The Speed of Light

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Abstract

The purpose of this experiment was to measure the speed of light using a Michelson Rotating-Mirror apparatus and a direct time of flight (TOF) measurement apparatus. In the Michelson Rotating-Mirror method, a beam of light reflected from a rotating mirror, through a lens to a stationary mirror, and back to a travelling microscope placed at the focal length of the lens. When the rotating mirror moves through an angle θ, the beam is deviated by an angle 2θ, corresponding to a displacement from the initial position of $2\theta/b$, where b is the distance of travel between the eyepiece and the rotating mirror. The initial frequency (and hence the position) was set to be $\sim 50Hz$, and was increased in steps of $\sim 50Hz$ to $\sim 600Hz$. In the TOF method, the speed of light was determined by measuring the time of flight for a beam travelling over three separate distances, distinguished by the placement of front-surface plane mirrors and a retro-reflective mirrors. The time of flight was determined from the change in time between pulses generated from the outgoing and incoming beams hitting photo-detectors.

With the Michelson Rotating Mirror Method the speed of light was determined to be $v_1 = (3.13 \pm 0.18) \times 10^8$ m/s. A consistency check with the accepted value of the speed of light, $v_{accept} = 299792458$, showed them to be consistent, with a percent error of 4.4%. With the Time of Flight method the speed of light was determined to be $v_2 = (2.991 \pm 0.002) \times 10^8$ m/s. A consistency check between $v_2 and v_{accept}$ showed them to be not consistent. The percent error was .218%.

I. Introduction

Michelson Rotating Mirror Method

In 1862, Léon Foucault measured the speed of light in order to obtain an improved value for an astronomical unit. [2] Foucault used a rotating mirror to reflect incoming light from a slit through a lens placed close to the rotating mirror. The lens created an image of the slit on a stationary spherical mirror, which reflected it back towards the slit. When rotating, the return beam of light is deflected away from the original source at angle 2θ. The speed of light can then be calculated by:

$$c = \frac{2\omega h}{\theta}$$

where ω is the frequency of the rotating mirror and h is the distance between the rotating and stationary mirrors. His value for the speed of light, 298000 km/s, was within 0.6% of the accepted value. Foucault was limited to a maximum distance of about 20m between the rotating mirror and stationary mirror, as the image of the slit became too dim if the distance was increased any more. [2]

In 1877 Albert Michelson improved Foucault's experiment through placing the rotating mirror at the focal point of the lens, so that the image produced on the stationary mirror was always visible, regardless of the distance between the two mirrors. In 1926 Michelson repeated the experiment with further refinements, obtaining a value of 299,796±4 km/s. [2]

In this experiment, Michelson's procedure was modified slightly in order to obtain a more measurable image. The focal length of the lens is approximately 5m, which is less than the distance from the lens to M_2. This means that the rays converge between the lens and M_2, then diverge until they hit the lens again, and finally converge into an image on the focal plane of the microscope. For the time t that it takes for the light to travel from the rotating mirror R to M_2 and back to R, during which R rotates by an angle θ, the returning light will be displaced by 2θ. For the corresponding displacement of the image y measured by the traveling microscope, and the distance b between the eyepiece and R, $2\theta = y/b$. Since R is rotating at a speed of f revolutions/second, $t = \theta/(2\pi f) = y/(4\pi b f)$. If a is the length of the light path RM_1M_2, then [1]

$$v = \frac{2a}{t} = \frac{8\pi abf}{y} \tag{1}$$

Time of Flight Method

In the time of flight (TOF) method, the speed of light is determined through measuring the time taken for a beam of light to travel various distances. The distances used to measure the TOF span the length of Elliott 128, Elliott 127, Elliott 125, and Elliott 123. The three mirrors mounted on the doors of Elliott 127 and the back wall of Elliott 123 provide the fixed distances for the light to travel. The mirrors M_1 and M_2 are standard front-surface plane mirrors, while the third mirror M_3 is a retro-reflecting mirror which reflects light along it's incident path.

A red diode laser that produces pulses of light according to an arbitrary function generator is split into two paths at a beam splitter. Path 1 reflects off the beam splitter through a lens which focuses it onto a photo-detector, PD1, triggering a reference trace (CH1) on the oscilloscope. Path 2 travels through the beam splitter to a stationary mirror M_0 on the top of the apparatus. It is then reflected down to one of the three distant mirrors, and back along it's original path to the beam splitter. Since the beam expands over distance and is dimmer when returning, the intensity is maximized through placing a front surface mirror, BSM, over a section of the beam splitter. This reflects half of the beam through a movable lens, ML, which in turn focuses the beam onto the photo-detector, triggering the second trace (CH2) on the oscilloscope. The change in time between the reference and secondary pulses is the time taken for the beam to travel, or the time of flight, for the given distance.[1] Given the time of flight and the distance travelled, the speed of light can be calculated simply using $v = d/t$.

II. Apparatus

Michelson Rotating Mirror Method

- Laser
- Beam splitter
- Rotating Mirror
- Variac
- Photocell
- Oscilloscope
- Lens
- 2 Mirrors
- Cross-Polarizing Intensity Filter
- Traveling Microscope

See Figure 9. in the Appendix.

Time of Flight Method

- Laser
- Beam splitter
- Beam splitter mirror
- 2 Photo-detectors
- 3 Standard front-surface plane mirrors
- Retro-reflective mirror
- Oscilloscope
- Function generator
- Range finder

See Figure 10. in the Appendix.

III. Procedure

Note: The apparatus for both methods is extremely delicate. Use caution when adjusting the components, and check which components are safe to adjust.

Michelson Rotating Mirror Method

Complete a preliminary alignment of the rotating mirror R by turning on the laser, plugging into the outlet as to not disturb the laser position, and ensuring that the beam hits the rotating mirror through the opening on the front of the housing. With the variac off, insert the adjustment key into the opening at the top of the housing and turn the mirror so that it deflects the beam through the center of the lens and towards the first stationary mirror M_1. If the beam is centered, a faint trace will be visible on the surface of the lens.

