What's happening today?

- Maths & Biodiversity (Robin)
- Morning/Afternoon Tea
- Modelling Queues (Sarah)

Modelling Queues

Dr Sarah Marshall

9 – 20 January 2023

Rotary Science and Technology Forum

Auckland University of Technology

A bit about me . . .

- Grew up in Napier
- Studied Conjoint BSc/BCA and MSc at Victoria University of Wellington
- Studied PhD at University of Edinburgh, UK
- Been at AUT since 2014
- I love using maths to model systems
- I teach on the **analytics** and **mathematical modelling and computation** majors on the BSc at AUT

Who has been stuck in this kind of queue?

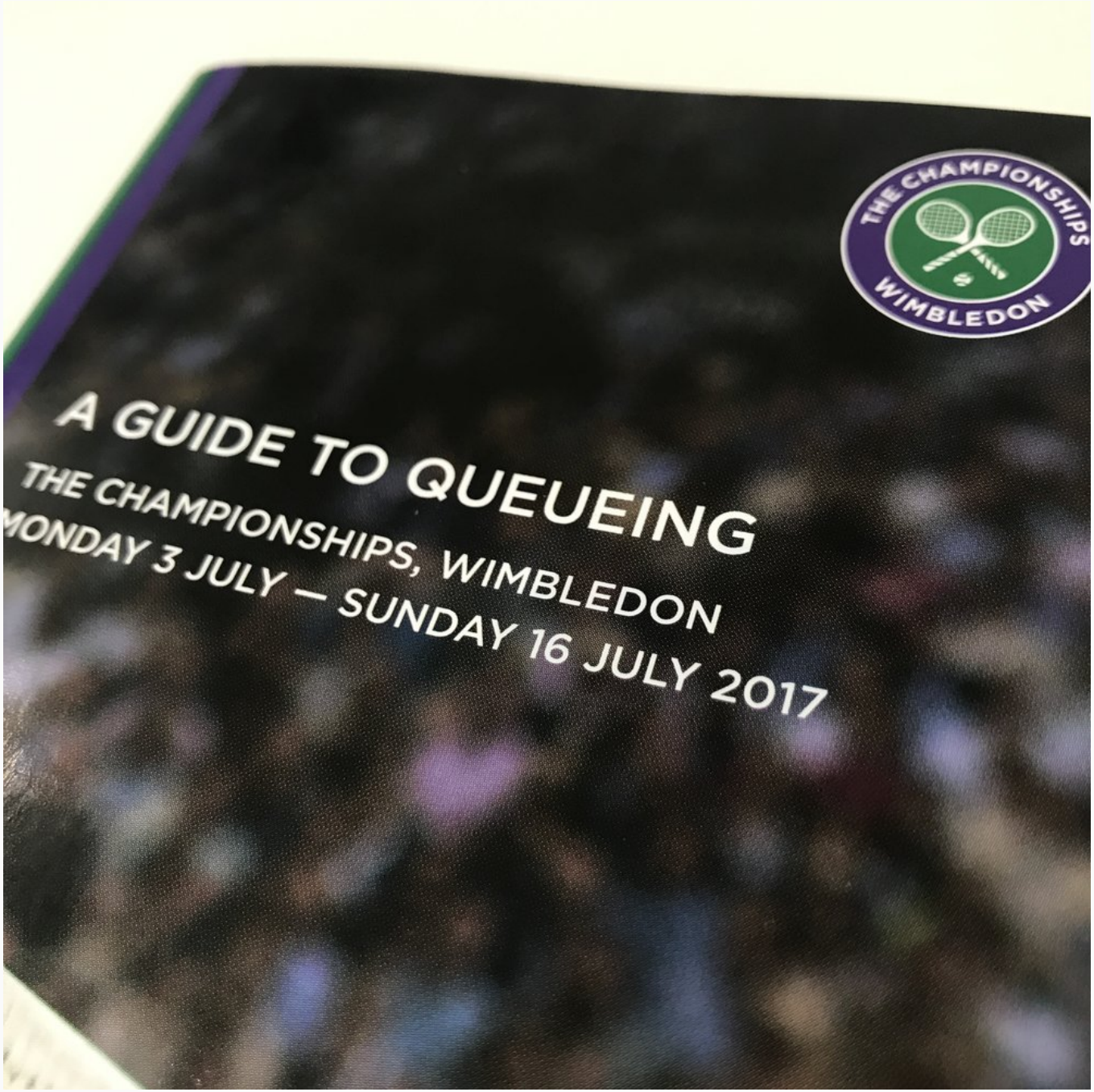
http://www.geograph.ie/photo/3414308

.... or this kind of queue?

Source: Sarah's Phone

.... or this kind of queue?

http://en.wikipedia.org/wiki/File:Waiting_in_line_at_a_food_store.JPG

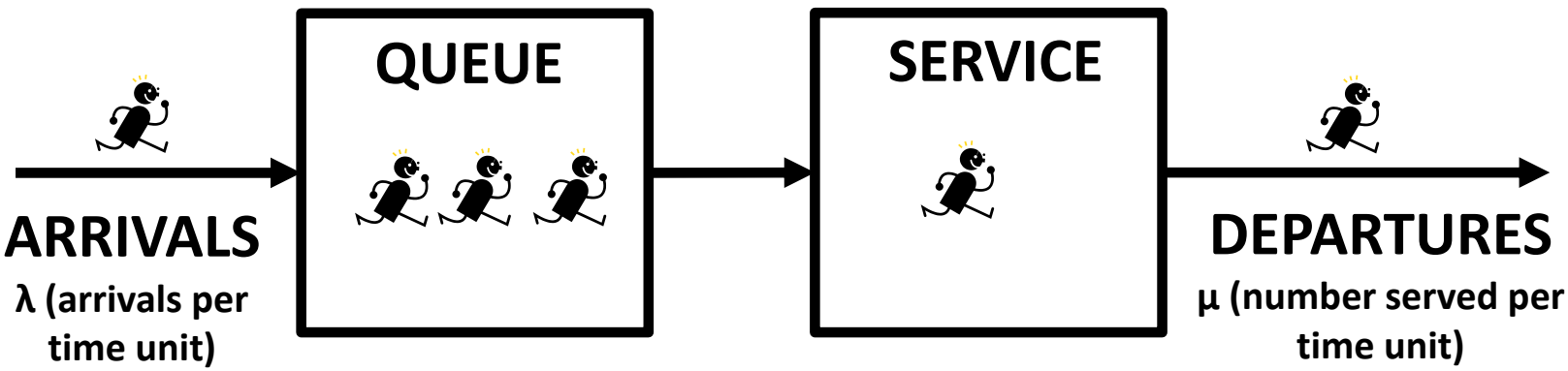


https://static1.squarespace.com/static/59d639183e00be6ee8e5633e/t/59e25331d55b41ace0cf28da/

How can we describe this queue?

