Institut d'Etudes Politiques Ecole Doctorale

Prices' dynamics: could covariances highlight the financialization of commodities?

May 30, 2013

Master thesis submitted by Sarah Dijols in partial fullfillment of the requirements for the Master's Degree (Master Economic and Public Policy, PhD track.)

Supervisor:
Nicolas Coeurdacier

Abstract

Excess comovements of the prices of commodities as studied in the early 1990s by Pindyck and Rotenberg have been observed more accurately in the last decade while a sharp increase in the popularity of commodity investing was also evidenced. In this paper, we first attest these excessive comovements have been exacerbated in the United States as compared to two selected emerging countries. To understand these distinguishing trends, we build a simple framework of financial assets comovements which introduces the covariances of commodities returns with a commodity index return as a key factor in tying up commodities prices.
This intuition is confirmed empirically, covariances with the index risk premium enter significantly as predictors of commodity specific risk premium. Further, returns on the non-indexed commodities are shown to be less responsive to covariances' changes as predicted by the model.

I am grateful to Nicolas Coeurdacier for his availabilility and encouragements while supervising this master thesis. I shall also be thankful to Yannick Lepen for having suggested an interesting pursuit of this work and Guy Laroque for a helpful discussion on this topic.

1 Introduction and Literature Review

The singularity of commodities markets lie in the environmental risk associated with production. Futures were introduced in the mid-19th century to cope with this systemic risk and provide insurance to producers and consumers of commodities. Some evidence suggest they have helped producers to take more broad-based decision on production, storage and marketing of products whether they were agricultural or energy-related. [1]

The benefice of futures commodities was twofold. Their growth has fuelled a steady development of agriculture infrastructure, which include storages, vaults, transport and logistics associated with the expansion of commodity markets. Further, commodities markets being characterized by high volatility of returns, some studies (for instance **?**) have evidenced that the introduction of futures contracts could account for reducing volatilities. This latter point being also consequential of the expansion of commodities markets fuelled by commodities futures.

Historically, futures commodities contracts were proeminently exchanged by commercial or institutional commodities producers or consumers, most participants were "hedgers" who traded futures to maximize the value of their assets, and to reduce the risk of financial losses from price changes. However, for the last decade new participants referred as "speculators" have entered those market in an attempt to profit from price changes in futures contracts.

Financialization of Commodities

The literature suggests that before 2000s, commodity markets were partly segmented from outside financial markets. Erb and Harvey (2006) had shown that commodities had only low positive returns correlations with each other, while **?** demonstrated that commodity returns had negligible correlations with the SP500 returns. However, the combination of various factors made commodities gradually appear as a new asset class.

A growing body of literature has acknowledged the increase in the amounts of transactions on commodities markets(**?**, **?**, **?**). According to BIS statistics, **?** report a 14 fold increase in the notional value of OTC commodity derivatives contracts outstanding between 1998 and mid-2006. In 2009, **?** reported that the number of futures and options contracts outstanding in commodity exchanges worldwide had rose more than threefold between 2002 and 2008; while they reported a 14-fold increase in the notional value to 13 trillion$ (BIS).

Generally large increases in open interests positions; and spread positions have been

[1] A commodity futures contract is an agreement to buy or sell a particular commodity at a future date, the price and the amount of the commodity are fixed at the time of the agreement. Most contracts contemplate that the agreement will be fulfilled by actual delivery of the commodity but most are liquidated before the delivery date. A commodity futures option gives the purchaser the right to buy or sell a particular futures contract at a future date for a particular price.

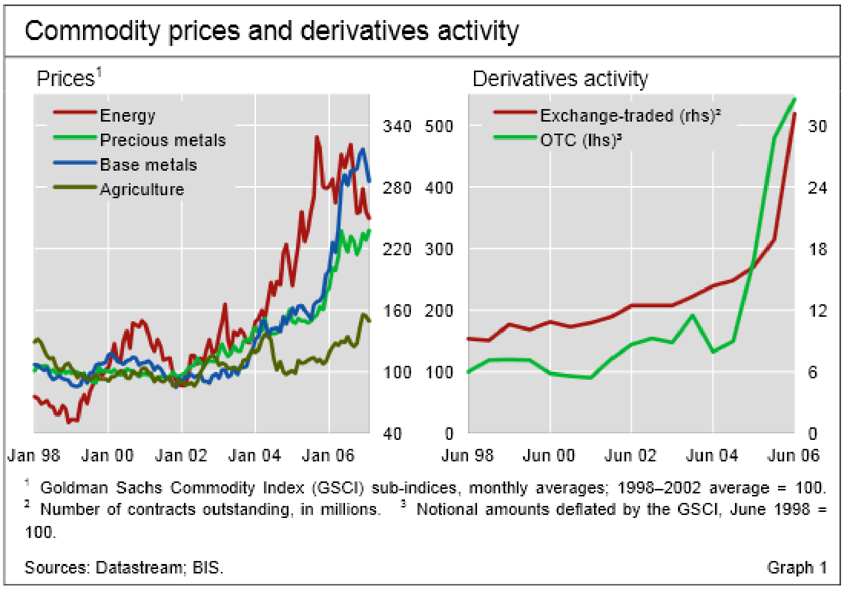
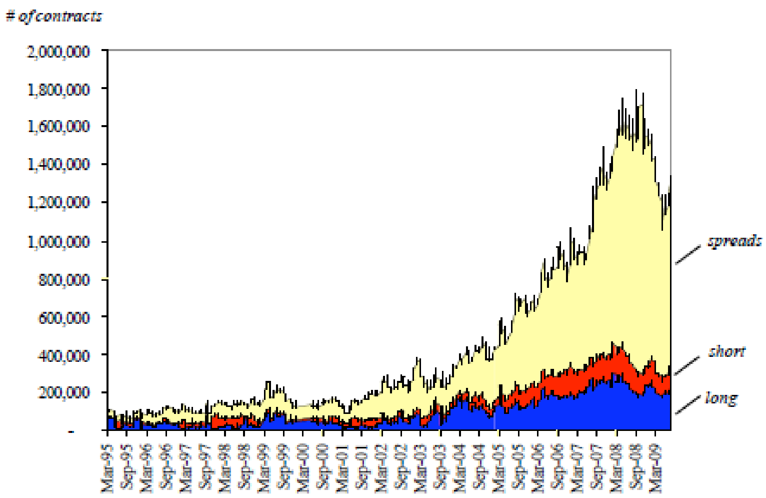


Figure 1: A strong and sustained increase in spread positions.



Source: CFTC Commitment of Traders Reports

According **?** spread positions is a natural measure of the relative importance of speculative activities. A spread is the purchase of a future contract regarding one delivery month, against the sale for another delivery month.

observed.

This process has coincided with a surge in prices comparable with the ones of the 70s; while inflation was by no mean comparable. This rise in prices preceded a sharp decline in the aftermath of the 2008 crisis, but the increasing trend has bounced back and has continued for most raw materials since then. Particularly noticeable was that this rise in prices was accompanied by a rise in volatilities of prices and comovement between commodities prices.

Excess comovements

Evolution and comovement of the prices of commodities may be determined by theirs relationship to the exchange rate and interest rate; many studies account for these relationships from **?** to **?**. The latter, using a VARs analysis discusses on what extent low real interest rates and the decline of the dollar can account for high commodity prices and whether commodity prices tend to display overshooting behavior in response to interest rate changes. His work consider the period ranging from 1990 to 2007. The former built a model in the same vein of Dornbush to account for the overshooting behaviors of commodities prices. Excessive behaviors when studying commodities seem the norm.

Highly volatile series, commodities also exhibit comovements. The puzzling phenomenon of comovement in prices of commodities is not new, **?** already in 1990 accounted for "excess comovement" in the sense that it is in excess of anything that could be explained by common effects of macro-economic shocks such as inflation; or changes in aggregate demand, interest rates and exchanges rates.
Recently, **?** have used a large approximate factor model to study commodities prices comovement. They gather 187 macro-economic variables which represent the main forces driving commodity prices. They are used to filter out the returns of a set of seemlingly unrelated [2]commodities. The residual correlation once accounting for these set of control variables is examined to investigate the issue of excess comovement.
They first regress commodity returns on these factors, and acknowledge the important role played by emerging countries in shaping commodity prices in recent years. Further, they show that measure of hedging and speculative pressure are able to explain a very significant part (around 60%) of their estimated time-varying excess comovements.

Increasing linkage with stocks markets

Different approaches have been used to account for these observations: **?** use the Dynamic Conditional Correlation (thereafter DCC) introduced by Engle to observe time-varying correlations between stocks and 25 different commodities. They acknowledge speculation phenomenon for oil, coffee, and cocoa. [3] They evidenced a tendency of rising correlations witht stocks for many raw materials. Both markets move upward in period of growing world demand for commodities, and as presented below and evidenced in our subsequent model, commodities as an asset class may offer new perspectives of investment when stocks markets get bearish.

Our focus in this paper is on the impact of institutional investors (who follow index investment strategies) on this rising trend in prices; and on the increase volatilities and comovements within this asset class and with other financial assets as evidenced by **?**. We will therefore first briefly present why news investors have

[2]In the sense given by Pindyck: their prices are unrelated meaning that their supply and demand cross-elasticities are almost null

[3]They use the term speculation for simplifying purposes to refer to a situation in which investors (i) engage in transactions to profit from short-term fluctuations in the market value of the considered asset or product, and (ii) focus only on price movements rather than on the fundamentals linked to the considered asset or product.

entered the commodities futures market and how they may have affected prices, their volatilities and their comovements.

Investors motives

"*The history of food took an ominous turn in 1991...the year Goldman Sachs decided our daily bread might make an excellent investment*"
Frederick Kaufman, The Food Bubble: How Wall Street Starved Millions & Got Away with it, Harpers Magazine, July 2010.

Investment in commodities has been perceived by financial investors as a way to diversify their portfolio and hedge against inflation. In the early 2000,**?** provide evidence that historically commodity prices have had a relatively low correlation with prices in other asset classes and a high correlation with inflation; further they show that the return on a diversified basket of long commodity futures has been comparable with the return on other asset classes with similar risk features, such as equities. Since then, institutional investors have promoted investment in commodities futures as an effective way to reduce portfolio risk.
Further, in periods of financial and economic distress, market conditions are often favorable to increasing commodity prices, making investment in commodities futures a profitable strategy.

Institutional investors follow built-up indexes. Each index is made up of its own uniquely weigthed basket of commodities. The large share devoted to energy futures reflect the quasi-uniqueness of the "backwardation symptom" in the oil market. The largest index, SP Goldman Sachs Commodity Index (SPGSCI) devotes a very large proportion to energy futures (roughtly 70%) while the second largest fund, the Dow Jones UBS had about 33% of its mix in the energy sector.

Trading commodities derivatives

A traditional view on market efficiency would make deviation of prices from their fundamentals rare and temporary. If institutional investors had any price impact and drove a wedge between market prices and fundamental values, the arbitrage opportunity would cause fundamental (rational) traders to trade against wrongly informed investors and bring market prices back to fundamental values . Proponents of this view explain the prices surge of the last decade by the increasing demand from emerging countries, especially China. (Kilian; **?**).

Another source of confusion relates to alleged "logical inconsistencies" in the view that financial investment can affect prices even though it only relates to futures market activity and does not concern spot market transactions. The causal link between the position taken on futures markets by institutional investors and the evolution of the cash prices remain complex and unclear. However, for most commodities markets, as acknowledged in the literature (Kilian,**?**,Gilbert) price discovery seldomly takes place at delivery, most transactions are executed according to futures prices with reference made to the price of the nearby futures contracts. Hernandez and Torero (2010) supported evidence that changes in futures prices lead changes in cash prices more often than not for their case study on wheat, maize and soybeans.

Many studies found a clear effect of trading strategies on the prices evolutions. **?** studies how returns (roll and spot returns) and diversification (proxied by the correlation between returns and SP500 or the dollar, and inflation) may influence positions taken by traders. This implies regressing the share of net long positions of index (or money-managers)traders on the explanatory variables just detailed. Further, using a Autoregressive Distributed Lag model suggested by Gilbert, he examines causal lead and lag dynamics between position taking and price developments. In the following equation:

$$r_t = \sum_j^3 \alpha_j r_{t-j} + \sum_j^3 \beta_j x_{t-j} + \sum_j^3 \gamma_j z_{t-j} + \epsilon_t$$

where z_t is the weekly change in the net long positions of "money managers" and x_t the weekly change in the net long positions of "index traders". Mayer tests for the null that Index positions (respectively Money Managers positions) do not Granger cause prices. His results indicate that index traders cause changes in prices of soybeans, soybean oil, copper and crude oil, while suggesting a causal impact only for maize. The tests on reverse causality reveal that it is only index position taking in gold that is affected by price development.

? focusing on the roll strategies of index traders, and using panel-data regressions documents a subtantial impact of roll strategies on the price of oil. Combining with Mayer's finding that roll returns from investment in oil have a strong and significant impact on index-trader investment; their results support the hypothesis that index-position changes Granger cause oil-price changes.

Finally, our model builds on the finding of **?**. These authors examine the comovements between indexed and off-index commodities. Their indentification strategy being built on the finding of **?** that after a stock is added to a SP 500 Index, the price comovement with the index increases significantly.

Considering that index investors are not particularly sensitive to prices of individual commodities because they tend to move in and out of all commodities in a given index at the same time on the basis of a strategic allocation of their capital to commodities versus other asset classes such as equities. As a result, any shock to their strategic allocation to the commodity asset class could cause commodities in the index to move together. Thus they were expecting price comovements of commodities in the SP GSCI and DJ-UBSCI to be greater than those of off-index commodities.

They indeed observe an increasing correlation of non-energy commodities with oil prices; this trend being more pronounced for commodities in two popular commodity indices.

To the best of our knowledge, few models were developed to explicit how index investment would tie together a set of commodities, and precipitate higher comovements between their prices; and therefore between investors returns on holding (usually) long positions on them.

The ambition of our model and its empirical assessment is to capture part of this controversial fact. Section 2 introduces the model built on the literature of comovements between financial assets; we propose a simple framework which introduces the covariances of commodities returns with a commodity index and explicit the effect of this additional term on prices dynamic. Section 3 is a comparative approach over three different regions: Brazil, China and the United States; We introduce the dynamic correlation framework recently proposed by Engle to compare commodities' prices dynamic in those three markets and present evidence of the specificity of the American market. In section 4, we assess empirically the intuitions presented in the model to evidence some specific effects of index investment on tying up commodities' prices movements.

2 The Model

The finance literature traditionally models derivatives and hence futures contracts with Brownian motion (as **?**). Since our focus is on comovement between commodities and equities prices; and since spot and futures prices for commodities tend to be highly dependant (Spot prices are the underlying asset upon which derivatives are based, as highlighted in **?**, we will not model derivatives explicitely. Deriving intuitions similar to ours from a mixed portfolio- mixed in the sense that it contains *regular financial assets* in positive supply and *derivatives*(futures on commodities) in zero net supply- would be a promising avenue of research.

In our simple framework, the economy contains a riskless asset, which we assume to have zero rate of return, and 2n risky assets in fixed supply. The first n assets are regular financial assets with a dividend process similar to Barberis. The second part of the assets are claims on commodities produced; whether the futures based on them is in an index or is not. We will refer to them as indexed or non-indexed commodities.

The risky asset is a claim to a future dividend $D_{i,T}$ to be paid at some later time T. As in Barberis, we assume

$$D_{i,T} = D_{i,0} + \epsilon_{i,1} + \epsilon_{i,2} + \epsilon_{i,3} + \epsilon_{i,4}...\epsilon_{i,T}$$

Where $D_{i,0}$ is known at time 0, whereas $\epsilon_{i,t}$ becomes known at time t. We assume

$$\epsilon_t = (\epsilon_{1,t}, \epsilon_{2,t}, \epsilon_{3,t}..., \epsilon_{n,t}) \sim \mathcal{N}(0, \Sigma_A)^4 \text{iid over time.}$$

The same idea lies for modeling the income received from holding the commodity. This assumption of a final payment at time T is even more accurate for modeling investment in a commodity (in a future backed on a commodity) since when an agent contracts long on a future as explained in Appendix, he will receive a payment at T of the form $F_{t0,t1} + b_T$, where $F_{t0,t1}$ stands for the price of the future contract at time t_0 for delivery at time t_1. The only assumption we relax

[4] The A superscript stands for Asset

while considering commodities *themselves* rather than futures, is the *limited and positive supply Q*. If we were to consider futures, we would need Q=0 (zero net supply) ; but this makes the model hardly tractable.
Rather, we now have a process similar to the regular financial assets for the commodities. To rationalize this simplification, one simply needs to assume investors mostly take long positions on commodities futures, hold this position until time T, and expect to collect $F_{t0,t1} + b_T$; long positions are in positive and limited supply hence we can assume $Q > 0$. For lisibility we will denote the profit made on the futures as D_t as well.

$$D_{i,T} = D_{i,0} + \epsilon_{i,1} + \epsilon_{i,2} + \epsilon_{i,3} + \epsilon_{i,4}...\epsilon_{i,T}$$

However, the process ϵ_t should not follow a normal distribution, as evidenced from the literature and from our empirical assessment. Commodities prices tend to be highly volatile, hence the normality of returns cannot be assumed. Rather we could assume ϵ_t follow a heavy-tail type of distribution centered at 0.

$$\epsilon_t = (\epsilon_{1,t}, \epsilon_{2,t}, \epsilon_{3,t}..., \epsilon_{n,t}) \sim H(0, \Sigma_C)\text{iid over time}^5$$

The matrix of all ϵ_t for all types of assets is therefore: $\Sigma_D = \begin{pmatrix} \Sigma_A & * \\ * & \Sigma_C \end{pmatrix}$

Further we assume the variation in price to represent the asset's return, this is essentially for tractability. That is

$$r_{i,t+1} = \Delta P_{i,t} = P_{i,t} - P_{i,t-1}$$

It first allow us to have a constant variance structure as detailed below (Adding dividend D_t in the return expression would yield a time-varying variance, and counter-arguments the view that comovement in returns are due to comovement in news about fundamental values). For commodities, it is more meaningful to presuppose a final payment as if investors hold a future contract on them.

