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Sarah Clapoff P325 A#3 03,03,2021

3.1 PARTIAL VISIBILITY

In lectures, showed that for perfectly coherent light, combining two fields $E_1 e^{i(kz-\omega t)} + E_2 e^{i(kz-\omega t+\phi)}$ gave an expression of the form: $2I(1+\cos\phi)$, when $E_1 = E_2$, and so the visibility was between 0 and 1.

a) Now assume that $I_1 \neq I_2$. Derive and expression for the intensity of $E_1 + E_2$ in this case. Verify that when $I_1 = I_2$ the expression reduces to the original form.

Not yet combining

$$E_1 = E_1 e^{i(kz-\omega t)} \qquad I = \tfrac{1}{2}\varepsilon_0 nc(|E_1^2 + E_2^2|)$$
$$E_2 = E_2 e^{i(kz-\omega t+\phi)}$$

$$E_1 + E_2 = E_1 e^{i(kz-\omega t)} + E_2 e^{i(kz-\omega t)} e^{-i\phi}$$

$$= e^{i(kz-\omega t)}(E_1 + E_2 e^{i\phi}) \quad \rightarrow \text{Take time avg. mag squared.}$$

$$|E_1+E_2|^2 = |e^{i(kz-\omega t)}|^2 |E_1 + E_2 e^{-i\phi}|^2 = (E_1 + E_2 e^{-i\phi})(E_1 + E_2 e^{i\phi})$$

$$= |E_1|^2 + E_1 E_2 e^{i\phi} + E_1 E_2 e^{-i\phi} + E_2^2 e^{-i\phi+i\phi}$$
$$= |E_1|^2 + |E_2|^2 + E_1 E_2 (e^{i\phi} + e^{-i\phi}) \qquad e^{i\phi} + e^{-i\phi} = 2\cos(\phi)$$
$$= |E_1|^2 + |E_2|^2 + 2|E_1||E_2|\cos(\phi)$$
$$= |E_1|^2 + |E_2|^2 + 2\sqrt{E_1 E_2}\cos\phi \quad , \quad I_{tot} = \langle \tfrac{1}{2}\varepsilon_0 nc |E_{tot}|^2 \rangle,$$

$\Rightarrow$ However,

$$I_{tot} = \tfrac{1}{2}\varepsilon_0 nc|E_1+E_2|^2 = \tfrac{1}{2}\varepsilon_0 nc(|E_1|^2 + |E_2|^2 + 2\sqrt{E_1 E_2}\cos\phi)$$

, Since considering ϕ and not $\phi_R(t)$, don't need to take time average.

Intensity of E_1+E_2 = $\boxed{I_{tot} = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi}$

- If $I_1 = I_2 = I$,

$$I_{tot} = I + I + 2\sqrt{I^2}\cos\phi$$
$$= 2I + 2I\cos\phi$$
$$I_{tot} = 2I(1+\cos\phi) \text{ , as expected.}$$

b) Determine an expression for visibility in terms of I_1 and I_2.

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi \quad \begin{cases} \text{This is at max when } \cos\phi = 1 \\ \text{This is at min when } \cos\phi = -1 \end{cases}$$

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$
$$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$

$$V = \frac{I_1 + I_2 + 2\sqrt{I_1 I_2} - I_1 - I_2 + 2\sqrt{I_1 I_2}}{I_1 + I_2 + 2\sqrt{I_1 I_2} + I_1 + I_2 - 2\sqrt{I_1 I_2}} = \frac{4\sqrt{I_1 I_2}}{2(I_1 + I_2)}$$

$$\boxed{V = \frac{\sqrt{I_1 I_2}}{I_1 + I_2}}$$

c) If fields are placed in Michelson Interferometer and interference fringes disappear after moving the mirror over a distance of $\pm 1mm$, estimate the bandwidth $\Delta\omega$ of the light that produced it.

