
Book Title



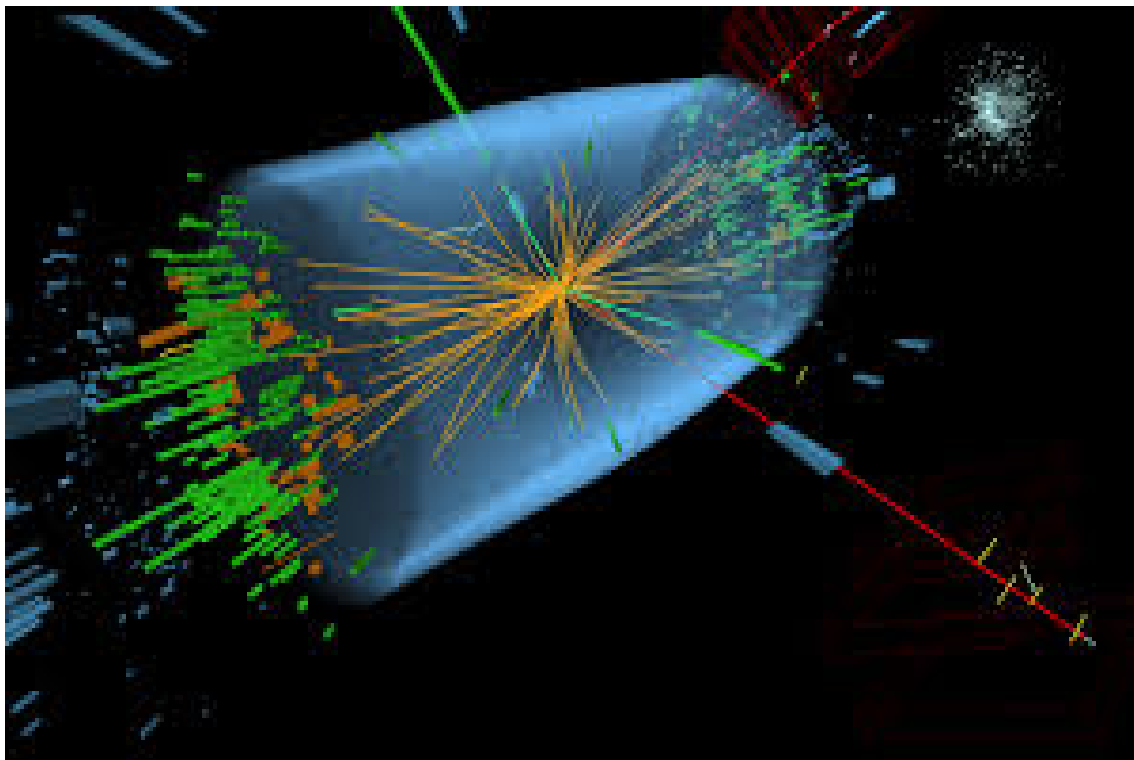
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Elementary Particles and Forces

1.1 Elementary Particles

One of the questions that we, as human beings, have been asking since we started thinking, is “What is the universe made up of and what holds it together?” A long time ago, Democritus tried to answer the first part by defining atoms although his definition of the atom was quite different from what we know about the atom today. In the last couple of centuries, we have come a long way in terms of answering the questions about the composition of matter and the glue that holds it together giving us the form of the universe we see today. The quest to answer the question “What is matter made of?” has been much like unraveling Russian nesting dolls. Everything, such as a chair, table, you, and me are made up of different kinds of molecules. Every molecule is made up of different kinds of atoms. All atoms, it turns out, even if they are different, have very similar structures they all have one nucleus around which electrons revolve. The nucleus is made up of protons and neutrons. Protons are positively charged and electrons are negatively charged particles. The number of protons and electrons in a neutral atom are exactly equal so that the positive and negative charges cancel each other out. All atoms are very similar in that they are all made up of protons, neutrons and electrons. However, a gold atom would be different from an iron atom or a hydrogen atom because they would have a different number of protons, neutrons and electrons.