2.1 Fundamental View on comovement

Under the fundamental view, comovement in returns are due only to comovement in news about fundamental values of assets and commodities. The traditional framework to present this view is that of a economy with identical fundamental traders that maximize CARA utility and take prices change to be normally distributed.

$$\max_{\omega_t^a, \omega_t^c} E(-exp(-\gamma(W_t + \omega_t'(r_{t+1}))^6$$

where $\omega_t = (\omega_t^{a1}, \omega_t^{a2}, ...\omega_t^{c1}, \omega_t^{c2})$ denotes the weights assigned to assets and commodities. As usual γ governs the degree of risk aversion, W_t is wealth at time t.

We depart from this maximization method because of the non-normality of our returns; yet assuming investors face a VaR constraint as in Etula (2010), we can

[5]The C superscript stands for commodities
[6]The T superscript refers to *Total* return

derive very similar expression for the shares and variances. Our investors maximize $E_t(r^T_{t+1})$ subject to a VaR constraint, where r^T_{t+1} is given by

$$\omega_t^{a1} r_{t+1}^{a1} + \omega_t^{a2} r_{t+1}^{a2} + ...\omega_t^{c1} r_{t+1}^{c1} + \omega_t^{c2} r_{t+1}^{c2} + ..$$

As in Barberis, we may assume the risk aversion of fundamentals traders remain constant over time, that is $\kappa\mu_t = \gamma$.

Following **?** and **?**, we suppose investors maximize expected return on equity subject to a VaR constraint [7] :

$$\max_{\omega_t^a, \omega_t^c} E_t(r^T_{t+1}) \text{ subject to} VaR_t < e_t$$

If we assume VaR_t is some multiple κ of the forward looking standard deviation of the equity returns $e_t\sqrt{Var_t(r^T_{t+1})}$, the constraint becomes $\sqrt{Var_t(r^T_{t+1})} \leq \frac{1}{\kappa}$.

Denote $\omega_t = (\omega_t^{a1}, \omega_t^{a2}, ...\omega_t^{c1}, \omega_t^{a2}...)$ the vector of shares, and r_{t+1} the vector of returns.

The Lagrangian is

$$L = E_{\omega_t^a, \omega_t^c}(r^T_{t+1}) - \mu_t(\sqrt{Var_t(r^T_{t+1})} - \frac{1}{\kappa})$$

Maximing over the share [8] ω_t gives:

$$\omega_t = \frac{1}{\gamma} Var_t(r_{t+1})^{-1} E_t(r_{t+1}) \tag{1}$$

We therefore derive the same very standard expression of the optimal share (or weight) as the ratio of expected return on the variance. $\omega_t = \frac{1}{\gamma} Var_t^{-1} E_t(P_{t+1} - P_t)$

If the total of assets and commodities available is given by a vector Q, then the market clearing condition gives: $P_t = E_t(P_{t+1}) - \gamma V_t Q$

We will roll this equation forward, and set $E_{T-1}(P_T) = E_{T-1}(D_T) = D_{T-1}$ where

$$D_t = (D_{1,t}, D_{2,t}, D_{3,t}, ...)$$

.

Notice that for the commodities, we therefore need to assume that the process followed by the ϵ_t will be a zero-mean process, even if it needs not be normal.

$$P_t = D_t - \gamma V_t Q - E_t(\sum_{k=1}^{T-t-1} \gamma V_{t+k} Q) \tag{2}$$

[7] In **?** and **?** investors risk appetite shifts endogenously with balance sheets constraints that fluctuate with market outcomes. The balance sheet constraints are imposed by a contracting setting which yields a value-at-risk rule: Investors (broke-dealers) leverage is limited by this value-at-risk constraint.
? provide a micro foundation for this constraint from a moral hazard problem between borrowers and lenders.

[8] Here we are assuming the shares dedicated by fundamental traders to assets or commodities are the same, this assumption could be relaxed yielding much more complex derivation

The crucial assumption comes here, fundamental traders set a constant for the variance, given by the above matrix:

$$V_t = \Sigma_D$$

Then equation reduces to

$$P_t = D_t - (T-t)\gamma\Sigma_D Q$$

This means that up to a constant, the price difference will be:

$$\Delta P_{t+1} = \Delta D_{t+1} = \epsilon_{t+1}$$

Therefore the variance, $V_t = Var_t(\Delta P_{t+1})$ is indeed constant, confirming the traders conjecture.

This equation is a very simple way to say that comovements in returns reflects comovements in the news process ϵ_t.

2.2 Introducing institutional investors

We now introduce a second type of investors [9], their total returns is given by

$$r^T_{t+1} = \omega^{a1}_t r^{a1}_{t+1} + \omega^{a2}_t r^{a2}_{t+1} + ...\omega^{c1}_t r^{c1}_{t+1} + \omega^{c2}_t r^{c2}_{t+1}... + q_t r^I_{t+1}$$

where $q_t r^I_{t+1}$ reflects the returns on some holdings q_t of index shares. The index is a commodity index such as Goldman Sachs GSCI or the Dow-Jones DJ-UBSCI. As in Etula, the returns on the index does not enter the market portfolio but it influences the portfolio choice. This specific term for institutional investors (those who invest in an index) carries all the specificities of our simple framework. While maximazing as previously, we now get a slightly modified version of the optimal share for those investors.

The investor has a total return given by: $r^T_{t+1} = \omega^{a1}_t r^{a1}_{t+1} + \omega^{a2}_t r^{a2}_{t+1} + ...\omega^{c1}_t r^{c1}_{t+1} + \omega^{c2}_t r^{c2}_{t+1}... + q_t r^I_{t+1}$
He maximizes $E_{\omega^a_t,\omega^c_t}(r^T_{t+1})$ for all assets a and all commodities c, subject to $VaR < e$ where VaR (Value-at-Risk) can be defined with trader equity e and $Var(r^T_{t+1})$.

We set-up a Lagrangian

$$L = E_{\omega^a_t,\omega^c_t}(r^T_{t+1}) - \mu_t(\sqrt{Var_t(r^T_{t+1})} - \frac{1}{\kappa})$$

Developing the Variance and deriving with respect to a specific weight ω^{ai}_t or ω^{ci}_t yields

$$\frac{d}{d\omega^{ci}_t} E_t(r^T_{t+1}) + \mu_t \frac{(2\omega^{ci}_t V(r^{ci}_{t+1}) + 2cov(r^{ci}_{t+1}, r^I_{t+1})q_t + 2cov(r^{ci}_{t+1}, \omega_i \sum_i r^{ai,ci}_{t+1}))}{2\sqrt{Var_t(r^T_{t+1})}} = 0$$

[9] For a clear understanding of the different type of investors and their behaviors, you can refer to **?** page 27

Since the derivation is inserted in the expectation, this yields:

$$E_t(r^{ci}_{t+1}) = -\mu_t \frac{2\omega^{ci}_t V_t(r^{ci}_{t+1}) + 2cov_t(r^{ci}_{t+1}, r^I_{t+1})q_t + 2cov(r^{ci}_{t+1}, \omega_i \sum_i r^{ai,ci}_{t+1})}{2\sqrt{Var_t(r^T_{t+1})}}$$

Using the binding constraint, we therefore have:

$$\omega^{ci}_t \mu_t \kappa V_t(r^{ci}_{t+1}) = E_t(r^{ci}_{t+1}) - cov_t(r^{ci}_{t+1}, r^I_{t+1})q_t - cov_t(r^{ci}_{t+1}, \omega_i \sum_i r^{ai,ci}_{t+1})$$

$$\text{That is } \omega^{ci}_t = \frac{V_t(r^{ci}_{t+1})^{-1}[E_t(r^{ci}_{t+1}) - cov_t(r^{ci}_{t+1}, r^I_{t+1})q_t - cov_t(r^{ci}_{t+1}, \omega_i \sum_i r^{ai,ci}_{t+1})]}{\mu_t \kappa}$$

The same formula holds for the assets ai.
The key term in this expression is $cov_t(r^{ci}_{t+1}, r^I_{t+1})q_t$, the second term $cov(r^{ci}_{t+1}, \omega_i \sum_i r^{ai,ci}_{t+1})$ which captures the covariance of the commodity ci (respectively the asset ai) considered with all other assets and commodities was implicitely present in the earlier derivation **??**.

Therefore the demands by institutional investors will be rewritten as some constant, plus a term capturing the time-varying covariance and the changing risk appetite of institutional investors.

$$\omega_i, t = \frac{1}{n}[A_A + \frac{q_t cov_t}{\mu_t \kappa}] \text{ for } i \in A \tag{3}$$

$$\omega_j, t = \frac{1}{n}[A_C + \frac{q_t cov_t}{\mu_t \kappa}] \text{ for} j \in C$$

$$\omega_j, t = \frac{1}{n}[A'_C + \frac{q_t cov_t}{\mu_t \kappa}] \text{ for } j \in C'$$

Where C denotes those commodities which are indexed while C' denote those which are not indexed. We might denotes cov_t for $cov_t(r^{ai}_{t+1}, r^I_{t+1})$ or $cov_t(r^{ci}_{t+1}, r^I_{t+1})$ to avoid notational clustering.

The demand for the assets and commodities by the institutional investors depend from now on their expected cov_t with the returns on the index.
This additional term is analogous to the noise introduced by some traders in Barberis model.
Let's observe now how the prices of assets and commodities may respond to changes in the expected covariance.

This economy still has fundamentals traders, their perception of the prices evolution will allow us to derive explicit formula for the returns, or prices' changes. Consider again the price as perceived by fundamentals traders, given their expectations about future prices, current prices are given by:

$$P_t = E_t(P_{t+1}) - \gamma(Q - \omega^I_t)^{10}$$

They treat the institutional traders demand as supply shock. where $\omega^I_t = (\omega^{a1}_t, \omega^{a2}_t .. \omega^{c1}_t, \omega^{c2}_t ...)$

[10]The I superscript stands for Institutional investors

Rolling the equation forward as previously and setting again $E_{T-1}(P_T) = E_{T-1}(D_T) = D_{T-1}$ yields

$$P_t = D_t - \gamma V_t(Q - \omega_t^I) - E_t(\sum_{k=1}^{T-t-1} \gamma V_{t+k}Q - \omega_{t+k}^I)$$

Now, we may distinguish between the commodities in and off index. Denote ci' the subscript for those commodities which are not indexed.Now we have:

$$\Delta\omega_t^I = \begin{pmatrix} \Delta\frac{q_t cov_t^A}{\mu_t\kappa} \\ \Delta\frac{q_t cov_t^C}{\mu_t\kappa} \\ \delta \end{pmatrix}$$

δ reflects a very small quantity. Indeed, for those commodities off index, since the cov_t above is negligible, the demand of institutional traders $\omega_t^{ci'}$ will be not vary much in time; one could assume it is constant and set $\delta = 0$ above.

The fundamental traders conjecture a structure of the variance matrix as:

$$V_t = \sigma^2 \begin{pmatrix} A_t & B_t & * \\ B_t & A_t & * \\ C & D & * \end{pmatrix}$$

Define the covariances between returns within a class as $\rho_{i,j,t}^w$ and the covariances between assets' and commodities' returns as $\rho_{i,j,t}^b$. Traders conjecture the matrix of variance-covariance for periods ulterior to $t+1$ as W, a constant [11]

Further, we will be imposing that the cash-flow shocks (the justifications are not central to our analysis and will be detailed in Appendix(**??**)) $\epsilon_{i,t+1}$ and $\epsilon_{j,t+1}$ will be related such that: $\Sigma_D^{i,j} \equiv cov(\epsilon_{i,t+1}, \epsilon_{j,t+1}) = \begin{cases} 1, i = j \\ \pi \text{ for i,j in the same category} \\ \upsilon \text{ for i,j in different categories} \end{cases}$

However, the matrix V_t might be time dependant, and they might conjecture the following form for A_t and B_t:

where A_t and B_t are defined as

$$A_t = \begin{pmatrix} \rho_{1,1,t}^w & \rho_{1,2,t}^w & \cdots & \rho_{1,n,t}^w \\ \rho_{2,1,t}^w & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \rho_{n-1,n-1,t}^w \\ \rho_{n,1,t}^w & \cdots & \cdots \rho_{n,n-1,t}^w & \rho_{n,n,t}^w \end{pmatrix}, B_t = \begin{pmatrix} \rho_{1,1,t}^b & \cdots & \cdots & \rho_{1,n,t}^b \\ \vdots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ \rho_{n,1,t}^b & \cdots & \cdots & \rho_{n,n,t}^b \end{pmatrix}$$

[11] We have $V_t = W$ constant for subsequent periods if the two $\Delta\omega_t$s which capture covariance and risk aversion are assumed to be jointly independant. That is :

$$\Delta\frac{cov_t(r_{i,t+1}^A, r_{t+1}^I)q_t}{\mu_t\kappa} \text{ is independant of } \Delta\frac{cov_t(r_{i,t+1}^C, r_{t+1}^I)q_t}{\mu_t\kappa}$$

for all subsequent periods, and the variance of any $\Delta\frac{cov_t(r_{i,t+1}^C, r_{t+1}^I)q_t}{\mu_t\kappa}$ is assumed to be constant. These necessary conditions can easily be recovered from equations **??** to **??**

for some σ^2, $\rho^w_{i,j,t}$ and $\rho^b_{i,j,t}$ where w and b, as supercript, refer to the the within or between classes covariances.

Recall that those matrices are only forecasts, therefore even if omitted to avoid excessive notation there should be *expectations* of variances-covariances matrices.

Traders may assume for simplicity that $\rho^w_{i,j,t+1} - \rho^w_{i,j,t} = \rho_{w,i,j} + \epsilon_{i,j,t}$; similarly $\rho^b_{i,j,t+1} - \rho^b_{i,j,t} = \rho_{b,i,j} + \epsilon_{b,i,t}$ for some constant depending on the class (assets or commodities) $\rho_{w,i,j}$ and $\rho_{b,i,j}$ for all i and j.

Therefore the matrix $E_t(\Delta V_t) = E_t(V_{t+1} - V_t)$ will rewrite:

$$V = \sigma^2 \begin{pmatrix} A & B & * \\ B & A & * \\ C & D & * \end{pmatrix}$$

where A and B are given by:

$$A = \begin{pmatrix} \rho_{w,1,1} & \rho_{w,1,2} & \cdots & \rho_{w,1,n} \\ \rho_{w,2,1} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \rho_{w,n-1,n} \\ \rho_{w,n,1} & \cdots & \rho_{w,n,n-1} & \rho_{w,n,n} \end{pmatrix}, B = \begin{pmatrix} \rho_{b,1,1} & \cdots & \cdots & \rho_{b,1,n} \\ \vdots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ \rho_{b,n,1} & \cdots & \cdots & \rho_{b,n,n} \end{pmatrix}$$

(4)

This hold for some $\rho_{w,i,j}$ and $\rho_{b,i,j}$ where the first index in the underscript *from now and thereafter* refers to the within or between classes covariances.

