d was found to be $d_{calc} = 3.164 \times 10^{-5}\,\mathrm{m}$. A consistency check between d_{obs} and d_{calc} shows that these values are consistent. However, we expect d_{calc} to be more accurate, as there were less uncertainties in the accepted value used for λ, compared to the measured value of the friction pin drive. The gear ratio method was subject to the inherent uncertainty of the worm gear train as well as error from counting the fringes and operating the interferometer. The human error from counting fringes could be remedied by placing a photodetecor on a small area of the screen, so that it would increment every time a fringe passed.

In part 2, the index of refraction of air was determined from Equation (2), through counting the fringes produced as the pressure in an air cell changed from a vacuum to room pressure. On average, 43 fringes were counted, and the refractive index of air was found to be $\mu = 1.0003 \pm 0.0010$. This value is consistent with the accepted value of refractive index of air at STP, $\mu_{accept} = 1.00029$. The primary source of uncertainty in part 2 is that the air cell was not exactly perpendicular to the mirror M_2, so there was additional refraction as light entered the air tube, resulting in a longer path length. This could be remedied through measured placement of the air cell. In addition, after the air cell had been fully evacuated, the vacuum pump would sporadically emit "popping" sounds, indicating that there was air being removed from the cell, so it may not have been exactly a vacuum. This could be remedied by letting the vacuum pump run for longer. There was also error in counting the fringes, similar to in part 1.

Equation (2) can be derived as follows:

The optical path length L of a beam inside of a cell of length l with refractive index n is: [3]

$$L = nl \tag{3}$$

In a vacuum, $n = 1$ and the optical path length doesn't change, as expected. However, for higher values of n, the optical path length increases. In this experiment, n is increasing from 1 to μ along a cell length l, and so the path difference is:

$$\mu l - l = (\mu - 1)l$$

Equation (1) gives us $2d = n\lambda$, where d is the path difference. Here, the path difference is $(\mu - 1)l$, so equation (1) becomes:

$$n\lambda = 2(\mu - 1)l$$

or Equation (2), as previously defined.

VII. Conclusion

The path length d for a laser with wavelength $632.8nm$ was found to be $d_{obs} = (3.11 \pm 0.10) \times 10^{-5}\,\mathrm{m}$, and $d_{calc} = 3.164 \times 10^{-5}\,\mathrm{m}$, when determined using the Michelson-Morley interferometer and Equation (1), respectively. These two values were found to be consistent. On average, 43 fringes were counted as the air cell went from a vacuum to room pressure. The refractive index of air was found to be $\mu = 1.0003 \pm 0.0010$. This value is consistent with the accepted value of refractive index of air at STP, $\mu_{accept} = 1.00029$.

References

[1] Physics 325 Laboratory Manual
Department of Physics and Astronomy, University of Victoria
2019

[2] 1. "The Michelson Interferometer", p. 428
2. "Table 4.1: Approximate Indices of Refraction of Various Substances", p. 103
Hecht, E., 2017, *Optics, 5th Edition*, Pearson Education, Essex, England, 730 p.

