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QUESTION # 1: Propositional Logic and Rules of Inference **[2 x 5 = 10 points]**

(i) Consider the following propositions:

p: Ali is a lawyer. *q*: Ali is ambitious. *r*: Ali is an early riser. *s*: Ali like chocolates.

Express these statements using the propositions *p*, *q, r and s* together with logical connectives (including negations).

(a) "If Ali is a lawyer, then he is ambitious." **Solution: $p \rightarrow q$**

(b) "If Ali is an early riser then he does not like chocolates." **Solution: $r \rightarrow \neg s$**

(c) "If Ali is ambitious then he is an early riser." **Solution: $q \rightarrow r$**

(ii) Write in English, the Converse of above statement (a), Contrapositive of statement (b) & Inverse of statement (c) from part (i).

Solution: **Converse(a): If Ali is ambitious then he is a lawyer.**
Contrapositive(b): If Ali like chocolates then he is not an early riser.
Inverse(c): If Ali is not ambitious then he is not an early riser.

(iii) Using the premises(statements) from part(i), apply rules of inference to obtain conclusion(s) from these premises.

Solution:

$P \rightarrow q$
$\underline{q \rightarrow r}$
$p \rightarrow r$
(Hypothetical syllogism)
$P \rightarrow r$
$\underline{r \rightarrow \neg s}$
$P \rightarrow \neg s$
(Hypothetical syllogism)

Conclusion: $p \rightarrow \neg s$: Then if Ali is a lawyer then he does not like chocolates.

(iv) Prove or disprove the following the logical equivalence using the laws of logic: ***$\neg(p \leftrightarrow q)$ and $p \leftrightarrow \neg q$***

Solution:

$$\neg(P \leftrightarrow Q) \equiv P \leftrightarrow \neg Q$$

$$\boxed{\begin{array}{l} ^{**}\quad P \rightarrow Q \equiv \neg P \vee Q \\ \#\#\quad P \leftrightarrow Q \equiv (P \rightarrow Q) \wedge (Q \rightarrow P) \\ \qquad\qquad\quad \equiv (\neg P \vee Q) \wedge (\neg Q \vee P) \end{array}}$$

$\neg(P \leftrightarrow Q)$

$\equiv \neg((\neg P \vee Q) \wedge (\neg Q \vee P))$ ##

$\equiv \neg(\neg P \vee Q) \vee \neg(\neg Q \vee P)$ *De Morgan's Laws*

$\equiv (P \wedge \neg Q) \vee (Q \wedge \neg P)$ *De Morgan's Laws*

$\equiv ((P \wedge \neg Q) \vee Q) \wedge ((P \wedge \neg Q) \vee \neg P)$ *Distributive Laws*

$\equiv ((P \vee Q) \wedge (\neg Q \vee Q)) \wedge ((P \vee \neg P) \wedge (\neg Q \vee \neg P))$ *Distributive Laws*

$\equiv (P \vee Q) \wedge T \wedge T \wedge (\neg Q \vee \neg P)$ *Negation Laws*

$\equiv (P \vee Q) \wedge (\neg Q \vee \neg P)$ *Identify Laws*

$\equiv P \leftrightarrow \neg Q$ ##

(v) Let *p, q* and *r* be statements. Determine, using a truth table, that if S1 is the statement **(p v q) ∧ (q v r) ∧ (r v p)** and if S2 is the statement **(p ∧ q) v (q ∧ r) v (r ∧ p),** then S1 and S2 are logically equivalent.

Solution:

p	q	r	p V q	q V r	r V p	S_1	p ∧ q	q ∧ r	r ∧ p	S_2
T	T	T	T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T	F	F	T
T	F	T	T	T	T	T	F	F	T	T
T	F	F	T	F	T	F	F	F	F	F
F	T	T	T	T	T	T	F	T	F	T
F	T	F	T	T	F	F	F	F	F	F
F	F	T	F	T	T	F	F	F	F	F
F	F	F	F	F	F	F	F	F	F	F

QUESTION # 2: Predicate Logic, Quantifiers and Sets **[2 x 5 = 10 points]**

Suppose R (x, y) is the predicate "**x understands y**", the universe of discourse for x is "the set of students in your discrete class", and the universe of discourse for y is "the set of examples in these lecture notes."

