Interference and Diffraction

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February 22, 2021

Abstract

The purpose of this experiment was to study examples of Fraunhofer interference and diffraction and show that they are consistent with the wave theory of light. Single and double slit arrangements were shown to meet the Fraunhofer criterion. Their diffraction and interference patterns were fit by their corresponding theoretical intensity curves, and the distance between fringes were shown to be within 5.2% of the theoretical value. The value of $b = 7.5 \times 10^{-5}$ m in the single slit theoretical curve was found to differ from the measured value of $b = 8 \times 10^{-5}$ m by 6.25%. The value of $b = 4 \times 10^{-5}$ m in the double slit theoretical curve did not differ from the measured value, however the theoretical value of $d = 2.3 \times 10^{-4}$ m differed from the measured value of $d = 2.5 \times 10^{-4}$ m by 8%.

I. Introduction

Theory

Huygens' principle states that each point along a wave front can itself be considered a new source of waves. When passing through an aperture of finite size, such as a slit, the constructive and destructive interference of these new waves produce a characteristic diffraction pattern. When the projection plane is sufficiently far enough from the aperture such that the Fresnel number, $F = b^2/L\lambda$, is much smaller than 1, Fraunhofer diffraction occurs and the diffraction pattern is easily visible. If the projection plane is too close, Fresnel diffraction is observable due to the significant curvature of the wave front. [1] The diffraction pattern of single and double slits are well known as evidence for the wave-nature of light, as the interference patterns produced resemble those of water flowing through obstacles.

The double slit experiment was first done by Thomas Young in 1801 to demonstrate the wave behaviour of light, contributing to classical physics and the wave theory of light. It also shows that diffraction and interference are different parts of the same phenomenon. [2] [1] Later, the particle-nature of light was demonstrated through the quantization of light and the photoelectric effect by physicists such as Max Planck and Albert Einstein, among others. These seemingly conflicting characteristics of light caused the debate of whether light was a particle or a wave, leading to the wave-particle duality. [3]

When a light source of wavelength λ is sent through a single slit of width b, the diffraction pattern produced has an intensity that is distributed angularly by

$$I = I_0 \frac{sin^2\beta}{\beta^2} \tag{1}$$

Here, $\beta = \frac{\pi b}{\lambda} sin\theta = \frac{\pi b}{\lambda} \frac{x}{\sqrt{L^2+x^2}}$, x is the distance from the central maximum, I_0 is the constant intensity at the central maximum, and θ is the angle of any diffracted ray with respect to the optical axis. The angular separation between successive minima is

$$sin\theta_{minimum} = m\frac{\lambda}{b},\ m = \pm1, \pm2, ... \tag{2}$$

and the x-separation between minima from the central maximum is approximately

$$\Delta x = L\lambda/b \tag{3}$$

where L is the distance from the slit to the projection plane.

The double slit interference pattern can only be seen when the light beam is monochromatic and spatially coherent. The bright zones observed occur when the cylindrical waves are in phase, or have a phase difference of a multiple of 2π. The dark zones are observed when they are out of phase, or have a phase difference of a multiple of $\pi(2n + 1)$. The intensity of the diffraction pattern is

$$I = I_0 \frac{sin^2\beta}{\beta^2} cos^2\alpha \tag{4}$$

where $\alpha = \frac{\pi d}{\lambda} sin\theta = \frac{\pi d}{\lambda} \frac{x}{\sqrt{L^2+x^2}}$, d is the separation between slits, and x, β, and I_0 are the same as before.

The intensity of the double slit is the product of the intensity of double slit interference ($cos^2\alpha$) and single slit diffraction. Since the central maximum has two beams constructively interfering in phase, its intensity I_0 is expected to be 4 times the single slit maximum intensity. The spacing between fringes in the interference pattern, δx, is given by

$$\delta x = \frac{L\lambda}{d} \tag{5}$$

Procedure

The slit wheel was set to the single slit and the mirrors were aligned so that the laser beam was visible at the projection plane. The distance L from slit to projection plane was measured using a laser range finder, so that the exact light path would be measured. 2mm was added to L to compensate for the position of the light sensor. The Fresnel Number formula was used to show the apparatus satisfied the Fraunhofer criterion for both single and double slits. The light probe and positioner were set up such that the voltage read by the LabPro voltage probe was proportional to the position. A +10V/-10V was connected to Channel 2 on the LabPro probe. The negative side of V1 was connected to the positive side of V2. Next, the positive side of V1 was connected to the red positioner socket, the negative side of V2 was connected to the black positioner socket, and the probe's black reference clip was connected to the common connection between V1 and V2. The probe's red clip was connected to the positioner's green wiper socket. Please see Figure 8 in Apparatus for a wiring diagram. Finally, the power supply was turned on.