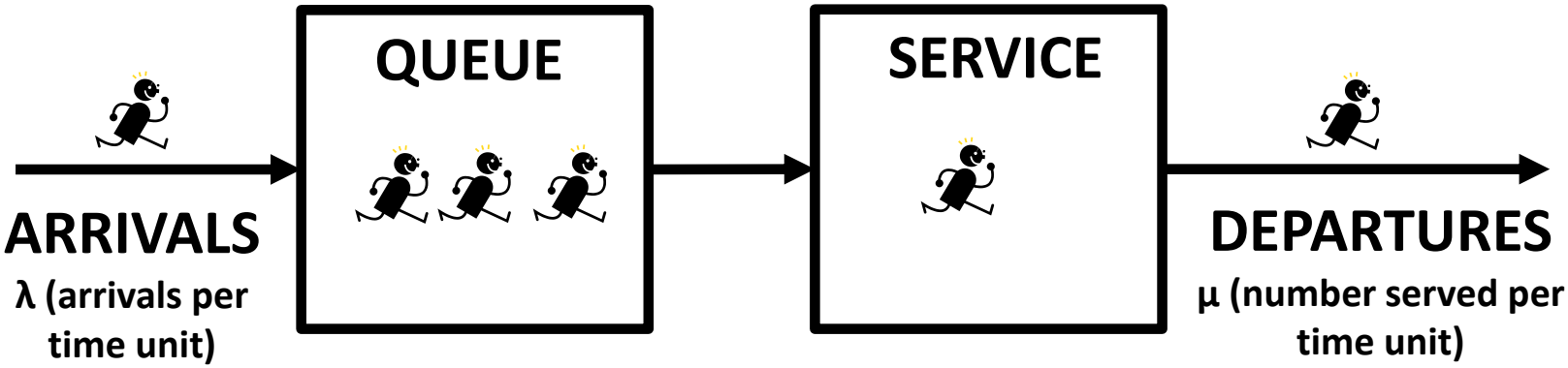
http://en.wikipedia.org/wiki/File:Waiting_in_line_at_a_food_store.JPG

Modelling a Queue



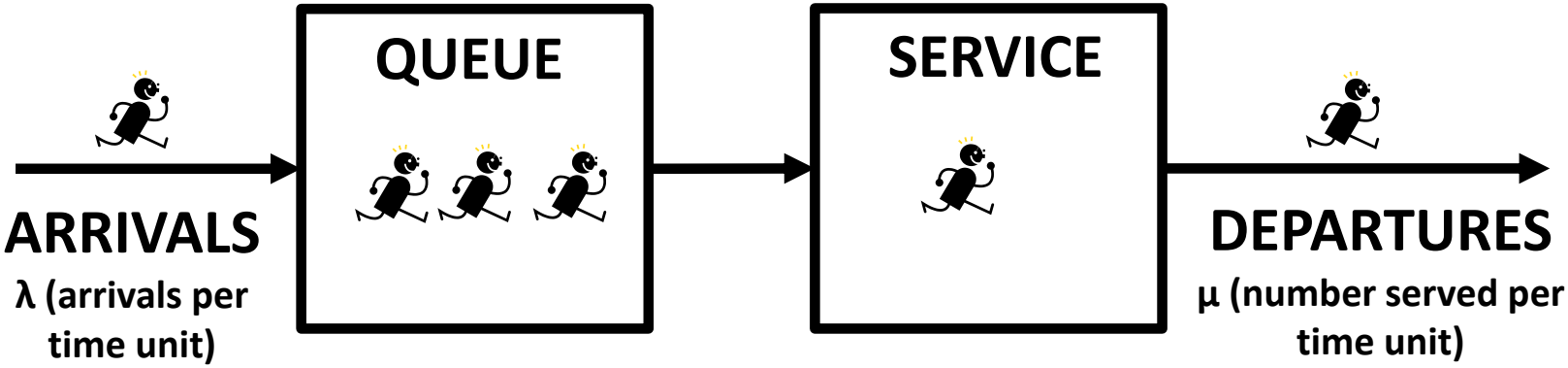
- Arrival Process
  - Probability distribution for time between customer arrivals (interarrival time)