(i) Write the following predicate expressions in good English without using variables in your answers:

(a) ∃x ∀y R (x, y)

Solution: There exists a student in this class who understands every example in these lecture notes.

(b) ∀y ∃x R (x, y)

Solution: For every example in these lecture notes there is a student in the class who understands that example.

(ii) Write the predicate expressions of the following statements using variables and any needed quantifiers:

(a) Every student in this class understands at least one example in these notes." **Solution: ∀x ∃ y R(x, y)**

(b) There is an example in these notes that every student in this class understands." **Solution: ∃ y ∀ x R(x, y)**

(iii) Most of the tallest buildings in the United States are in Chicago and New York City. Here's a table (table 1) of the 10 tallest buildings in the U.S. and some data about them (Source: Wikipedia).

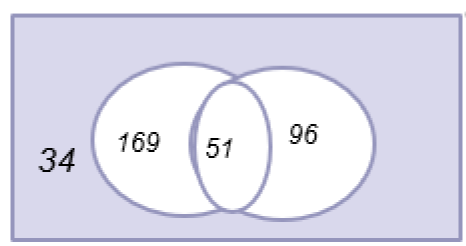
Define *B* to be the set of all buildings in the table above, *C* to be the set of buildings in the table above that are in Chicago, and *N* to be the set of buildings in the table above that are in New York. Furthermore, define *T* to be the set of buildings in the table above that are exactly 1046 ft. tall. Use the information given to find the truth value (and explain) of each statement below (you may use reasonable abbreviations for building names).

(a) ∃x ∈ N such that ∀y ∈ C, x is taller than y.

Solution:

1WTC ∈ *N* and Willis ∈ *C* and 1WTC is taller than Willis,
Trump ∈ *C* and 1WTC is taller than Trump,
Aon ∈ *C* and 1WTC is taller than Aon, and
Hancock ∈ *C* and 1WTC is taller than Hancock,
so this claim is true.

(b) $\forall x \in C, \forall y \in T$, x has more stories than y.
Solution:



Willis $\in C$, New York Times $\in T$, and Willis has more stories than New York Times,
Chrysler $\in T$, and Willis has more stories than Chrysler,
Trump $\in C$, New York Times $\in T$, and Trump has more stories than New York Times,
Chrysler $\in T$, and Trump has more stories than Chrysler,
Aon $\in C$, New York Times $\in T$, and Aon has more stories than New York Times,
Chrysler $\in T$, and Aon has more stories than Chrysler,
Hancock $\in C$, New York Times $\in T$, and Hancock has more stories than New York Times,
Chrysler $\in T$, and Hancock has more stories than Chrysler,
so this statement is true

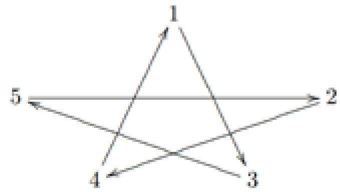
Building Name	Location	Height (ft.)	Number of Stories	Year Built
One World Trade Center	New York	1776	104	2104
Willis Tower	Chicago	1451	108	1974
432 Park Avenue	New York	1396	88	2014
Trump International Hotel and Tower	Chicago	1389	98	2009
Empire State Building	New York	1250	102	1931
Bank of America Tower	New York	1200	55	2009
Aon Center	Chicago	1136	83	1973
John Hancock Center	Chicago	1127	100	1969
Chysler Building	New York	1046	77	1930
New York Times Building	New York	1046	52	2007

Table 1: Tallest Buildings

(iv) FAST-NUCES, Department of Computer Science received 350 applications. Suppose that 220 majored in Computer Science, 147 majored in Software Engineering, and 51 majored in both. How many of these applicants majored neither in Computer Science nor in Software Engineering? Draw the Venn diagram.
Solution:
Let A: Applicants majored in Computer Science
B: Applicants majored in Software Engineering
Now,
$|A \cup B| = |A| + |B| - |A \cap B| = 220 + 147 - 51 = 316$

350 – 316 = 34 applicants majored neither in Computer Science nor in Software Engineering.

