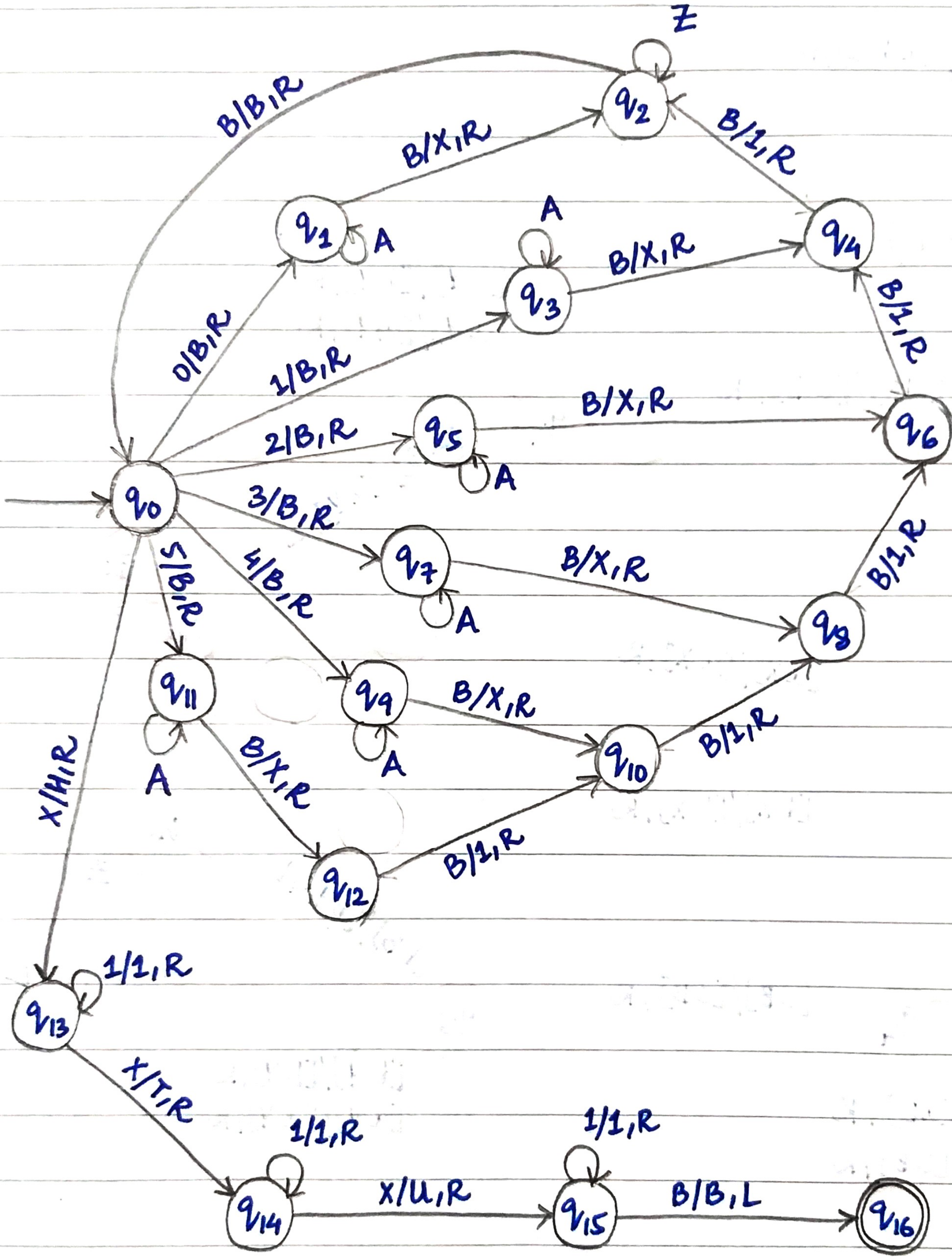
K180268
Section : A

"Assignment #05"

Q1 : 3 digit decimal number to unary.

Ans : let string : | B | 4 | 0 | 5 | B |



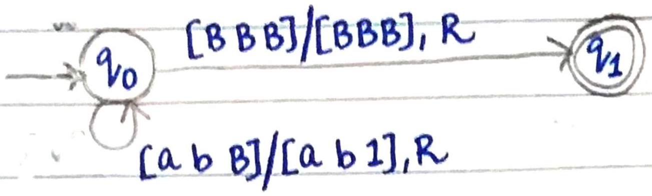
let :