Once the beam is centered, ask the lab instructor to verify that it's coaxial with the lens. When the rim of the lens is tapped lightly, the back-reflection of the beam should wiggle. If it doesn't, the beam may be missing the lens, and the lens should be readjusted. Re-align the mirrors so that the beam follows the same path from R to the second stationary mirror M_2 and back again, and the two traces are superimposed in the center of the lens. If only one trace is visible, then the return beam may be bypassing M_1, M_2, or the lens, and can be corrected by following the path of the beam and adjusting the mirrors as needed. If both traces are visible but not superimposed, turn the x- and y- adjustment screws on the back of M_1 until they are superimposed.

Align the microscope by moving the polarizing filter away from the beam's path. Do not look into the microscope at this point! Locate the beam by holding a sheet of paper between the beam splitter and the microscope, and follow it to the microscope. Adjust the horizontal and vertical AP5 and AP6 dials so that the beam enters the microscope. Move the polarizing filter back in the beam's path, and hold the paper between the filter and the microscope. Adjust the filter until the laser spot on the paper appears to fade completely. Now, it's safe to look into the microscope. Re-adjust AP5 and AP6 such that the reticule is centered on the laser image, and bring the reticule into focus by adjusting the black rim of the eyepiece. Adjust the filter further to reduce the intensity (and thus the diameter) of the image. If necessary, shine a desk lamp towards the back wall to illuminate the reticule and further reduce the intensity.

To perform the experiment, turn on the oscilloscope. Ensure that the adjustment key is removed from the rotating mirror and that the variac dial is set to minimum. Turn on the variac and turn the dial until R starts to rotate audibly. Since only a fraction of the beam's original intensity reaches the eyepiece due to the rotation, re-adjust the filter to create a visible image. Adjust the oscilloscope until 5-10 pulses are displayed, and adjust the trigger level to 1/2 the pulse amplitude to display a stable trace, then select *Frequency* from the *Measure* menu. Adjust the variac frequency to start at about 50Hz. Since R is silvered on both sides, two pulses are sent to the oscilloscope per turn, so the frequency of the variac is actually $2f$. Turn on the position readout on the microscope, recenter the reticule on the trace, and zero the readout.

Increase the frequency in steps of $\sim$50Hz, up to 600Hz. At each step, adjust the traveling microscope so that the image is re-centered, and record the frequency and displacement. Plot a linear graph of the displacement as a function of frequency and determine the slope. Repeat the experiment 4 times for accuracy, and determine the mean slope value. Use this mean slope, and distances a and b to determine the speed of light. Compare with the accepted value.

Time of Flight Method

Configure the function generator used to create the pulses of light for the laser by recalling the configuration stored in memory. Turn on the function generator, press *Recall*, and select Setup1 by pressing the button beside *Recall* on the main screen. The function generator should a *Run Mode* of *continuous*, and the *Function* should be *pulse.* The waveform should have the following parameters: *Frequency - 10 Hz, Delay - 0 ns, Amplitude - 4.5 Vpp, Offset - 2.5 V, Duty - 20%.* Once this is displayed, turn *Channel* on. When the laser is on, it should be visibly pulsing.

Check that the system is aligned by holding a paper in front of the beam and ensuring that the Path 1 beam enters the lens over the photo-detector. Turn on the power supply to the photo-detectors and oscilloscope. Once the oscilloscope is set up, press *Auto.* CH1, the signal from the Path 1 photo-detector is seen as a square pulse. If CH2 is not visible, hold a piece of paper in front of the Path 2 beam and adjust the lens/BSM so that the beam enters the photo-detector. If the beam is not visible on the paper, adjust the far mirrors so that the beam returns along it's original path. On the oscilloscope, adjust the CH1 trace so that it has the same baseline as CH2, and adjust the time base to 100 ns per div. Two separate signals should now be visible.

Press the *Run/Stop* button to freeze the image. Press the *Cursor* button, and set the type of measurement to time. Measure the time of arrival (TOA) by placing the cursors for CH1 and CH2 at the points where the signals are detected. Measure the TOA for each distance, and collect an image of the oscilloscope screen.

Measure the distances that the light travels using the laser range finder. Place the range finder on a box so that it's above the eyeline of anyone passing by, and measure the distances from the box to the mirror's wall, as well as the distance from the box to the laser's wall. Note that the range finder measures from the back of the unit. Plot a graph of distance vs. time for the three distances and determine the speed of light from the slope. Compare to the accepted value.

IV. Data

Michelson Rotating Mirror Method

Light Path Distances:
$M_1M_2 = (8.67 \pm 0.01)\,\mathrm{m}$
$RM_1 = (6.79 \pm 0.01)\,\mathrm{m}$
R to Beam Splitter $= (5.75 \pm 0.01)\,\mathrm{m}$
Beam Splitter to Focal Plane$= (0.830 \pm 0.005)\,\mathrm{m}$
a, b from Equation 1:
$a = (15.460 \pm 0.014)\,\mathrm{m}$
$b = (6.580 \pm 0.011)\,\mathrm{m}$

TABLE I. Frequency and Displacement of Travelling Microscope (Trial 1)

Frequency (Hz, $\pm 2\%$)	**Displacement (mm, $\pm 0.025mm$)**
48.0	0
100.7	-0.3
152.2	-0.31
201.0	-0.65
250.9	-0.86
301.2	-1.15
352.1	-1.35
396.8	-1.54
454.5	-1.68
500.0	-1.87
555.6	-2.16
595.8	-2.35

TABLE II. Frequency and Displacement of Travelling Microscope (Trial 2)

Frequency (Hz, $\pm 2\%$)	**Displacement (mm, $\pm 0.025mm$)**
50.1	0
102.5	0.6
151.1	-0.15
201.0	-0.43
255.1	-0.56
304.2	-0.74
357.1	-0.86
403.3	-0.96
448.7	-1.43
500.0	-1.77
555.6	-1.93
609.8	-2.15

TABLE III. Frequency and Displacement of Travelling Microscope (Trial 3)

Frequency (Hz, $\pm 2\%$)	**Displacement (mm, $\pm 0.025mm$)**
47.8	0
101.2	-0.15
150.4	-0.39
204.9	-0.67
255.1	-0.67
301.2	-0.83
352.1	-1.08
403.3	-1.28
454.5	-1.46
500.0	-1.65
555.5	-1.89
609.8	-2.15