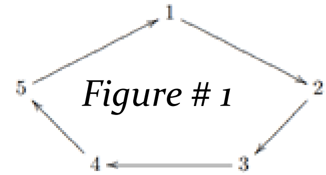
(v) Let ***X*** and ***Y*** be two sets. Prove or disprove using set identities that $X - (X \cap Y) = (X - Y)$.
Solution:



$= X \cap (X \cap Y)^c$ A – B = A ∩ B^c Law

$= X \cap (X^c \cup Y^c)$ De-Morgan

$= (X \cap X^c) \cup (X \cap Y^c)$ Distributive Law



$= \emptyset \cup (X \cap Y^c)$ Complement Law

$= X - Y$ Hence Proved A – B = A ∩ B^c

QUESTION # 3: Relations [2 x 5 = 10 points]

Abdul Karim is teaching Relational Algebra to Database Systems students. Before he starts teaching, he wants to check the basic concepts of relation of his students. He has designed a simple quiz as shown below:

(i) Find the matrix that represents the given relation. Use elements in the order given to determine rows and columns of the matrix.

(a) R1 on {−2, −1, 0, 1, 2} where a R b means $a^2 = b^2$.
Solution:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

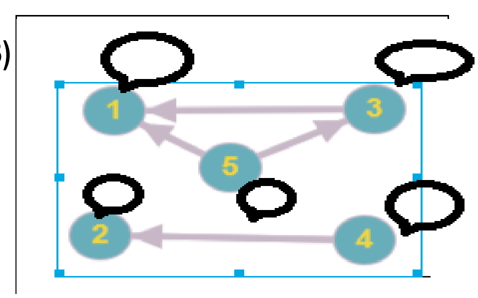
(b) R2 on {1, 2, 3, 4, 6} where a R b means a | b.
Solution:

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(ii)
(a) Now using the matrix obtained from part (i) (a), determine whether R1 is an equivalence relation?
Solution: Yes, its an equivalence relation since it holds Reflexive, Symmetric and Transitive properties.

(b) Now using the matrix obtained from (i) (b), determine whether R2 is a partial-order relation?
Solution: Yes, its an Partial Order relation since it holds Reflexive, Antisymmetric and Transitive properties.

(iii) Let *A* = {1, 2, 3, 4, 5}. Define a relation *R* on set *A* as follows:
{ ∀ (x, y) ∈ A, x R y ↔ 2 divides (x – y)}. Draw the diagraph of relation *R*.
Solution:
R= {(3,1),(4,2),(5,1),(5,3),(1,1),(2,2),(3,3) (4,4),(5,5)}



(iv) Let *R* be the relation on {1, 2, 3, 4, 5} represented by the digraph shown in figure 1. What is $R \circ R$? (draw digraph).
Solution: R= {(1,3), (2,4), (3,5), (4,1), (5,2)}

(v) Determine whether the relation R on the set of all Web pages is reflexive, asymmetric, and/or transitive, where $(a, b) \in R$ if and only if: "Everyone who has visited Web page a has also visited Web page b."

Solution:

(I) Reflexive: Everyone who has visited web page a has also visited web page a.

(II) NOT Asymmetric: Because it's not irreflexive. i.e. Asymmetric = Irreflexive + Antisymmetric

(III) Transitive: If everyone who has visited web page a also has visited web page b and everyone who has visited web page b also has visited web page c, then it follows that everyone who has visited web page a has also visited web page c.

QUESTION # 4: Functions [2 x 5 = 10 points]

Abdullah is a 2nd semester student. He missed the lectures for the topic "Functions". He asked his friend to help him in understanding the topic and asked the questions given below:

(i) Define g: R→Z by the rule $g(x) = \lceil 2x - 1 \rceil$ for all integers x.

(a) Is g one-to-one (injective)? Prove or give a counterexample.

Solution: No, Because g (0.3) = g (0.4) = 0

(b) Is g onto (surjective)? Prove or give a counterexample.