A = 0/0,R
1/1,R
2/2,R
3/3,R
4/4,R
5/5,R
X/X,R

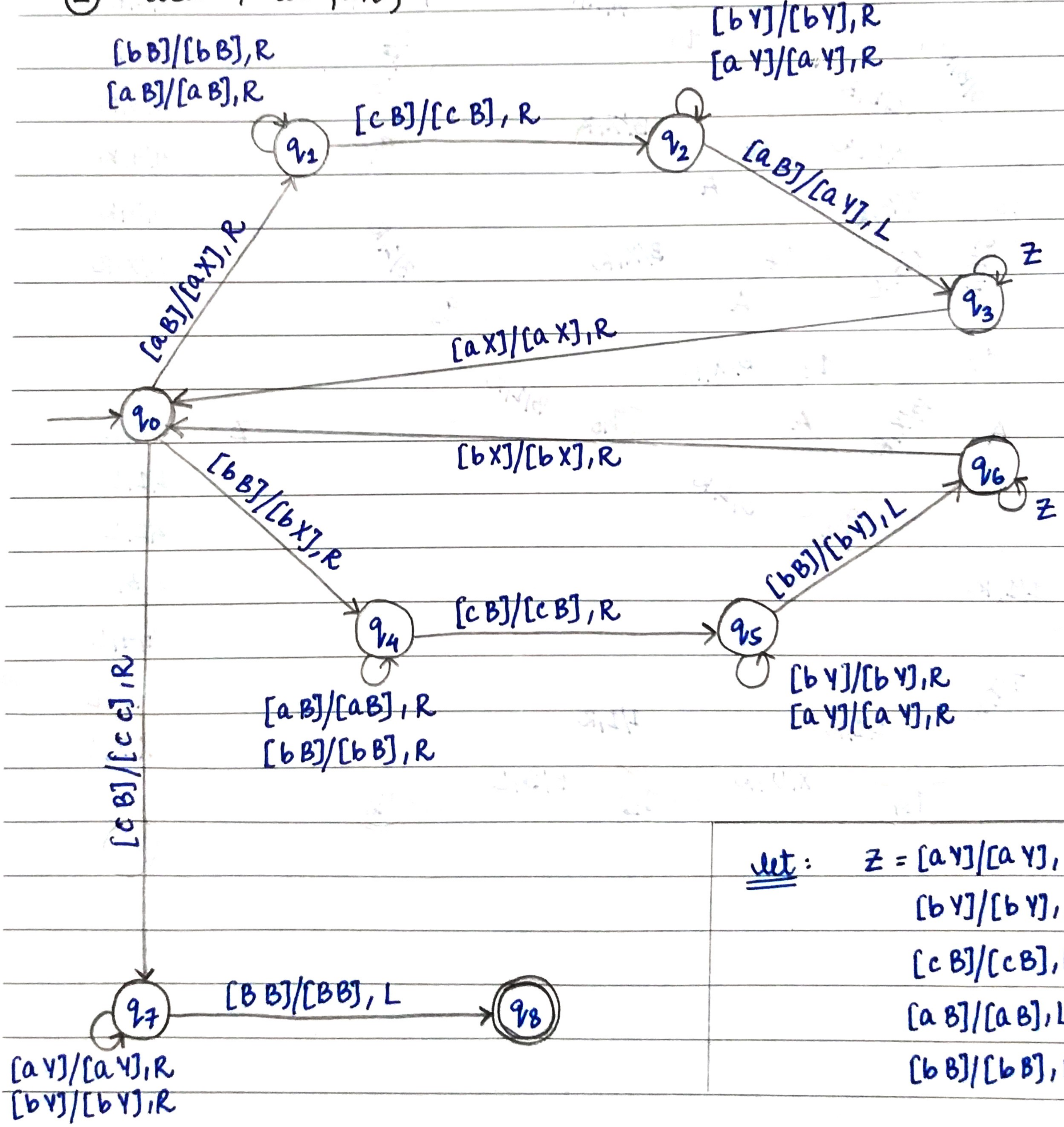
Z = 0/0,L
1/1,L
2/2,L
3/3,L
4/4,L
5/5,L
X/X,L

Q2: ① $a^n b^n$, $n \geq 1$

Ans:



② wcw , w = {a, b}



let: Z = [a Y]/[a Y], L
[b Y]/[b Y], L
[c B]/[cB], L
[a B]/[a B], L
[b B]/[b B], L

