Phys 342 Lab#9 Due: Nov 30, 13:00

Consider a system that produces an oscillating voltage for several seconds after a laser flash strikes it. The best model for its response is that the voltage as a function of time is given by:

$$v = \alpha t \cos \beta t$$

with unknown parameters, α and β. You set up a device to measure that voltage every half-second for 4 seconds after the laser flash. Your device response is well modelled by a Gaussian distribution about the true value, but its standard deviation grows with each measurement. You can model your set of measurements v_i as outcomes of Gaussian random variables V_i whose expectation values are $E[V_i] = \alpha t_i \cos \beta t_i$ and standard deviations are σ_i.

A skeleton python notebook has been created to analyse the data from this experiment. It assumes that the true values for the two parameters are $\alpha = 1.5$ and $\beta = 2.0$.

1. Finish cell#3 to properly calculate the χ^2 function. As a check, with the reference data and the assumed true values for the parameters, you should find that $\chi^2 = 6.47$.
2. Run cell#4 to find the parameter estimates for the reference data. These are found by finding the parameter values that minimize the χ^2 function completed in step 1. You should find that the estimated values for the parameters are $\hat{a} = 1.75 \pm 0.16$ and $\hat{b} = 2.03 \pm 0.03$.
3. Run cell#5 to compare visually the model with the true parameter values (labelled "truth"), the model with the estimated parameter values (labelled "fit") and the data points with error bars showing the assumed standard deviations for the measurements (as is conventionally shown).
4. Add a new cell (#6) to consider 1000 replications of the experiment. For each, generate a new dataset by calling get_data() as done at the bottom of cell#2. Plot the "pull distributions" for the two parameter estimates, and overlay these with a standard Gaussian (mean 0, standard deviation 1) scaled appropriately:

$$\mathrm{pull}_\alpha = \frac{\hat{\alpha} - \alpha}{\sigma_{\hat{\alpha}}} \qquad \mathrm{pull}_\beta = \frac{\hat{\beta} - \beta}{\sigma_{\hat{\beta}}}$$

   What are the means and standard deviations of the two pull distributions? Explain why these values are close to 0 and 1.

5. Include in cell #6 the code to display the distribution of $\chi^2_{\min}$ and compare that to a χ^2 distribution with 6 degrees of freedom. Explain in words how $\chi^2_{\min}$ could be used for a goodness of fit test for this study.

Report answers to questions in markdown cells and submit your .ipynb file to the course website for grading as usual.