Figure 1: Scale of fundamental particles However the atom is not the last of our Russian dolls.

We know now that every proton and neutron itself is made up of even smaller particles, called quarks. Electrons, on the other hand, appear to be fundamental particles, that is, they cannot be further decomposed. So, as of now, we can divide fundamental particles into two categories - Quarks and Leptons. There are 6 types of quarks and 6 types of leptons. Quite symmetric, don't you think? But the symmetry doesn't end here. They seem to form 3 pairs (we call them families or generations) as you can see from Fig.2.

Three generations of quarks (and leptons) are very similar in that they have same charge, spin etc. They differ in some internal quantum properties, but the most apparent difference is in mass which increases as we go from 1st generation to 3rd. As you can see from Table. 2 below, this difference is up to 5 orders of magnitude.

For every particle, there is also a corresponding anti particle. Usually they have simple names, such as an anti-up quark or anti-neutrino. The anti-particle for the electron, however, has the special name positron. The existence of anti-matter was predicted when Paul Dirac unified quantum mechanics and Einstein's theory of special relativity. The anti-electron was discovered a few years later by Anderson.

1.2 Forces

There are few other fundamental particles which don't make up matter directly, but instead are exchanged in interactions between matter particles. These force carrier particles, when exchanged between two matter particles, make these particles interact through the force which they are carriers of.

There are four known fundamental forces through which particles can interact and each has its own carrier particles.

1. Strong nuclear force Only quarks can feel this force through the exchange of force carrier particles called gluons. This force is much stronger compared to electromagnetic force of repulsion between the same charge protons and keeps the atomic nucleus stable.

2. Weak nuclear force This is the force responsible for radioactive decay of atomic nuclei. It has three force carrier particles W^+ , W^- , and Z boson. It allows one type of quark to turn into another, usually within the same generation. So a top quark can turn into a bottom quark by emitting a W boson. It also allows charged leptons to change into a neutrino. So a muon can decay to a muon neutrino by emitting a W boson. The weak force is also very important in the fusion reactions that power the sun.

3. Electromagnetic force The force between charged particles is mediated through the exchange of electromagnetic force carrier particles called photons. This is the same photon that makes up the light you see from sun or a bulb. This is the force of attraction (repulsion) between opposite (same) electric charges or magnetic poles.

4. Gravitational force The everyday force that keeps us safely on the surface of the earth and is felt by all particles with mass. Graviton, the theorized particle proposed to be the force carrier for gravity has not been discovered yet.

The forces and corresponding carrier particles are listed in Table.1. Table 1: Fundamental forces and corresponding force carrier particles.

1.3 Standard model of particle physics

Going back to the question what is the universe made up of and what holds it together? our current understanding is that the matter in our visible universe is made up of quarks and leptons which interact with one another through force carrier particles. There is a theory called the standard model of particle physics which describes this visible universe and, partly, the forces that hold it together. This theory, in principle, can describe all the physical process in chemistry and biology in

terms of fundamental interactions The standard model of particle physics describes phenomenon concerning three out of the four known fundamental forces electromagnetic force, weak force and strong force. The formal name for the mathematical structure of this theory is gauge-invariant quantum field theory. Gauge-invariant means that the properties of the bosons (force mediators) are determined by symmetries relating the matter fields (quarks and leptons).

The fundamental particles and some of their basic properties are listed in Table. 2.

Table 2: Fundamental particles and their properties.

1.4 Properties of fundamental particles

Every fundamental particle, for example a u quark, has exactly the same properties no matter where or how it is produced. The quarks and leptons have similar properties like charge, mass, spin etc., but quarks have one extra property called color. Just like particles with electromagnetic charge can only interact through the electromagnetic force, only the particles with color charge can interact through the strong force. This color has nothing to do with the color of things we see, but is a quantum (or internal) property of these particles. Every quark comes in three colors red, blue and green. This is a useful analogy because the particles we observe directly don't seem to have this property. In other words, they are colorless. According to the quark theory, just like combining these three colors gives white, combining three quarks with these three quantum properties gives a particle which does not have any color. So every particle that we can directly observe should be made up of a combination of either three quarks or a quark and an anti-quark, giving us the color-less particles we observe.