TABLE IV. Frequency and Displacement of Travelling Microscope (Trial 4)

Frequency (Hz, $\pm 2\%$)	**Displacement (mm, $\pm 0.025mm$)**
50.0	0
102.5	-0.19
155.3	-0.40
200.0	-0.61
257.7	-0.85
301.2	-1.06
357.1	-1.31
403.3	-1.59
446.4	-1.70
500.0	-1.98
555.6	-2.13
609.7	-2.38

Time of Flight Method

Distance from wall to laser $= (0.145 \pm 0.005)\,\mathrm{m}$
Distance from diode plane to mirror:
$DM_0 = (0.745 \pm 0.024)\,\mathrm{m}$
Average distance to laser's wall: $d_l = (2.852 \pm 0.008)\,\mathrm{m}$
Length of range finder $= (0.190 \pm 0.005)\,\mathrm{m}$

TABLE V. Time of Flight and Distances Travelled for Retro-Reflecting, Far, and Close Mirrors

	Retro Reflecting Mirror	**Far Mirror** M_2	**Close Mirror** M_1
$\Delta t(ns) \pm 2\%$	340.0	164.0	118.0
Distance to laser's wall $(m) \pm 0.002m$	2.856	2.851	2.849
Distance to mirror's wall $(m) \pm 0.002m$	46.292	19.935	13.102
Distance from wall to mirror surface (m)	$0.12 \pm .01$	$.107 \pm .002$	$.125 \pm .002$
Total distance travelled (m)	98.868 ± 0.057	46.180 ± 0.053	32.478 ± 0.053

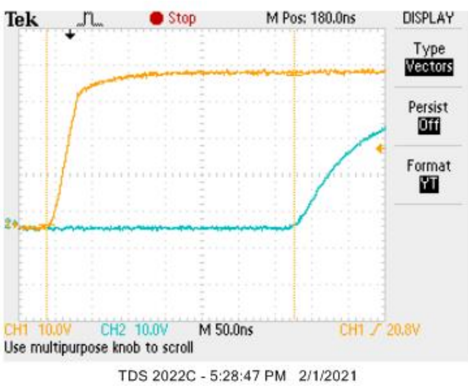

Figure 1. Time of flight for retro-reflecting mirror

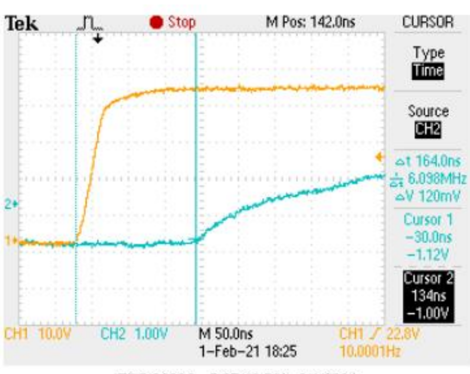

Figure 2. Time of flight for far front-surface mirror

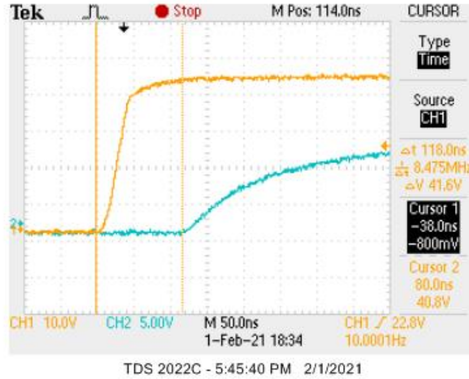

Figure 3. Time of flight for close front-surface mirror

V. Analysis

See the Appendix for sample calculations.

Michelson Rotating Mirror Method

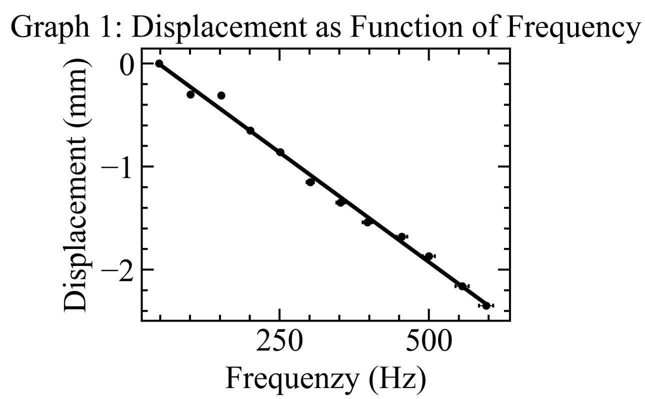

Figure 4. Graph 1: Displacement (mm) as a Function of Frequency (Hz), using data from Table 1. Error bars for the displacement and frequency measurements are included. See Appendix for a larger image.
Slope = $(-4.25 \pm 0.11) \times 10^{-3}$ mm/Hz

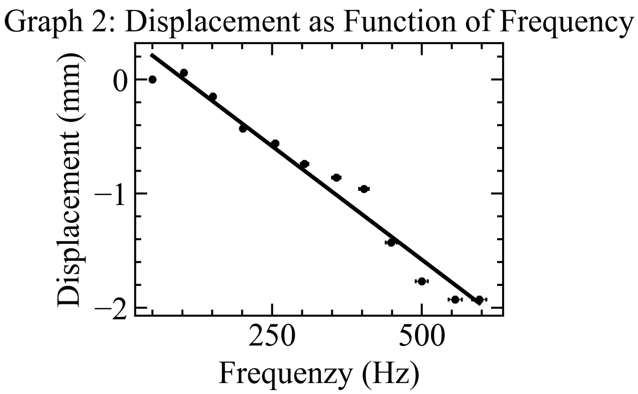

Figure 5. Graph 2: Displacement (mm) as a Function of Frequency (Hz), using data from Table 2. Error bars for the displacement and frequency measurements are included. See Appendix for a larger image.
Slope = $(-3.98 \pm 0.23) \times 10^{-3}$ mm/Hz