Fringe disappears after moving mirror $\pm 1mm$

$\Rightarrow \lambda/2 = \pm 1mm$

$\Rightarrow \lambda = 2mm$

$$\Delta\omega = \frac{2\pi c}{\lambda} = \frac{\cancel{2}\pi(3\times10^8 m/s)}{\cancel{2}\times10^{-3} m} = 3\pi\times10^{11} s^{-1} \approx \boxed{942 GHz \approx \Delta\omega}$$

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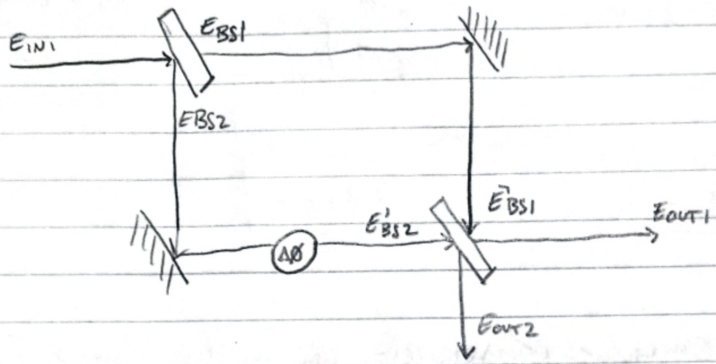
3.2 MACH ZEHNDER INTERFEROMETER
- Measure very small phase shifts
- Assume symmetric beam splitter



a) Ch 1 input light is $E_0 e^{ikz}$, find state of $\underbrace{\text{Ch } 1, 2}_{=E_{BS1},\, E_{BS2}}$ directly after BS #1.

$$E_{IN1} = E_0 e^{ikz}$$

$$\begin{pmatrix} E_{BS1} \\ E_{BS2} \end{pmatrix} = \begin{pmatrix} t & -r \\ r & t \end{pmatrix} \begin{pmatrix} E_{IN1} \\ 0 \end{pmatrix}$$

Since Beam splitter is symmetrical, $\begin{pmatrix} t & -r \\ r & t \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$

$$\begin{pmatrix} E_{BS1} \\ E_{BS2} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} E_0 e^{ikz} \\ 0 \end{pmatrix} = \boxed{\frac{1}{\sqrt{2}} \begin{pmatrix} E_0 e^{ikz} \\ E_0 e^{ikz} \end{pmatrix} = \begin{pmatrix} E_{BS1} \\ E_{BS2} \end{pmatrix}}$$

b) Top + Bottom paths identical, but bottom has aditional phase shift ϕ, show ~~a~~ field directly before BS #2 can be written as $E_2 = \pm E_1 e^{i\phi}$

Take L to be the length of the top path (and thus the bottom path w/out ϕ).

Along Top path: $E'_{BS1} = \frac{1}{\sqrt{2}} E_0 e^{ik(z+L)}$

Along Bottom path: $E'_{BS2} = \frac{1}{\sqrt{2}} E_0 e^{ik(z+L+\phi)} = \frac{1}{\sqrt{2}} E_0 e^{ik(z+L)} e^{i\phi}$

$$\vec{E} \text{ before BS\#2} = \begin{pmatrix} E'_{BS1} \\ E'_{BS2} \end{pmatrix} = \frac{1}{\sqrt{2}} E_0 e^{ik(z+L)} \begin{pmatrix} 1 \\ e^{i\phi} \end{pmatrix}$$

$\Rightarrow E'_{BS2} = E'_{BS1} e^{i\phi}$

Since $\frac{1}{\sqrt{2}} E_0 e^{ik(z+L)}$ is a global phase on $\vec{E}$, it's sign can be disregarded, thus $\boxed{E'_{BS2} = \pm E'_{BS1} e^{i\phi}}$

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c) Fields at output channels as func of ϕ.