The charge of electrons and protons is equal but opposite. We say electrons and protons have 1 unit of charge ($-1e$ for an electron and $+1e$ for a proton, where e is the charge of an electron). When quark theory was formulated, the charge of a proton equaling $+1e$ had to be accounted for. In order to make sure that protons charge comes out to be equal to $+1e$, the constituent quarks were given fractional charges, as shown in Table. 2.

The everyday matter (atoms) is made up of protons, neutrons and electrons. The proton is made of two u quarks and one d quark (does the sum of their charges work?), Thus, in terms of fundamental particles, everyday matter is made up of u and d quarks and the electron. These particles belong to the first family of quarks and leptons. One of the big questions physicists are trying to answer is Why do we need three families of particles? The standard model also does not describe phenomenon concerning force of gravity. The recent cosmological discoveries of dark matter and dark energy are also open questions not addressed by this theory.

Lets count:

Fermions: There are 6 leptons - electron, muon, tau and the corresponding three neutrinos. There are 6 quarks - up, down, charm, strange, top and bottom. Every quark comes in 3 colors. Leptons and quarks are both fermions, as they all have half integer spin $1/2$.

Now multiply the total number of particles by 2 all these particles have their anti-particles. For every quark, there is an anti-quark, and for every lepton there is an anti-lepton. These anti-particles are almost identical to their corresponding particles except for a very few properties. For example, an electrons anti-particle has the same mass but a positive charge (called positron).

Bosons: There are force carriers corresponding to four forces photon, W^+ , W^- , Z and gluons. These are all bosons with integer spin 1.

By 1995, all of the particles predicted by the standard model were discovered except one the Higgs boson with spin 0. In 2012, a new boson was discovered at the Large Hadron Collider. The studies done so far seem to indicate that this newly observed particle is very similar to the Higgs boson predicted by the standard model.

1.5 Non-Fundamental (composite) particles

Apart from fundamental particles, more than 200 subatomic particles (made up of known fundamental particles) have been discovered using particle accelerators and detectors (see <http://pdg.lbl.gov> for a listing of the known particles). These composite particles are made up of quarks, and are of two types: Baryons: are made of three quarks, for example neutron and proton. Mesons: are made of quark pairs for example pions. Baryons and mesons are collectively called Hadrons. Quantum Mechanics and Particle Physics One generally only attempts to learn about particle physics after studying quantum mechanics. Quantum mechanics is indeed fundamental to particle physics. However, if a student's goal is to become involved in particle physics research, it is only strictly necessary to learn a small fraction of this fascinating subject. In this section, we will try to describe the minimum amount of quantum mechanics you need to understand particle physics. To really learn quantum mechanics, you need first to study differential equations, and many of your class mates are just not there yet. Quantum mechanics is strongly related to one of the fundamental constants of nature, called h or \hbar () where \hbar has the numeric value of 1.054×10^{-34} Js. If you see this constant in an equation, you know quantum mechanics is involved. Often quantum mechanical systems involve quantization of energies. The classic example is the energy states of the hydrogen atom (or any atom or molecule). The electrons can only have certain energies. If an atom absorbs energy, it can only do this if the energy is the right amount to move the electron between one of these discrete states (approximately there are corrections to this that are important only when being very precise). Likewise when an atom that is in one of the higher energy states de-excites, it can only emit photons with energies For example, photons, the fundamental packet of electromagnetic radiation, can only have energies that are integer multiples of the product of its angular frequency (where f is the frequency of the photons oscillation and where c is the speed of light and λ is the photons wavelength) and \hbar . Thus where n is an integer. When you combine this with the energy levels of atoms, you get the characteristic absorption/emission spectra of atoms/molecules which can be used to identify them.

