Phys 342 Lab#7 Due: Nov 19, 13:00

Suppose an experiment is designed in order to test whether a long standing theory is correct (call that hypothesis H_0) or if a new theory is correct (call that H_1).

The experiment is difficult and the result cannot be definitive. The test statistic that is formed from the measurements can take on a value between 0 and 1. If H_0 is correct, the test statistic can be modelled by a random variable, T, whose pdf is given by:

$$g(t|H_0) = a\, e^{-4t} \quad \text{for} \quad 0 \leq t \leq 1 \text{ and } 0 \text{ otherwise}$$

where a is the normalization constant. On the other hand, if H_1 is correct, the pdf would be given by:

$$g(t|H_1) = 2\sin^2(\pi t) \quad \text{for} \quad 0 \leq t \leq 1 \text{ and } 0 \text{ otherwise}$$

Determine the critical value, t_{cut}, such that if the experiment reports a value t above that value, hypothesis H_0 is rejected at the 95% confidence level.

Using a Jupyter notebook, investigate this situation. Create two methods:

```
def nextValueForH0():     and
def nextValueForH1():
```

each of which return a single random number generated according to the corresponding pdfs. The first one should use the transformation method, the second should use the accept/reject method.

Do the following to determine the fraction of experiments for which H_0 would be falsely rejected or falsely accepted:

(a) Simulate 10000 experiments when H_0 is true, by repeated calls to the first method, and count how many experiments result in rejecting H_0 at the 95% confidence level. Statisticians call these Type-I errors.
(b) Simulate 10000 experiments when H_1 is true, by repeated calls to the second method, and count how many experiments result in accepting H_0 (yes, H_0 !) at the 95% confidence level. Statisticians call these Type-II errors.

Show a histogram for t for each of these samples (20 bins from 0-1) and overlay the curve that shows the expected number in each bin (in other words – the properly scaled pdf).

Report your results in markdown cells and submit the .ipynb file to the course website for grading as usual.