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Convergent cognition

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Abstract In an attempt to address shortcomings revealed in international assessments and lamented in legislation, many schools are reducing or eliminating elective courses, applying the rationale that replacing "non-essential" subjects with core subjects, such as mathematics and language arts, will better position students in the global market. However, there is evidence that systematically pairing a core subject with another, complementary subject, may lead to greater overall learning in both subjects. In this paper, we analyze two subject area pairs—first and second language, and computer programming and mathematics—to demonstrate in what ways two subjects might complement each other. We then analyze the relationships between these pairs to better understand the principles and conditions that encourage what we call *convergent cognition*, the synergistic effect that occurs when a learner studies two complementary subjects.

Keywords Cognition · Convergence · Transfer · Learning · Mathematics · Computer programming · L1 · L2 · SLA second-language acquisition

Convergent cognition

The concept of transfer, the influence prior learning has on acquiring new knowledge, was initially studied and subsequently rejected by Thorndike and Woodworth (1901) when they

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analyzed research on the effect of learning “the classics” (i.e., Latin, Greek, and Geometry) on general intelligence. It has since been studied repeatedly by researchers in Gestalt (Wertheimer 1959), behavioral (Gagné 1974), cognitive (Campione and Brown 1981), and situated (Säljö 2003) traditions. Although transfer research has examined the influence of prior learning on the acquisition of new knowledge in very specific ways, there is an important consideration missing. Namely, the reciprocal effect of new knowledge in one domain on prior knowledge of another domain. In this paper, we propose that there is a more specific cross-domain influence on learning that has the potential to transform the way we currently think about the structure of learning. We call this more specific cross-domain influence *convergent cognition.*

We first discuss various ways transfer has been conceptualized as a cross-domain phenomenon. We then propose convergent cognition as a beneficial way to capitalize on the similarities between these approaches, while shedding some of the unwanted baggage inherent in transfer research. We discuss underlying principles and conditions of convergent cognition and conclude with implications of this construct for both educational policy and research.

A cross-domain phenomenon

Researchers have often investigated the effect of learning one subject on another through the study of transfer. The form of transfer that most closely addresses the issue of cross-domain influence may be lateral transfer (Haskell 2001). In this section, we examine what is meant by lateral transfer, how that conceptualization is problematic, and why, despite (or perhaps in spite of) these problems, there remains a need to better examine and understand the reciprocal effect of learning two complementary subjects.

Campione and Brown (1981) conceptualized lateral transfer as the sharing of an underlying conceptual base between domains:

> A subject can learn to say “seb” to “dax” or to consistently pick the red object, but she cannot understand why that choice is correct; it is correct only because the experimenter has designated it so. Academic domains, in contrast, tend to have an internal structure…. It is possible for learners to identify “new” problems as sharing essential features with the prior ones. (p. 271)

Lateral transfer, thus conceptualized, suggests a shared underlying structure between different domains of knowledge. Robert Gagné (1974) proposed that lateral transfer was the degree to which an individual was able to broadly generalize what he had learned previously to a new situation. Given this definition, it would appear that the concept of lateral transfer might be able to explain the reciprocal influence of one domain of knowledge on another.

Unfortunately, most subsequent references to lateral transfer fall into one of two extreme camps, neither of which are actually supported by much empirical research. One camp is too specific, focusing on Gagné’s (1974) claim of equal “level of complexity” (p. 335) within the same domain. Researchers in this camp study student learning by focusing on apparently related tasks “at the same level” of skill in the same domain (Zheng 2009). An oft-cited example of this overly-specific view of lateral transfer is Haskell’s (2001) description of a person learning to drive a car and then later learning to drive a truck. This skills-based approach frequently relies on researchers’ arbitrary decisions of which skills are at the same “level” of complexity. As Carraher and Schliemann (2002) pointed out, researchers have repeatedly judged transfer not to occur in such situations when, “they merely showed that the participants did not draw upon the particular learning situations [the researchers] had hoped they would draw upon" (p. 3), a relatively common theoretical pitfall of education research (c.f. Leatham 2006). A same-domain, skills-based approach to lateral transfer often fails to account for influences of prior knowledge not anticipated by the researcher.

The second way of interpreting lateral transfer is too broad, equating it with the concepts of general, far or analogic transfer. Bassok and Holyoak's (1993, 1989) work with specific interdomain transfer in mathematics, Lave's (1989) seminal critique of general transfer, and Barnett and Ceci's (2002) examination of near and far transfer have all been mischaracterized as elaborating on lateral transfer even though the word "lateral" never appears in the manuscripts. Campione and Brown (1981) characterize the confusion here:

> The problem of a domain is an old one, intimately tied to the problem of general or specific factors in intelligence… The key problem with such positions is how broad one would expect the generalization effect to be. What is the domain or context within which transfer is to be constrained? (p. 462)

As a general domain-based phenomenon, there is little continuity amongst lateral transfer research that would indicate how to determine the bounds of one domain or another. General transfer, the notion that learning in one domain will result in broad generalizations that help a learner in another domain, has long been problematic in educational research due to the difficulty in establishing domain boundaries and degrees of generality. Analogical (or high road) transfer research has more positively reflected the benefits of dual perspectives from different domains. Yet, the personal nature of analogies requires a familiarity and expertise in a domain that is often not easily shared amongst learners (Salomon and Perkins 1989). What constitutes a shared underlying structure for one learner may not be as clear for another, introducing a difficult-to-teach personal element when advocating analogical transfer as a means for cross-domain influence (Campione and Brown 1984).