Figure 3: figure stolen from facultu.admiramar.edu Another important thing to know about quantum mechanics is that it is probabilistic, in the sense often for a given initial state (say, an electron aimed at a piece of metal) there may be many possible final states (say, amount of energy that the electron loses, and the deflection of the electron from its initial direction). In quantum mechanics, you can only calculate the probabilities of the different possible outcomes; you cannot ever, even with perfect information about the initial state, predict exactly what the final state will be. Finally, there is the Heisenberg uncertainty principal.

1.6 Feynman Diagrams

Feynman diagrams began as a mnemonic to help particle theorists do the very complicated calculations required by gauge-invariant quantum field theories. These calculations are done using something which is very important to practicing physicists, but rarely mentioned in undergraduate curriculum: perturbation theory. In perturbation theory, you first calculate an approximate answer (the leading order calculation or LO). You then calculate a correction to this answer (the next to leading order calculation or NLO). You then calculate a correction to this correction (NNLO). The Feynman diagrams represent aids to help in each order of the calculation. However, they are also useful for just helping to visualize how bosons are exchanged between quarks/leptons to create interactions.

The figure below shows the LO diagram for annihilation of an up quark and an anti-up quark to a Z boson to an electron-positron pair.

The figure below shows a NNLO (in the strong force) diagram for the same process.