$$\begin{pmatrix} E_{OUT1} \\ E_{OUT2} \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \vec{E}_{BS1} \\ \vec{E}_{BS2} \end{pmatrix} = \frac{E_0 e^{ik(z+L)}}{\sqrt{2}\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ e^{i\phi} \end{pmatrix}$$

$$= \frac{1}{2} E_0 e^{ik(z+L)} \begin{pmatrix} 1 - e^{i\phi} \\ 1 + e^{i\phi} \end{pmatrix}$$, $\frac{1}{2} E_0 e^{ik(z+L)}$ is constant, hence:

$$\boxed{\vec{E}_{OUT} = \begin{pmatrix} E_{OUT1} \\ E_{OUT2} \end{pmatrix} = \frac{1}{2} E_0 e^{ik(z+L)} \begin{pmatrix} 1 - e^{i\phi} \\ 1 + e^{i\phi} \end{pmatrix}}$$

d) Show Energy is conserved - specifically show $I_{OUT1} + I_{OUT2} = I_{IN}$,

$$I = \frac{1}{2} \varepsilon_0 n c \langle |E_1|^2 \rangle$$

$$I_{IN} = \frac{1}{2} \varepsilon_0 n c \langle |E_0 e^{ikz}|^2 \rangle = \frac{1}{2} \varepsilon_0 n c \langle |E_0|^2 \rangle$$

$$I_{OUT1} = \frac{1}{2} \varepsilon_0 n c \langle |E_{OUT1}|^2 \rangle = \frac{1}{2} \varepsilon_0 n c \langle | \tfrac{1}{2} E_0 e^{ik(z+L)} (1 - e^{i\phi}) |^2 \rangle$$

$$= \frac{1}{2} \varepsilon_0 n c \langle \tfrac{1}{4} (1 - e^{i\phi})^2 (E_0 e^{-ikz} e^{-ikL})(E_0 e^{ikz} e^{ikL}) \rangle = \frac{1}{2} \varepsilon_0 n c \langle |E_0|^2 \rangle \langle \tfrac{1}{4} (1 - e^{i\phi})^2 \rangle$$

$$I_{OUT1} = \frac{1}{4} I_{IN} \langle (1 - e^{i\phi})^2 \rangle$$

$$I_{OUT2} = \frac{1}{2} \varepsilon_0 n c \langle |E_{OUT2}|^2 \rangle = \frac{1}{2} \varepsilon_0 n c \langle | \tfrac{1}{2} E_0 e^{i(kz + kL)} (1 + e^{i\phi}) |^2 \rangle$$

$$= \frac{1}{2} \varepsilon_0 n c \langle |E_0|^2 \rangle \langle \tfrac{1}{4} (1 + e^{i\phi})^2 \rangle = \frac{1}{4} I_{IN} \langle (1 + e^{i\phi})^2 \rangle$$

$(1 - e^{i\phi})(1 - e^{-i\phi})$

$1 - 2e^{i\phi} + e^{i\phi - i\phi}$

$$I_{OUT1} + I_{OUT2} = \frac{1}{4} I_{IN} \left(\langle (1 - e^{i\phi})^2 \rangle + \langle (1 + e^{i\phi})^2 \rangle \right)$$

$$= \frac{1}{4} I_{IN} \left(1 - \cancel{2e^{i\phi}} + 1 + 1 + \cancel{2e^{i\phi}} + 1 \right) = \frac{1}{4} I_{IN} (4)$$

$\therefore$ $\boxed{I_{OUT1} + I_{OUT2} = I_{IN}}$

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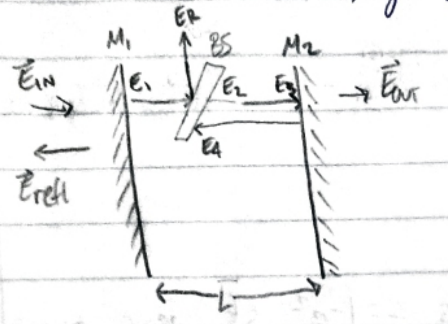
3.3: LOSY FABRY PEROT CAVITY:

a) We can model a Fabry-Perot Interferometer w/ imperfect (lossy) mirrors by placing BS w/ transmitivity η b/w mirrors. Determine an expression for phase + loss per round trip. (In lectures, this term was $r^2 e^{2ikL}$, yours will be similar but will involve η)