To set up the LabPro data collection, the light intensity probe was connected to Channel 1 on the LabPro probe and the LabPro was connected to the lab computer. The time range and sampling rate were selected to be 15s and 150/s, respectively. The positioner was calibrated by first zeroing the positioner and ensuring the probe uses 95% of it's +10V/-10V range, then establishing the conversion factor. The positioner was zeroed by sliding it to the close end of its range, at position A (See Figure 7 in the Appendix), and V2 was increased until LoggerPro displayed -9.5V. Then, the positioner was moved to the other end of its range, at position B, and V1 was increased until LoggerPro displayed +9.5V. The conversion factor was established in the *Calibrate* box, by first moving the positioner to position A and measuring the distance from an arbitrary reference point on the chassis to the positioner, entering the distance in the first calibration box. Then the positioner was moved to position B, the distance was measured again, and was entered in the second calibration box.

The same procedure for data collection and analysis was followed for both the single and double slit, and is as follows. The light sensor and slit wheel position were adjusted so that at the brightest fringe the sensor read between 60,000 and 120,000 lux. Data acquisition was initiated, and the carriage was moved across the positioner, creating the single/double diffraction pattern of the respective splits. The peak's position was subtracted from all positions to create symmetry about x=0, and the data was cropped to consider only the part with significant signal-to-noise ratio. I_0 was estimated at the central peak, and was used with λ to make a calculated column for the theoretical intensity curve. A good fit was found by varying b through trial and error. The goodness of fit of the data was compared to the theoretical curve, and the estimated value of b was compared to the measured value. Finally, the distance from minima to the central maximum was compared to Equation 3 and Equation 5, for single and double slits.

II. Apparatus

- 632.8 nm Laser
- Slit Wheel with Single and Double Slits
- Various Optical Stands and Mirrors
- LabPro ADC and Software
- Light Intensity Probe Assembly/Positioner
- Range Finder

See Figure 7 in the Appendix

IV. Data

TABLE I. Slit Dimensions, Projection Distance, and Laser Wavelength

	Single Slit	**Double Slit**
b	.08 mm	.04 mm
d		.25mm
	L	$(1.116 \pm 0.002)\,\mathrm{m}$
	λ	$632.8 \times 10^{-9}\,\mathrm{m}$

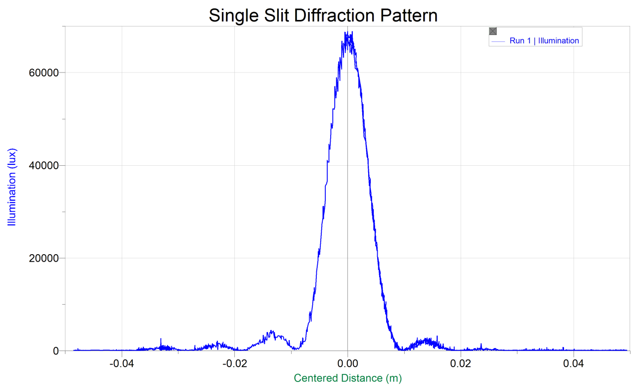

Figure 1. Single Slit Diffraction Pattern.

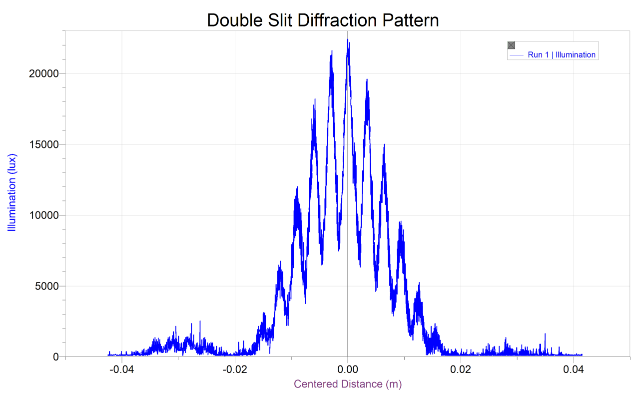

Figure 2. Double Slit Diffraction Pattern.

V. Analysis

See Appendix for sample calculations.

Single Slit

Fresnel Number:

$$F = (9.0625 \pm 0.0002) \times 10^{-3}$$

$F << 1$, thus the single slit arrangement meets the Fraunhofer criterion.