Solution: Yes, Because all integers in co-domain will be mapped.

Suppose $g: A \to B$ and $f: B \to C$ where $A = \{1, 2, 3, 4\}$, $B = \{a, b, c\}$, $C = \{2, 7, 10\}$, and f and g are defined by $f = \{(a, 10), (b, 7), (c, 2)\}$ and $g = \{(1, b), (2, a), (3, a), (4, b)\}$.

(ii) Is Function f and g invertible? If yes find f^{-1} and g^{-1} or if not why?

Solution: f is invertible since it's a bijective function. $f^{-1}(10) = a, f^{-1}(7) = b, f^{-1}(2) = c.$

g is not invertible since it's not a bijective function.

(iii) Find $f \circ g$ and $g \circ f$ (if any does not exist, give reason).

Solution:

$f \circ g$: $f(1) = 7, f(2) = 10, f(3) = 10, f(4) = 7.$

$g \circ f$ is not defined, because the range of f is not a subset of the domain of g.

(iv) Prove or disprove the statement $\lceil a + b \rceil = \lceil a \rceil + \lceil b \rceil$ for real numbers a and b.

Solution: It's a disproof.Here is counter example.

$\lceil 0.5 + 0.5 \rceil = \lceil 0.5 \rceil + \lceil 0.5 \rceil$

$\lceil 1 \rceil = \lceil 1 \rceil + \lceil 1 \rceil$

$1 \neq 2$

(v) How many functions are there from a set with three elements to a set with four elements?

Solution:

Hence 4 * 4 * 4 = 64 different functions from a set with three elements to a set with four elements.

QUESTION # 5: Number Theory [2 x 5 = 10 points]

Ali and Fatima are cousins. Both are good in Number Theory. They challenge one-another to solve different problems.

(i) Ali gifted Fatima a book entitled "Discrete Mathematics and Its Applications". Suppose that first 9 digits of ISBN-10 of the textbook are ***125973128***. How can Fatima find the check digit to validate the originality of the book?

Solution:

$X_{10} \equiv 1{\cdot}1 + 2{\cdot}2 + 3{\cdot}5 + 4{\cdot}9 + 5{\cdot}7 + 6{\cdot}3 + 7{\cdot}1 + 8{\cdot}2 + 9{\cdot}8 \pmod{11}$
$\equiv 1+4+15+36+35+18+7+16+72 \pmod{11} \equiv 204 \bmod 11$
$X_{10} \equiv 6$.
Yes it's a original copy since 204 +(10*6) ≡ 264 mod 11 ≡ 0.

(ii) Fatima has asked Ali to encrypt the message ***Exam*** using the RSA system with n = 11 · 3 and e = 3. translate each letter into integers and write in the form of Cipher text equation.
Solution:
E: C = 04^3 mod 33. X: C = 23^3 mod 33. A: C = 00^3 mod 33. M: C = 12^3 mod 33.

(iii) Now it's Ali turn to ask a question. He asks Fatima to show him that 15 is an inverse of 7 modulo 26.
Solution:
a = q d + r
26 = 3. 7 + 5
7 = 1.5 + 2
5 = 2.2 + 1 **Now, 1 = 1.5 – 2.2;** **2 = 1.7 – 1.5;** **5 = 1.26 – 3.7**
1 = 1.5 + 2(1.7 – 1.5) = 1.5 – 2.7 + 2.5
1 = 3.5 – 2.7 = 3(1.26 – 3.7) – 2.7
1 = 3.26 – 9.7 – 2.7 = 3.26 – 11.7
1 = (3)(26) + (– 11)(7)
Inverse can't be negative so – 11 +26 = 15.

(iv) But suddenly their cousin Bilal comes, Fatima tells Ali "***XYTU YFQPNSL***". This message is encoded using function ***f(p) = (p + 5) mod 26.*** Decode the message.
Solution:
Encrypted Message: "STOP TALKING"

(v) Bilal asks Ali to determine the remainder of 5^{119} upon division by 59?
Solution:
By Fermat's little theorem, $5^{58} \equiv 1 \pmod{59}$, so $5^{116} \equiv 1 \pmod{59}$, so $5^{119} \equiv 5^3 \equiv 7 \pmod{59}$.