Consider parallel mirrors, reflectivity $R = |r|^2$, w/ transmitivity η b/w mirrors.
→ Seperated by length L,
→ With input field $\vec{E}_{IN} = E_0 e^{i(ky - \omega t)}$

- Once passing through M_1, $E \to tE_{IN}$ $\Rightarrow \vec{E}_1 = t\vec{E}_{IN}$
- At Beam spliter, the reflected beam leaves the interferometer.

$$\begin{pmatrix} E_t \\ E_R \end{pmatrix} = \begin{pmatrix} \eta & -r \\ r & \eta \end{pmatrix} \begin{pmatrix} tE_{IN} \\ 0 \end{pmatrix} = \begin{pmatrix} \eta t E_{IN} \\ r t E_{IN} \end{pmatrix} \Rightarrow \vec{E}_2 = E_t = \eta t \vec{E}_{IN}$$

In general, $\begin{pmatrix} E_t \\ E_R \end{pmatrix} = \begin{pmatrix} \eta & -r \\ r & \eta \end{pmatrix} \begin{pmatrix} \vec{E} \\ 0 \end{pmatrix} \Rightarrow \eta \vec{E}$

- At surface of M_2, wave will have propegated L, gaining phase shift of e^{ikL}

$$\Rightarrow \vec{E}_3 = \eta t e^{ikL} \vec{E}_{IN}$$

- Light transmitted out gains another factor of t: $\Rightarrow \vec{E}_{OUT} = \eta t^2 e^{ikL} \vec{E}_{IN}$

- On second round, $\vec{E}_{OUT} = \eta t^2 e^{ikL} \vec{E}_{IN} + (\eta t^2 e^{ikL} \vec{E}_{IN})(r^2 \eta^2 e^{2ikL})$

r^2 → reflect of M_1, M_2
η^2 → pass through beam splitter twice,
e^{2ikL} → Travel twice the length of cavity.

∴ At each round trip, the phase and loss expression is $r^2 \eta^2 e^{2ikL}$

b) Using this expression, show that we can write transmission profile as

$$T(\omega) = \frac{1}{\sigma^2 + \frac{4R}{(1-R)^2}\sin^2(L\omega/c)}$$

where $\sigma = \frac{(1-\eta^2R)}{\eta(1-R)}$. Ensure the expression reduces to lossless mirrors expression as $\eta \to 1$.

$\sigma = \frac{(1-\eta^2R)}{\eta(1-R)}$, as $\eta \to 1$, $\sigma \to \frac{(1-R)}{(1-R)} = 1$, $T(\omega)$ reduces to lossless mirrors.

c) If $R=0.999$ what is max loss $L=1-\eta^2$ st peak transmission is at least 10%?

$$T(\omega) = 10\% = \frac{1}{10} = \frac{1}{\sigma^2 + \frac{4(.999)}{(.001)^2}\sin^2(L\omega/c)}$$

Peak $T(\omega)$ at least 10% $\Rightarrow$ Max($T(\omega)$) at least 10% $\Rightarrow$ Max($T(\omega)$) $\geq \frac{1}{10}$

Max($T(\omega)$) occurs when $\sin^2(L\omega/c) = 0$, since $\sigma < \frac{4(.999)}{(.001)^2}\sin^2(L\omega/c)$

$$\sin^2(L\omega/c) = 0 \Rightarrow \frac{4(.999)}{(.001)^2}\sin^2(L\omega/c) = 0$$

$$\Rightarrow \frac{1}{10} = \frac{1}{\sigma^2} \Rightarrow 10 = \sigma^2 \Rightarrow \sqrt{10} = \sigma = \frac{1-.999\eta^2}{\eta .001}$$

$$.00316\eta = 1 - .999\eta^2 \Rightarrow 0 = .999\eta^2 + .00316\eta - 1$$

$$\eta = \frac{1}{2(.999)}\left(-.00316 \pm \sqrt{(.00316)^2 + 4(.999)}\right)$$

$$\eta = .998919$$

$$L = 1-\eta^2 = 1-(.998919)^2 = .00216083$$

$$\boxed{L = 2.16\times10^{-3}}$$

into