Double Slit

Double Slit Fresnel Number:

$$F = (2.226\,56 \pm 0.000\,04) \times 10^{-3}$$

$F << 1$, thus the double slit arrangement meets the Fraunhofer criterion.

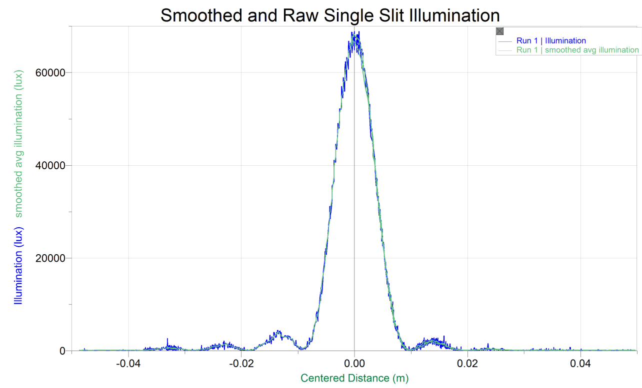

Figure 3. Smoothed illumination curve to estimate I_0 at central peak.
$I_0 = 68237.058\ lux$.

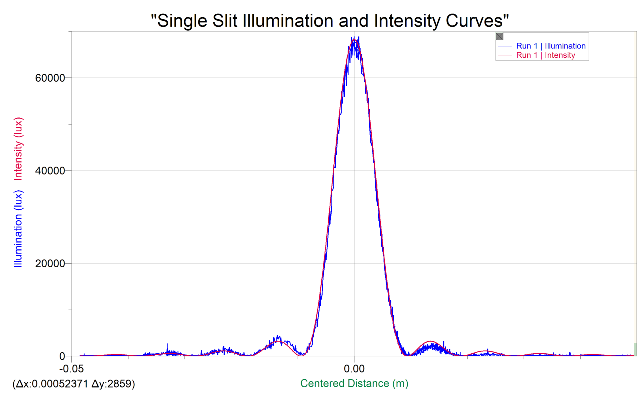

Figure 4. Single Slit Illumination and Theoretical Intensity.
Through trial and error, b was determined to be 7.5×10^{-5} m. When compared to the given value of $b = 8 \times 10^{-5}$ m, the percent difference was 6.25%.

TABLE II. Distance between successive minima from the central maximum, compared to Δx from Equation 3

	Δx from Eq. 3	9.4×10^{-3} m
	Δx measured	**Percent Difference**
center to min$_1$	9.5×10^{-3} m	.89%
min$_1$ *to min*$_2$	9.2×10^{-3} m	2.7%
min$_2$ *to min*$_3$	9.8×10^{-3} m	4.2%
min$_3$ *to min*$_4$	9.5×10^{-3} m	.94%

VI. Discussion

For the single and double slits, the Fresnel number was calculated to be $F = (2.226\,56 \pm 0.000\,04) \times 10^{-3}$ and

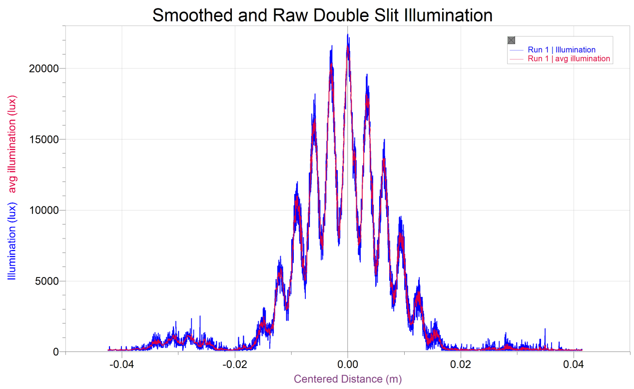

Figure 5. Smoothed illumination curve to estimate I_0 at central peak.
$I_0 = 21602.801\ lux$.

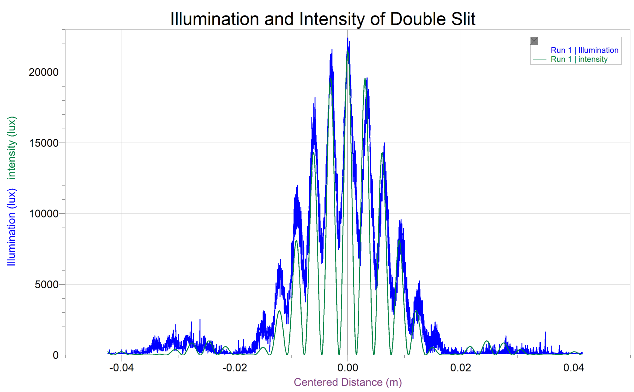

Figure 6. Double Slit Illumination and Theoretical Intensity.
Through trial and error, b and d were determined to be 4×10^{-5} m and 2.3×10^{-4} m, respectively. When compared to the given values of $b = 4 \times 10^{-5}$ m and $d = 2.5 \times 10^{-4}$ m, the percent differences were 0% and 8%, respectively.