Notice the zeros on the diagonal, they will make the covariance of a specific asset with r^I_t disappears from the returns expressions (5) and (6).
Further they may assume a negligible covariance of returns between assets and non-indexed commodities, and between indexed and non-indexed commodities, therefore C and D would be null. Assuming investors can only conjecture covariance and risk aversion one period ahead, they might infer that the posterior demand of institutional investors on the assets will have a similar expression as in Barberis. That is they will assume that $\omega = \frac{1}{n}[A + u]$ where u follow a law centered at 0. In this case, the simplification below holds. Further, remember traders were assuming the matrix of variance-covariance V_t to be constant after t+1, and set $V_t = W$ The price expression is

$$P_t = D_t - \gamma V_t(Q - \omega^I_t) - (T - t - 1)\gamma W(Q - A)$$

$$\text{where } A = (\frac{A_A}{n}, ... \frac{A_A}{n}, ... \frac{A_C}{n} ... \frac{A'_C}{n})$$

Therefore, this means that up to constant:

$$\Delta P_{t+1} = \epsilon_{t+1} - \gamma Q E(V_{t+1} - V_t) + \gamma E(V_{t+1} - V_t)\Delta\omega^I_t$$

Since the second term on the right hand side is now fixed, we have up to constant:

$$\Delta P_{t+1} = \epsilon_{t+1} + \gamma V \Delta\omega^I_t$$

$$\text{This writes: } \begin{pmatrix} \Delta P^A_{t+1} \\ \Delta P^C_{t+1} \\ \Delta P^{C}_{t+1}{}' \end{pmatrix} = \begin{pmatrix} \epsilon^A_{t+1} \\ \epsilon^C_{t+1} \\ \epsilon^{C}_{t+1}{}' \end{pmatrix} + \gamma V \begin{pmatrix} \Delta \omega^A_{t+1} \\ \Delta \omega^C_{t+1} \\ \Delta \omega^{C}_{t+1}{}' \end{pmatrix}$$

$$\Delta P_{i,t+1} = \epsilon_{i,t+1} + \sum_i \rho_{w,i} \Delta \frac{cov_t(r_{i,t+1}, r^I_{t+1}) q_t}{\mu_t \kappa} + \sum_j \rho_{b,j} \Delta \frac{cov_t(r_{j,t+1}, r^I_{t+1}) q_t}{\mu_t \kappa} \text{ for } i \in A \tag{5}$$

$$\Delta P_{j,t+1} = \epsilon_{j,t+1} + \sum_j \rho_{w,j} \Delta \frac{cov_t(r_{j,t+1}, r^I_{t+1}) q_t}{\mu_t \kappa} + \sum_i \rho_{b,i} \Delta \frac{cov_t(r_{i,t+1}, r^I_{t+1}) q_t}{\mu_t \kappa} \text{ for } j \in C \tag{6}$$

Where C denotes those commodities which are indexed.

$$\Delta P_{j,t+1} = \epsilon_{j,t+1} \text{ for } j \in \text{ C' where C' denotes those commodities which are not indexed} \tag{7}$$

For expository purpose, we previously considered that traders were holding n standard financial assets, and n claims on commodities, this can easily be modulated. We provide a simple example. Consider we have two risky assets, two indexed commodities and two non-indexed commodities. Next suppose the trader assumes the following V matrix.

$$V = \begin{array}{c} \\ a1 \\ a2 \\ c1 \\ c2 \\ c1' \\ c2' \end{array} \begin{array}{c} \begin{array}{cccccc} a1 & a2 & c1 & c2 & c1' & c2' \end{array} \\ \begin{pmatrix} \rho_{w,1,1} & \rho_{w,1,2} & \rho_{b,1,1} & \rho_{b,1,2} & 0 & 0 \\ \rho_{w,2,1} & \rho_{w,2,2} & \rho_{b,2,1} & \rho_{b,2,2} & 0 & 0 \\ \rho_{b,1,1} & \rho_{b,1,2} & \rho_{w,1,1} & \rho_{w,1,2} & 0 & 0 \\ \rho_{b,2,1} & \rho_{b,2,2} & \rho_{w,2,1} & \rho_{w,2,2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

where the first index in the underscript refers to the within or between classes (1 or 2). Then:

$$\Delta P_{a1,t+1} = \epsilon_{a1,t+1} + \rho_{w,1} \Delta \frac{cov_t(r_{a1,t+1}, r^I_{t+1}) q_t}{\mu_t \kappa} \rho_{w,2} \Delta \frac{cov_t(r_{a2,t+1}, r^I_{t+1}) q_t}{\mu_t \kappa} + \rho_{b,1} \Delta \frac{cov_t(r_{c1,t+1}, r^I_{t+1}) q_t}{\mu_t \kappa} + \rho_{b,2} \Delta \frac{cov_t(r_c}{} \tag{8}$$

$$\Delta P_{c1,t+1} = \epsilon_{c1,t+1} + \rho_{w,1} \Delta \frac{cov_t(r_{c1,t+1}, r^I_{t+1}) q_t}{\mu_t \kappa} \rho_{w,2} \Delta \frac{cov_t(r_{c2,t+1}, r^I_{t+1}) q_t}{\mu_t \kappa} + \rho_{b,1} \Delta \frac{cov_t(r_{a1,t+1}, r^I_{t+1}) q_t}{\mu_t \kappa} + \rho_{b,2} \Delta \frac{cov_t(r_a}{} \tag{9}$$

Those equations imply there can be a common factor in the returns of a group of assets; in particular any shock to the expected covariance cov_t may affect the returns on the assets and indexed commodities while leaving the non-indexed commodities unchanged.

As evidenced in the above formula and from the shares expressions (equation **??** and subsequents) shocks from institutional traders demand may emerge from three different components.
Consider first a change in μ_t the Lagrangian, reflecting how binding is the VaR constraint; As the balance sheet binds harder, the shadow price μ_t increases, and leverage must be reduced. This impacts necessarily the investor portfolio and will affect therefore commodities prices.
Secondly, one can think of a shock to cov_t. As the cov_t decreases, the share allocated to some asset i responds positively; since all shares must sum to 1; this implies rebalancing portfolio. Therefore the shares allocated by institutional traders evolve across time depending on the perceived cov_t of the return of the index with other assets.
Eventually, a shock to the parameter q_t representing the relative importance of the index return in institutional investors portfolio will necessarily impact commodities prices.

Another observation needs to be made. We are talking about an updating process, where shares and prices evolve through time. Each period, the institutional trader rebalances his portfolio taking into account the covariance of returns. Following formula **??**, the returns on those indexed commodities may change provided they are rebalanced by large number of traders. Therefore, their covariances with the returns on the index will change itself, leading to further rebalancing. Large changes in prices and returns may therefore be observed. This model therefore accounts for large change in prices as observed in the literature and detailed in the introduction. If institutional investors are enough (or big enough) to bid up on prices (rebalance their portfolio toward specific commodities), fundamental traders might not be willing or capable of countering their effects on prices.
The fact that institutional investors whose investment capacity may go beyond the normal absorption capacity of other market participants can severely impact prices' evolution was behind Hong and Yogo's intuition. In a 2012's paper, they have shown that open interest positions can be a reliable determinant of future economic activity and asset prices.

Further, the covariances between commodities (whether they are or not in the index) and between commodities and other financial assets enter as a key feature of the model. It enters as an input to account for the change in prices, and as an output by the process explained above. Observing how conditional covariances of returns have evolved through time; and how we may observe spillovers from and to financial and commodities markets will be detailed in the empirical section.

3 Geographic-based comparison

The argument justifying the recent price surge for various commodities, particularly marked for metals and energies was the growing demand from emerging countries.

An interesting study to implement therefore is a geographic-based comparison to observe if those price evolutions have occured similarly in emerging markets. This first descriptive approach helps us grasp the singularity of the US markets while considering the behaviors of commodities prices.

We use Brazil and China as counterfactual to observe prices' evolution and dynamic correlations between various commodities. Both are large producers and consumers of commodities, China being particularly keen on metals, while Brazil is a large exporter of exotic, softs and livestocks (cattle). It is also needless to mention that if futures exist for these markets, to the best of our knowledge commodity index investment does not.

For Brazil, we select 8 commodities whose time series draw back to 2004. They include Wheat, Soybeans, Corn, Sugar, Feed Cattle, Gold, Natural Gas and Diesel. The data are extracted from Datastream are presented in Appendix.We therefore have a set of diverse commodities where we can implement pairwise comparison between supposedly unrelated (in the sense given by Pindyck) commodities. The time range selection is justified mostly by the time range availability. Monthly US data are extracted from various sources: EIA, IMF, and UCSA. We select grower or spot prices, with the exception of sugar in the US where the futures' price is used.

The same process is repeated for China whereas we rely this time on a different set of commodities. The time range for certain commodities in China (Wheat, Copper, and Aluminium) is also larger. The commodities are: Wheat, Soybeans, Cotton, Sugar, Aluminium, Oil and Copper. The prices used are from futures contracts. This is both due to unavailability of data and the fact that the government does not report monthly prices consistently and always changes the methods and reporting of growers prices. All prices series have been plotted and displayed in **??** and subsequents.
The returns are defined as

$$R_t = ln(\frac{P_{t+1}}{P_t}) \approx \frac{P_{t+1} - P_t}{P_t}$$

We use montlhy log-returns, they can easily be interpreted as a percentage change from month to month. Interestingly, log returns exhibit more erratic behaviors on the US markets as compared to Brazil.[12]. This is particularly true for gold, cattle, natural gas, sugar, and soybeans. The magnitude of changes is generally ten time larger for the US commodities. Similarly the magnitude of changes of prices for China (see the figure**??**) is much comparable to Brazil than to the US.

3.1 Dynamic Conditional Correlations

Our first investigation rely on the DCC approach as followed recently by **?**. However, if these authors documents a substantial increase in the correlations between SP500 and various commodities; our analysis extend to across commodities dynamic correlations and especially dynamic correlations with oil, the central composite of indexes.

[12]The plots of log returns are not presented in this report but available upon request

3.1.1 Methodology

Consider n time series of returns and make the usual assumption that returns are serially uncorrelated. Then, we can define a vector of zero-mean white noises $\epsilon_t = r_t - \mu$, where r_t is the $n \times 1$ vector of returns and μ is the vector of expected returns. Despite of being serially uncorrelated, the returns may present contemporaneous correlation. That is:

$$E_{t-1}[(r_t - \mu)(r_t - \mu)'] = \Sigma_t$$

may not be a diagonal matrix. Moreover this comtemporaneous correlation may be time-varying depending on past information such as presented in the model.

The GARCH-DCC involves two steps. The first step accounts for the conditional heteroskedasticity. It consists in estimating, for each one of the n series of returns r_t^i, its conditional volatility σ_t^i using a GARCH model. Let D_t be a diagonal matrix with these conditional volatilities, i.e. $D_{i,it} = \sigma_t^i$ if $i = j$, $D_t^{i,j} = 0$ if $i \neq j$. Then the standardized residuals are:

$$\nu_t = D_t^{-1}(r_t - \mu)$$

Therefore these standardized residuals have unit conditional volatility. Now, define the matrix:

$$\bar{R} = \frac{1}{T}\sum_{t=1}^{T} \nu_t \nu_t'$$

This is the Bollerslev's Constant Conditional Correlaton Estimator.

The second step consists in generalizing Bollerslev's CCC to capture the dynamics in the correlation, hence the name. The DCC correlations are:

$$Q_t = \bar{R} + \alpha(\nu_{t-1}\nu_{t-1}' - \bar{R}) + \beta(Q_{t-1} - \bar{R})$$

So, $Q_t^{i,j}$ is the correlation between r_t^i and r_t^j at time t. This is what is plotted on the presented graphics.

For the second step, which is the DCC estimation per se, standard softwares estimates both parameters, α and β simultaneously, by maximizing the log likelihood. The standardized residuals are assumed to be jointly Gaussian. The DCC model captures a stylized facts in financial time series: correlation clustering. The correlation is more likely to be high at time t if it was also high at time t-1. Another way of seeing this is noting that a shock at time t-1 also impacts the correlation at time t. However, if $\alpha + \beta < 1$, the correlation itself is mean reverting, and it fluctuates around $\bar{R}$, the unconditional correlation.
Usual restrictions on the parameters are $\alpha, \beta > 0$. Though, it is possible to have $\alpha + \beta = 1$; the conditional correlation is then an integrated process.

3.1.2 Results

The DCC approach allows us to evaluate the evolutions of correlations between equities and various commodities as in **?**. We extend their analysis to various commodities, with a particular focus on the dynamic of oil with other commodities.

The results are presented in below ??[13] and in Appendix.
On the figure ?? which considers the US market we can observe a break around 2008-2009 in the dynamic correlations of the SP500 with grains, industrial metals and crude oil. The dynamic correlation of SP500 with grains tends to fade out more recently.
On figure 3, the dynamic correlations of oil are presented. If the effects of the crisis on these dynamics tend to weaken for grains and softs, the correlation of industrial metals with oil remains high.
Similarly, for wheat and copper (Figure 4 and 5) the clear effects of the 2008 crisis can be observed, but dynamic correlations tend to revert to the mean in more recent time (which is also due to the specification of the DCC model).

To examine if this increasing correlation between oil and other commodities is characteristic of the US market, we plot the DCC of various commodities in China with oil ??[14]. The price evolution of oil in China displayed in picture ?? was subdued and less erratic than in the US; in particular we could not observe the intense increase characteristic of the US market.
However, as presented on the graph ??, one can also contemplate an increased correlation of oil with industrial metals and cotton or soybeans in China. The correlation of oil with Aluminium and Copper has climbed to almost 0.5, and this with Copper has remained very high since then. This is comparable to the evolution of the GSCI Copper Index with oil as in ??, but the magnitude remains lower in China than in the United States, whereas the demand for both energies and metals is larger in the former.

Figures ?? and ?? permit to continue the comparison while considering the dynamic correlation of various other commodities.
Wheat, aluminium and copper time range extends to 1993 in China; one can contemplate no particular evolution in an historically low correlation between wheat and those metals. This is in striking contrast with the evolution observed in the United States as presented in figure ??.
Similarly, the correlations of soybeans or cotton with wheat, aluminium and copper tend to be higher and mark a brutal increase around 2008 in the United States, these behaviors are not observed in China except for the correlation of soybeans with copper.

Pairwise DCC are plotted for Brazil versus United States for various commodities. Mainly, the dynamic correlations of commodities in Brazil are lower, except for the correlation of natural gas and cattle which surprisingly reaches the top (on the graphic ??). An observation of the price dynamics given in picture ?? in Appendix confirms that those two commodities have followed a very similar trend since 2004.
As a conclusion, the correlations of commodities prices tend to be higher in the US as compared to two large commodities consumers. An observation which justifies the accuracy of our problematic: could this pattern be partially explained by the financialization of commodities in the United States?

[13] The graphs for the United States when compared to China were made using Vlab.

[14] As robustness check, the DCC were also computed using daily returns rather than monthly. The results were not quite similar evidencing that the dynamic correlations tend to be sensitive to the selected period. Yet, on a broader basis they exhibited the same dynamic.

Figure 2: DCC of the SP500 with grains, crude oil and industrial metals

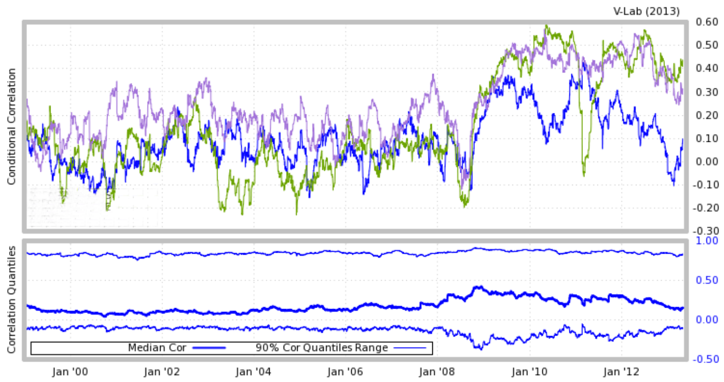


Figure 3: DCC of the all crude GSCI with grains, industrial metals, and softs

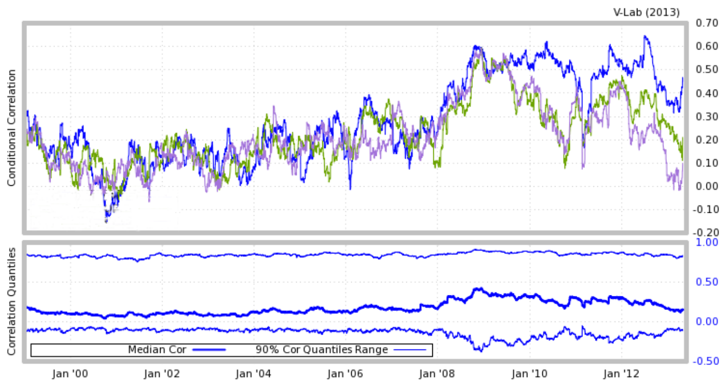


Figure 4: DCC of Copper with Coffee, Cotton, and Soybeans

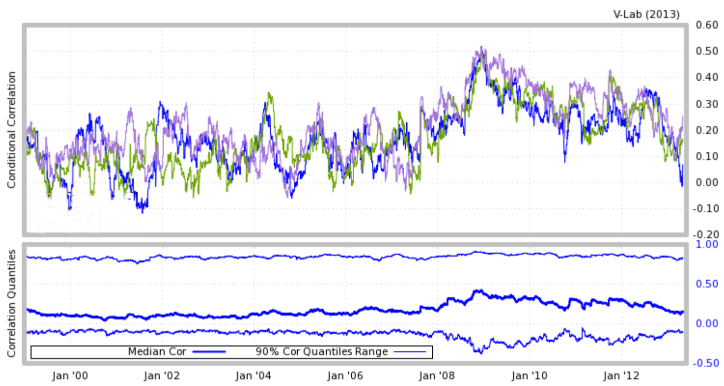
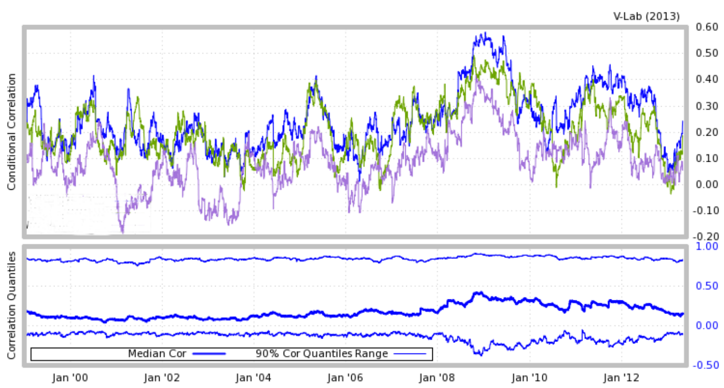