[3] "Optical path length"
Wikipedia, the free encyclopedia
2021

Appendix

1. Apparatus

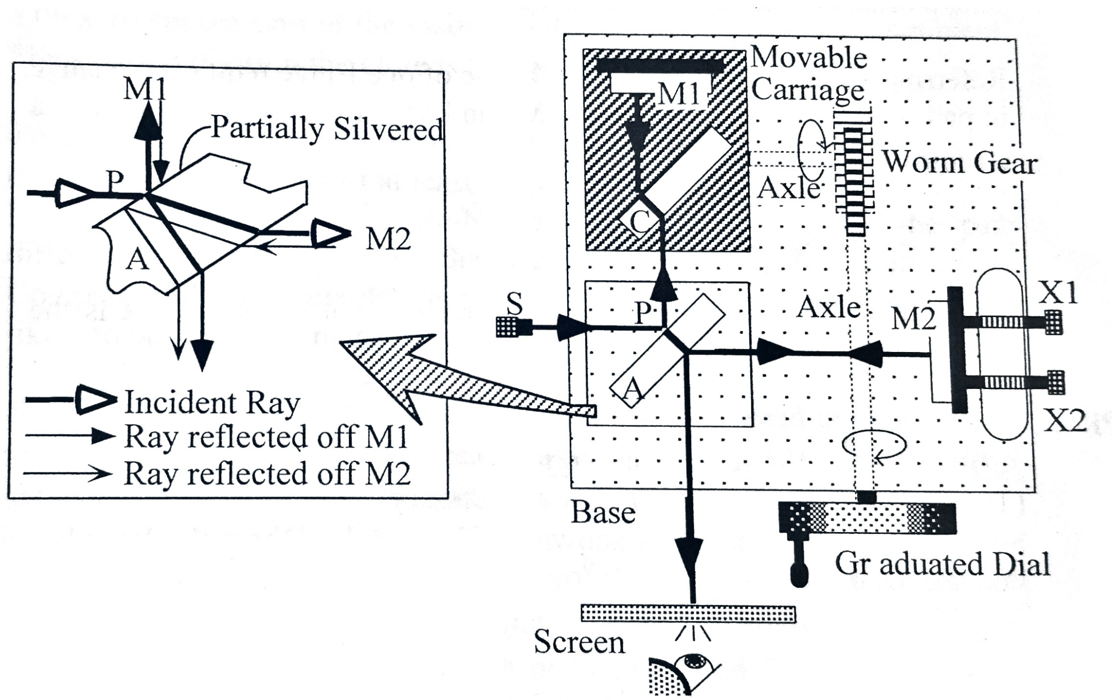

Figure 1. The Michelson-Morley Interferometer [1]

2. Sample Calculations

Mean and Standard Deviation of Number of Dial Increments

```
In [25]:  import numpy as np

In [26]:  start_pos = np.array([0, 0, 0, 5., 0])
          end_pos = np.array([58.8, 58.9, 62.6, 68.4, 61.9])
          distance = -1*(start_pos - end_pos)
          inc = np.mean(distance)
          inc_sig = np.std(distance)/diff
          print("Average number of dial increments = ", inc, " +- ", inc_sig*100, "%")

          Average number of dial increments =  61.11999999999999  +-  3.130776780069374 %
```

Figure 2. Python code used to calculate mean and standard deviation.

Smallest Graduated Dial Division (including uncertainties

$$div = \frac{\pi(pin - drive - diameter)}{10000} = \frac{\pi(1.62 \pm 0.01) \times 10^{-3}\,\mathrm{m}}{1000} = 5.09 \times 10^{-7}\,\mathrm{m} \pm .86\%$$

Path Difference From Gear Ratio

$$d_{obs} = (inc) \times (div) = (61.1 \pm 3.1\%)(5.09 \times 10^{-7}\,\mathrm{m} \pm .86\%) = 3.11 \times 10^{-5}\,\mathrm{m} \pm 9.9 \times 10^{-7}\,\mathrm{m} = (3.11 \pm 0.10) \times 10^{-5}\,\mathrm{m}$$

Path Difference From Equation (1)

$$2d_{calc} = n\lambda$$
$$d_{calc} = \frac{n}{2\lambda}\frac{100(632.8 \times 10^{-9}\,\mathrm{m})}{2} = 3.164 \times 10^{-5}\,\mathrm{m}$$

Refractive Index of Air

$$n\lambda = 2(\mu - 1)l$$
$$\mu = \frac{n\lambda}{2l} + 1 = \frac{43(632.8 \times 10^{-9}\,\mathrm{m})}{2((48.260 \pm 0.005) \times 10^{-3}\,\mathrm{m})} + 1 = 1.0002819 \pm 1.0363467 \times 10^{-4} = 1.0003 \pm 0.0010$$

Consistency Check for Refractive Index of Air

$$|(\mu - 1) - (\mu_{accept} - 1)| \leq \Delta\mu + \Delta\mu_{accept}$$
$$|0.0002819 - 0.00029| \leq 1.036 \times 10^{-4}$$
$$8.085 \times 10^{-6} \leq 1.036 \times 10^{-4}$$
$$\therefore \mu \text{ and } \mu_{accept} \text{ are consistent.}$$