Despite these problems with our current understanding of transfer in general and lateral transfer in particular, the notion of a shared underlying base of knowledge that Campione and Browne (1984) proposed is an important distinction in understanding the relationship that may account for the original benefit seen from study of "the classics." Through his study of the influence of a second language (L2) on one's first language (L1), Cook (1992) proposed a theory of *multicompetence* which proposes that, instead of one language merely influencing the other, prior knowledge converges with new knowledge to change a learner's underlying understanding of a concept related to both L1 and L2. In the past 15 years, second-language acquisition researchers have demonstrated how speakers of multiple languages possess an underlying knowledge base that simultaneously changes, and is changed by, each language they learn. Jarvis and Pavlenko (2008) referred to this relationship as a *common underlying conceptual base*, where a learner's understanding of first and second languages combines to inform both. Pavlenko (2003) described this relationship as, "a unified, complex, coherent, interconnected, interdependent ecosystem" (p. 58). In Carraher and Schliemann's (2002) critique of transfer, they proposed that the common view of transfer as prior learning influencing new learning fails to account for these changing schema. They suggested we do away with the idea of transfer and instead utilize Piaget's (1954) conception of adaptation, where new knowledge helps to either modify existing schema or to form new schema completely. Gestalt theorists (e.g., Wertheimer 1959) promoted a similar view of transfer, where the key was to tease out the underlying structure of related problems rather than to identify specific elements of related tasks. The traditional view of transfer does not account for this effect, but rather views transfer as a uni-directional, skills-based, phenomenon.

There may be several elements at work in the former notion of mental discipline that remain unaccounted for in a single existing construct. First, as second-language acquisition researchers have noted, learning a skill in a new domain (e.g., a second language) may actually influence former knowledge, thereby affecting one's underlying understanding of a concept shared between the first and second domain. Second, this mutual influence suggests a shared set of core concepts between domains. Third, the phenomenon at play may actually be the manifestation of several forms of transfer simultaneously. Far, analogical, positive, bi-directional or lateral transfer may converge to create a synergy wherein former and new knowledge combine to change one's underlying conceptual knowledge.

Campione and Brown (1981) further noted that, "the ability to correctly sort superficially different problems in terms of their underlying structure is one of the features that distinguish expert[s] from novice[s]" (pp. 271–272). Thus, a view of knowledge in which new and old knowledge converge to change an underlying conceptual base may be the key to cultivating expertise. Because various forms of transfer may be at play, it is difficult to identify the phenomenon as one type or another. In order to avoid this difficulty, we refer to the synergistic effect wherein learning topics in two related domains combine to change one's underlying conceptual understanding as *convergent cognition*.

Evidence of convergent cognition

We now provide evidence of convergent cognition by reviewing research on two subject area pairs—first and second language learning, and computer programming and mathematics. These subject pairs were chosen due to the broad extant literature base comparing performance in one on the other, and are not meant to be a comprehensive review of all possible complementary subject pairings. Indeed, there may be many such convergent relationships between multiple domains of knowledge. We then analyze these relationships, extracting possible principles and conditions that might explain and lead to sustained convergent cognition.

Correlations between first and second language learning

Those who learn a second language (L2) often report increases in their understanding of their first language (L1) as a result of learning the L2. Commonplace claims abound such as, "I never really understood grammar until I studied a foreign language" (Eddy 1981, p. 6), and "foreign language study enhances a student's achievement in English" (Weatherford 1986, p. 4). In fact, the American Council on the Teaching of Foreign Language (2009) has maintained that one important reason for studying a foreign language is that research has demonstrated how L2 study increases L1 literacy. In this section, we discuss research spanning a century that supports such claims. The students in these studies are mostly those learning a language foreign to their country of residence. Research on second-language learners who are learning the language of the country they reside in, and whose native language is foreign to that country, is beyond the scope of this paper.

Latin study

In 1914, teachers noted that those studying Latin tended to have more extensive vocabularies than those who did not. Scores of 42 high school students were compared using simple percentages, revealing an advantage in spelling (82.5 vs. 72.6 %), use of words in sentences (57.5 vs. 40.6 %), definitions and parts of speech (69.5 vs. 33.3 %), meaning of words and spelling (57 vs. 27.5 %), and excellence in vocabulary (36 vs. 6.8 %) (Barber 1985). Although a simple comparison of percentages does not indicate statistical significance, these observations highlight perceived differences between the performance of students that studied Latin and those that did not, on tests of native English ability.

Contemporary statistical measures corroborate these early claims. Sheridan (1976) reported on 400 sixth grade students who received 30 min of Latin a day. On the pre-test, there was no significant difference between the Latin-studying students' scores with a control group on Form F of the Metropolitan Achievement Test. On the end-of-year post-test, Latin-studying students significantly outperformed control group students in word knowledge ($p < 0.05$), reading ($p < 0.1$) and language ($p < 0.01$). Likewise, Offenberg (1971) found that the scores from 4,000 Latin-studying students on the Vocabulary portion of the Iowa Test of Basic Skills (ITBS)—who were matched with a control group in terms of prior ITBS scores, socio-economic status, and grade level at the beginning of the year—outgained control students by an entire year ($p < 0.05$).

Masciantonio's in-depth (1977) review of the "Latin in the Schools" projects repeatedly found similar results, demonstrating greater gains for students participating in Latin-studying programs than control group counterparts. Over 1,100 randomly selected sixth-grade Washington, D.C., students who were not reading at grade level before studying Latin, showed post-test scores a full five months ahead of students without foreign language instruction. Fifth and sixth-grade students in Massachusetts, who were also reading below grade level, demonstrated pre-post gains of two years on the vocabulary sub-test of the ITBS, as opposed to a gain of one year for control group students. In Los Angeles, fifth and sixth-grade Latin studying students' pre/post-test scores on the comprehensive test of basic skills again demonstrated greater gains on English vocabulary and comprehension than a non-Latin-studying control group. Fifth-grade students had a mean gain in scores of eight months, compared to a mean gain of six months for the control group. Likewise, sixth graders studying Latin showed a nine-month gain, whereas control group students gained only sixth months between pre and post-tests.

In short, studying Latin has repeatedly correlated with, or resulted in, increased English ability, as demonstrated by varied measures, across several studies, irrespective of age, geography, or socio-economic status.

Foreign language study

In addition to the benefits of studying a related "dead" language, research has demonstrated that studying contemporary sister foreign languages can be positively related to one's L1. Cunningham and Graham (2000) studied 30 fifth and sixth grade immersion students (i.e., students who spend at least half of the school day learning in a foreign language) and 30 monolinguals to determine the extent to which students' L2 vocabulary aided their understanding of their L1 in terms of cognates (i.e., words that share the same root meaning). Scores on the Peabody Picture Vocabulary Test and the Spanish–English cognate test, analyzed using an ANCOVA method, revealed a significant difference ($p < 0.05$) in favor of immersion students. This study highlights the importance of cross-linguistic influence, or the positive impact of studying languages that share common ancestry.