Figure 5: DCC of Wheat with Cotton, Softs, and Cocoa



4 Uncovering a channel: the covariances of commodities returns with the index return

Tying together a set of seemlingly unrelated commodities through financial instruments could explain the increasing linkage within the commodities class, and between this class and standard financial assets. To test for this hypothesis, recall our equations that were relating commodities prices with the covariances of price changes and the financial return on a commodity index:

$$\Delta P_{a1,t+1} = \epsilon_{a1,t+1}+\rho_{w,1}\Delta\frac{cov_t(r_{a1,t+1},r^I_{t+1})q_t}{\mu_t\kappa}\rho_{w,2}\Delta\frac{cov_t(r_{a2,t+1},r^I_{t+1})q_t}{\mu_t\kappa}+\rho_{b,1}\Delta\frac{cov_t(r_{c1,t+1},r^I_{t+1})q_t}{\mu_t\kappa}+\rho_{b,2}\Delta\frac{cov_t(r_{c2,t+1},r^I_{t+1})q_t}{\mu_t\kappa} \quad (10)$$

$$\Delta P_{c1,t+1} = \epsilon_{c1,t+1}+\rho_{w,1}\Delta\frac{cov_t(r_{c1,t+1},r^I_{t+1})q_t}{\mu_t\kappa}\rho_{w,2}\Delta\frac{cov_t(r_{c2,t+1},r^I_{t+1})q_t}{\mu_t\kappa}+\rho_{b,1}\Delta\frac{cov_t(r_{a1,t+1},r^I_{t+1})q_t}{\mu_t\kappa}+\rho_{b,2}\Delta\frac{cov_t(r_{a2,t+1},r^I_{t+1})q_t}{\mu_t\kappa} \quad (11)$$

Our main target is not on the parameters $\kappa\mu_t$ and q_t, the former being investigated in **?** and the latter being an inviting pursuit to complement our study. Rather, we focalize on the covariances'changes terms in the above expressions. The above formula could be reformulated as:

$$(ExcessFuturesReturn_j) = b_0+b_1cov_t(ER'_j, ER_{index})+b_2cov_t(ER_{SP500}, ER_{index})+b_3cov_t(ER_{oil}, ER_{index})+...$$

$$(ExcessSpotReturn_j) = b_0+b_1cov_t(ER'_j, ER_{index})+b_2cov_t(ER_{SP500}, ER_{index})+b_3cov_t(ER_{oil}, ER_{index})+...$$

In this section, we will first present our strategy and methodology to empirically assess these equations and further comment on the results obtained.
The tedious part in the empirical investigation was to measure investors returns. From holding a commodity we defined previously the return as the price difference from t to t+1. However, investors' returns when investing in an index is threefold. The price difference is the spot return, but roll yield and collateral return adds up to it. The roll yield reflects the return (positive or negative) from rolling one contract into the next.[15] [16]

Spot return is defined in Tang and Xiong's or Etula's are simply the price difference over a day, month or quarter [17]. Futures excess returns however are usually computed as in **?** or **?** by constructing a return from rolling the first-month futures contract of the commodity on a fully collateralized basis. [18]

[15]For example, the S&P GSCI total return index measures a fully collateralized commodity futures investment that is rolled forward from the fifth to the ninth business day of each month. Excess return comprises both spot return and roll yield. The S&P GSCI excess return is the measure of commodity returns that is completely comparable to returns from a regular investment in SP 500 (with dividend re-investment) or a government bond.

[16]For further clarification on these different type of returns adcquired by investors in commodity indexes refer to**?**

[17]Excess spot returns are usually generated by subtracting the Treasury bill rate from it but we allowed ourselves to use simply spot returns

[18]The excess return of this hypothetical investment would therefore be defined as $r_{i,t} = ln(F_{i,t,T}) - ln(F_{i,t-1,T})$ where $F_{i,t,T}$ is the price of the future contract held on date t with maturity T.

This means accessing data (Bloomberg provides some) procuring expiry date of contracts, since those were not available we therefore rely on a different strategy.

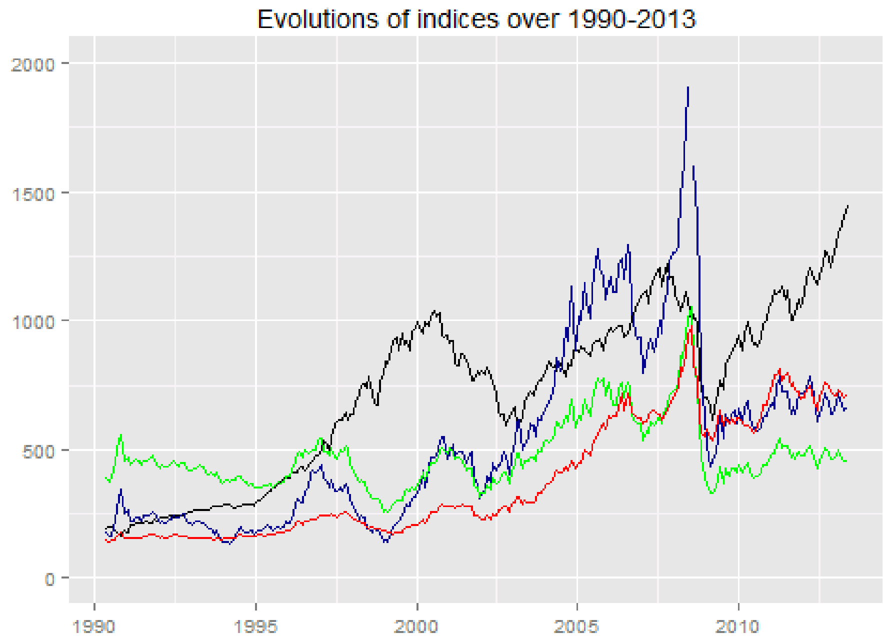
We use Datastream series to apprehend *effective* returns. Datastream series take the form of prices, total or excess returns indices. Those two latter follow the performances of commodities both for spot and futures, in particular they capture the price's change and the change in roll yield and collateral return. Excess and total return usually follow the exact same trend, at slighty different levels while the underlying price series is also often following the same trend.
Since our focus is on comovement of returns, using indices *or* effective returns should not greatly affect the computation of covariances.
Our covariances will capture how the trend in different returns co-move, that is how the average returns of investors holding such or such assets or commodities co-move. In particular, the key element entering each covariance is the return on the index, r_t^I, this return is computed from the excess commodity returns of Goldman Sachs defined as GSCIEXR.

Datastream proposes numerous series accouting for the evolution of total and excess returns for different commodities and investment institutions such as Goldman Sachs Commodity Index (GSCI) Merill Lynch (MC), or the Dow Jones UBS Commodity Index (DJCI).We will also use those series to reflect the effective returns of investors from holding *futures*.The main series used are displayed in Table **??** in Appendix.
The figure below graphs the evolutions of some financial indices, clearly they exhibit a trend and are non-stationary. Our methodology consists in considering those indices series as if they were prices series, this procedure is applied for both r_t^I and excess futures returns. [19]

Once those series in hand [20] we can calculate the time dependant covariances of these with r_t^I.
Finally, we regress $\frac{\Delta_{Y\,t}}{Y_t}$ where Y_t is either a spot price, either a futures'excess return on a set of covariances'changes.
As a methodological note, notice that if the covariances were calculated using only a first difference on the series far more coefficients on covariances'changes would remain significant. A striking example was wheat which was extremely responsive if the covariances were computed using a first difference, while not particularly (it is therefore not included in the first tables) if the covariances are computed using returns.

The examination was conducted with a sample of 20 futures and 25 spot prices [21] including various metals; grains and energies. The earliest series start in the seventies, the latest begin in the early 2000s. Most series especially cover the last 15 years when the financialization of commodities could have been observed.
Our examination involved a major pitfall, commodities covariances with the GSCI (especially for metals) tended to exhibit very similar trends, generally with a pick around 2007-2008 resulting in a multicollinearity issue and potentially unconsistent

[19] We therefore compute $\frac{\Delta_{Y\,t}}{Y_t}$ when Y_t is defined as an excess returns index.
[20] or the spot returns $\frac{P_{t+1}-P_t}{P_t}$ when prices series were available.
[21] or excess returns series



black: SP500 price index divided by 2, Goldman Sachs crude oil excess return, GS Commodity excess return, Long only excess return.

coefficients estimates. To insure the robustness of the results, a careful check of the plot of covariances was conducted when including them in a regression. Thereafter, a thorough analysis conducted by adding, dropping covariances as predictors was implemented for all commodities; usually when not significant the covariances were dropped to avoid unexpected interactions between covariances as predictors. The results while quite promising tended to be very sensitive to the choice of window to compute the covariances(windows of 2,5 and 10 months were used, eventually leading to a choice of a 5 months window), and to the addition or deletion of covariances. This latter process sometimes had strong impact on the significance of the coefficients.

4.1 Results

We start our analysis considering futures. According to the literature, the effects are usually more pronounced while considering futures'excess returns rather spot excess returns.
All predictors have been scaled to facilitate interpretation and all results are displayed in percentage points. In table **??**, for instance if the level of the covariance of the return on corn with the GSCI index return is one standard deviation above average investors will accept about -2% percentage point lower returns on their long positions in aluminium or zinc *over the following month*, while -4% for tin, similarly they would accept about -1% on their returns on the Dow Jones or SP500 representing their equities positions in the model.

Many futures' prices did not (or only marginally) respond to covariances' changes,

this includes wheat, coffee, cattle, but also some metals such as silver; only the most striking results are presented in Appendix. Nevertheless, their covariances with the return on the index r_t^I might impact other commodities' futures'prices as evidenced with wheat which enter significantly and positively for aluminium, tin, zinc and corn.
As expected, within a same commodity class (ie metals, grains or energies), the effects observed were more frequently significant, highlighting some common underlying factors which could be unrelated to index investment. However, the effects of wheat, corn or soybeans on metals would hardly be justified by any common external factors, the cross-elasticities of those commodities being quasi-null. This lends support to the view that commodity index investing could be this common denominator.

Since many futures contracts started in the mid-late 90s, the full sample includes only from 166 to 199 observations. While keeping the same set of explanatory covariances, the results for the subperiod 2003-2013 are also presented in table **??**, the effects on prices (returns) are usually larger for this subperiod acknowledging the peculiarity of the crisis period and the boom and bust in energies commodities. If the covariance of wheat with the GSCI index is one standard deviation above average, investors will require 1.5 to 3% higher returns on aluminium, tin, zinc and corn; while if the the covariance of corn with GSCI index is one standard deviation above average, they will accept -2 to -6% lower returns for these metals *over the following month*; while they would accept about -1% lower returns on their equities positions.

The covariances were inserted both in levels and in differences, interestingly and in concordance with the model the significance of coefficients was usually greater for the change in covariances [22].

Notice also that, if inserted as a control variable, the covariance of any specific asset with the return on the GSCI is generally non significant. We may also assume $\rho^w_{i,i,t+1} - \rho^w_{i,i,t} = \epsilon^w_{i,i,t}$ and $\rho^b_{i,i,t+1} - \rho^b_{i,i,t} = \epsilon^b_{i,i,t}$ for some epsilon centered at 0. [23]

[22] Therefore, except if mentioned differently all covariances enter in changes.

[23] Then the matrix A and B would be given by:

$$A = \begin{pmatrix} \rho_{w,1,1} & \rho_{w,1,2} & \cdots & \rho_{w,1,n} \\ \rho_{w,2,1} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \rho_{w,n-1,n} \\ \rho_{w,n,1} & \cdots & \rho_{w,n,n-1} & \rho_{w,n,n} \end{pmatrix}, B = \begin{pmatrix} \rho_{b,1,1} & \cdots & \cdots & \rho_{b,1,n} \\ \vdots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ \rho_{b,n,1} & \cdots & \cdots & \rho_{b,n,n} \end{pmatrix}$$

(12)

and our equation would be:

$$\Delta P_{i,t+1} = \epsilon_{i,t+1} + \sum_{i' \neq i} \rho_{w,i'} \Delta \frac{cov_t(r_{i,t+1}, r^I_{t+1}) q_t}{\mu_t \kappa} + \sum_{j' \neq j} \rho_{b,j'} \Delta \frac{cov_t(r_{j,t+1}, r^I_{t+1}) q_t}{\mu_t \kappa} \text{ for } i \in A \tag{13}$$

$$\Delta P_{j,t+1} = \epsilon_{j,t+1} + \sum_{j' \neq j} \rho_{w,j'} \Delta \frac{cov_t(r_{j,t+1}, r^I_{t+1}) q_t}{\mu_t \kappa} + \sum_{i' \neq i} \rho_{b,i'} \Delta \frac{cov_t(r_{i,t+1}, r^I_{t+1}) q_t}{\mu_t \kappa} \text{ for } j \in C \tag{14}$$

That is the return on the asset ai or the commodity ci essentially depends on others commodities or assets covariances with the GSCI excess return, the covariance change $\Delta cov_t(r_{ci,t}, r^I_t)$ seldomly enter as a significant predictor of $r_{ci,t+1}$. This is consistent with the observation that many commodities returns (most indexed), during the last decade, have experienced unexpected prices' fluctuations.
Even if their prices may only be affected by the covariances of other indexed commodities with the return on the index (r^I_t); by a feedback effect, the covariance $cov_t(r_{ci,t}, r^I_t)$ will a period later for instance also affect $r_{ci,t+1}$.
Let us detail this intuition. Suppose you have three commodities $c1$ to $c3$ and one risky asset $a1$. Our expression for the returns are :

$$\Delta P_{c2,t+1} = \epsilon_{c2,t+1} + \rho_{1,1}\Delta\frac{cov_t(r_{c1,t+1}, r^I_{t+1})q_t}{\mu_t\kappa} + \rho_{1,3}\Delta\frac{cov_t(r_{c3,t+1}, r^I_{t+1})q_t}{\mu_t\kappa} \tag{15}$$

$$\Delta P_{c1,t+1} = \epsilon_{c1,t+1} + \rho_{1,2}\Delta\frac{cov_t(r_{c2,t+1}, r^I_{t+1})q_t}{\mu_t\kappa} + \rho_{1,3}\Delta\frac{cov_t(r_{c3,t+1}, r^I_{t+1})q_t}{\mu_t\kappa} + \rho_{2,1}\Delta\frac{cov_t(r_{a1,t+1}, r^I_{t+1})q_t}{\mu_t\kappa} \tag{16}$$

Suppose some shift of the perceived covariance $cov_t(r_{c1,t+1}, r^I_{t+1})$, it does not enter directly in equation **??**, but it affects the returns $\Delta P_{c2,t+1}$, and similarly could affect $\Delta P_{a1,t+1}$, therefore the perceived covariances $cov_t(r_{c2,t+2}, r^I_{t+2})q_t$ and $cov_t(r_{a1,t+2}, r^I_{t+2})q_t$ could shift impacting in the subsequent period the return on commodity $c1$. [24]

In table **??** we present the results of a similar analysis using spot prices. As expected the results using spot prices were feebler, previously responsive series such as aluminium or corn were responding to very few covariances' changes. The results for cotton and cocoa previously omitted since they were respondent to no more than three covariances (while using futures) are presented through this spot analysis. Using futures, cocoa was also responsive to the covariance change of gold with energy(a 2 month forward contract on energies-SPGSLPI). Notice that if tin is not presented, it was one of the most respondent commodity in spot price, as evidenced from table **??** given in Appendix.