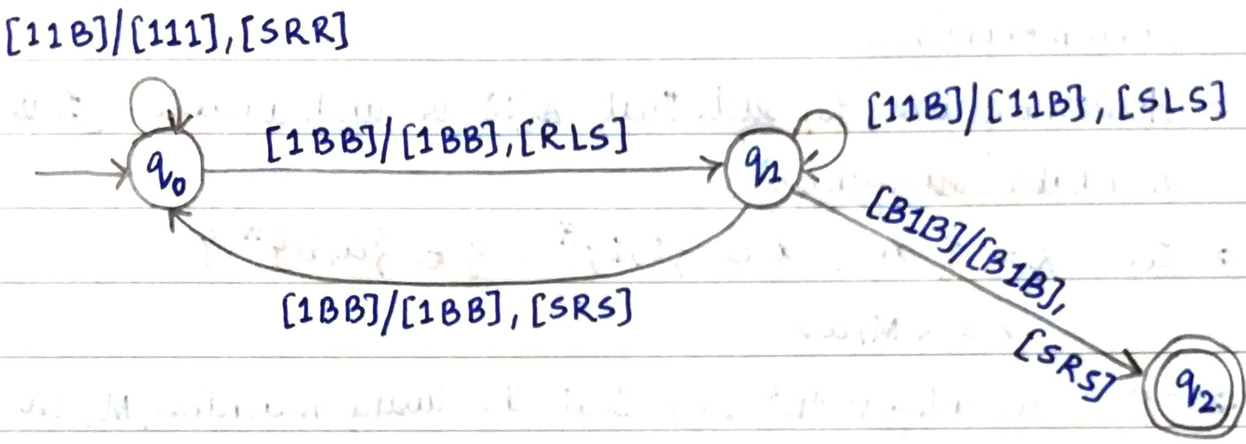
③ m^2

Ans: $m^2 = m \times m$, so

m : | B | 1 | 1 | 1 | 1 | B |

m : | B | 1 | 1 | 1 | 1 | B |

output : | B | B | B | B | B | B |



Q3: RECURSIVE :

✓ Concatenation : If L_1 and L_2 are two recursive languages, their concatenation $L_1 \cdot L_2$ will also be recursive.

⇒ e.g: $L_1 = \{a^n b^n \mid n \geq 0\}$

$L_2 = \{c^m d^m \mid m \geq 0\}$

$L_1 \cdot L_2 = \{a^n b^n c^m d^m \mid n \geq 0 \ \& \ m \geq 0\}$ is also recursive

⇒ input w is divided into 2 parts: w_1 and w_2. Then M_1 is run on w_1 and M_2 is run on w_2. The string is accepted if both accept it, else it is rejected if any one of these rejects.

✓ Kleene's Closure (star) : If L is recursive, its kleene's closure is also recursive.

⇒ eg: $L = \{a^n b^n \mid n \geq 0\}$

$L^* = \{a^n b^n \mid n \geq 0\}^*$

⇒ if $\Sigma = \{0,1\}$

$\Sigma^* = \{\lambda, 0, 1, 00, 11, \dots\}$

⇒ On input x, if $x = \varepsilon$, accept. Else run M_1 (machine that decides L_1) on each string, w_i, and accept if M_1 accepts all, else reject.

✓ Homomorphism: Recursive languages are not closed under homomorphism.

⇒ Homomorphism, h, such that $h(L)$ is undecidable. (L is a decidable language).

⇒ Let $L = \{ xy \mid x \in \{0,1\}^*, \ y \in \{a,b\}^* \}$

$x = \langle M, w \rangle$

⇒ "y" is an integer "n" such that the turing machine, M, on input, w, will halt in "n" steps.

⇒ L is decidable : can stimulate M on input "w" with "n" steps.

⇒ Consider homomorphism $h : h(0) = 0$, $h(1) = 1$, $h(a) = h(b) = \varepsilon$

⇒ $h(L)$ = halt, which is undecidable.

✓ Inverse Homomorphism: Turing machine, M_1 that decides L_1, a turing machine to decide $h^{-1}(L_1)$ is on input x, compute $h(x)$ and run M_1 on $h(x)$; accept if and only if M_1 accepts.

7. RECURSIVE ENUMERABLE:

- ✓ **Union**: Recursively enumerable languages are closed under union.
  - ⇒ Given turing machines, TM_1 and TM_2, with languages, L_1 and L_2.
  - ⇒ If TM_1 halts, the whole system halts.
  - ⇒ If TM_1 doesn't halt, but TM_2 halts, then system halts.
  - ⇒ A system that recognizes $L_1 \cup L_2$ on input x, run M_1 and M_2 on x in parallel and accepts if and only if either accepts.

- ✓ **Intersection**: Similar as union, except that there is no need for parallel simulation.

- ✓ **Concatenation**: A turing machine to recognize L_1L_2. On input "x", in parallel, for each $|x|+1$ ways to divide x as yz, run M_1 on y and M_2 on z, accepts if both accept, else reject.

- ✓ **Kleene's Closure**: To determine if $w \in L_1^*$, on input "x", if $x \in$, accept. We need to break the inputs into parts and figure out where to put the breaks. Then, M_1 is run on each part. If 'w' is the language, then each of these will accept.