Cross-linguistic influence appears in multiple studies. For example, in Canada, researchers found that, "intensive exposure to French facilitated the comprehension of certain English syntactic structures" (Cummins 1978, p. 867). Bastian (1980) reported that graduating high school seniors with two or more years of foreign language study significantly outperformed their peers on English achievement tests. Andrade et al. (1989) demonstrated that students representing a wide spectrum of socioeconomic levels exceeded national norms in reading when they participated in a foreign language program. And in New York, 114 elementary school students received French instruction for 15 min a day, resulting in higher mean scores on English portions of the Stanford achievement test (Lopato 1963). Eddy's (1981) review confirmed these results, repeatedly reporting significantly higher mean scores for foreign language studying students.

Presented with the opportunity to corroborate decades-old studies, researchers in the province of Ontario, Canada compared the scores on mandatory English-language tests of third and sixth-grade students in immersion programs with the scores of monolingual students (Turnbull et al. 2001). Given the data from nearly 70 % of participating schools, researchers confirmed several findings from earlier studies. First, immersion students tend to 'catch up' to their monolingual peers within 1–2 years of study. By third grade, a greater proportion of immersion over English-only students achieved in the high and exceeding expectations categories on the reading (+6.7 %) and writing (+4.9 %) English tests, with the authors concluding that, "at Grade 6, immersion students' literacy test scores were notably better than their peers' in English programs" (p. 23).

Non-sister language correlations

One might argue that linguistic transfer from sister languages is to be expected, yet further evidence confirms that learning an L2 is often positively related to L1 ability, regardless of whether those two languages derive from the same language family (Chen et al. 2010). A rare two-way Korean–English program in the United States studied the English skills of 192 first and second graders (Bae, 2007). The English-only classes chosen as comparison groups were statistically comparable in their writing skills at the beginning of the study (i.e., before first and second grade). At the end of the study, students in the English-only programs "were ranked in the upper levels compared against the national norms" (p. 319). All students were given a series of pictures and asked to write stories about the pictures. Responses were graded for several components of writing fluency; namely, coherence, content, grammar, and text length. MANOVAs demonstrated that there was no statistical difference between immersion and English-only students. Thus, consistent with prior studies, there is evidence that within the first few years, immersion students achieve on par with monolingual students.

In a study by Kesckes and Papp (2000) Hungarian children studied English for an hour each week. They found that English-studying students were able to use more complex sentence structure in Hungarian than their non-English studying counterparts. In a German study, 225 students received French instruction daily for eight months, while 161 control students did not. The experimental group increased L1 vocabulary, made fewer grammatical errors, and used complete verbs to a greater extent than the control group (Doye 1977). Swedish students that participated in the English in the Primary Schools project, which borrowed time from other subjects to teach English, performed on par with their peers in different aptitude tests (Eddy 1981).

Perhaps the most important finding from these studies is that students who studied a second language consistently scored on par or higher on informal and standardized measures of their native language than their monolingual peers. Furthermore, taking time away from other subjects to study a foreign language did not decrease performance in other areas. One implication of such studies, as we discuss later in the paper, is that rather than merely increasing the time students spend in language arts, it may be more beneficial to supplement this time with study of a foreign language, increasing overall language ability in both subjects.

Correlations between computer programming and mathematics

Similar to the connection between L1 and L2, there exists substantial evidence of a complementary relationship between learning mathematics and learning computer programming. The president's National Mathematics Advisory Panel (U.S. Department of Education 2008) recommended "that computer programming be considered as an effective tool, especially for elementary school students, for developing specific mathematics concepts and applications, and mathematical problem-solving ability" (p. 58). In the following section, we analyze the research evidence for this claim.

Logo—connecting mathematics and programming

The Logo programming environment is the clearest attempt to integrate forms of computer programming into mainstream school curriculum. Although arguably controversial, Logo's successes and failures have important implications for how learning to program might affect students' mathematical understanding.

Logo was created in 1966 as part of a constructionist learning initiative by Seymour Papert (Agalianos et al. 2001). Papert originally designed Logo to teach particular mathematical concepts to children (Feurzeig et al. 1970). Despite mixed reception, the Logo experiments provided educators with hints of the possibilities learning to program could offer. Several studies demonstrated that Logo had a substantial impact on students' performance in mathematics, indicating increased skills in understanding geometry and variables (McCoy 1996), and spatial and symbolic mental representations (Hoyles and Noss 1987). In research conducted in elementary algebra, students that learned to program in Logo constructed algebraic meaning in a non-computational context better than those without Logo experience (Noss 1986). More recently, Subhi (1999) compared the impact of computer-assisted-learning games and learning to program in Logo for 286 gifted students, finding that students who learned to program increased their problem solving abilities more than the computer-assisted-learning group. More specifically, programming in Logo has been shown to increase students' understanding of shape, measurement, symmetry, and arithmetic processes (Clements et al. 2001).

Despite hundreds of studies pointing to a positive increase in mathematical understanding for Logo-trained students, a large number of studies point to no significant difference on mathematics tests (Pea and Kurland 1984). Researchers later investigated these stark differences, concluding that pedagogy and time involved with Logo were markedly different between studies with positive and neutral results (Palumbo 1990). Thus, the manner and purpose of teaching programming, as well as the academic time allotted, influenced the degree to which learning computer programming impacted learning mathematics (see also Hoyles and Noss 1992). The Logo experience therefore offers a tentative, but incomplete, view of how learning to program might positively influence students' mathematical abilities.

Computer programming influences on mathematics

Computer programming's influence on mathematical ability has been heavily investigated in research on transfer of problem solving ability. In one experiment (Oprea 1988), a group of students were randomly selected to study the BASIC computer programming language in 60–90 min sessions two to three times per week for a total of 20 h of instruction. Programming students' scores on the Understanding of Variables instrument were compared in a pre-post test design with a control group. Students were matched with control students in terms of math ability, age, gender, use of a computer, or length of computer ownership. Tasks ranged from evaluating algebraic expressions to translating verbal statements to symbolic equations. A multivariate analysis revealed no statistically significant differences on the pre-test; post-test results showed significantly greater gains for the computer programming group over the control group ($p < 0.05$). Researchers concluded that learning to program enhanced sixth-graders' understanding of variables. Similarly, Soloway et al. 1982) found that students with experience in learning BASIC computer programming improved their word-solving ability on algebra problems. Other research examining students' understanding of the concept of variable after learning to program have yielded similarly positive results (Soloway et al. 1982; Milner 1973).