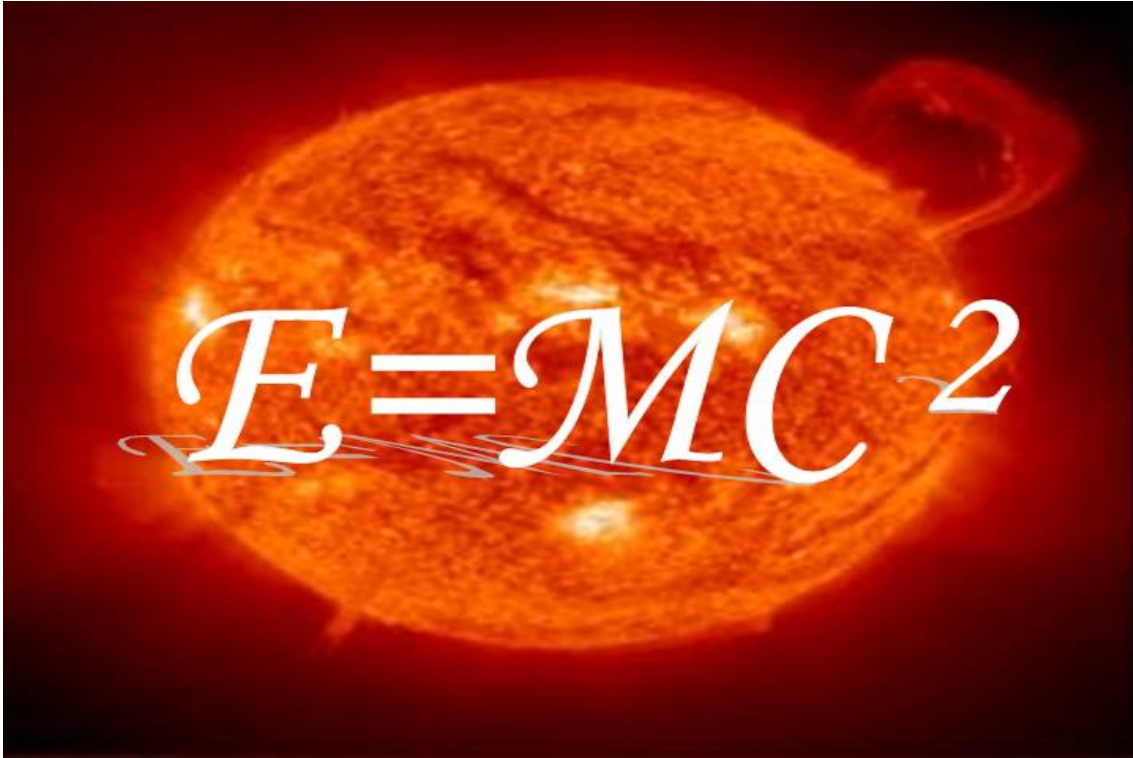
1.7 More on Mesons

Mesons are bound states of 2 quarks. These are very important for particle physics because, although the proton is the lightest stable particle, mesons are the lightest states containing quarks. They are unstable, meaning that they decay, sometimes very quickly, to other types of particles. The lightest mesons are called pions. There are three of them, one with a positive charge, one neutral, and one with a negative charge. The charged pions decay via the weak force to a muon and a neutrino. The neutral pion usually decays to two photons. The naming conventions for all the mesons are very strange because the standard model and the existence of quarks was not yet understood when they were discovered. Thus there are kaons that contain strange quarks, D mesons that contain charm quarks.

As with the hydrogen atoms, there are also excited state of the two bound quarks. While with hydrogen, we just say excited hydrogen, with the mesons, the excited states often have separate names, as it was not understood at the time of their discovery that they were essentially excited pions. Thus rho mesons, eta mesons, etc are in a sense excited states of bound states of the up and down quark.

1.8 Further reading

pdg.lbl.gov is always a great site for all things related to particle physics Modern Particle Physics by Mark Thomson Introduction to Elementary Particle Physics by Alessandro Bettini



Special Relativity

2.1 Disclaimer

Einstein's theory of special relativity is one of the most fascinating, elegant, surprising, and powerful theories in modern physics. The derivation of its laws, beginning from the simple postulate that the speed of light is a constant, independent of the velocity of the person measuring it, along with the resulting profound implications in electromagnetic theory and the nature of space and time is astounding. We, unfortunately, do not have the time to cover everything in this course. And you, as freshmen, for the most part are just not ready. You will study this theory in depth when you take 300-level E&M.

However, to understand what is going on at the LHC, you do need to understand some relativity. The goal of this tutorial is to teach you the minimum needed to understand the Higgs discovery. We will approach it from a practical point of view. Although this is anathema to the physicist in you, we will give you formulas without telling you where they come from. You need to just accept them (For now! Until you are older!) as experimental reality, just as you accept as a fact that you can use as a tool to work other problems.

2.2 4-vectors

When doing Newtonian mechanics, you are taught about vectors. You typically work with vectors that have three components, associated with three spatial directions (x, y, z). Often, then you

parameterize these components as a function of a time. You might calculate the height of a particle above the ground (z) as a function of time, its z -component of velocity as a function of time, or its momentum as a function of time.

With special relativity, instead of these 3-vectors, we will work with 4 component vectors (4-vectors). The fourth component for our position vector will be time. However, time and position have different units (seconds for time and meters distance). Mixing time and distance is thus mixing apples and oranges, so what can be done?

In special relativity, the speed of light is very special. It is a constant, and all observers, regardless of their relative velocity, will get the same result when they measure it. We'll discuss this more later. Since this is a special number, one of the fundamental constants of nature, along with the fundamental constant of quantum physics, and a few other fundamental numbers. We can use c to convert time to a distance and write our 4 vector for an event (something that has a time and a position) as

$$d = (ct, x, y, z)$$

Every 3-vector will be augmented this way, although the associated time-like component may not be obvious to you at this stage. Most importantly, momentum becomes 4-momentum, defined

$$p = (E, cp_x, cp_y, cp_z) \quad (2.1)$$

where E is the particles energy. Again, we use c to make sure all components have the same units.

We will use red to denote 4 vectors and the usual vector notation to denote 3 vectors. Thus we can write:

$$p = E, cp_x, cp_y, cp_z \quad (2.2)$$

$$p = (E, \vec{c}p) \quad (2.3)$$

Exercise 2.1 show that E and cp have the same units. Remember that the i th component of force is related to energy by and to momentum by....