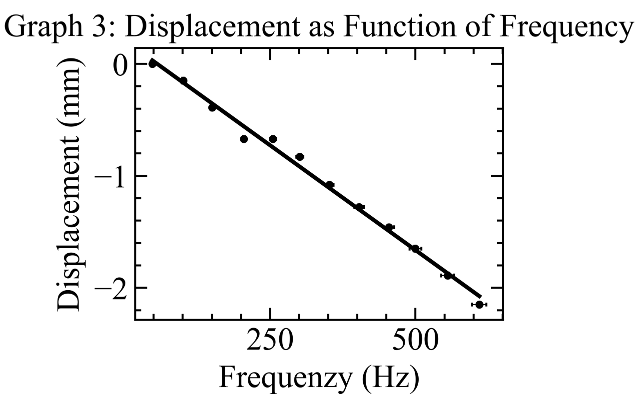

Figure 6. Graph 3: Displacement (mm) as a Function of Frequency (Hz), using data from Table 3. Error bars for the displacement and frequency measurements are included. See Appendix for a larger image.
Slope = $(-3.75 \pm 0.10) \times 10^{-3}$ mm/Hz

Average slope of Graphs 1-4:

$$slope_{avg} = (-4.08 \pm 0.24) \times 10^{-3}\,\text{mm/Hz}$$
$$= (-4.08 \pm 0.24) \times 10^{-6}\,\text{m/Hz}$$

Speed of Light from Equation (1):

$$v_1 = (3.13 \pm 0.18) \times 10^{8}\,\text{m/s}$$

Accepted speed of light:

$$v_{accept} = 299792458$$

A consistency check shows that v_1 and v_{accept} are consistent.

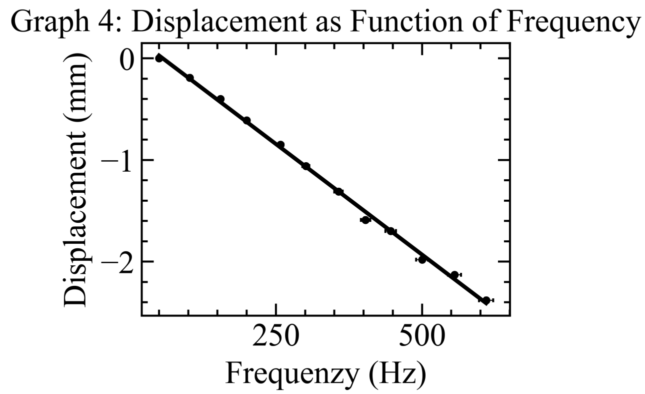

Figure 7. Graph 4: Displacement (mm) as a Function of Frequency (Hz), using data from Table 4. Error bars for the displacement and frequency measurements are included. See Appendix for a larger image.
Slope = $(-4.36 \pm 0.06) \times 10^{-3}$ mm/Hz

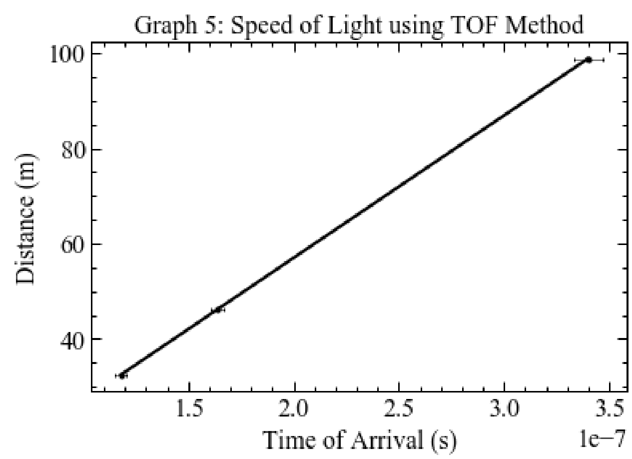

Figure 8. Graph 5: Speed of light using TOF method. Error bars for the distance and time measurements are included. Slope = $(299\,140\,040 \pm 254\,332)$ m/s

Time of Flight Method

Speed of light from the slope of Graph 5:

$$v_2 = (299\,140\,040 \pm 254\,332)\,\text{m/s}$$
$$v_2 = (2.991 \pm 0.002) \times 10^{8}\,\text{m/s}$$

A consistency check between v_2 and $Vaccept$ show that they are not consistent. The percent error is .218%.

VI. Discussion

Michelson Rotating Mirror Method

The speed of light was determined by measuring the displacement of the image of a laser as the frequency of the rotating mirror was increased from $\sim 50Hz$ to $\sim 600Hz$; this was done a total of four times. The position of the image at the initial frequency ($\sim 50Hz$) was zeroed, and used as a starting position for the rest of the measurements. The arbitrary zero position is acceptable because the experiment focuses on the deflection of the beam as the frequency of rotation increases. This also means that all further displacement measurements must be measured with the reticule centered at the same place on the image.

The displacement of the image was plotted against the frequency of rotations (Graphs 1-4), and the slope of each graph was determined through linear regression. The error bars for the individual measurements of displacement and frequency were included; since they did not exceed the degree of deviation from the line of best fit, the standard deviation of the slope was computed with a least-squares method, as outlined in the lab manual. [1] Graph 1 and Graph 4 produced similar slopes of $(-4.25 \pm 0.11) \times 10^{-3}\,\mathrm{mm/Hz}$ and $(-4.36 \pm 0.06) \times 10^{-3}\,\mathrm{mm/Hz}$, respectively, and Graph 2 and Graph 3 produced similar slopes of $(-3.98 \pm 0.23) \times 10^{-3}\,\mathrm{mm/Hz}$ and $(-3.75 \pm 0.10) \times 10^{-3}\,\mathrm{mm/Hz}$, respectively.

The mean slope value was calculated to be $slope_{avg} = (-4.08 \pm 0.24) \times 10^{-6}\,\mathrm{m/Hz}$. Using Equation (1), with $f = 2f$ since the rotating mirror is silvered on both sides, and $f/y = |slope_{avg}|$ where y is the displacement, the speed of light was calculated to be $v_1 = (3.13 \pm 0.18) \times 10^{8}\,\mathrm{m/s}$. A consistency check with the accepted value of the speed of light, $v_{accept} = 299792458$, showed them to be consistent, with a percent error of 4.4%.