Till now we have not mentioned any result regarding equation **??** which stated that the returns (respectively prices) of non-indexed commodities should not respond to changes in the covariances terms. This equation was certainly empirically verified for prices series such as lumber, oats, rough rice among others which are supposed to carry no weight in the GSCI index, these commodities were not responding to any covariances change. As a counterfactual exercise, we present the results of regression specifications for some of these supposedly independant and unsensitive commodities. We use the exact same specifications implemented in table **??**. Crude oil for covariances in level and in changes is replaced by palladium, which still show some responses; soybeans is replaced by soybean meal, copper (in changes and in level) is replaced by titanium. Corn is replaced by sugar which we had noticed was not responding in differently specified regressions. We also add electricity and gas, two totally 'indifferent' commodities.
For electricity for instance, only the coefficient of the covariance of gold with energy enters significantly in the second specification.

[24] We did not test for this lagged reponse of a return $r_{c1,t+k}$ to a shift in the perceived covariance $cov_t(r_{c1,t+1}, r^I_{t+1})$, but it would be an engaging pursuit of research.

As said previously, other commodities sometimes included in a commodity index such as the GSCI were not either responding; this could also be due to our choice of contracts or the series time range. This work could be improved by testing for many different series of the same commodity to observe the differences in responses.

4.2 Robustess Check

Our first estimation demonstrates that covariances of commodities returns with the GSCI return are statistically significant predictors of many commodity futures returns. In this subsection we investigate the extent to which controlling for others common predictors of commodity returns might affect those results.

In table **??**, we first introduce common predictors such as the VIX volatility index, the US consumer price index both lag by one month, and the 3 months Treasury Bill rate; for copper, soybeans and oil the total open interest positions is also inserted (denoted OI) [25]

For the latter, we mentioned previously the work of Hong and Yogo (2012) who have shown that total open interest positions could be a significant predictor of returns, and indeed for all three commodity (oil, soybean and copper) open interests enter significantly.

Our results are robust to the insertion of those control variables. For instance, one may observe that the lag covariance of copper has an as strong effect that the volatility proxy on the zinc returns but in the opposite direction; however the current covariance change of lead has a negative and larger effect (about -4%). If these effects within the metals class may not be surprising since they are somehow related in their use as an industrial input, our results also show that there is a link with unrelated commodities. Zinc being responsive to wheat and corn as mentioned previously.

In table **??**, we also add the one month lagged Han and DeRoon indexes. The Han index measures speculators sentiment by considering their relative long and short positions, it is defined as:

$$HanIndex : \frac{\text{number of long speculative positions - number of short speculative positions}}{\text{total open interest}}$$

and is contructed using CFTC data.
The DeRoon index is a measure of trading activity, roughtly it is an estimate of hedging pressure in commodities markets and is defined as :

$$RNV : \frac{\text{number of short hedge positions - number of long hedge positions}}{\text{total number of hedge positions}}$$

The idea is to consider the positions of hedgers, ie traders who have a cash business for the commodity. Notice however, than these variables didn't enter significantly for oil and soybeans and were dropped in the interest of space; they are are significant for copper and are respectively denoted 'copper S and R'. [26]

[25] I would like to thank Yannick LePen for providing me the data on open interest positions and the Han and DeRoon index used thereafter

[26] Similarly the constant and the F stat were not included to save space.

A final interesting perspective to mention in this study was to observe the effect of lags covariances (indeed we use only lag covariances rather than the lag of covariances'change) on the returns; this would suppose a slight modification of our theorical model. Investors would rather than anticipate the covariances'change (which would affect their positions, and therefore prices and returns), react to covariances levels. Inserting those levels along with the previously considered predictors yield significant results. Particularly noticeable was the lag level of silver (first line of the table) which was a effective predictor for various metals and oil. Interestingly however, others lags such as soybean or zinc would also enter significantly especially for equities (SP500 and Dow Jones Industrials Index).

Essentially our study evidences three points: many commodities covariances with the index influence marginally, if at all, other commodities returns; they are the off-index commodities (as expected), the softs and exotic (coffee, cocoa) and cotton; grains which usually carry higher weights in the index such as corn or wheat interact more frequently with metals and energies, the major components of commodity indexes corroborating the intuitions carried over by the model.
The returns on those unsensitive commodities are usually also independant from equities proxies such as the SP500 and theirs comovements with oil for example. This seems to be not only due to their nature (they do not enter industrial inputs as energies and metals do hence would be less related with financial and industrial proxies such as SP500 and DJ indexes) since palladium and titanium (which carry no weight in majors commodity index, see Table 1 of **?**'s) as opposed to copper and zinc are less responsive as evidenced in our last table. Further, comparing respondant grains such as corn or wheat (indexed) to roughrice and oats lead to the same conclusion.

In our theorical framework we have also introduced the time dependant risk aversion of institutional investors $\kappa\mu_t$ and the parameter q_t representing the relative weight of commodities in financial institutions; a further step in completing our empirical investigation would be to account for these two parameters. Considering risk aversion, an entire study of its relative importance as predictor of excess returns for both spot and futures commodities is the work **?** previously mentioned.

Our main equations takes a very familiar form for someone introduced to the (I)CAPM framework; somehow the usual covariance with the market portfolio is in our framework the covariance with the commodity indexes. However, empirical assessment of the CAPM model has shown mixed results until recently, essentially due to technical restrictions according to **?**.
In this paper, he shows that using the dynamic correlation set up to evaluate the conditional covariances of assets with the market portfolio would yield promising results, in particular in the standard equation:

$$R_{i,t+1} = C_i + Acov_t(R_{M,t+1}, R_{i,t+1}) + \epsilon_{i,t+1}$$

where $R_{M,t+1}$ denotes the excess return on the market portfolio; two essential tests of the ICAPM are passed: the positive common slope $A > 0$ and the condition that the intercepts are null $C_i = 0$.
The singularity of his success lies in the larger set of assets' returns considered. Assessing the CAPM model requires a forecast of the variance-covariance matrix

of assets returns, and could necessitate the estimation of very large covariance matrices. Using DCC has clear computational advantages over multivariate GARCH models for example, in that the number of parameters to be estimated in the correlation process is independent of the number of series to be correlated. Therefore, potentially very large correlation matrices can be estimated. In the present study we have simply implemented estimation of pairwise rolling covariances, the DCC framework would allow to measure conditional covariances of larger set of assets and commodities and considerably enrich the robustness of our findings.
Further, a subsequent extension of the model and empirical analysis would consist in systematically estimating the parameters $\rho^1_{i,j,t}$ and $\rho^2_{i,j,t}$ rather than infering a relationship between $\rho_{i,j,t+1}$ and $\rho_{i,j,t}$.

5 Conclusion

Correlating prices and returns throught index investment could be an explanation of how a tendency of rising prices for some commodity (due to a growing demand and relative inelasticity of supply as for crude oil for instance) would have impacted other commodities prices (respectively returns) such as observed in the United States. This domino effect would reinforce itself since while the covariance of commodity ci with the return on the index may influence commodity cj' return and its covariance with the index, it might affect again the effective return on commodity ci and so on. This model and its empirical assessment seizes this mechanism.

6 Appendix

Table 1: regression for tin as spot commodity

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	0.0094	0.0048	1.94	0.0541
gas	-0.0079	0.0049	-1.60	0.1116
soybean	0.0109	0.0073	1.49	0.1381
tin	0.0165	0.0048	3.45	0.0007
lead	-0.0152	0.0066	-2.30	0.0222
corn	-0.0234	0.0077	-3.02	0.0028
copper	-0.0130	0.0065	-1.99	0.0480
wheat	0.0075	0.0054	1.40	0.1619

6.1 Hedging via futures

Hedgers can be short or long. One takes a short hedge position in the futures market when he expects to sell an asset he already owns and wants to guarantee the price. Notice the asset we are trying to hedge may not be exactly the same as the asset underlying the futures, and the time at which we sell the asset (which could be random) might not be exactly be the same as the delivery date of the futures.

The basis is defined as the spot price of asset to be hedged minus the futures price of asset being used for the hedge : $b_t = S_t - F_t$. If the asset being hedged and used for the hedge are the same, then the basis will be zero at the expiration of the futures contract.
Consider a short hedger who will sell an asset at time T and who takes a short futures position at $t_0 < T$ for delivery at some time $t_1 > T$. Income at T will be the price of asset at time T: S_T plus the profit on futures position:

$$(F_{t_0,t_1} - F_{T,t_1})e^{-r(t_1-T)} \approx F_{t_0,t_1} - F_{T,t_1}$$

[27] if $T - t_1$ is small, as is usually the case.
Total income is therefore $\approx S_T + F_{t_0,t_1} - F_{T,t_1}$

$$= F_{t_0,t_1} + (S_T - F_{T,t_1})$$

$$= F_{t_0,t_1} + b_T$$

The basis b_T collects all the uncertain terms therefore represents all the risk.

The same formula for basis applies to a long hedge; if you consider an investor who knows he will buy an asset at T. The total payment at T (approximately, as before) will be the spot price S_T plus the loss on the future $F_{t_0,t_1} - F_{T,t_1}$.
Note that if b_T were known at we would have a perfect hedge (because if b_T is known, then b_T is fixed. This means that S_T and F_T must always change by equal amounts, leaving income unchanged).
When b_T is unknown at time t_0 the uncertainty about the period T income, captured by the uncertainty about the value of b_T, hence called basis risk.

In our model, we therefore assume investors hold mostly long positions, and therefore expect a payment $F_{t0,t1} + b_T$, the b_T has no reason to be a sum of normal laws, one may posit a different distribution.

[27] where r denotes some constant interest rate of returns

6.2 Complement on the cash-flow covariance matrix

The literature usually decomposes the cash flow shock to an asset with three components: a market-wide cash-flow factor which will affect assets in all categories, commodities futures and financial standard assets; a category specific cash-flow factor which affects assets in one category but will leave the other category unchanged and finally a a idiosyncratic cash-flow shock. You could therefore write:

$$\epsilon_{i,t} = \lambda_M f_{M,t} + \lambda_S f_{A,t} + \sqrt{(1 - \lambda_M^2 - \lambda_S^2)} f_{i,t} \text{ for i in A}$$

and

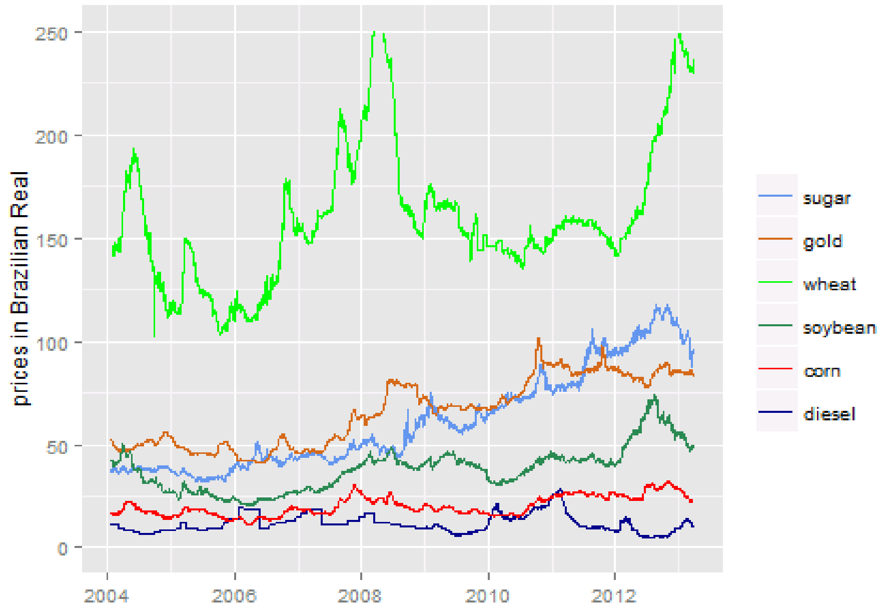
$$\epsilon_{j,t} = \lambda_M f_{M,t} + \lambda_Y f_{C,t} + \sqrt{(1 - \lambda_M^2 - \lambda_S^2)} f_{j,t} \text{ for j in C}$$

Where λ_M and λ_S are constant which control the relative importance of the three components. You may further assume that each factor has unit variance and is orthogonal to the other factors. This implies the following structure for the matrix of covariances of shocks: $\Sigma_D^{i,j} \equiv cov(\epsilon_{i,t+1}, \epsilon_{j,t+1}) = \begin{cases} 1, i = j \\ \lambda_M^2 + \lambda_S^2 = \pi \text{ for i,j in the same category} \\ \lambda_M^2 = \upsilon \text{ for i,j in different categories} \end{cases}$

Table 2: Main series used

Series	Starting date	Code	Use
Crude oil WTI	1986	CRUDOIL	Spot
Wheat No.2,Soft Red Cts/Bu	1982	WHEATSF	Spot
Soybeans No1 yellow	1979	SOYBEAN	Spot
Cocoa-ICCO Daily Price US$/MT	1971	COCINUS	Spot
Cotton New York Average Price	2002	COTNYAV	Spot
Copper	1961	LCPCASH	Spot
Corn	1979	CORNUS	Spot
Aluminium	1961	LAHCASH	Spot
Natural Gas, Henry Hub U$/MMBTU	1990	NATGHEN	Spot
Nickel	1993	LNICASH	Spot
Sugar Daily Price	1983	WSUGDLY	Spot
Zinc	1993	LZZCASH	Spot
Silver	1970	SLVCASH	Spot
LME-Lead Cash U$/MT	1993	LEDCASH	Spot
Tin	1961	TITNDIO	Spot
Palladium	1987	PALLADM	Spot
Electricity: Nordpool-Electricity Avg Reference	1990	NPXAVRF	Spot
Soyameal USA 48% Protein $/MT	2001	SOYMUSA	Spot
Rhodium: CIF NWE U$/Ounce - DS MID PRICE	1993	RHODNWE	Spot
Oil, Brent 1 Mth Forward	1999	SG1MBRE	Futures
S&P GSCI 1 Mth Fwd Soybeans Ind ER -	1995	SG1MSOE	Futures
Kansas Wheat 2 Month ER	1990	DJUKW2E	Futures
S&P GSCI 1 Month Fwd Cocoa Index - PRICE INDEX	1995	SG1MCCS	Futures
S&P GSCI 1 Month Fwd Corn Index - ER	1995	SG1MCNE	Futures
DJ UBS Tin 3 Mth Fwd TR - RETURN IND. (OFCL)	1996	DJUBTN3	Futures
S&P GSCI 1 Month Fwd Coffee Index ER - EXCESS RETURN	1995	SG1MKCE	Futures
S&P GSCI 1 Month Fwd Copper Index ER - EXCESS RETURN	1995	SG1MICE	Futures
S&P GSCI 1 Month Fwd Cotton Index ER -	1995	SG1MCTE	Futures
S&P GSCI 1 Month Fwd Nickel Index - PRICE INDEX	1995	SG1MIKS	Futures
DJ UBS Lead 3 Mth Fwd ER -	1996	DJUBLD3	Futures
S&P GSCI 1 Month Fwd Aluminium Index ER -	1995	SG1MIAE	Futures
SP500 PRICE INDEX	1970	S.PCOMP.PI.	Financial index
DOW JONES INDUSTRIALS - PRICE INDEX	1970	DJINDUS.PI.	Financial index
S&P GSCI Energy Excess Return - RETURN IND. (OFCL)	1980	GSENEXR	Financial index
S&P GSCI Commodity Excess Return - RETURN IND. (OFCL)	1980	GSCIEXR	Financial index
S&P GSCI Gold Excess Return - RETURN IND. (OFCL)	1980	GSGCEXR	Financial index
S&P GSCI 2 Mnth Fwd Light ENE Spt - PRICE INDEX	1980	SPGSLPI	Financial index
CYD Long Only Excess Return - EXCESS RETURN	1980	CYDLOER	Financial index
VIX			Source: CBOE Historical series
US CPI - ALL URBAN: ALL ITEMS SADJ		USCONSPCE	
US TREASURY BILL RATE - 3 MONTH (EP)		USGBILL3	
SHFE-ALUMINIUM CONT. INDEX - SETT. PRICE - CH/TE		SHACS04	China DCC
ZCE-COTTON 1 CONTINUOUS - SETT. PRICE - CH/TE		ZCTCS04	China
SHFE-COPPER CONT. INDEX - SETT. PRICE - CH/TE		SCUCS04	China
SHFE-FUEL OIL CONT. INDEX - SETT. PRICE - CH/TE		SFUCS04	China
DCE-NO.1 SOYBEAN CONT. INDEX - SETT. PRICE - CH/TE		DA.CS04	China
ZCE-HARD WHITE WHEAT CONT.INDEX - SETT. PRICE - CH/TE		ZWTCS04	China
ZCE-SUGAR CONTINUOUS - SETT. PRICE - CH/TE		ZSACS04	China
Diesel Brazil North Cons. BRL/LTR		ANPDCNT	Brazil DCC
Natural Gas Brazil Cons. BRL/CBM		ANPCCBR	Brazil
Fed Cattle Triangulo Mneiro BRL/15KG		FEDCABR	Brazil
Gold-Brazil Adjusted BM&F (250g) BR/G		GOLDBRA	Brazil
Sugar Amour Crystal Daily BR		CASUCBR	Brazil
Soybean Grower Exp Triangulo Mneiro		SBGTMBR	Brazil
Wheat (grower) Soft Brazil BRL/60KG		WHGPABR	Brazil
Corn Wholesale SudoesteBRL/60KG		CORWSBR	Brazil