Recent attempts to teach programming to middle and high school students has continued to reveal a strong positive correlation between student programming experience and performance on mathematics assessments. This correlation is evident, for example, in the work with Dr. Scheme, a scaffolded programming environment that gradually introduces students to core programming concepts (Bloch 2007). In a Texas study, 134 pre-Advanced Placement algebra eighth grade students at one high school passed a statewide math test at a 90 % rate (North 2007). In contrast, all 17 students enrolled in the programming course passed the exam. The following year, only 85 % ($N = 161$) of the control group passed the test, with 100 % ($N = 20$) of the Dr. Scheme group once again passing the exam. Furthermore, after one year of study, 12 % of the Dr. Scheme students were commended for high scores (compared to 10 % for the control group), but after two years of programming, 35 % were commended (compared to 19 % of the control group). Thus, prolonged experience with a scaffolded programming environment correlated with increased student performance in algebra.

Programming predictors

An analysis of higher education computer science programs helps to further corroborate the claim of a positive relationship between mathematics and computer programming learning. This corroboration is found in predictive success factors of computer science programs. Research of this sort typically creates statistical models using multiple linear regression, which invariably include prior mathematics experience and achievement, including high school mathematics achievement scores, mathematics scores on standardized national tests, and enrollment in prior mathematics courses. Byrne and Lyons (2001) suggested that mathematics is usually included in these statistical models because of "a belief that the concepts which a student has to comprehend in order to master mathematics problems are similar to those for programming. Mathematics aptitude is thus often a prerequisite for acceptance into computer science" (p. 51).

In one such study, Leeper and Silver (1982) used data from computer science students' SAT scores (verbal and math), high school class ranking, and high school grades in mathematics, English, language arts, and science and compared these to their programming course scores. A multivariate regression analysis revealed that wholly 55 % of the variance accounted for in the model was due to mathematics. Mathematics performance, experience, or both, appear consistently in these predictive models, often accounting for the strongest positive variance (Austin 1987; Honour-Werth 1986; Hostetler 1983).

Today, scholars employ increasingly advanced statistical methods to create models that better predict student performance in higher education programming courses. For example, Bergin and Reilly (2006) conducted a multi-institutional study of beginning programming students to identify predictors of different levels of success. Using logistic regression methods, they generated a 3-factor model that included high school mathematics score, programming self-esteem, and number of hours playing video games. The model accounted for 79 % of the variance, with mathematics being the strongest positive predictor of success in computer science courses. Wilson and Shrock (2001) used hierarchical linear modeling to examine the predictive power of 12 variables for 105 students enrolled in an introductory programming course. A background questionnaire and a computer programming self-efficacy scale were administered at the beginning of the course and at mid-term. The proportion of variance on the midterm from the model was 0.44 ($p < 0.0001$). Mathematics background proved significant even when placed last in the model ($p < 0.0002$), and was thus considered the most important factor. Finally, Byrne and Lyons (2001) conducted a study with 110 introductory computer science students and found similar results—that students' Leaving Certificate mathematics scores (a high school exit exam taken in Ireland) accounted for approximately 35 % of the variance in their success in the course ($p < 0.01$).

The mere correlation of mathematics with computer programming does not in itself indicate causation. As Deary et al. (2007) suggested, many subject pairs have correlations, suggesting that a general intelligence factor may be at play. However, the repeated appearance of mathematics as one of the strongest positive predictors of success in computer programming studies (over other factors such as performance in science courses, prior language scores, and time spent playing videogames) warrants serious consideration of its potential relationship with and effect on computer programming (and vice versa).

Principles and conditions

Experimental, correlational, descriptive, and anecdotal evidence repeatedly corroborate the notion that learning a complementary subject positively benefits the learning of another, related subject, but they do not indicate *why or under what conditions* such convergence might occur. In order to better understand convergent cognition as a cross-domain phenomenon, we now analyze the relationship between the presented complements to extract the principles and conditions that might help explain this phenomenon. This analysis is our own interpretation of the aforementioned research through a deductive process. It is possible that other principles and conditions of convergent cognition exist that we failed to elucidate in this initial review of the topic. We posit the following principles and conditions and welcome further discussion and evidence for or against the proposed claims.

Convergent principles

Convergent principles are fundamental relationships that help to explain why convergent cognition might occur. The two principles we saw most readily in our review we have

Fig. 1 Diagram of convergent principles represented by a bridge

termed core attributes and duality (see Fig. 1). Using the metaphor of a bridge, we see how these principles might support the bringing together of ideas from two distinct, but closely related shores (i.e., domains of knowledge).

Core attributes

Core attributes are essential elements shared by complementary domains. These attributes are used in each subject and are integral to applying or understanding that type of knowledge. We categorize core attributes into concepts and processes. Concepts are the basic building blocks of a subject. *Processes* are the "big-ideas" one uses when engaging with the subject (see Fig. 1). In the following section, we establish and validate these core attributes through our example pairs: (a) L1 and L2, and (b) mathematics and computer programming.

Core attributes: concepts

Examples of core linguistic concepts may include the many different parts of speech, such as gerunds, direct and indirect objects, definite and indefinite articles, and so on. A once-common lesson in middle-grades English in the U.S. was to teach students how to diagram a sentence. This process consists of breaking the sentence into its constituent parts and identifying their grammatical function. The practice has largely become the butt of many jokes about the uselessness of learning such information. Yet, when second language learners begin to study another language, their early lessons on grammar take on new meaning, and not only are they are able to see the importance of learning such information, but they are able to begin to apply the rules of their L2 to explain the rules of their L1 (Cisero and Royer 1995; Manis et al. 2004). It is because first and second languages share core structural attributes that one is able to complement the other.