Solving for y/f, or the slope, in Equation (1) produces a value of $4.26 \times 10^{-6}\,\mathrm{m/s}$. This indicates that the slopes from Graph 2 and 3, which used the results of Trials 2 and 3, were less accurate. When looking at the measurements from Trial 2 and 3, there were some instances where the displacement did not change, or increased between steps in frequency. Thus, one of the largest sources of uncertainty in this experiment was in the measurement of displacement. As mentioned above, each measurement was relative to the initial position when the traveling microscope readout was zeroed, so if the reticule was not centered at the same place on the image the measurement was not accurate. At low voltages, the rotational frequency also increased quickly, contributing to a shift in position between centering the reticule and reading the frequency, which added to the uncertainty of the initial position. This may be partially remedied by using a more powerful microscope so that the displacement is easier to observe.

Time of Flight Method

In this method, the speed of light was determined by measuring the time of flight for a beam travelling over three distances. The distance travelled was plotted against the time of arrival (Graph 5), and the slope was obtained as the speed of light, $v_2 = (2.991 \pm 0.002) \times 10^{8}\,\mathrm{m/s}$. A consistency check between $v_2 and v_{accept}$ showed them to be not consistent. The percent error was .218%. A significant source of error was from only collecting the TOF data once, as well as using three distances. This could be remedied by repeating the measurements.

The distances were determined by the placement of two standard front-surface mirrors and one retro-reflecting mirror. The total distance travelled was calculated using the distances between the diode plane, the top mirror M_0, a box placed on a table near the apparatus, and the far mirrors M_1, M_2, M_3 (See sample calculations in Apparatus), as well as the distances between the mirrors and the walls they were mounted on. Since the photodetectors were of equal distance from the beam splitter, they cancelled out in the total distance. That is, beam 2 had travelled the distance between the beam splitter and the photo-detector before the first pulse was detected, however it travels the same distance when it returns before hitting the second photo-detector.

The distances between the box and the mirrors' walls was measured using a range-finder placed on top of the box. While the range-finder is precise, there was uncertainty in the distance measurements, as the range-finder did not follow the exact path of the beam. In addition, the exact reflective surface inside the retro-reflecting mirror was estimated as half the length of the unit. Since the range finder is a laser, the uncertainty in path distance may be remedied by placing a solid surface over each mirror, inserting the range-finder directly above the laser in the apparatus, and measuring the path distance to the solid surface (i.e. the same path distance that the beam takes in the experiment). This would reduce the uncertainties in the distance measurements and alignment of M_0, however would not reduce the uncertainties in the alignment of the other mirrors, or the distance to the reflective surface in M_3. The cable lengths from each photo-detector affects the measurements of pulses since there is an additional time for the signal to travel through the cable to the oscilloscope probes. If the cables are the same length, this problem is reduced, however the resistance of the individual cables vary and adds to this effect.

Between the Michelson rotating mirror method and the time of flight method, the time of flight method is more accurate. While the Michelson method provided a consistent result in this experiment, it had a much larger final uncertainty and a larger percent difference (4.4% compared to .218% from the TOF method). With improved measurements of distances travelled, or alternatively higher uncertainties to reflect the error in distance measurement, the TOF method would produce results consistent with the accepted value for the speed of light.

VII. Conclusion

In this experiment, the speed of light was determined using the Michelson Rotating Mirror Method to be $v_1 = (3.13 \pm 0.18) \times 10^8\,\mathrm{m/s}$. A consistency check with the accepted value of the speed of light, $v_{accept} =$ 299792458, showed them to be consistent, with a percent error of 4.4%. The speed of light was also determined using the Time of Flight method to be $v_2 = (2.991 \pm 0.002) \times 10^8\,\mathrm{m/s}$. A consistency check between $v_2 and v_{accept}$ showed them to be not consistent. The percent error was .218%.

References

[1] Physics 325 Laboratory Manual
Department of Physics and Astronomy, University of Victoria
2019