Figure 6: Prices' evolutions over 2004-2013 in Brazil



Wheat price was rescaled- divided by three

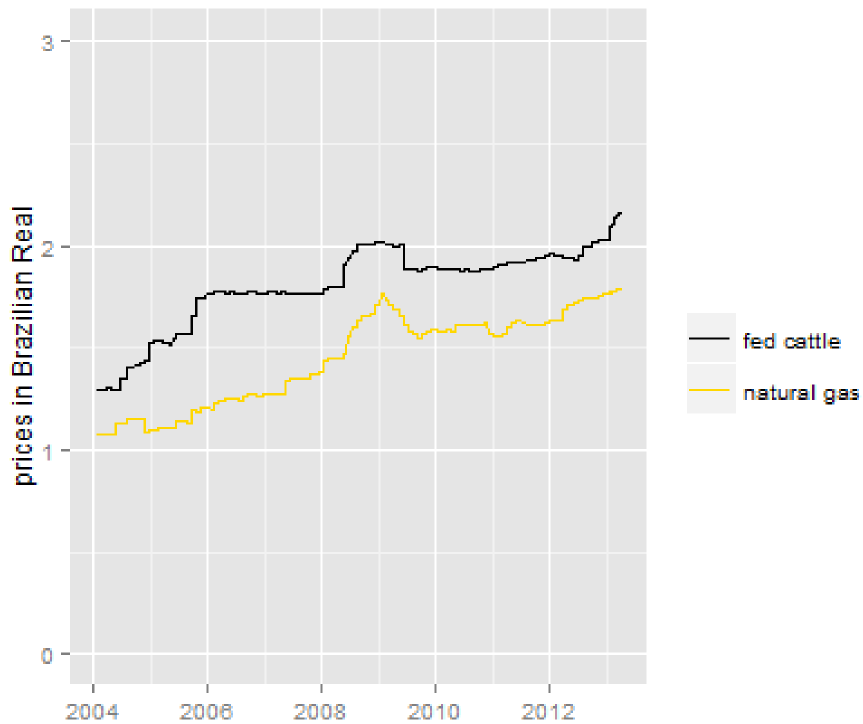
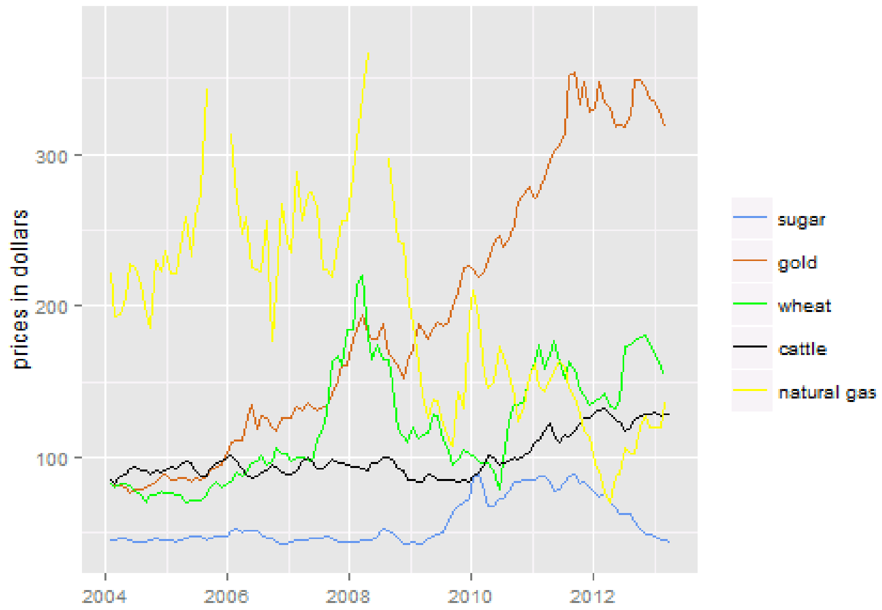