As you may remember from your high school physics, there are several important mathematical operations that are used with vectors. One is the dot product a way of making a scalar out of two vectors. You may remember that:

$$C = \vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

The dot product of a vector with itself gives the square of the magnitude of the vector.

$$|\vec{A}|^2 = \vec{A} \cdot \vec{A}$$

The dot product, in Newtonian physics, is used in calculating work from force and displacement. There is a dot-like product associated with 4 vectors, and it has some very interesting, useful, and sometimes bewildering properties.

$$c = \vec{a} \cdot \vec{b} = a_0 b_0 - \vec{a} \cdot \vec{b}$$

More on this useful operation later.

2.3 Frames (center-of-mass, lab) and transforming between frames

Imagine two people, (A and B) with rulers and stop watches. A is standing in the sidewalk on Route One, while B is in a white car in the left hand lane going south on Route One. Both see a red car in the right hand lane going south which passes the car containing B. However, person A sees the space between her and red car increasing at a much faster rate than person B sees the space between him and the red car increasing. They each measure a different velocity for the red car relative to themselves. Mathematically, what we have in 1 spatial dimension; it is easy to extend to 3 spatial dimensions.

let v_B, x_B be the velocity and relative to person A let $v_{RA}, x_{RA}, v_{RB}, x_{RB}$

Then,

$$v_{RB} = v_{RA} - v_B \quad x_{RB} = x_{RA} - v_B t$$

This set of equations that transform a variable as it is measured in one frame to the value it will have when measured in another frame is called Galilean Relativity.

However, in relativity, if the red car were moving at the speed of light both A and B would see the distance between them and the car increasing at the same rate. In other words, all observers will measure the same speed of light regardless of their relative velocities. Obviously, the equations given above do not predict this. Einstein developed new equations that give us a measurement in one frame relative to another.

$$ct_{RB} = \gamma(ct_{RA} - \beta x_{RA}) \quad (2.4)$$

$$x_{RB} = \gamma(-\beta ct_{RA} + x_{RA}) \quad (2.5)$$

where

$$\beta = v_B/c \quad (2.6)$$

$$\gamma = \frac{1}{1 - \beta^2} \quad (2.7)$$

Note that β is a number between 0 and 1, and is the fraction of the speed of light the other measured frame has with respect to your frame of reference. γ is a number that is greater than 1.

It is called the relativistic boost and will be very useful.

In general, any 4-vector will transform this way. Thus, the energy-momentum 4-vector will transform in this way. This set of transformation equations is called Special Relativity or the Lorentz transformation.

Note that these are extremely weird equations. They imply that observers that are not at rest with respect to each other will not agree on the time when something occurred. They will not agree if two things happening at two different positions happened at the same time or not. There are lots of fascinating implications to this, but you'll just have to take more advanced physics to learn about them. We are going to concentrate on the minimum you need to look for new particles at the LHC.

2.4 Length of 4 vectors

A 4 vector has the interesting property that all observers, regardless of their relative velocities, will agree on the length of a 4-vector. Lets prove this:

Lengths of 4 vectors

$$\begin{aligned} |a|^2 &= a_0 a_0 - a_x a_x \quad |a'|^2 = a'_0 a'_0 - a'_x a'_x = \gamma(a_0 - \beta a_x) \gamma(a_0 - \beta a_x) - \gamma(-\beta a_0 + a_x) \gamma(-\beta a_0 + a_x) \\ &= \gamma^2(a_0^2 - 2\beta a_0 a_x + \beta^2 a_x^2 - \beta^2 a_0^2 + 2a_x \beta a_0 - a_x^2) = \gamma^2(a_0^2(1 - \beta^2) - a_x^2(1 - \beta^2)) \\ &= a_0^2 - a_x^2 = |a|^2 \end{aligned}$$

In particular, this is true for the energy-momentum 4 vector $|p| = E^2 - (pc)^2$. (working in 1 spatial dimension you can extend this to 3) In fact,

$$(mc^2)^2 = E^2 - (pc)^2 \quad (2.8)$$

In the rest frame of a particle (the frame where $|\vec{p}|$ is zero), you get the famous equation that appears on T-shirts everywhere.