Modelling a Queue



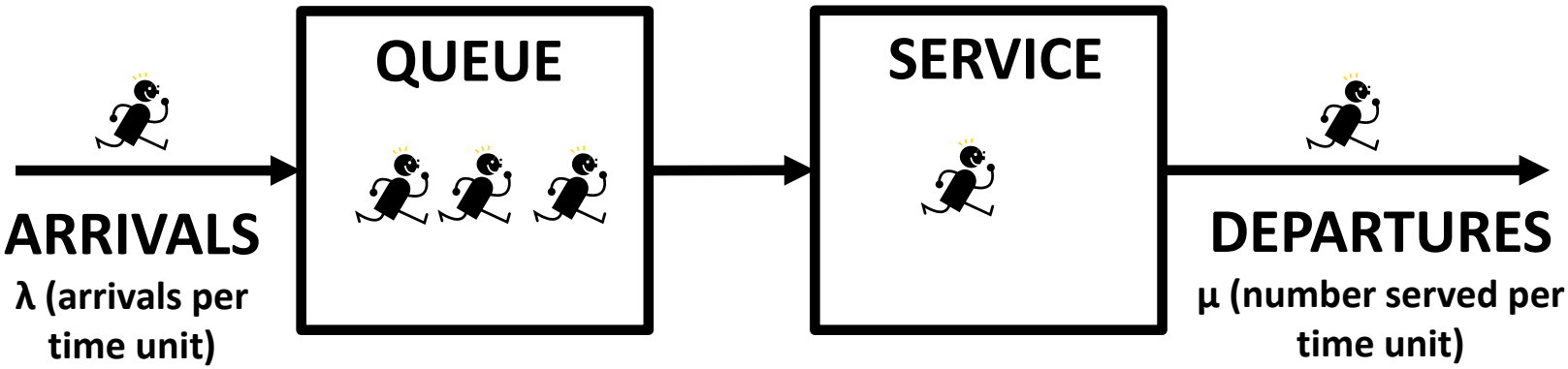
- Service Process
  - Number of Servers
  - Probability distribution for service time

Modelling a Queue



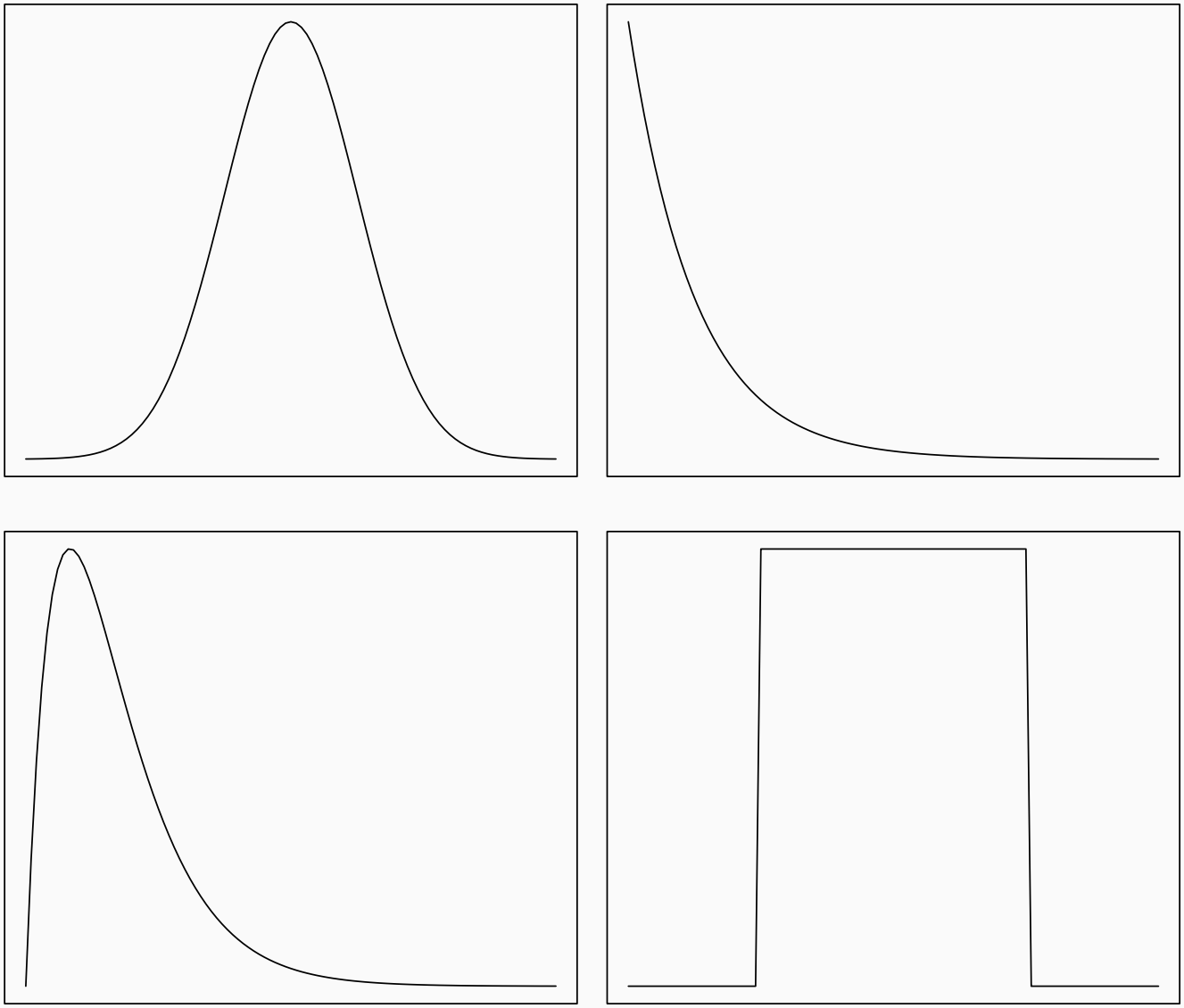
- Queueing Discipline
  - First-come-first-served
  - Last-come-first-served
  - Priority-based service
  - Service in random order