[2] "Fizeau–Foucault apparatus"
Wikipedia, the free encyclopedia
2021

Appendix

1. Apparatus

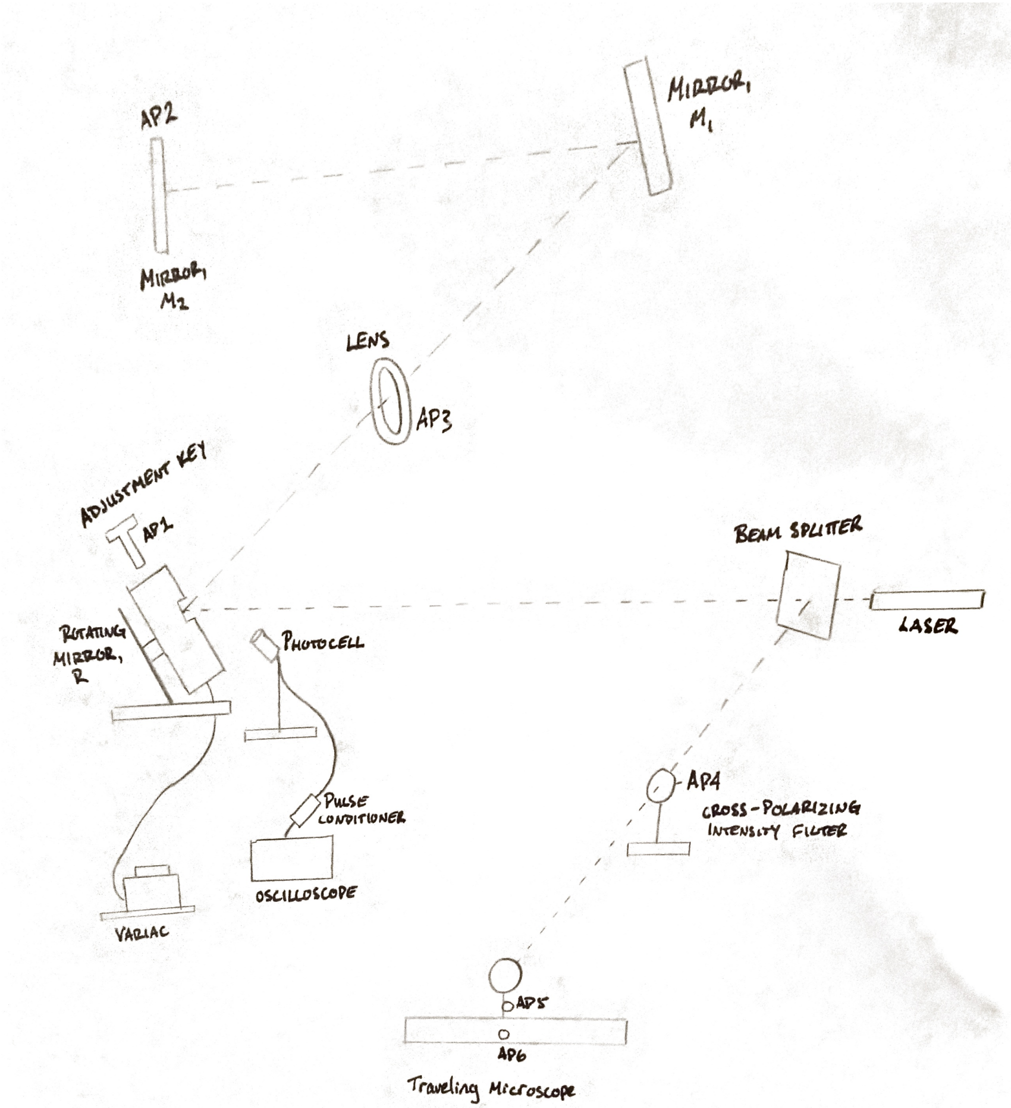

Figure 9. Schematic Diagram of the Michelson Rotating-Mirror apparatus.

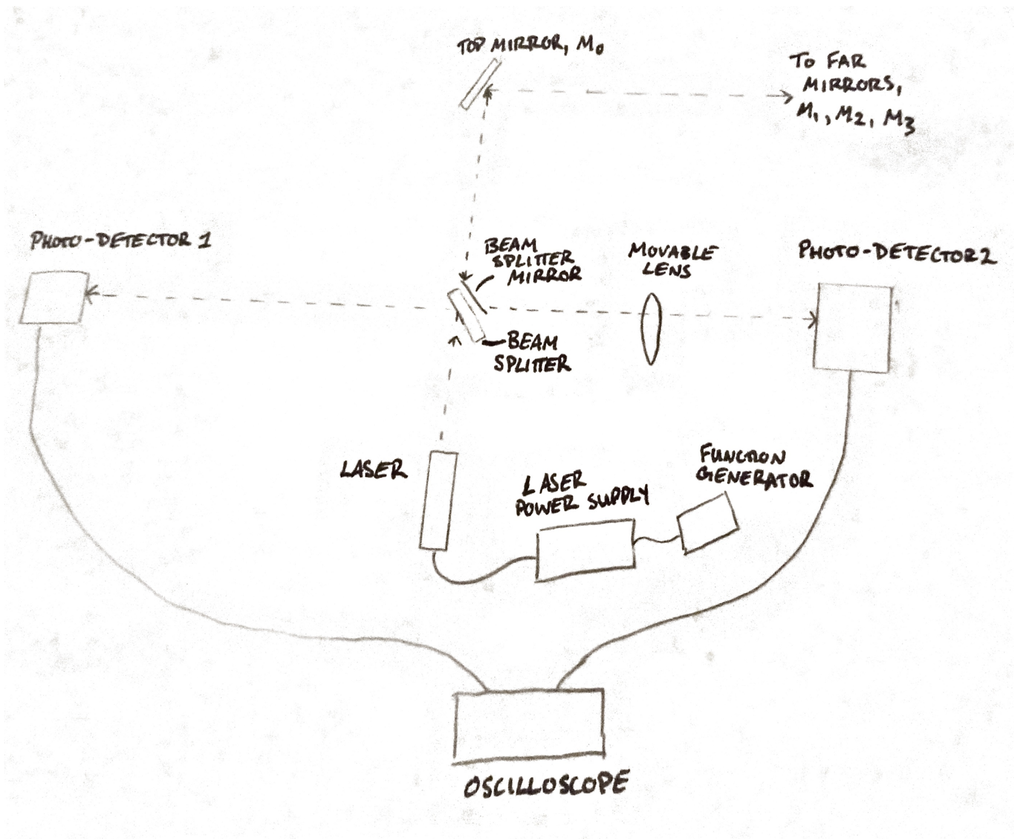

Figure 10. Schematic Diagram of the Time of Flight apparatus.

2. Sample Calculations

Calculation of v_1

$$v = \frac{8\pi abf}{y}$$

$$f = 2f$$

$$f/y = |slope_{avg}|$$

$$v_1 = 4\pi ab|slope_{avg}| = 4\pi((15.460 \pm 0.014)\,\mathrm{m})((6.580 \pm 0.011)\,\mathrm{m})(4.08 \pm 0.24)\,\mathrm{m/Hz} = (313\,036\,097 \pm 18\,283\,690)\,\mathrm{m/s}$$

Plot Graphs and Determine Slope

```
slopes=[]; slopes_std=[]
count=1
for f,d in zip([f1,f2,f3,f4],[d1,d2,d3,d4]):
    p, V = np.polyfit(f,d, 1, cov='True') #polynomial approx and covariance matrix(for slope uncertainty)
    print(p, np.sqrt(V[0][0]))
    d_fit = np.polyval(p,f) #linear fit, to plot
    #uncertainties of measurements, to compare data to best-fit line
    d_err = 0.025 #mm, uncertainty of displacement measurements
    f_err = .02*f #Hz, uncertainty of freq measurements
    plt.plot(f, d_fit)
    plt.errorbar(f,d,yerr=d_err, xerr=f_err, fmt='.k', elinewidth=.5, capsize=1)
    slopes.append(p)
    slopes_std.append(np.sqrt(V[0][0]))
    plt.ylabel("Displacement (mm)")
    plt.xlabel("Frequenzy (Hz)")
    plt.title("Graph %d: Displacement as Function of Frequency" %(count))
    #plt.savefig('graph'+str(count)+'.png')
    #since uncertainty due to scatter of data points dominates, use std dev of slope.
    print("Slope of Graph {:d}, using data from Table {:d}: {:.5f} +/- {:.2f}".format(count, count, p[0], np.sqrt(V[0][0])*100))
    count += 1
    plt.show()
```

Figure 11. Python code used to plot Graphs 1-5, and determine slope and slope error.