Figure 7: Prices' evolutions over 2004-2013 in the US



Wheat and Gold price were rescaled- divided by two and five respectively

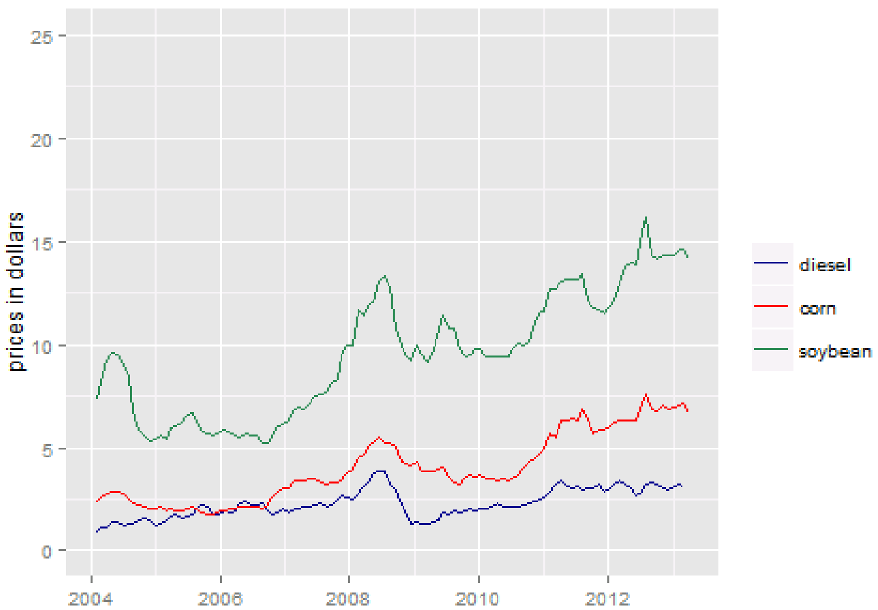


Figure 8: Prices' evolutions over 1993-2013 in China

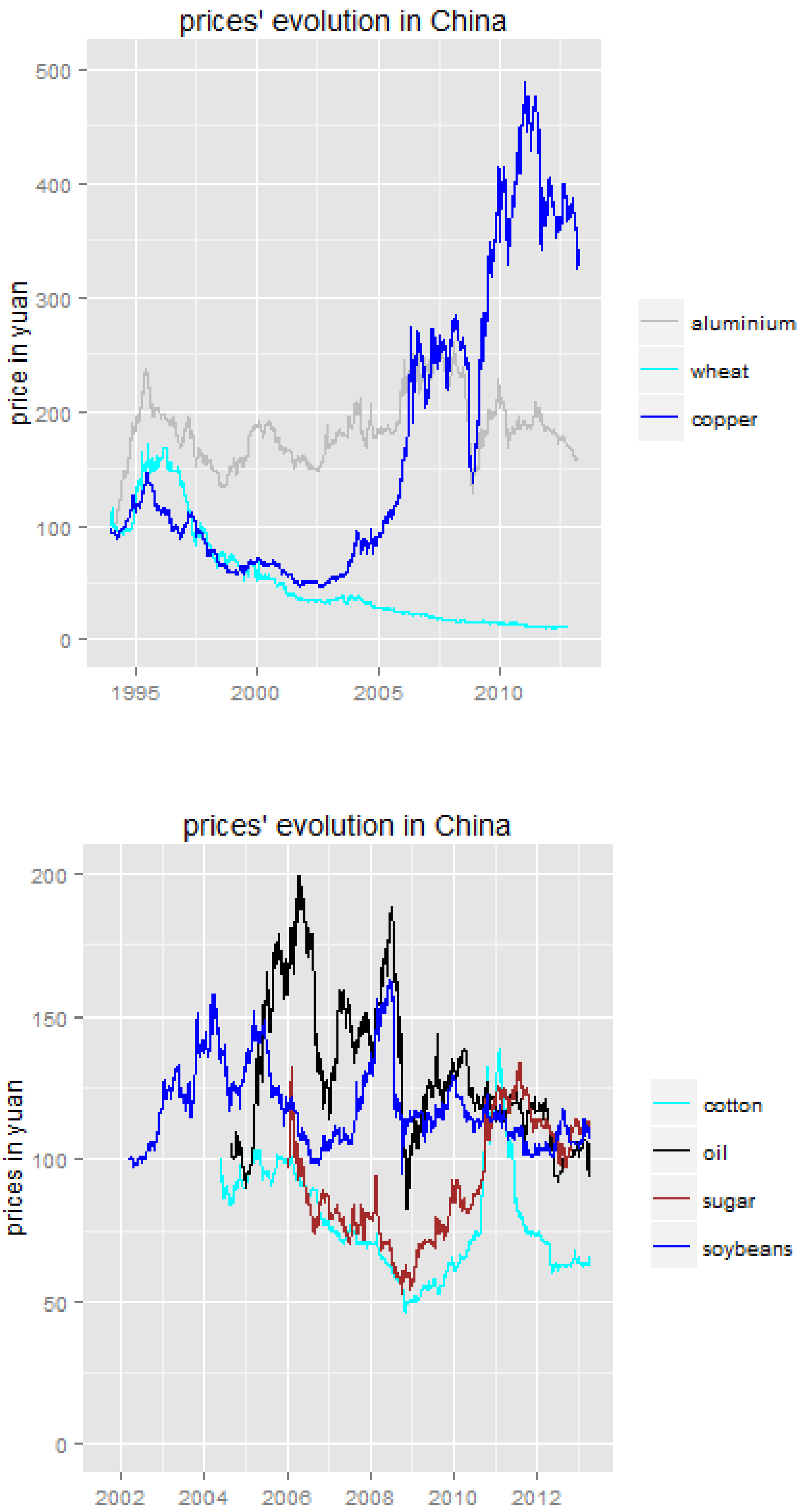


Figure 9: DCC over 1993-2013 in China

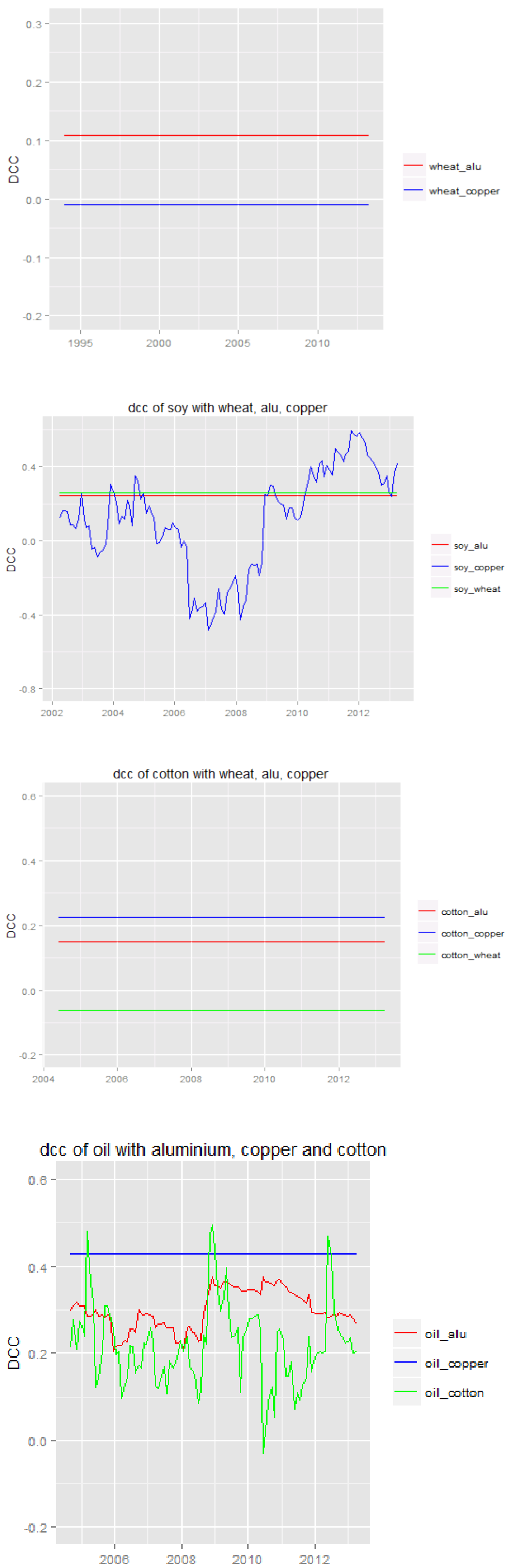


Figure 10: DCC of wheat with aluminium and copper

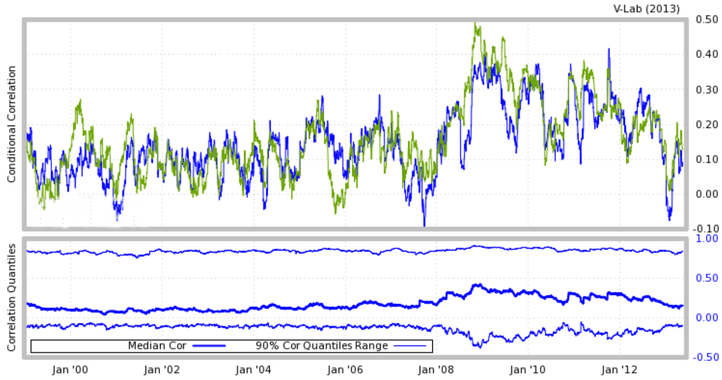


Figure 11: DCC of soybeans with wheat, aluminium and copper

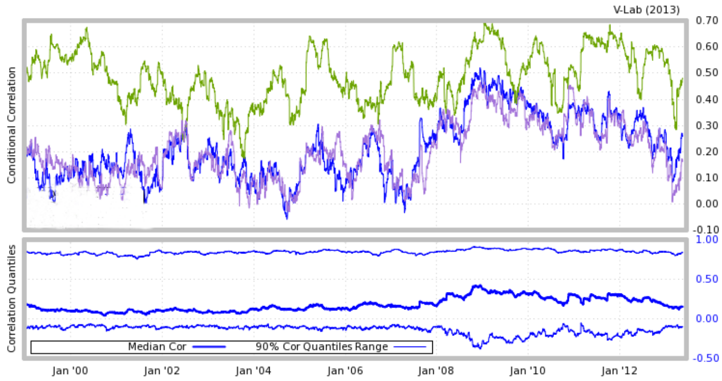


Figure 12: DCC of cotton with wheat, aluminium and copper

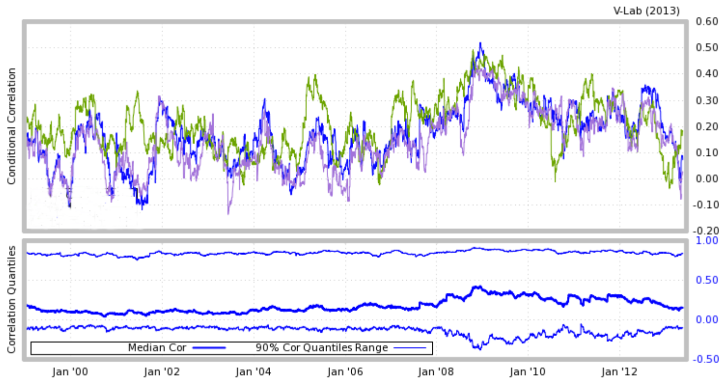


Figure 13: DCC of oil with aluminium, copper and cotton

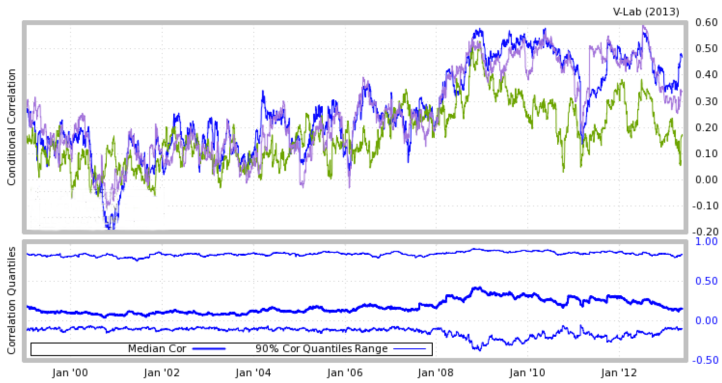


Figure 14: DCC in Brazil and US over 2004-2013

Figure 15: DCC in Brazil and US over 2004-2013

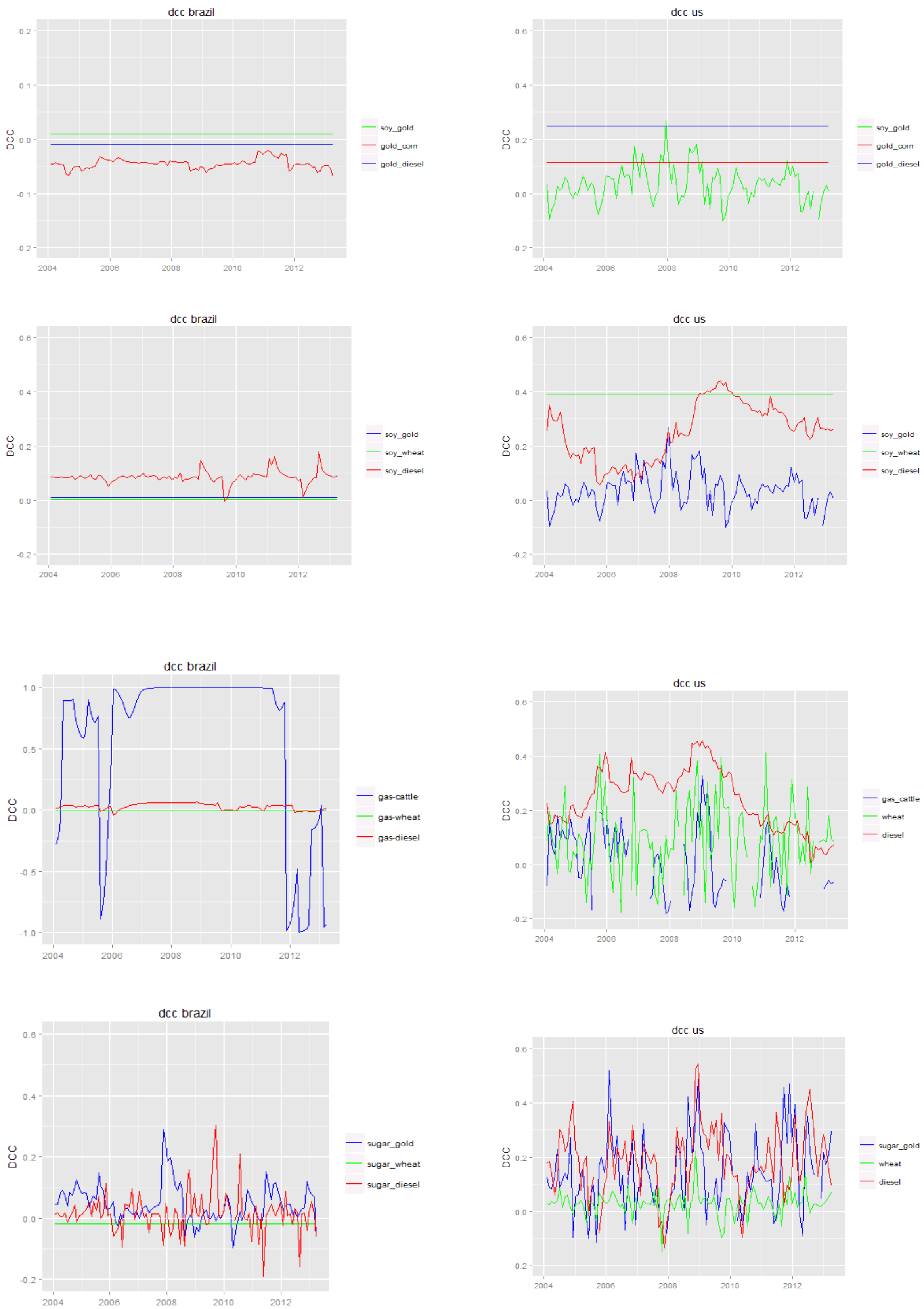
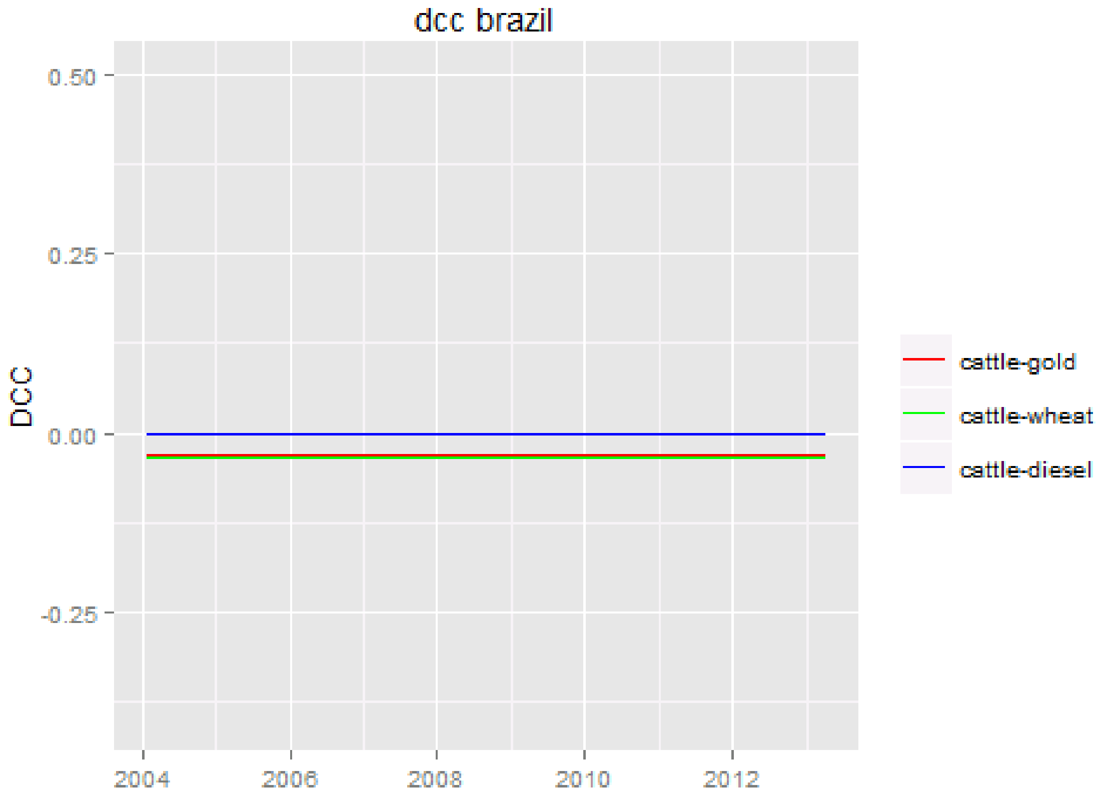


Figure 16: DCC in Brazil and US over 2004-2013



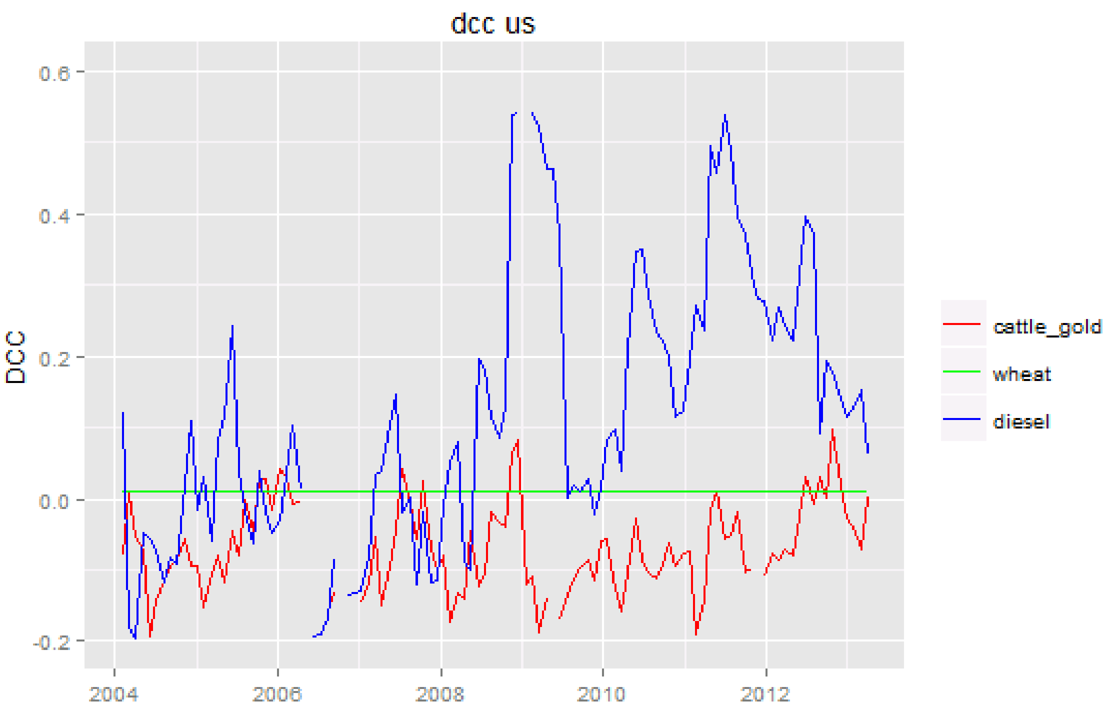


Table 3: Regression Results for Commodity Futures. Full sample: 1990-2013

Note: The dependant variables are displayed on the top row, with regressions number (1) to (9) and are returns for commodities c_i, where the return is defined as $\frac{\Delta_{Pt}}{P_t}$
Except if differently mentioned all commodities c_i presented below refer to the covariance'change $\Delta cov_t(r_{c_i}, r_t^I)$ where r_t^I is the excess return on the GS commodity index

	crude oil	soybean	copper	aluminium	tin	zinc	corn	Dow Jones	SP500
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
tin	0.003	0.012^{*}	0.019^{**}	0.015^{**}	0.019^{**}	0.002	−0.003	0.011^{***}	0.016^{***}
	(0.009)	(0.007)	(0.008)	(0.006)	(0.008)	(0.008)	(0.008)	(0.004)	(0.004)
gas	0.019^{**}			0.010	0.012	0.017^{***}	0.015^{**}		
	(0.008)			(0.006)	(0.008)	(0.006)	(0.007)		
(aluminium, energy)	0.019^{*}				0.013				
	(0.010)				(0.010)				
(SP500, gold)		0.010^{*}				0.012^{*}			
		(0.006)				(0.006)			
copper			0.034^{***}		−0.013		-0.015^{*}		
			(0.012)		(0.011)		(0.008)		
(gold, energy)		-0.017^{***}	-0.021^{***}	-0.015^{***}		-0.025^{***}			
		(0.007)	(0.007)	(0.005)		(0.008)			
(soybean, r_t^I)		0.009			0.011				
		(0.007)			(0.009)				
(SP500, r_t^I)	0.039^{***}	0.012		-0.010^{*}					0.0002
	(0.013)	(0.011)		(0.005)					(0.003)
(crude oil, r_t^I)	-0.024^{**}	-0.016^{**}	−0.012	−0.003	−0.002		−0.008	0.006	
	(0.010)	(0.007)	(0.009)	(0.007)	(0.009)		(0.010)	(0.004)	
zinc			0.015			0.012^{*}			
			(0.011)			(0.007)			
(SP500, energy)	-0.037^{***}	-0.023^{**}	-0.017^{**}			-0.011^{*}			
	(0.013)	(0.011)	(0.007)			(0.007)			
lead	-0.015^{*}		-0.048^{***}		-0.026^{***}			-0.017^{***}	-0.018^{***}
	(0.009)		(0.011)		(0.010)			(0.004)	(0.004)
corn				-0.018^{***}	-0.039^{***}	-0.019^{***}		-0.007^{*}	-0.007^{**}
				(0.006)	(0.009)	(0.007)		(0.004)	(0.003)
wheat	0.008	0.008	0.008	0.010^{**}	0.016^{**}	0.011^{*}	0.011^{*}	0.0001	
	(0.006)	(0.005)	(0.006)	(0.005)	(0.006)	(0.006)	(0.006)	(0.003)	
Constant	0.017^{**}	0.012^{**}	0.014^{**}	0.001	0.013^{**}	0.004	0.001	0.003	0.005^{*}
	(0.007)	(0.006)	(0.006)	(0.005)	(0.006)	(0.006)	(0.007)	(0.003)	(0.003)
Observations	166	166	166	166	166	199	166	166	199
R^2	0.148	0.106	0.156	0.117	0.198	0.116	0.098	0.137	0.119
Adjusted R^2	0.105	0.060	0.113	0.078	0.152	0.078	0.070	0.111	0.100
Residual Std. Error	0.086($df = 157$)	0.072($df = 157$)	0.081($df = 157$)	0.066($df = 158$)	0.079($df = 156$)	0.082($df = 190$)	0.084($df = 160$)	0.042($df = 160$)	0.044($df = 194$)
F statistic	3.421^{***}($df = 8; 157$)	2.318^{**}($df = 8; 157$)	3.632^{***}($df = 8; 157$)	3.002^{***}($df = 7; 158$)	4.284^{***}($df = 9; 156$)	3.103^{***}($df = 8; 190$)	3.490^{***}($df = 5; 160$)	5.101^{***}($df = 5; 160$)	6.522^{***}($df = 4; 194$)

*p<0.1; **p<0.05; ***p<0.01

Table 4: Regression Results 2003-2013

Note:The dependant variables are displayed on the top row, with regressions number (1) to (9) and are returns for commodities c_i, where the return is defined as $\frac{\Delta_{Pt}}{P_t}$
Except if differently mentioned all commodities c_i presented below refer to the covariance'change $\Delta cov_t(r_{c_i}, r_t^I)$ where r_t^I is the excess return on the GS commodity index

	crude oil (1)	soybean (2)	copper (3)	aluminium (4)	tin (5)	zinc (6)	corn (7)	Dow Jones (8)	SP500 (9)
(aluminium, energy)					0.015				
					(0.015)				
gas				0.012	0.019*	0.026**	0.023**		
				(0.009)	(0.011)	(0.010)	(0.010)		
tin		0.016	0.026*	0.019*	0.029**	0.001	−0.005	0.013**	0.019***
		(0.011)	(0.013)	(0.010)	(0.013)	(0.013)	(0.011)	(0.005)	(0.005)
(SP500, gold)		0.014				0.017			
		(0.009)				(0.011)			
copper			0.049***		−0.020		−0.022*		
			(0.018)		(0.017)		(0.012)		
(gold, energy)		−0.024**	−0.031***	−0.021***		−0.035***			
		(0.010)	(0.010)	(0.008)		(0.013)			
soybeans		0.016*			0.012				
		(0.009)			(0.013)				
(SP500,r_t^I)	0.061***	0.018		−0.015*					0.001
	(0.017)	(0.016)		(0.008)					(0.004)
(crude oil,r_t^I)	0.008	−0.023**	−0.016	−0.001	−0.001		−0.007	0.007	
	(0.009)	(0.010)	(0.013)	(0.010)	(0.014)		(0.014)	(0.005)	
zinc			0.020			0.015			
			(0.017)			(0.012)			
(SP500, energy)	−0.054***	−0.034**	−0.026**			−0.015			
	(0.018)	(0.016)	(0.011)			(0.011)			
lead	−0.020**		−0.062***		−0.030**			−0.021***	−0.024***
	(0.009)		(0.017)		(0.014)			(0.005)	(0.005)
corn				−0.025***	−0.057***	−0.027**		−0.008*	−0.007*
				(0.009)	(0.013)	(0.012)		(0.004)	(0.004)
wheat	0.018**	0.011	0.013	0.015*	0.026***	0.018*	0.018**	−0.002	
	(0.008)	(0.008)	(0.009)	(0.008)	(0.010)	(0.011)	(0.009)	(0.004)	
Constant	0.013*	0.013*	0.021**	0.003	0.019**	0.010	0.005	0.005	0.005
	(0.008)	(0.007)	(0.008)	(0.007)	(0.008)	(0.009)	(0.008)	(0.004)	(0.004)
Observations	119	119	119	119	119	119	119	119	119
R^2	0.147	0.145	0.174	0.140	0.251	0.152	0.153	0.206	0.206
Adjusted R^2	0.109	0.083	0.114	0.086	0.189	0.090	0.115	0.170	0.178
Residual Std. Error	0.085($df = 113$)	0.076($df = 110$)	0.092($df = 110$)	0.074($df = 111$)	0.087($df = 109$)	0.097($df = 110$)	0.089($df = 113$)	0.038($df = 113$)	0.040($df = 114$)
F statistic	3.887***($df = 5; 113$)	2.335**($df = 8; 110$)	2.893***($df = 8; 110$)	2.591**($df = 7; 111$)	4.049***($df = 9; 109$)	2.461**($df = 8; 110$)	4.081***($df = 5; 113$)	5.848***($df = 5; 113$)	7.392***($df = 4; 114$)

