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S.1 COUPLING IN A FIBRE OPTIC CABLE

a) What is the internal critical angle, where $\theta_1 = \theta_c$ in terms of n_1 and n_2?

Internal critical angle: reflection angle is 90°

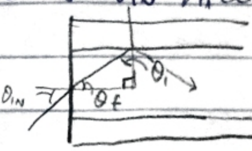
SNELL'S LAW:

$n_1 \sin\theta_1 = n_2 \sin\theta_2$ $\Rightarrow$ @ Internal critical angle: $\theta_2 = 90°$, $\theta_1 = \theta_c$

$n_1 \sin\theta_c = n_2 \sin(90)$ $\Rightarrow$ $\boxed{\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)}$

b) Using Snell's Law, show that this corresponds to $\sin\theta_{IN} = n_1 \cos\theta_c$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$ $\begin{cases} n_{IN} \sin\theta_{IN} = n_1 \sin\theta_f \\ n_1 \sin\theta_1 = n_2 \sin\theta_2 \end{cases}$



$n_{IN} \sin\theta_{IN} = n_1 \sin\theta_f$, $n_{IN} = 1$, $\sin\theta_f = \cos\theta_1$

$\theta_f = 180 - 90 - \theta_1 = 90 - \theta_1$

$\sin\theta_f = \sin(90 - \theta_1) = \cos\theta_1$

$\sin\theta_{IN} = n_1 \cos\theta_1$

@ critical angle, $\theta_1 = \theta_c$:

$\boxed{\therefore \sin\theta_{IN} = n_1 \cos\theta_c}$

c) Using (a) and (b), show that $NA = \sqrt{n_1^2 - n_2^2}$

NA = Numerical Apeture (property of fibre), $NA \equiv \sin\theta_{IN}$

$NA = \sin\theta_{IN}$, from (b) $\sin\theta_{IN} = n_1 \cos\theta_c$ $\cos^2\phi = 1 - \sin^2\phi$

$NA = n_1 \cos\theta_c$

$NA^2 = n_1^2 \cos^2\theta_c$ $\cos^2\phi + \sin^2\phi = 1 \Rightarrow \cos^2\theta_c = 1 - \sin^2\theta_c$

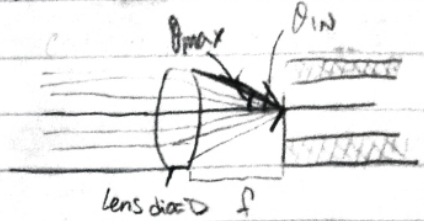
$NA^2 = n_1^2(1 - \sin^2\theta_c) = n_1^2 - \sin^2\theta_c$, from (a) $\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) \Rightarrow \sin\theta_c = \frac{n_2}{n_1} \Rightarrow \sin^2\theta_c = \frac{n_2^2}{n_1^2}$

$NA^2 = n_1^2 - n_1^2 \frac{n_2^2}{n_1^2}$

$NA^2 = n_1^2 - n_2^2 \Leftrightarrow \boxed{NA = \sqrt{n_1^2 - n_2^2}}$

S1 cont'd:

d) The numerical Aperature of a lens is the ratio of its radius $r = D/2$ to its focal length f: $NA_L = D/2f$. Use this fact and fig. 1b to show that in order to couple light into a fiber, the NA of the lens should not exceed that of the fiber.



$NA_L = \frac{D}{2f}$

$NA_F = \sin\theta_{IN} = \sqrt{n_1^2 - n_2^2}$

Consider the maximum angle of light entering fiber, θ_{max}

For total internal reflection, $\theta_{max} \leq \theta_{IN} \Rightarrow \sin\theta_{max} \leq \sin\theta_{IN}$

where θ_{IN} is the input angle for the critical angle (as in parts (a)+(c))

$NA_L = \frac{D}{2f} = \tan\theta_{max} \approx \sin\theta_{max}$ (small angle approx)

$NA_F = \sin\theta_{IN}$

Since $\sin\theta_{max} \leq \sin\theta_{IN}$, $\boxed{NA_L \leq NA_F}$

→ If $NA_L > NA_F$, not all light has total internal reflection ($\theta_{max} > \theta_{IN}$) and there is coupling loss.

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5.2

Laser diode: $L \times W \times H \approx 100\mu m \times 5\mu m \times 15\mu m$

~~The~~ Material provides gain over a freq. of range 5 THz, centered @ $\lambda = 650nm$

Front + rear faces have reflective coating $R = 90\%$ (forms resonator)

Gain material: $n \approx 3$

a) Given length of material, how many individual modes can laser emit?

→ material = Fabry-Perot cavity. "Mode" = resonant freq. of cavity.

$$FSR = \frac{c}{2nL} = \frac{3\times10^{8} m/s}{2(3)(100\times10^{-6} m)} = 5\times10^{11}\ Hz \Rightarrow \text{spacing b/w modes}$$

Gain from material: $G = 5\times10^{12}\ Hz$

$$\#modes = \frac{G}{FSR} = \frac{5\times10^{12}}{5\times10^{11}} = 10$$

THERE ARE 10 MODES

b) In reality, each mode has spread in freq. What is freq width of each ~~m~~ of these modes?

For Fabry-Perot: $\Delta f = \frac{FSR}{\mathcal{F}}$, $\mathcal{F} = \frac{\pi\sqrt{R}}{1-R}$

$$\Delta f = \frac{5\times10^{11}(1-.9)}{\pi\sqrt{.9}} = \boxed{1.678\times10^{10}\ Hz}$$

c) Given operating in precisely one mode, what is coherence length of laser?

$$l_c = c\,T_c,\quad T_c = \frac{1}{\Delta f},\quad l_c = \frac{c}{\Delta f} = \frac{3\times10^{8} m/s}{1.678\times10^{10} Hz} = \boxed{.01788m = l_c}$$

d) Given dimentions (width x dir $= 15\mu m$, height y dir $= 5\mu m$), what is divergence angle of beam in each direction?
→ Apeature rectangle

FROM WIKIPEDIA (Beam Divergence):

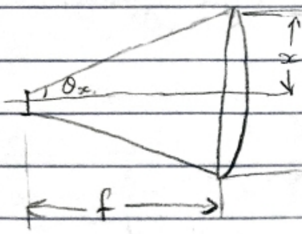
Gaussian Beam divergence is from optimized laser cavities:

$\theta = \frac{\lambda}{\pi W}$, W = beam width @ smallest point (here, it's when leaving aperature)

$$\Rightarrow \theta_x = \frac{650 \times 10^{-9}}{\pi (15 \times 10^{-6})} = \boxed{.01379° = \theta_x}$$

$$\Rightarrow \theta_y = \frac{650 \times 10^{-9}}{\pi (5 \times 10^{-6})} = \boxed{.04138° = \theta_y}$$

e) Dim of beam if f = 10mm collimating lens placed 10mm away?



$$\tan \theta_x = \frac{x}{f} \Rightarrow x = f \tan \theta_x$$
$$= (10 \times 10^{-3})(\tan(.03179°))$$
$$= 2.407 \times 10^{-6} m$$

→ width in x dir $= 2x = 4.814 \times 10^{-6} m$

$$\tan \theta_y = \frac{y}{f} \Rightarrow y = f \tan \theta_y$$
$$= (10 \times 10^{-3}) \tan(.04138°)$$
$$= 7.222 \times 10^{-6} m$$

→ height in y dir $= 2y = 1.444 \times 10^{-5} m$

DIMENSION of BEAM: $\boxed{4.814 \times 10^{-6} m \times 1.444 \times 10^{-5} m \approx W \times H}$