<u>QUESTION # 6: Proofs</u> **[2 x 5 = 10 points]**
Prove the following theorems:

(i) If $A = \{x \in \mathbb{Z} \mid x$ is a multiple of 5$\}$ and $B = \{x \in \mathbb{Z} \mid x$ is a multiple of 7$\}$. Then $A \cap B = \{x \in \mathbb{Z} \mid x$ is a multiple of 35$\}$.
Solution:

Proof: Part 1. $A \cap B \subseteq \{x \in \mathbb{Z} \mid x$ is a multiple of 35$\}$

Let $x \in A \cap B$. Then $x \in A$ and $x \in B$. This implies that x is a multiple of 5 and it is a multiple of 7. Therefore, $x = 5n$ and $x = 7m$ with n and m integer numbers.

If we combine these two equalities, we obtain $5n = 7m$. As 5 and 7 are prime numbers, $5n$ is divisible by 7 only if n is divisible by 7. Thus, $n = 7k$ for some integer number k. Therefore, $x = 5n = 5(7k) = 35k$ for some integer number k. This means that x is a multiple of 35.

Part 2. $\{x \in \mathbb{Z} \mid x$ is a multiple of 35$\} \subseteq A \cap B$

Let x be a multiple of 35. Therefore, $x = 35t$ for some integer number t. Thus, x is divisible by 5 (so $x \in A$) and it is divisible by 7 (so $x \in B$). This implies that $x \in A \cap B$. Therefore, the two sets are equal. ■

(ii) Let x be an integer and P is the following statement. P: "If $x^2 - (x-2)^2$ is not divisible by 8, then x is even." Prove by contraposition.
Solution: **Contraposition: If x is odd then $x^2 - (x-2)^2$ is divisible by 8.**
Let x =2k + 1 be an odd number.
$x^2 - (x-2)^2 = x^2 - (x^2 - 2x + 4) = x^2 - x^2 + 4x - 4) = 4(x-1) = 4(2k+1-1) = 8k$
which is an integer multiple of 8. Therefore $x^2 - (x-2)^2$ is divisible by 8.

(iii) Suppose that $w^2 + x^2 + y^2 = z^2$, where w, x, y, and z always denote positive integers.
Prove the proposition: "z is even if and only if w, x, and y are even."

Solution:

Let w, x and y are even

Hence, w = x = y = 2k

$z^2 = (2k)^2 + (2k)^2 + (2k)^2 = 4k^2 + 4k^2 + 4k^2 = 12k^2 = (3)\,4k^2 = 3(2k)^2$

Hence z is also even.

(iv) Prove the following statement by contradiction: "For all integers n, if n is even then $n^3 + 5$ is odd."

Solution:

Suppose For all integers n, if n is even then $n^3 + 5$ is even.

Let n = 2k

$n^3 + 5 = (2k)^3 + 5 = 8k^3 + 5 = 2k\,(k^2) + 5$ **which is odd.**

Hence our supposition is false so above statement "For all integers n, if n is even then $n^3 + 5$ is odd." Is true.

(v) Prove using mathematical induction that $1^3 + 2^3 + 3^3 + \dots + n^3 = (\frac{n(n+1)}{2})^2$ whenever n is a nonnegative integer.

Solution:

$1^3 + 2^3 + 3^3 + \dots + n^3 = (n(n+1)/2)^2$

STEP 1: We first show that p (1) is true.

Left Side = $1^3 = 1$

Right Side = $1^2 (1+1)^2 / 4 = 1$

hence p (1) is true.