*p<0.1; **p<0.05; ***p<0.01

Table 5: Regression Results with control variables

Note:The dependant variables are displayed on the top row, with regressions number (1) to (9) and are returns for commodities c_i, where the return is defined as $\frac{\Delta_{P\,t}}{P_t}$
Except if differently mentioned all commodities c_i presented below refer to the covariance'change $\Delta cov_t(r_{c_i}, r_t^I)$ where r_t^I is the excess return on the GS commodity index

	crude oil	soybean	copper	aluminium	tin	zinc	corn	Dow Jones	SP500
US CPI(lag1)	-0.179^{***}	-0.077^{**}	-0.005	-0.012	0.001	-0.006	0.013	0.0008	-0.008
	(0.056)	(0.032)	(0.015)	(0.010)	(0.012)	(0.012)	(0.013)	(0.006)	(0.006)
OI(open interests) oil	0.074^{***}								
	(0.027)								
OI soybeans		0.039^{**}							
		(0.016)							
OI copper			0.017^{*}						
			(0.009)						
VIX(lag15days)	-0.021^{***}	-0.005	-0.018^{***}	-0.021^{***}	-0.014^{**}	-0.023^{***}	-0.011	-0.006^{*}	-0.009^{**}
	(0.008)	(0.007)	(0.007)	(0.005)	(0.006)	(0.006)	(0.007)	(0.003)	(0.003)
Interest rate	-0.034^{**}	-0.015	-0.003	-0.009	-0.007	-0.006	-0.001	0.003	-0.004
	(0.014)	(0.010)	(0.010)	(0.007)	(0.009)	(0.009)	(0.010)	(0.005)	(0.005)
tin	0.003	0.014^{*}	0.010	0.015^{**}	0.018^{**}	0.001	-0.004	0.011^{***}	0.016^{***}
	(0.010)	(0.008)	(0.008)	(0.006)	(0.008)	(0.008)	(0.008)	(0.004)	(0.004)
gas	0.019^{**}			0.006	0.010	0.014^{**}	0.013^{*}		
	(0.009)			(0.006)	(0.008)	(0.006)	(0.008)		
(aluminium,energy)	0.016				0.013				
	(0.010)				(0.010)				
(SP500, gold)		0.010				0.013^{**}			
		(0.006)				(0.006)			
copper			0.023^{**}		-0.015		-0.014^{*}		
			(0.010)		(0.012)		(0.008)		
(gold, energy)		-0.019^{**}	-0.025^{***}	-0.016^{***}		-0.026^{***}			
		(0.007)	(0.007)	(0.005)		(0.007)			
soybeans		0.007			0.009				
		(0.007)			(0.009)				
(SP500, r_t^I)	0.046^{***}	0.013		-0.010^{**}					0.0002
	(0.014)	(0.012)		(0.005)					(0.003)
(crude oil, r_t^I)	-0.029^{***}	-0.018^{**}		-0.003	-0.001		-0.008	0.006	
	(0.010)	(0.008)		(0.006)	(0.009)		(0.010)	(0.004)	
(SP500, energy)	-0.043^{***}	-0.021	-0.015^{**}			-0.011^{*}			
	(0.014)	(0.012)	(0.007)			(0.006)			
zinc			0.021^{**}			0.015^{**}			
			(0.011)			(0.007)			
lead	-0.004		-0.038^{***}		-0.022^{**}			-0.017^{***}	-0.018^{***}
	(0.010)		(0.011)		(0.010)			(0.004)	(0.004)
corn				-0.020^{***}	-0.039^{***}	-0.021^{***}		-0.007^{*}	-0.007^{**}
				(0.006)	(0.009)	(0.007)		(0.004)	(0.003)
wheat	0.006	0.006	0.010^{*}	0.011^{**}	0.016^{**}	0.012^{*}	0.011^{*}	0.0001	
	(0.007)	(0.006)	(0.006)	(0.005)	(0.006)	(0.006)	(0.006)	(0.003)	
Constant	0.083^{***}	0.037^{***}	0.021^{***}	0.009	0.012	0.011	-0.005	0.003	0.005^{*}
	(0.020)	(0.013)	(0.008)	(0.007)	(0.009)	(0.007)	(0.010)	(0.003)	(0.003)
Observations	128	128	147	162	162	195	162	166	199
R^2	0.289	0.172	0.245	0.216	0.225	0.184	0.123	0.137	0.119
Adjusted R^2	0.215	0.086	0.183	0.164	0.162	0.135	0.077	0.111	0.100
Residual Std. Error	0.087(df = 115)	0.074(df = 115)	0.080(df = 135)	0.063(df = 151)	0.079(df = 149)	0.079(df = 183)	0.084(df = 153)	0.042(df = 160)	0.044(df = 194)

$^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Table 6: Regression Results with control and lags variables

Note:The dependant variables are displayed on the top row, with regressions number (1) to (9) and are returns for commodities c_i, where the return is defined as $\frac{\Delta_{P}t}{P_t}$
Except if differently mentioned all commodities c_i presented below refer to the covariance'change $\Delta cov_t(r_{c_i}, r_t^I)$ where r_t^I is the excess return on the GS commodity index

	crude oil	soybean	copper	aluminium	tin	zinc	corn	Dow Jones	SP500
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
lag(silver)	-0.015^{*}		-0.025^{**}	-0.015^{**}	-0.016^{*}	-0.016^{*}			
	(0.009)		(0.010)	(0.006)	(0.009)	(0.009)			
oil OI	0.071^{**}								
	(0.029)								
lag(lead)				0.020^{***}					
				(0.006)					
gas	0.018^{**}			0.001			0.010^{*}		
	(0.008)			(0.006)			(0.006)		
(aluminium,energy)	0.017^{*}								
	(0.009)								
lag(aluminium)			-0.035^{**}						
			(0.015)						
lag(soybean)								-0.006^{*}	
								(0.003)	
lag(zinc)									-0.013^{**}
									(0.007)
lag(copper)			0.045^{***}		0.013	0.033^{***}		0.012^{***}	0.022^{***}
			(0.015)		(0.009)	(0.008)		(0.004)	(0.007)
copper(Han index)			-0.111^{*}						
			(0.060)						
copper(De Roon index)			-0.115^{*}						
			(0.064)						
copper OI			0.019^{*}						
			(0.010)						
corn		-0.035^{***}	-0.016^{**}	-0.020^{***}	-0.031^{***}	-0.023^{***}		-0.008^{**}	-0.006^{*}
		(0.009)	(0.008)	(0.006)	(0.007)	(0.007)		(0.004)	(0.003)
lead			-0.027^{**}		-0.025^{***}	-0.042^{***}		-0.013^{***}	-0.015^{***}
			(0.010)		(0.008)	(0.010)		(0.004)	(0.004)
US CPI	-0.150^{**}	-0.123^{***}	0.036	-0.005	0.007	-0.016	0.013	-0.008	-0.023^{***}
	(0.063)	(0.040)	(0.029)	(0.012)	(0.015)	(0.015)	(0.012)	(0.007)	(0.007)
soy OI		0.056^{***}							
		(0.019)							
VIX	-0.025^{***}	-0.003	-0.026^{***}	-0.026^{***}	-0.018^{***}	-0.032^{***}	-0.010	-0.009^{**}	-0.013^{***}
	(0.008)	(0.007)	(0.009)	(0.005)	(0.007)	(0.007)	(0.006)	(0.004)	(0.003)
interest rate	-0.026^{*}	-0.019^{*}	-0.005	-0.002	-0.006	-0.011	-0.0002	-0.008	-0.014^{**}
	(0.015)	(0.010)	(0.012)	(0.008)	(0.010)	(0.010)	(0.009)	(0.005)	(0.006)
silver							-0.008		
							(0.007)		
copper							-0.016^{**}		
							(0.007)		
tin		0.015^{*}		0.015^{**}	0.019^{**}		-0.002	0.011^{***}	0.016^{***}
		(0.008)		(0.006)	(0.009)		(0.007)	(0.004)	(0.004)
(SP500, gold)		0.013^{**}	-0.003			0.009			
		(0.006)	(0.007)			(0.006)			
(gold,energy)		-0.018^{***}	-0.017^{**}	-0.019^{***}		-0.027^{***}			
		(0.007)	(0.008)	(0.005)		(0.007)			
soybeans		0.030^{***}							
		(0.009)							
(SP, r_t^I)	0.047^{***}	-0.010	-0.004	-0.011^{**}	0.0004	-0.004			-0.001
	(0.014)	(0.006)	(0.007)	(0.005)	(0.006)	(0.006)			(0.003)
(crude oil,r_t^I)	-0.033^{***}	-0.017^{**}	0.006	0.003	0.003	0.014^{*}		0.006	
	(0.010)	(0.007)	(0.009)	(0.006)	(0.008)	(0.008)		(0.004)	
(SP, energy)	-0.045^{***}								
	(0.014)								
zinc			0.029^{**}			0.048^{***}			
			(0.011)			(0.010)			
wheat	0.004	0.007	0.012^{*}	0.012^{**}	0.016^{**}	0.017^{***}	0.012^{**}	0.0001	
	(0.007)	(0.006)	(0.007)	(0.005)	(0.006)	(0.006)	(0.005)	(0.003)	
Observations	128	128	128	162	162	162	195	162	195
R^2	0.331	0.269	0.331	0.283	0.217	0.330	0.122	0.210	0.211
Adjusted R^2	0.255	0.178	0.227	0.220	0.159	0.271	0.084	0.158	0.173
Residual Std. Error	0.085($df = 114$)	0.070($df = 113$)	0.081($df = 110$)	0.061($df = 148$)	0.079($df = 150$)	0.077($df = 148$)	0.080($df = 186$)	0.041($df = 151$)	0.043($df = 185$)

*p<0.1; **p<0.05; ***p<0.01

Table 7: Regression Results Spot

Note: The dependant variables are displayed on the top row, with regressions number (1) to (9) and are returns for commodities c_i, where the return is defined as $\frac{\Delta P t}{P_t}$ where P_t is a spot price.
Except if differently mentioned all commodities c_i presented below refer to the covariance'change $\Delta cov_t(r_{c_i}, r_t^I)$ where r_t^I is the excess return on the GS commodity index
level refers to the cov_t introduced in level rather than in changes

	crude oil	crude oil (level)	copper	copper (level)	rhodium	zinc	cotton	cocoa	soybeans
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
(aluminium, energy)	0.017^{**}								
	(0.008)								
tin	-0.008								-0.004
	(0.007)								(0.005)
rhodium	-0.010								
	(0.007)								
cocoa								-0.011	
								(0.007)	
(SP500, r_t^I)	0.036^{***}							0.002	0.007
	(0.012)							(0.005)	(0.009)
corn			-0.003						-0.021^{***}
			(0.005)						(0.008)
soybeans							0.014^{**}	0.008^{*}	0.023^{***}
							(0.006)	(0.005)	(0.007)
copper			0.014^{*}				-0.030^{***}		
			(0.008)				(0.007)		
(crude oil,r_t^I)	-0.007		-0.005			-0.018^{*}			-0.014^{**}
	(0.008)		(0.007)			(0.010)			(0.005)
zinc						0.021^{**}			
						(0.009)			
(SP500, energy)	-0.051^{***}		-0.015^{**}			-0.021^{***}	-0.002		-0.016
	(0.013)		(0.007)			(0.007)	(0.007)		(0.010)
silver						0.022^{**}			
						(0.010)			
lead	-0.021^{***}		-0.011			-0.018^{**}		-0.009^{*}	
	(0.007)		(0.007)			(0.008)		(0.005)	
(gold, energy)			-0.012^{**}			-0.029^{***}		-0.007	-0.009^{*}
			(0.006)			(0.008)		(0.006)	(0.005)
wheat	0.005		0.004						0.002
	(0.006)		(0.006)						(0.006)
silver		-0.011							
		(0.007)							
wheat		0.014^{**}			0.018				
		(0.007)			(0.011)				
sugar				-0.009^{**}	-0.023^{**}				
				(0.004)	(0.010)				
rhodium					0.021^{*}				
					(0.012)				
zinc					0.003				
					(0.011)				
(SP500,r_t^I)		0.011^{*}		0.006	-0.0004				
		(0.006)		(0.004)	(0.012)				
(crude oil,r_t^I)		0.008			-0.008				
		(0.006)			(0.014)				
corn		-0.023^{***}		-0.013^{***}	-0.042^{***}				
		(0.007)		(0.004)	(0.010)				
(SP500, gold)						0.015^{**}		0.017^{***}	0.011^{**}
						(0.007)		(0.005)	(0.005)
Constant	0.013^{*}	0.011^{**}	0.007	0.007^{*}	0.017^{*}	0.010	0.012	0.005	0.007
	(0.006)	(0.005)	(0.005)	(0.004)	(0.010)	(0.007)	(0.008)	(0.005)	(0.005)
Observations	214	324	214	360	231	130	135	214	214
R^2	0.128	0.066	0.055	0.048	0.100	0.182	0.136	0.092	0.114
Adjusted R^2	0.094	0.051	0.018	0.040	0.071	0.128	0.116	0.066	0.075
Residual Std. Error	0.094($df = 205$)	0.097($df = 318$)	0.080($df = 205$)	0.077($df = 356$)	0.143($df = 223$)	0.085($df = 121$)	0.091($df = 131$)	0.074($df = 207$)	0.072($df = 204$)
F statistic	3.766^{***}($df = 8; 205$)	4.480^{***}($df = 5; 318$)	1.484($df = 8; 205$)	5.948^{***}($df = 3; 356$)	3.526^{***}($df = 7; 223$)	3.364^{***}($df = 8; 121$)	6.860^{***}($df = 3; 131$)	3.496^{***}($df = 6; 207$)	2.919^{***}($df = 9; 204$)

*p<0.1; **p<0.05; ***p<0.01

Table 8: Regression Results Spot: counterfactual

Note: The dependant variables are displayed on the top row, with regressions number (1) to (9) and are returns for commodities c_i, where the return is defined as $\frac{\Delta_{Pt}}{P_t}$ where P_t is a spot price. All first six predictors are covariances in level, while after they are in changes as usually specified.Except if differently mentioned all commodities c_i presented below refer to the covariance'change $\Delta cov_t(r_{c_i}, r_t^I)$ where r_t^I is the excess return on the GS commodity index

	palladium level (1)	palladium (2)	soy meal (3)	titanium level (4)	titanium (5)	sugar (6)	gas (7)	electricity (8)	electricity 2 (9)
silver(level)	−0.003 (0.007)								
wheat	0.006 (0.007)								
sugar				−0.001 (0.002)					
(SP500,r_t^I)	0.010 (0.006)			0.002 (0.002)					
(crude oil,r_t^I)	−0.004 (0.006)								
corn(level)	−0.016** (0.007)			−0.0001 (0.002)					
(aluminium,energy)		−0.015 (0.010)					−0.019 (0.016)	0.041 (0.028)	
corn			−0.019* (0.011)		0.001 (0.002)			0.026 (0.032)	0.018 (0.020)
tin		−0.003 (0.007)	0.004 (0.007)				−0.007 (0.013)	0.008 (0.021)	0.010 (0.020)
rhodium		0.019** (0.008)							
cocoa						0.008 (0.008)			
(gold,SP500)			0.020** (0.008)			0.001 (0.006)			−0.020 (0.020)
copper					0.001 (0.002)			−0.033 (0.031)	
(SP500,energy)					0.001 (0.002)				0.011 (0.023)
(gold, energy)			−0.005 (0.007)		0.002 (0.002)	−0.001 (0.007)			0.050** (0.022)
soybean			0.018* (0.010)			−0.009 (0.006)		−0.010 (0.031)	
gas							−0.040*** (0.012)	0.022 (0.021)	0.015 (0.019)
(SP500,r_t^I)		0.0002 (0.008)	−0.003 (0.009)			−0.003 (0.007)	0.014 (0.013)		
(crude oil,r_t^I)		−0.022** (0.009)	−0.026*** (0.009)		−0.001 (0.002)		0.006 (0.015)	−0.013 (0.026)	
sugar					−0.003* (0.002)				
lead		0.012 (0.009)			−0.0002 (0.002)	−0.007 (0.006)	0.003 (0.014)	−0.007 (0.026)	
wheat		−0.0005 (0.007)	−0.003 (0.008)		−0.001 (0.002)		−0.007 (0.011)	−0.018 (0.022)	−0.022 (0.022)
Constant	0.011** (0.005)	0.013* (0.007)	0.012 (0.008)	0.0002 (0.002)	0.001 (0.002)	0.006 (0.006)	0.019 (0.012)	0.041** (0.020)	0.041** (0.020)
Observations	316	214	143	280	214	214	214	214	214
R^2	0.041	0.053	0.158	0.009	0.030	0.027	0.074	0.030	0.039
Adjusted R^2	0.026	0.021	0.108	−0.001	−0.008	−0.001	0.042	−0.013	0.007
Residual Std. Error	0.095(df = 310)	0.107(df = 206)	0.092(df = 134)	0.028(df = 276)	0.024(df = 205)	0.093(df = 207)	0.179(df = 206)	0.294(df = 204)	0.291(df = 206)
F statistic	2.668**(df = 5; 310)	1.639(df = 7; 206)	3.147***(df = 8; 134)	0.880(df = 3; 276)	0.795(df = 8; 205)	0.953(df = 6; 207)	2.335**(df = 7; 206)	0.693(df = 9; 204)	1.209(df = 7; 206)

*p<0.1; **p<0.05; ***p<0.01