Consistency Check

```
[6]: #where a, b are uncertainty.ufloat objects
     def consistencycheck(a,b):
         if(abs(a.n-b.n) <= a.s+b.s):
             return True
         else:
             return False
```

Figure 12. Python code used to perform a consistency check on v_1 and v_2 with v_accept.

Mean and Standard Deviation

```
[22]: #mean slope
      slopes = np.array([i[0] for i in slopes])
      print(slopes)
      slopes = slopes/1000
      slope_avg = np.mean(slopes)
      slope_std = np.std(slopes)
      print("Average slope = {:.3} +/- {:.3} m/Hz".format(slope_avg, slope_std))
      slope_avg, slope_std
```

Figure 13. Python code used to calculate mean and standard deviation of slopes.

TOF Total Distance Travelled

```python
[26]: #average distance from box to laser's wall:
dl = uc.ufloat(np.mean([2.856, 2.851, 2.849]), np.std([2.856, 2.851, 2.849])*np.mean([2.856, 2.851, 2.849]))
#distance from diode plane to M0 = (desk to M0) - (desk to diode plane):
dm0 = uc.ufloat(1.143, .02*1.143) - uc.ufloat(.398, .02*.398)
#distance from wall to laser:
wall_laser = uc.ufloat(.145,.005)
#distance from M0 to box = (box to laser's wall) - (wall to laser)
m0b = dl - wall_laser
#length of range finder:
rf = uc.ufloat(.190, .005)

#distance from box to mirror's wall for M1, M2, M3:
bm1_wall = np.array([uc.ufloat(46.292, .002), uc.ufloat(19.935, .002), uc.ufloat(13.102, .002)])
#distance from wall to mirror for M1, M2, M3:
wm1 = np.array([uc.ufloat(.12, .01), uc.ufloat(.107, .002), uc.ufloat(.125, .002)])
#distance from box to mirror for M1, M2, M3:
bm1 = bm1_wall - wm1
#total distance travelled = 2*((wall to M0) + (M0 to box) + (box to mirror) - (length range finder) + (distance from beam splitter to PD2) - (beam splitter to PD1))
# since distance from beam splitter to PD1 and PD2 are the same, they cancel out, and total distance is:
d_tot = 2*(dm0 + m0b + (bm1 - rf))
#get numerical value of total distance (no uncertainty)
d_totn = [i.n for i in d_tot]
d_totn.sort()
#time array:
t = np.array([118.0, 164.0, 340.0])*10**(-9)
```

Figure 14. Python code used to the total distance travelled by beam for M_1, M_2, M_3.

3. Graphs

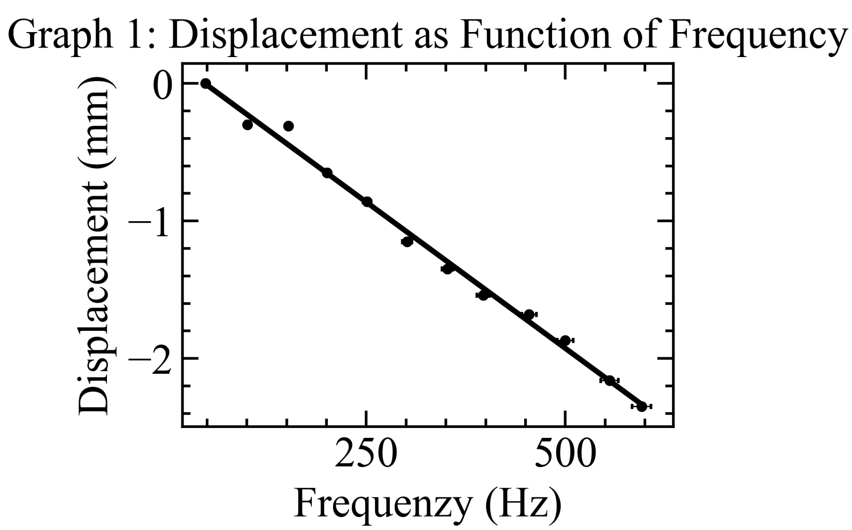

Figure 15. Graph 1: Displacement (mm) as a Function of Frequency (Hz), using data from Table 1. Error bars for the displacement and frequency measurements are included. Slope = $(-4.25 \pm 0.11) \times 10^{-3}$ mm/Hz

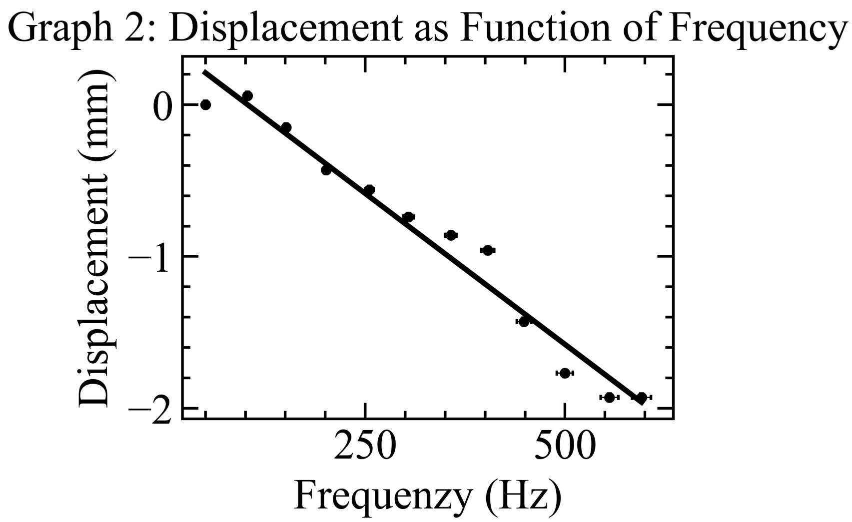

Figure 16. Graph 2: Displacement (mm) as a Function of Frequency (Hz), using data from Table 2. Error bars for the displacement and frequency measurements are included. Slope = $(-3.98 \pm 0.23) \times 10^{-3}$ mm/Hz