$F = (2.226\,56 \pm 0.000\,04) \times 10^{-3}$, respectively. Since these were much less than 1, both arrangements met the Fraunhofer criterion. In order to estimate the intensity at the central peak, I_0, the illumination curves were smoothed using the "smoothAve" function on LoggerPro, which is a moving average function. The peak intensity for the single slit diffraction pattern was estimated to be $I_0 = 68237.058\ lux$, and $I_0 = 21602.801\ lux$ for the double slit. The peak intensity for the double slit was expected to be approximately 4 times the single slit,

TABLE III. Distance between successive fringes, compared to Δx from Equation 5

	Δx from Eq. 5	3.1×10^{-3} m
	Δx measured	**Percent Difference**
$fringe_{center}$	3.2×10^{-3} m	5.1%
$fringe_1$	2.9×10^{-3} m	4.1%
$fringe_2$	3.2×10^{-3} m	2.7%
$fringe_3$	3.1×10^{-3} m	1.8%
$fringe_4$	3.1×10^{-3} m	.39%

however it was much lower. This may have been due to incorrect alignment of the double slit, so that part of the light beam did not pass through the slits and thus did not reach the sensor.

The theoretical intensity curve was calculated using Equations 2 and 4 for both arrangements, and values of b and d were determined through trial and error. For the single slit, b was determined to be 7.5×10^{-5} m. When compared to the given value of $b = 8 \times 10^{-5}$ m, the percent difference was 6.25%. For the double slit, b and d were determined to be 4×10^{-5} m and 2.3×10^{-4} m, respectively. When compared to the given values of $b = 4 \times 10^{-5}$ m and $d = 2.5 \times 10^{-4}$ m, the percent differences were 0% and 8%, respectively. Both theoretical intensity curves fit their respective data well at the center peak, however the fit worsened towards the ends of the diffraction patterns. The single slit fit better than the double slit, which may also be due to the previously mentioned errors from data collection. From equation 3 and equation 5, the respective distances between fringes were determined to be 9.4×10^{-3} m, and 3.1×10^{-3} m. The measured distances confirmed this, with percent differences below 5.1%.

The double slit experimental minima did not have zero intensity like the theoretical minima due to the finite nature of the light probe sensor, meaning that light from either side of the sensor is additionally collected. When measuring the narrow minima of the diffraction pattern, light from the much brighter maxima was also detected by the sensor, increasing the intensity. In other words, the minima is too narrow for the collection width of the sensor. The noise in the raw data was partly from the background light, as well as the unstable movement of the carriage across the positioner, as it was moved by hand. This could be remedied by a completely dark room, a motorized positioner or graduated wheel to smooth carriage movement, and a thinner sensor to lower the intensity of local minima.

VII. Conclusion

Single and double slit diffraction patterns were observed, and were fit with their respective theoretical intensity curves. For both arrangements, the Fresnel numbers were determined to be $F = (2.226\,56 \pm 0.000\,04) \times 10^{-3}$ and $F = (2.226\,56 \pm 0.000\,04) \times 10^{-3}$, showing that the arrangements met the Fraunhofer criterion (F ¡¡ 1). The estimated peak intensity for the single and double slits was determined to be $I_0 = 68237.058\ lux$, and $I_0 = 21602.801\ lux$, respectively. The peak intensity for the double slit was expected to be approximately 4 times higher than for the single slit, however this was not the case due to misalignment of the slit and laser. The theoretical curves fit both datasets well, with the single slit theoretical and given values of b differing by 6.25% and the double slit theoretical and given values of d differing by 8%. The distances between fringes for both arrangements were within 5.4% of the theoretical values. These examples of Fraunhofer interference and diffraction were consistent with the wave theory of light.