STEP 2: We now assume that p (k) is true

$1^3 + 2^3 + 3^3 + \dots + k^3 = (k(k+1) / 2)^2$ **(1)**

add $(k+1)^3$ **to both sides**

$1^3 + 2^3 + 3^3 + \dots + k^3 + (k+1)^3 = [(k+1)(k+1+1) / 2)]^2$

$1^3 + 2^3 + 3^3 + \dots + k^3 + (k+1)^3 = [(k+1)(k+2) / 2)]^2$ **(2)**

put eq(1) in eq(2)

$\Rightarrow (k(k+1)/2)^2 + (k+1)^3 = [(k+1)(k+2)/2)]^2$

$\Rightarrow k^2 (k+1)^2 /4 + (k+1)^3$

$\Rightarrow (k+1)^2 [k^2 + 4k + 4] / 4$

$\Rightarrow (k+1)^2 [(k+2)^2] / 4$

$\Rightarrow [(k+1)(k+2) / 2)]^2 = [(k+1)(k+2) / 2)]^2$

LHS = RHS, Hence proved!

<u>QUESTION # 7: Graph Theory</u> **[2 x 5 = 10 points]**

Usaid is an undergraduate student. The following graph map (figure 2) shows different path from his home to the school.

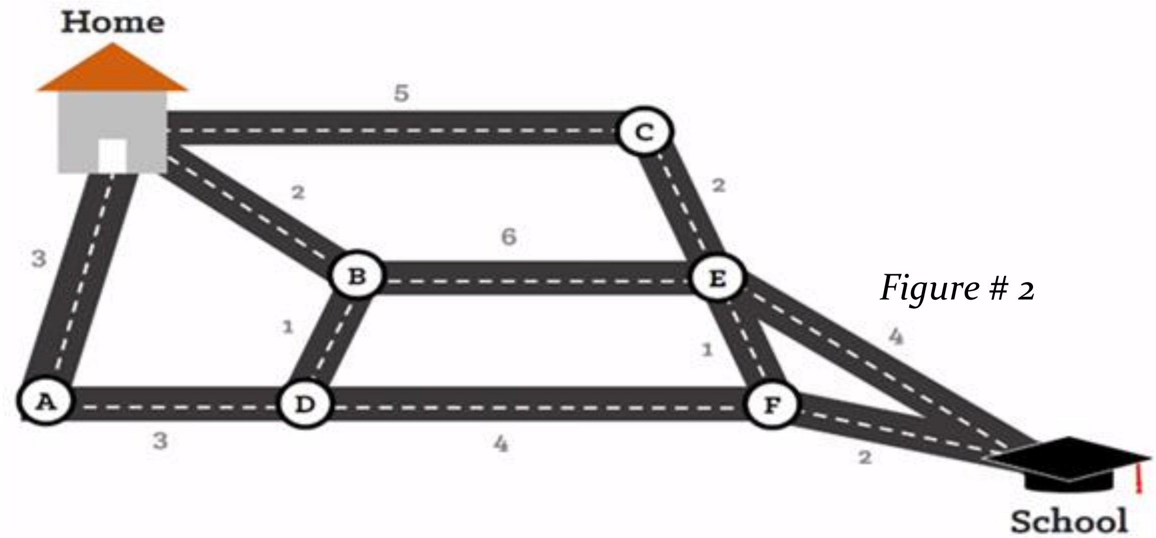


Figure # 2

(i) Usaid wants to determine the shortest path from home to school using Dijkstra Algorithm. What will be the cost of that path? Show all steps.

Solution:

The shortest path, which could be found using Dijkstra's algorithm, is Home →B→D→F→ School

Cost: 2 + 1 + 4 + 2 = 9.

N	D(A)	D(B)	D(C)	D(D)	D(E)	D(F)	D(S)
H	3,H	2,H	5,H	∞	∞	∞	∞
HB	3,H		5,H	3,B	8,B	∞	∞
HBA			5,H	3,B	8,B	∞	∞
HBAD			5,H		8,B	7,D	∞
HBADC					7,C	7,D	∞
HBADCE						7,D	11,E
HBADCEF							9,F
HBADCEFS							

(ii) Determine the Euler Circuit and Path in the above graph for Usaid? Show all of your steps with reasoning.

Solution: No Euler Circuit possible because some nodes have odd vertices. No Euler path is possible because there are more than two vertices with odd degree.