Modelling a Queue



- Type of queues (one vs several lines)

Probability distributions

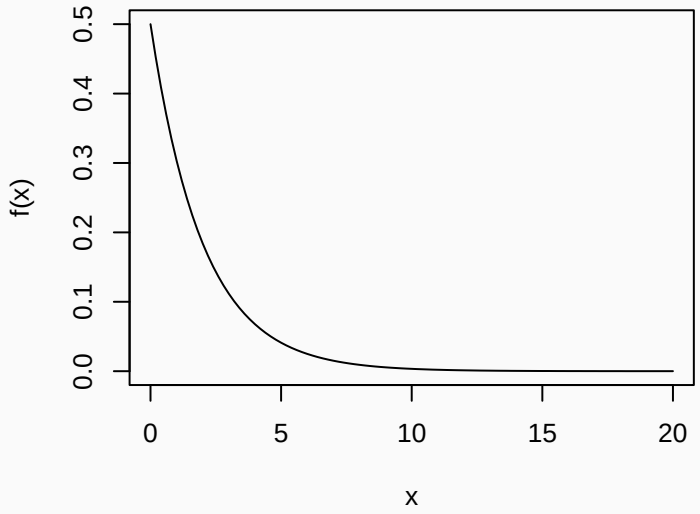


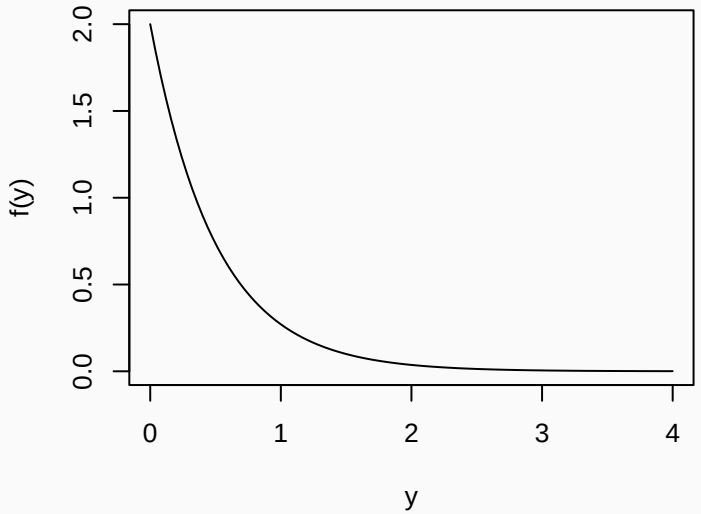
In queueing: needs distributions with values that are ≥ 0.