References

[1] Physics 325 Laboratory Manual
Department of Physics and Astronomy, University of Victoria
2019

[2] "Double Slit Experiment"
Wikipedia, the free encyclopedia
2021

[3] "Wave-Particle Duality"
Wikipedia, the free encyclopedia
2021

Appendix

1. Apparatus

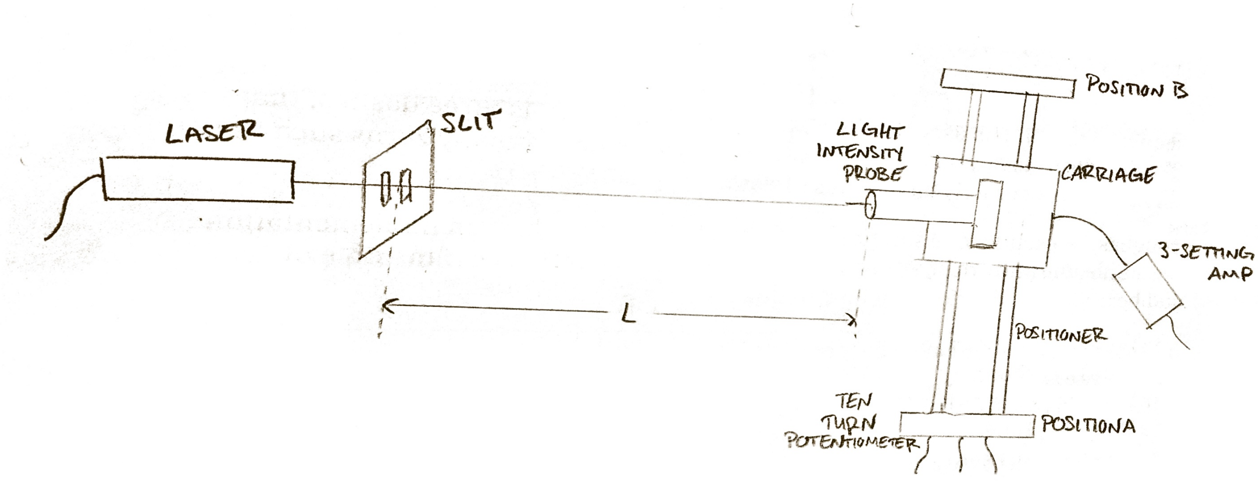

Figure 7. Light Intensity Probe Assembly

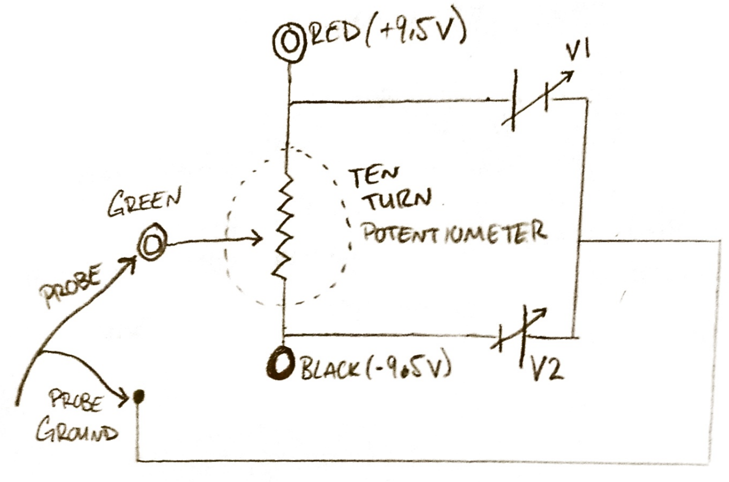

Figure 8. Positioner Wiring

2. Sample Calculations

Verify Single Slit Meets Fraunhofer Criterion using Fresnel Number

$$F = \frac{b^2}{L\lambda} = \frac{(0.08 \times 10^{-3}\,\mathrm{m})^2}{((1.116 \pm 0.005)\,\mathrm{m})(632.8 \times 10^{-9}\,\mathrm{m})} = (9.062\,526 \pm 0.000\,002) \times 10^{-3} < 0.5$$

Since $F < 0.5$, the single slit arrangement meets the Fraunhofer criterion.

Δx from Equation 3, and Percent Difference Between Δx and Measured Distance

```
In [8]:
dx1 = 1.116*632.8e-9/.000075
li = np.array([.0095, .0095044, .0098140, .0091639])
diff = abs((li - dx1)/dx1)*100
print("Delta x = {:.6f}".format(dx1))
for j,k in zip(li, diff):
    print("Measured delta x = {:.6f} || % Difference = {:.6f}%".format(j,k))
```

```
Delta x = 0.009416
Measured delta x = 0.009500 || % Difference = 0.891413%
Measured delta x = 0.009504 || % Difference = 0.938141%
Measured delta x = 0.009814 || % Difference = 4.226139%
Measured delta x = 0.009164 || % Difference = 2.678019%
```

Figure 9. Python code used to calculate Δx and percent differences