Shared core *concepts* also exist between computer programming and mathematics. One of the basic building blocks of both programming and mathematics is the notion of variable, a mathematics concept that is difficult for many students to grasp (Knuth et al. 2005), despite the fact that variables appear as early as first grade (often in the form of squares or triangles in "math sentences"). Even the most basic programming language requires the programmer to create and manipulate variables. Students who understand variable from one perspective are at an advantage when it is presented from the other. Another core concept is that of function. Research on students' understanding of function has established both the importance of function in mathematics and the many difficulties students have developing a robust understanding of it (e.g., Leinhart et al. 1990; Thompson 1994). As with variables, functions are central to learning to program. An understanding of function in one discipline can support an understanding of function in the other. Concomitantly, as

Paz and Leron (2009) have found in their work, misconceptions in one discipline can similarly be illuminated in the other—providing pedagogical providence or pitfall depending on the nature of the classroom.

Concepts such as variable and function play important roles in understanding core principles in each complement; understanding their role from one perspective facilitates understanding it from another, despite differences in the way each discipline perceives that concept. To illustrate how a core concept might be shared, but different, consider the case of algebraic functions verses programmatic functions. In algebra, a function might be expressed as $f(x) = x^2 + 3/x$. When $x = 3$, the output of the function is 10 (i.e., $f(3) = 10$). In a procedural programming language, a function might be defined in the following way:

```
var someOutput = 0;
function calculate(x) {someoutput = someoutput + 1; return someoutput;}
```

If I were to execute the "calculate" function as calculate(3), the function would return a one. However, if I then ran that same function again as calculate(3), I would get a two. In other words, the output of the function does not depend on its input. Situations like this, where the value of the function for a given input changes based on "when" you evaluate it violates the definition of function in mathematics. Nonetheless, this alternative view of function may help the learner to better understand what a function is and how it might vary.

It is common for core concepts to vary among a number of dimensions. Even within mathematics, there are multiple uses of the concept of variable (Usiskin 1988). Likewise, the concept of function is expressed differently depending on the programming language employed. Rather than hinder learners' understanding of function (i.e., negative transfer), these differences may actually enhance a learner's understanding of the underlying concept of function, particularly if teachers use such contrasts as pedagogical levers (cf. Schwarz and Hershkowitz 1999) to build understanding. To visualize this possible cognitive benefit, imagine each type of function represented by a different colored square (see Fig. 2). Taken individually, a person sees two different squares.

Now overlap these two representations. Doing so reveals that there may exist a relationship between these two objects. Helping the learner to connect the two forms maintains their individual shape as squares, while simultaneously revealing that they also exist as one face of a shape with greater dimensionality, such as a cube (see Fig. 3).

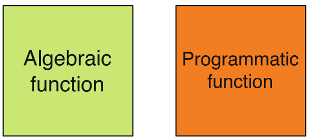

Fig. 2 Visual representation of the concept of "function" in two different domains

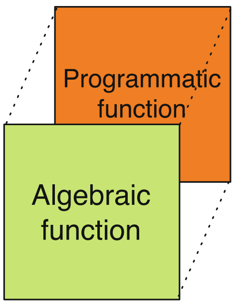

Fig. 3 Connecting the squares reveals they are two sides of a cube

This synergistic relationship wherein combining two objects reveals a more complex object is what we believe may occur when a learner connects a core concept from two different domains.

Core attributes: processes

An example of a core *process* in language learning is manifest through verb tense and person. Verb tense suggests to learners that there exists a notion of time (i.e., tense) and different ways to express that time (e.g., present, past, past perfect). Likewise, the concept of "person" suggests that there are different ways to address people depending on their physical and affective relationship to the speaker. In Spanish, for example, the person "vos" once indicated a class difference between the speakers. This distinction is present in many other languages (e.g., "thou" in old English, "tu" in French). The process occurring in determining time and place deals with positioning. The speaker positions him/herself in relation to actions, people, places, and other verbs. As L2 learners become exposed to such processes, they begin to recognize the different forms they might take in their L1, possibly altering their underlying notion of what "person" means (Jarvis and Pavlenko 2008) and how to express relationships between speakers. Learners use their L1 knowledge to make sense of L2 processes and, concomitantly, their L2 knowledge sheds new light on their understanding of their L1, making processes apparent that may have formerly been obscure, thereby changing their underlying understanding of that particular process—across both L1 and L2 (Chen et al. 2010).

Problem solving, representation, and reasoning are examples of Processes shared by mathematics and computer programming. Problem solving is central to both mathematics and computer programming and has been studied extensively, demonstrating an almost universal acceptance that these two subjects share this process (Palumbo 1990). In fact, both mathematics (Schöenfeld 1992) and computer programming (Raiaravivarma 2005) have been characterized as essentially problem-solving activities. Another process—representation—was deemed by The National Council of Teachers of Mathematics (2000) (NCTM) as the mathematics focus for 2006–2007 and is one of NCTM's five central process standards. Mathematically, representation is defined as the variety of forms or ways of communicating a given concept, or of understanding the various isomorphs of a given problem (Kotovsky et al. 1985). For example, data may be represented graphically, such as in a pie chart, a histogram, a plot, etc. That same data might be represented algebraically, numerically or verbally. According to NCTM (2006),

> Representations should be treated as essential elements in supporting students' understanding of mathematical concepts and relationships; in communicating mathematical approaches, arguments, and understandings to one's self and to others; in recognizing connections among related mathematical concepts; and in applying mathematics to realistic problem situations through modeling. (p. 67)

Representations are therefore a way of thinking about, reasoning through, and communicating understanding of mathematical concepts. Representing information in a pie chart immediately communicates specific information (e.g., amount in relation to other amounts), while also limiting the interpretable scope of that data (e.g., data must represent 100 %).

In computer programming, representation is manifest through data structures. At the most basic level, data structures are composed of data types. For example, a programmer can create a variable called "mySavings." The programmer then determines what data type "mySavings" represents. Even though the "mySavings" variable might be assigned the same value of

1,000,000, whether it is represented as a regular number, an integer (no decimals), a floating point number (rational number with decimals), or even a string (numbers and letters) will affect the manner the system interprets and limits that data. At the more complex level, data types are part of larger data structures, such as a stack, a list, a queue, a dictionary, a heap, a matrix, a vector, a probability distribution, etc. Certain structures work best with specific algorithms, and permit the execution of dedicated functions. Thus, as in mathematics, the way the programmer chooses to "represent" information affects the way that information may be processed and interpreted by the computer.