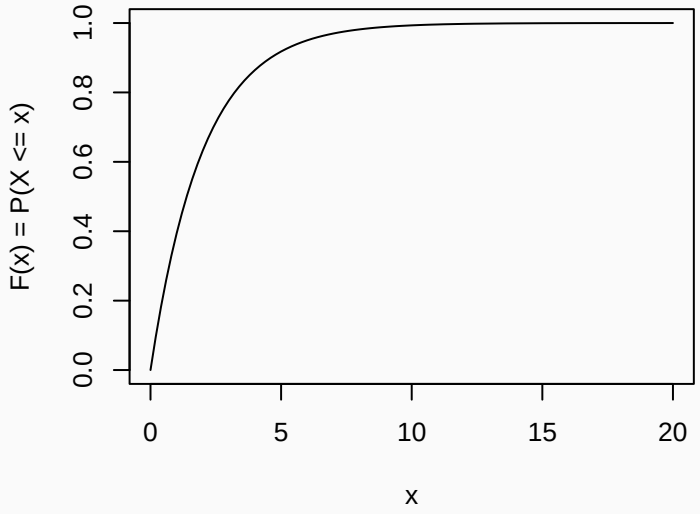
M/M/1 Queue

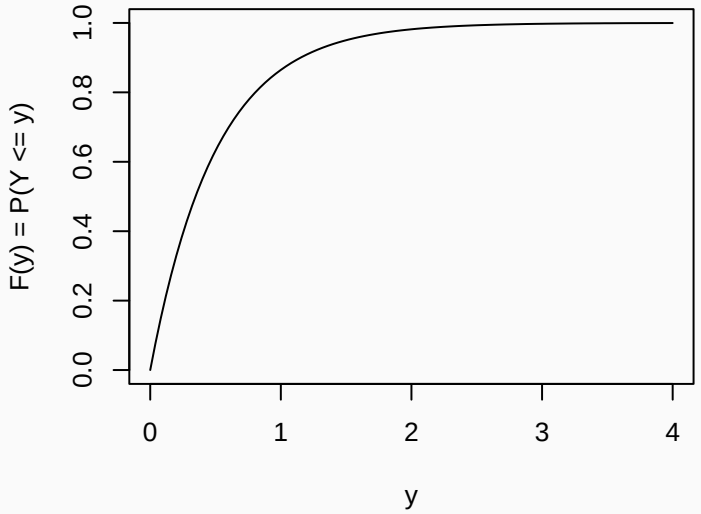
- Arrival: interarrival time is exponentially distributed
- Service: service time is exponentially distributed
- Number of servers is 1

Exponential distribution









Measuring Performance of a Queue

- Number of customers in the:
  - Queue, L_q
  - Service, L_s
  - System, L
- Time spent in the:
  - Queue, W_q
  - Service, W_s
  - System, W

Questions of interest

- What happens if customers arrive faster than they are being served?
- If a queue has 3 servers, is it better to have individual queues or one queue?

R Simulation Activity

See handout

Designing your own supermarket checkout

- Two types of customers
  - Express (12 items or less)
    - Arrival rate 4 per minute
    - Service rate 1 per minute (service time $= 1$ min)
  - Regular (more than 12 items)
    - Arrival rate 1 per minute
    - Service rate 0.2 per minute (service time $= 5$ min)
- Two types of checkouts
  - Self-checkout (12 items or less) cost $= \$10$
  - Staffed-checkout (any number of items) cost $= \$100$

Designing your own supermarket checkout

Find a configuration of self and staffed checkouts which:

- Costs $700 or less
- Average waiting time in the queue is less than the average service time
    - Express customers – average time in queue less than 1 minute
    - Regular customers – average time in queue less than 5 minutes

R Simulation Activity

See handout