This equation is very important to the LHC. When we look for particles, we recognize them by their mass. W bosons have a mass of 80 GeV. Z bosons have a mass of 91 GeV. Top quarks have a mass of 173 GeV. Obviously, we cannot put particles on a scale to weigh them. This equation tells us that, if we want to see the mass of a particle made in a collision, we just need to know its energy and momentum. Then, from that, we can determine its mass. We can also show that it doesn't matter what our velocity is with respect to the particle. This equation will always give us the mass of that particle, and that all observers will agree on the mass.

2.5 HEP units

Masses in GeV? What kind of mass unit is that? You may be more used to kg or g or lbs. Remember that the eV is a unit of energy, defined as the potential energy gained when an electron moves from one position to another whose potential is higher by 1 V.

We know from Einstein that mass is related to energy through c^2 . So eV/c^2 is a unit of mass. GeV/c^2 is then $10^9 \text{eV}/c^2$. For scale, the proton has a mass of $1 \text{ GeV}/c^2$.

Particle physicists are lazy. They hate to type even a single extra letter. Particle physicists are lazier than most. They have defined a whole unit system designed to help them avoid typing c and $/h$. Unit systems need a way of defining a time unit, a distance unit, and a weight unit. c is related to both the time and distance unit. The units of $/h$ are Energy \times time. We know from Einstein that energy is related to mass, so we can use it instead of mass as our third necessary unit. The eV, which is the potential energy an electron gains when it transverses a potential difference of 1 V, is chosen to be the unit of energy. Length and time units are then chosen so that both c and $/h$ are 1.

Once we do that, GeV/c^2 just becomes $\text{GeV}/1^2$ or GeV.

Not only that, but we can also measure both length and time in GeV.

For example $/h/\text{GeV}$ is a unit of time. But $/h=1$ in this unit system, so GeV^{-1} is a unit of time. c/GeV is a unit of length. So GeV^{-1} is also a unit of length.

In this unit system, the length of the energy-momentum 4-vector becomes (in 1 dimension):

$$m^2 = E^2 - p^2 \quad (2.9)$$

2.6 Lifetimes of particles

An interesting consequence of special relativity is that the lifetime you measure for a particle will depend on its velocity relative to you. Let τ_0 be the lifetime you measure for the particle when the

particle is at rest with respect to you. Then, using equation 1.4, we can see that the lifetime for a particle moving with a velocity β is

$$\tau = \gamma \tau_0 \quad (2.10)$$

The lifetime is longer for a moving particle than it is for one at rest by a factor γ .

2.7 Breit-Wigner

We have learned that most heavy particles are not stable, and decay. Above, we said that particles are identified by their mass, and that particles have a well-defined mass characteristic of their type. However, this is not quite true. As you will learn when you take quantum mechanics, a quantum state cannot have a well defined time and a well defined energy (mass). This is expressed through the Heisenberg uncertainty principle:

$$\delta E \delta t \geq \hbar \quad (2.11)$$

Because of this, when we measure the particles mass, we get a range of values. How big is that range? Remember that we can express the lifetime of a particle, using our new funny units, in GeV, using:

$$\Gamma = \frac{\hbar}{\tau_0} \quad (2.12)$$

where Γ is the lifetime in GeV. When the lifetime is expressed in GeV, it is referred to as the particles width.

The actual functional form of the mass distribution is called a Breit-Wigner. The form is:

$$\propto \frac{1}{(E - M)^2 + \Gamma^2} \quad (2.13)$$

where M is the value of the peak of the mass distribution and E is the observed mass.

Exercise 2.2 For the Z boson, M is 91.2 GeV and Γ is 2.5 GeV. What is the corresponding lifetime? Use root to plot this function versus E , the observed mass.