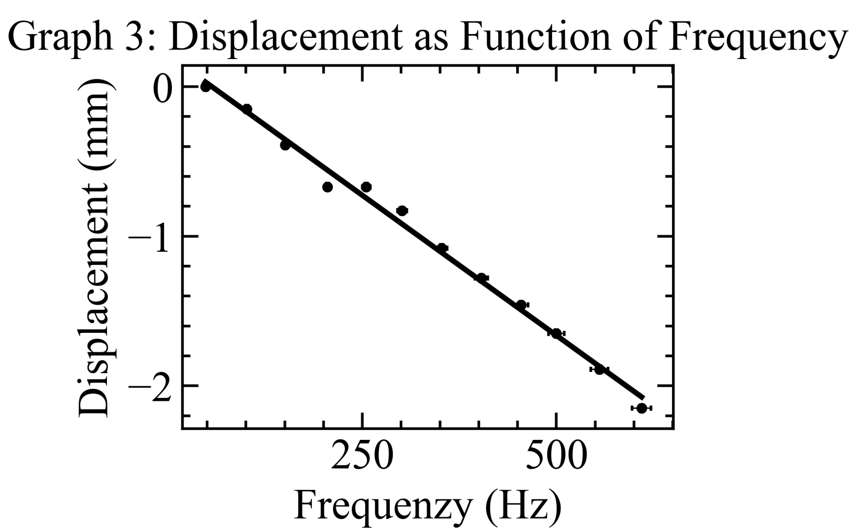

Figure 17. Graph 3: Displacement (mm) as a Function of Frequency (Hz), using data from Table 3. Error bars for the displacement and frequency measurements are included. Slope = $(-3.75 \pm 0.10) \times 10^{-3}$ mm/Hz

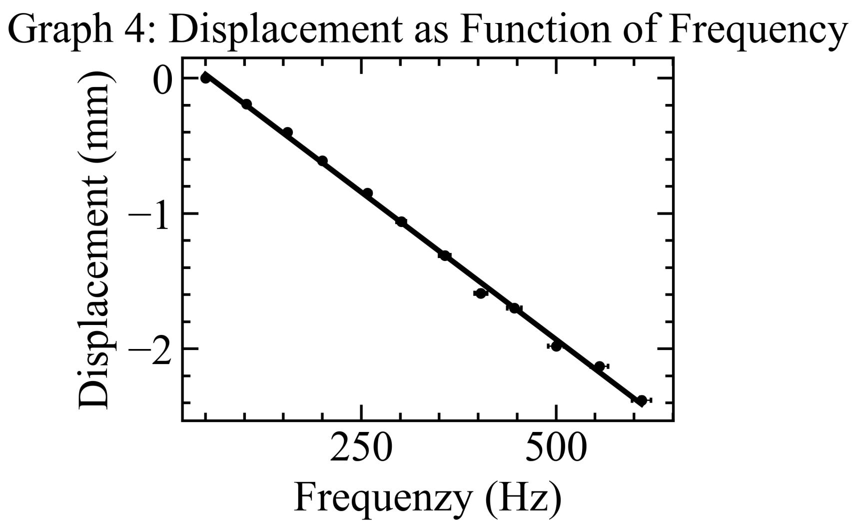

Figure 18. Graph 4: Displacement (mm) as a Function of Frequency (Hz), using data from Table 4. Error bars for the displacement and frequency measurements are included. Slope = $(-4.36 \pm 0.06) \times 10^{-3}$ mm/Hz

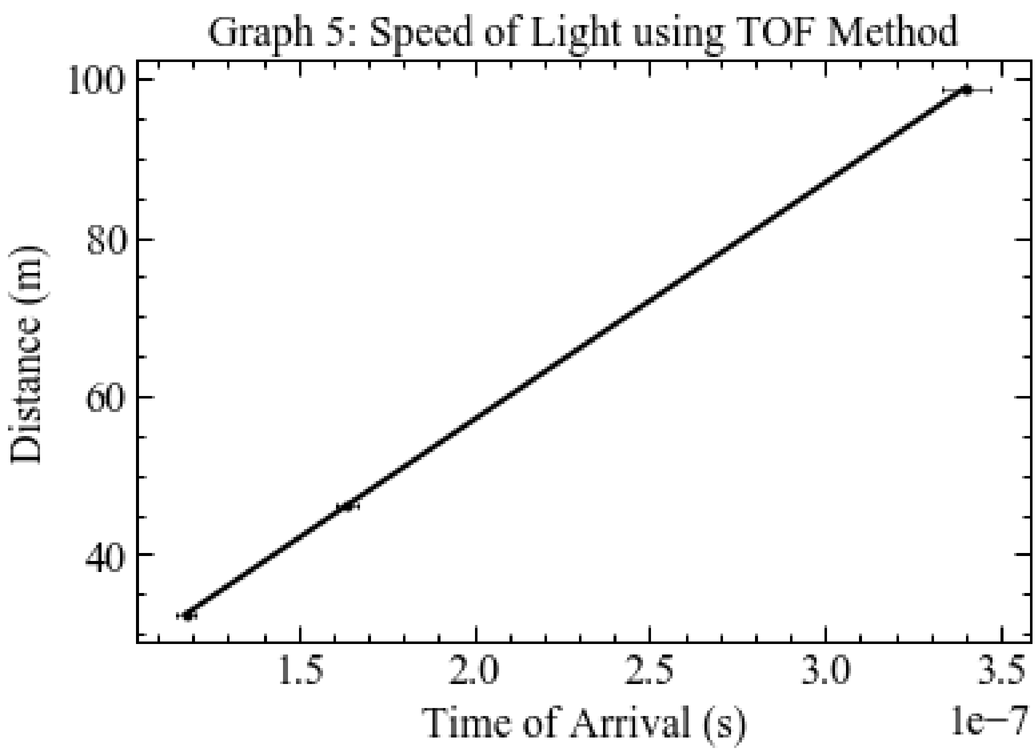

Figure 19. Graph 5: Speed of light using TOF method. Error bars for the distance and time measurements are included. Slope = $(299\,140\,040 \pm 254\,332)$ m/s