Certainly not all concepts or process attributes of mathematics and computer programming are shared; the same is true with languages. Yet, there exists a significant overlap of core attributes in each complementary pair. Learning and manipulating core attributes from one complement reinforces and enhances an underlying comprehension of that attribute from the other.

Duality

The second principle of convergent cognition that we propose deals with an important relationship between the applied and abstract interpretations of the two complements; one of the two complementary subjects tends to be interpreted in learners' minds in a more abstract manner, while the other encourages learners to focus on application (see Fig. 1). It is important to understand, however, that there is nothing inherent in the nature of a subject that makes it more abstract or more applied; both share applied and abstract qualities. Learners, however, tend to interpret one or the other as either more abstract or more applied. This duality emerges from the ways students interact with complementary subjects.

Kotovsky et al. (1985, 2011) recently demonstrated the importance of the relationship between abstract and concrete visual representations on students' near transfer of problem solving tasks in a series of experiments. In the first experiment, an electrical circuit unit was taught to 71 high school students using either abstract, concrete, or abstract and concrete visuals and instructions. The group receiving both concrete and abstract instructions and visuals outperformed the other groups in problem solving ($p < 0.005$). In the second experiment, 128 high schools students with no electrical engineering experience were given similar learning materials, the only difference being that students were not told to explicitly draw either a concrete or abstract visual to solve the problem. As in the first experiment, the group exposed to both concrete and abstract learning materials outperformed the concrete-only group ($p < 0.05$). The authors propose that these studies "indicate that problem solving is fostered when learners experience concrete visual representations that connect to their prior knowledge and are enabled to use abstract visual representations" (p. 32).

In the case of languages, the learners' understanding of their L1 tends to be rather abstract. It is not necessary to be able to identify verbs and nouns to be able to form a basic sentence in one's native tongue; after all, students learned how to speak the language long before learning to identify its constituent parts and functions. Learning about a language is in a large degree independent of the way learners actually use that language. Learning about language is learning about its modularity. However, learning an L2 is usually a process of consciously applying specific linguistic rules and principles, regardless of pedagogy (e.g., direct instruction or immersion). L2 learners are able to use these modules to reason through and construct novel sayings. The applied nature of needing to consciously create an L2 clarifies and often makes explicit abstract or implicit L1 understanding.

In mathematics and computer programming, mathematics most often assumes the role of the more abstract subject. Mathematics in itself is a way of abstracting relationships observed in the natural world. On the other hand, while programming involves many abstract concepts itself, the act of programming is a consciously creative act wherein a learner implements and tests learned material. As with learning an L2, learners are involved in the conscious creation and execution of programming. They implement a concept and then verify to see if it is applied correctly, receiving quick feedback on their application. Thus, in computer programming, one must apply (and often make explicit) mathematical concepts, the understanding of which may be somewhat hidden or implicit to the learner. An apparently important aspect of an applied complement is the ability to test and quickly receive feedback in its application.

Both the abstract and applied nature of the complements are essential to our understanding of convergent cognition. There is a certain myth in today's education that application is more important than abstraction. This is a false and dangerous dichotomy. The ability to think in the abstract has been the impetus for innovations throughout centuries. The importance of the applied and abstract relationship might well be understood through Krashen's theory of second language acquisition. Krashen (1987) postulated two types of language learning—acquired language and learned language. Acquired language is gained through experience, by being immersed in and practicing the language. Learned language is the didactic instruction learners receive about that language. Both are important. An L2 learner that has gained his/her new tongue by immersion is often able to converse quite fluently and more quickly than those that only learn that language didactically, suggesting to some that applied language learning is better than abstract language learning. However, as in the cases of concrete and abstract visual representations discussed above (Kotovsky et al. 1985, 2011), learners that possess both acquired and learned language are much more adept at creating and reasoning about novel utterances. Immersive experiences are strengthened, not weakened, by didactic instruction, and vice versa. Likewise, the strength of convergent cognition lies in the duality of the applied and abstract relationship between complements.

Setting the stage: creating convergent conditions

The preceding section discussed key principles that help to explain why and how convergent cognition might occur. In analyzing these principles through our literature review, we noted other considerations that, although not apparently requisite for convergence, nonetheless appear important to cultivating an effective and lasting relationship between complements. To better understand the nature of these conditions, it is helpful to return to our bridge metaphor. Over time, a beam bridge will tend to bow due to the forces of tension and compression (see Fig. 4a). Trusses can be added to reinforce the bridge against these forces, increasing the longevity of the bridge and strengthening it against possible storms (see Fig. 4b). The conditions we noted that may add strength and longevity to convergent cognition are (a) independent utility and (b) intellectual maturity.

Independent utility

Independent utility is the notion that each complement must be an end unto itself and not learned simply for the sake of improving capacity in its complement. When one complement is relegated to a pedagogy, it is subject to the curriculum pendulum that will

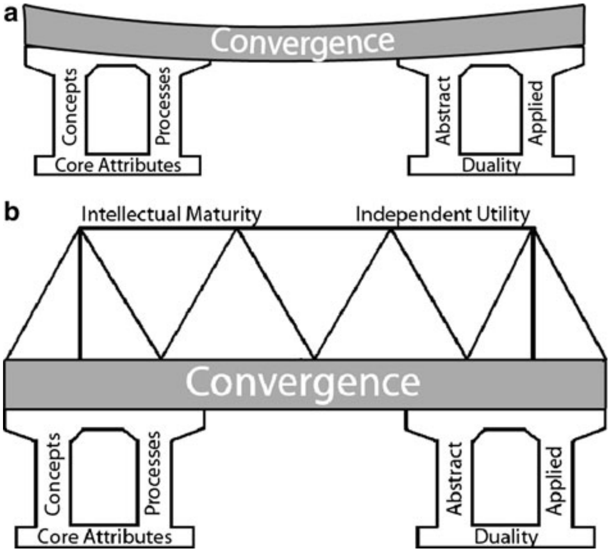

Fig. 4 **a** A bridge that has bowed under pressure. **b** A bridge reinforced with trusses is more able to withstand (political) storms and pressures

eventually swing in the opposite direction, extinguishing the study of that subject. This subservience further reduces the relevance of that subject in learners' minds, and the intellectual need necessary to motivate learning, phenomena that have been manifest through both of our example pairs, L1 and L2 and computer programming and mathematics.