2.8 The coordinate system for a collider experiment

The CMS detector, whose data we will be using in this course, uses a right-handed coordinate system, with the origin at the nominal interaction point, the x axis pointed to the center of the LHC, the y axis pointing up (perpendicular to the LHC plane), and the z axis along the anticlockwise-beam direction. The polar angle is measured from the positive z axis and the azimuthal angle ϕ is measured in the x-y plane.

2.9 Collisions (conservation of 4-momentum)

When two protons appear to collide in a detector, what really collides is the partons (either quarks or gluons) inside the protons. A sketch of a collision that produces a Z boson is shown below.

Figure 1: Feynman diagram for production of a Z boson in a proton-proton collision, with subsequent decay to electrons

The total momentum of the proton is divided among the partons in an unequal way that varies collision by collision. Sometimes most of the momentum is held by a single parton. Sometimes it is divided among a very large number of partons. We can know the distribution of probabilities versus fraction of proton momenta, but cannot know, for a specific collision, what fractions the colliding partons had. It will be very rare, however, for both partons to have equal but opposite momenta in the lab frame.

It is usually easiest to understand this collision if we transform to the frame where the partons do have equal but opposite momenta. This special frame is called the center-of-mass frame. In this frame, the partons have equal but opposite 3-momenta. The total initial state 2-momenta, therefore, is zero. Since momentum is conserved, this means the Z boson produced must have zero 3-momenta and therefore zero kinetic energy. Its only energy will be that due to its mass. Since energy is also conserved, this means the initial energies of the partons must be half the Z mass (ignoring for now the finite width of the Z). Since the partons are approximately massless, this means that the initial momenta of the partons must be:

$$p_{e^+} = (\frac{M_Z}{2}, \frac{M_Z}{2} \cos \theta \cos \phi, \frac{M_Z}{2} \cos \theta \sin \phi, \frac{M_Z}{2} \sin \theta)$$

$$p_{e^-} = (\frac{M_Z}{2}, \frac{M_Z}{2} \cos(\pi - \theta) \cos \phi, \frac{M_Z}{2} \cos(\pi - \theta) \sin \phi, \frac{M_Z}{2} \sin(\pi - \theta))$$

What about the electrons that are produced? Again, due to conservation of momentum, their 3-momenta must be equal and opposite. And, from conservation of energy, their energies must be half the Z mass. Because they are approximately massless, the magnitude of their 3-momenta must be equal to their energy. However, their 3 momenta do not have to be along the z axis. So, in general:

The probability distribution for the polar and azimuthal angles are predicted by the standard model, but is beyond the level of this course.

What will this look like in the lab frame?

Since to boost back into the lab frame, we do a boost along the z axis, only the z components of 3-momentum will change. The x and y components will stay the same. The energy will change as well, since (in the massless, highly relativistic approximation)

$$E^2 = p_x^2 + p_y^2 + p_z^2 +$$

The polar angle changes, but the azimuthal angle does not. Because of this, transverse variables are very important to collider physicists; they are most sensitive to the particle produced, and not sensitive to the boost.

The component of momentum transverse to the beam axis is called the transverse momentum. Its magnitude is calculated:

$$p_T = \sqrt{p_x^2 + p_y^2} \quad (2.14)$$

You will see this variable over and over when you work on hadron collider physics.

2.10 Rapidity and Pseudorapidity

Another variable related to boosts and the polar angle is the rapidity, defined as:

$$y = \frac{1}{2} \ln \left(\frac{E + p_z}{E - p_z} \right) \quad (2.15)$$

Exercise: show that the difference in rapidity between two particles is independent of the boost in the z direction.

A related variable is the pseudorapidity, defined as:

$$\eta = -\ln \left(\tan \left(\frac{\theta}{2} \right) \right) \quad (2.16)$$

Exercise 2.3 show that rapidity and pseudorapidity are equal for massless particles.

2.11 Further reading

http : //en.wikibooks.org/wiki/SpecialRelativity