We presented evidence that studying Latin for 30 min/day has been shown to improve students' English ability all across the United States. The studies providing this evidence were largely conducted using randomized controlled trials. Why then, are we not still teaching Latin in the schools en masse? The answer may be that the study of Latin for the sake of improving English was just that—Latin for English improvement. In the early 1980s, the *whole language* movement began to edge out Latin as a pedagogy for English language arts curricula. Another contributing factor to the demise of Latin programs was the interpretation of the 1983 report, *A Nation at Risk: The Imperative for Educational Reform*, which initiated a "back to basics" movement, essentially eradicating Latin as an English language arts pedagogy. Despite the proven benefits of studying Latin, there was no general outcry from students or parents about the loss of Latin. Indeed, the promise of whole language was exactly the missing element from the Latin curriculum—relevance. The same could be said of the back to basics movement; people saw a purpose for studying the parts of English they thought to be important and proposed to do so in a manner that tied content to meaning. Even in the face of statistically significant and overwhelmingly positive results, there was no reason to continue to study Latin without the notion of independent utility. Latin was not seen as relevant for its own sake.

Similar to the disappearance of Latin, Logo is now rarely found in the common curriculum, despite it once having a strong following the world over. Although Papert's intention was for Logo to be a fundamental rethinking of educational practices and the role of technology, the influx of computers in school classrooms in the early 1980s saw a wide variety of application. In response to apparently misguided usage, Papert disappointedly commented: "It started a route that made it [Logo] a school thing, whereas originally… it was *instead* [italics in original] of school" (Agalianos et al. 2001, p. 483). For many students and teachers, there was no purpose in learning Logo other than to learn a particular mathematical concept. As with Latin, the subservience of one subject (computer programming) to another (mathematics) relegated it to pedagogy. As new methods and strategies of teaching mathematics emerged, including more complex and interactive computer software, the use of Logo waned (see Sinclair 2008 for more insights into "the decline of Logo").

The fact that each subject needs to stand on its own does not mean that the connection between the two need remain occulted. In fact, our review suggests that just the contrary is warranted; when the connection between complements is deliberately made, convergence appears to be greater. For example, Masciantonio (1977) reported on a study that compared the English skills of Latin-studying, foreign language studying, and non-foreign language studying students. The students that performed the best were those that had studied Latin. Likewise, those that studied a foreign language significantly outperformed those without any L2 training. Latin-studying students benefited from the deliberate connection between the two languages, while foreign language studying students also benefitted, but not to the same extent. Similarly, the curriculum followed by the Dr. Scheme approach deliberately emphasizes algebraic concepts and provides a supportive environment for repeated practice, perhaps helping to explain the success of Dr. Scheme students in algebra. This observation suggests that *mindful abstraction* (Salomon and Perkins 1989), a core component of high-road transfer, may be an important element to fostering effective convergent cognition.

Thus, while there appears to be a benefit in making explicit connections between complements, a subject must be an end unto itself in order to maintain relevance, or independent utility.

Intellectual maturity

Another important condition for maintaining and fostering convergent cognition is *intellectual maturity*. It appears that there must be some level of cognitive competence in one subject to begin to see convergence in a complementary domain. This phenomenon is manifest in L1 and L2 learning as well as mathematics and computer programming learning.

A longitudinal assessment of Australian aborigines schooled in both their L1 and English (an L2) compared yearly academic achievement in oral English (using the peabody picture vocabulary test, ready to read series, and the story retelling test), English reading comprehension (using the Dolch sight words test, the Schonell reading age test, and a Cloze test), and English composition, to their English-only peers. Students were tested in their fifth, sixth, and seventh years of study. Initial results placed bilingual learners behind their English-only counterparts. In sixth grade, *t*-tests revealed no difference between groups, and by seventh grade, bilingual students significantly outperformed English-only counterparts in several tests ($p < 0.001$) (Gale et al. 1981). Recent analyses confirm this observation (Chen et al. 2010). A study of bilingual students in the London school system that used detailed statistical regression to match student achievement with L1 and L2 language ability found that lower English (L2) ability correlated strongly with low achievement, but that fully bilingual students outperformed even their English-only peers ($p < 0.001$) (Demie and Strand 2006).

The question raised by these studies is not whether L2 instruction is beneficial, but at what point does it become a boon to supporting L1 development? In a study of over 700,000 students in different types of bilingual education programs in the U.S., Collier and

Thomas (2002) determined the two most important features of successful bilingual programs. First, the length of time is critical. Specifically, it takes four to seven years for L2 students to catch up to and eventually surpass their peers academically in their L1. Second, the type of dual language program makes a vital difference. Only two-way immersion and 50/50 one-way immersion programs resulted in students performing consistently above grade level. Immersion programs are those in which students learn subject material in the language they are learning. For example, a child in a Chinese immersion program might receive mathematics and science instruction in Chinese. In their review of bilingual programs, Howard et al. (2003) declared, “Only those groups that received strong, grade-level cognitive and academic support in both their first and second languages for many years were found to be succeeding at the end of high school” (p. 12). Thus, we cannot reasonably expect to see positive convergence occur until a minimum competence has been gained in one’s L2 and L1.

In mathematics and programming, a common criticism of early Logo studies was that they did not deliver the promised levels of transfer to other seemingly related subjects (de Corte and Verschaffel 1986, 1989). Linn and Dalbey (1985) suggested that adequate training, practice, and time between experiences are essential to foster transfer of programming skills to other problem-solving contexts. When this recommendation was followed, the results were positive. For example, third-graders programming in Logo for 1 h a week performed significantly higher on a test of rule learning than those with a half hour a week of programming exposure (Gorman and Bourne 1983). In another study (Miller et al. 1988), benefits of increased geometric understanding did not emerge until students had at least a year of experience in working with Logo. Some early studies failed because they did not account for intellectual maturity with the first subject before testing for transfer in the complementary subject (Palumbo 1990). Therefore it seems that exposure to a complementary subject alone is insufficient, and that we must test for intellectual maturity in one domain before testing for convergence between the two (de Corte et al. 1992). There is a need to ensure not only sufficient exposure, but also some level of skill mastery in a subject in order to begin to see the positive effects of convergent cognition.

Implications

Convergent cognition as a bi-directional, cross-domain phenomenon presents implications for both policy and research in current educational systems. We first discuss the policy implications and conclude with recommendations for future research.

Policy

Current efforts to improve educational quality have sometimes led to the winnowing of curriculum. For example, in California, the requirement that all eighth grade students pass an algebra test has resulted in the phenomenon of “shadow classes” wherein students who need strategic intervention to succeed on the test are given an extra class hour of support (California Department of Education 2010). Increasingly, schools require students that underperform in language arts and mathematics to spend extra time studying these subjects, whereas students that perform well in core areas may choose the time for other activities (Paris and McEvoy 2000). By necessity, this extra time focusing on required subjects diminishes time spent in other elective courses, and likely will lead to an increased Matthew effect (Stanovich 1986). Likewise, in an effort to promote mathematics and language arts, many elementary schools have eliminated afternoon recess (Friel 2003), and reduced or eliminated social studies, science, foreign languages, art or music (Rosenthal 2004; Stewart 2005).

Despite increased efforts to support the core subjects, recent international testing demonstrates no gain in student scores. In 2003, the U.S. participated in the Program for International Student Assessment (PISA), which tested 15-year-olds' science and mathematics skills from 41 different countries. U.S. students placed below average internationally in both categories (OECD 2007). Three years later, in another PISA study ($N = 57$), U.S. students' scores were statistically the same, demonstrating no gains country-wide. Results from 2009 also suggest no significant change. Perhaps narrowing curriculum to focus on "the basics" may not be the most effective educational approach. The presented evidence suggests that rather than reduce curriculum to focus on a few core subjects, systematically pairing related subjects may lead to increased learning in both subjects.

There is a need to diversify curricula. However, this does not imply that such diversification should be haphazard (as in the phenomenon of the "shopping mall high school" so eloquently lamented by Powell et al. (1985)). Rather, there ought to be a deliberate union of subjects, paired in such a way as to reap the benefits of the synergy of convergent cognition. The complementary pairs of L1 and L2 and mathematics and computer programming are likely good places to start, given the extant research. In addition, researchers need to identify other complementary subjects and in what specific ways they might converge. Effective diversification, however, must also account for the conditions of independent utility lest the learning of one subject become merely a pedagogy to teach another subject, reducing its overall relevance, and leading to its eventual dismissal.

Research

The research we have presented is considerable, but certainly not conclusive. Many of the studies were correlational or descriptive, and do not indicate causation. There are many variables that contribute to the success or failure of learners and we do not mean to suggest that convergent cognition is some sort of panacea that will resolve them all.

Despite the individual blemishes of the presented research, however, the repeated preponderance of correlational, experimental, and anecdotal evidence between complementary subjects suggests a relationship worthy of attention. Inasmuch as these relationships exist, it is incumbent upon researchers to examine the principles governing these relationships and to experiment with differing conditions to determine learning configurations that can foster and capitalize on convergent cognition.

Beyond the obvious need to identify complementary pairs, research needs to be conducted to identify and understand core attributes that exist between pairs. It is insufficient to know that subjects X and Y complement each other. Rather, we need to understand how and in what ways complements might improve a student's underlying conceptions. As Campione and Brown (1981) pointed out, experts often possess this knowledge. Thus, teasing out core attributes may be a matter of capitalizing on the many studies conducted on expert thinking. Experts also have informed opinions on which core attributes are most important. Identifying such connections and arguing their relative value will enable educators to design curricula that purposefully maximize the potential synergy of convergent cognition. Understanding the core attributes of a complementary pair will equip educators with multiple ways of explaining and manipulating important concepts within and across disciplines. This understanding may also better inform curricular decisions of how best to find a home for complementary subjects within programs of study (see Johnson 2000, for such a suggestion with respect to programming and mathematics).

We also acknowledge the research that indicates that students perform better when the connection between complements is intentional. We need to know more about the importance of learners being explicitly aware of the relationship between concepts, as well as the degree to which convergence occurs with mere implicit awareness. Is it sufficient to make connections to a subject's complement deliberate (i.e., the curriculum specifically chooses tasks where there is a connection to its complement, but does not explicitly point out this connection to students)? Or must we explicitly point out and make connections between complements in order for students to gain the greatest benefit and to experience convergent cognition? Salomon and Perkin's (1989) research on high road transfer, wherein learners engage in *mindful extraction* may be a key to understanding how and to what extent to make connections either deliberate or explicit.

In these efforts to make connections more explicit, however, we must take into account the conditions discussed previously. We need to respect the condition of *independent utility*, lest we fall into the same trap of relegating one subject to a lesser status (e.g., Latin and Logo), thereby losing the important condition of relevance. And it would be imprudent to believe that anything but a sustained effort at teaching both complements would yield significant convergence. Intellectual maturity must be attained, but to what extent? At what stage do learners begin to use their knowledge of one subject to comprehend or explain concepts in its complement? Language learners, for example, seem to begin to make connections between L2 and L1 rather soon, especially where cognates are concerned. Likewise, the converse may also be true; there may be a saturation point at which there is no longer a positive increase from learning in one subject to another. Or, is there a point at which too much knowledge of one subject leads to negative transfer to its complement? Intellectual maturity is important, but to what extent? And when does it begin?

In sum, *convergent cognition* is a theory that seeks to explain why the learning of one subject often correlates with improved performance in another, possibly accounting for the simultaneous occurrence of multiple forms of transfer. Systematically pairing certain subjects may not only diversify educational curricula, but may also lead to greater improvements in core areas. There remains much to investigate about convergent cognition, such as which subjects complement each other and in what ways, how we should teach complementary subjects, and how much understanding of one subject leads to convergence with another. We propose that answering such questions could be extremely valuable to the educational community in our ongoing efforts to help all students learn better. As attributed to Arthur Aufderheide, "All knowledge is connected to all other knowledge. The fun is in making the connection." The lens of convergent cognition is an attempt to explicitly leverage such connections.

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