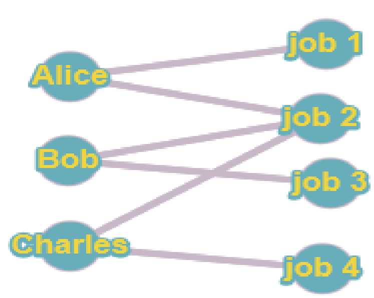
(iii) Determine the Hamilton Circuit and Path in the above graph for Usaid? Show all of your steps with reasoning.

Solution: No Hamilton Circuit But path is possible. E.g: C-> E-> U-> F-> D-> B-> H-> A

(iv) Suppose there are three employees Alice, Bob, and Charles and there are four jobs to be done. Alice can do job 1 or 2, Bob can do job 2 or 3, and Charles can do jobs 2 or 4. Which type of graph Usaid can plot representing this situation, and draw this graph?

Solution:

It's a bipartite Graph.



(v) Usaid wants to determine that whether given pair of graph G and G1 in figure 2 are isomorphic or not. Now you are supposed to help him. If they are, give function g: V(G) → V(G1) that define the isomorphism. If they are not, give reason.

Solution:

G is isomorphic to G1.

Hence,

g(a) = t, g(b) = x, g(c) = u, g(d) = y, g(e) = v, g(f) = z, g(g) = w.

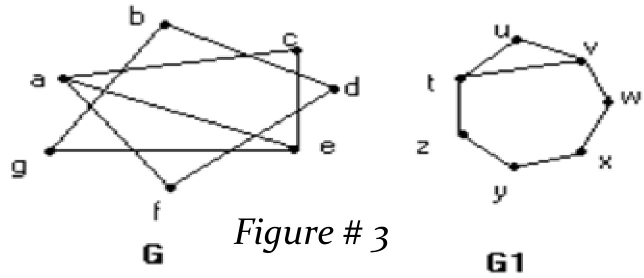


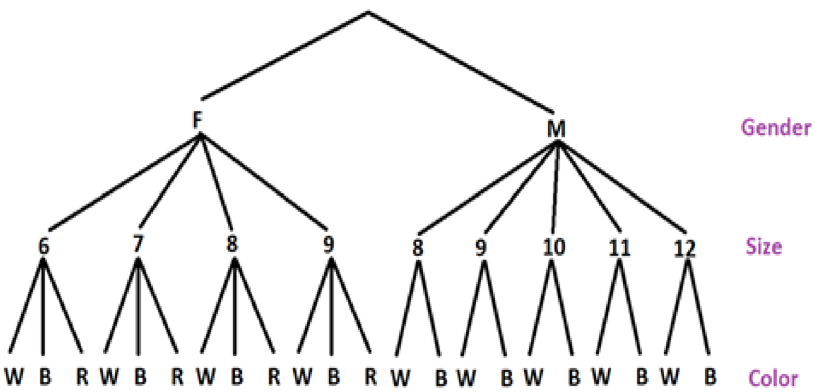
Figure # 3

<u>QUESTION # 8: Combinatorics and Trees</u> **[2 x 5 = 10 points]**

Suppose that a popular style of running shoe is available for both men and women. The woman's shoe comes in sizes 6, 7, 8, and 9, and the man's shoe comes in sizes 8, 9, 10, 11, and 12. The man's shoe comes in white and black, while the woman's shoe comes in white, red, and black.

(i) Use a tree diagram to determine the number of different shoes that a store has to stock to have at least one pair of this type of running shoe for all available sizes and colors for both men and women.

Solution: 22 pairs of shoes in total.



Gender: Women (F) and Men (M)

Women shoe sizes: 6, 7, 8, 9

Men shoe sizes: 8, 9, 10, 11, 12

Women shoe colors: white (W), black (B), red (R)

Men shoe colors: white (W), black (B),

(ii) Consider the tree obtained in part (i).

Show how many internal vertices and leaves the tree has? Determine the height of the tree?

Solution:

Internal vertices= 12, leaves=22, Height=3

(iii) Consider the tree obtained in part (i).

(a) Determine whether it is a Full m-ary tree or not? Give reason.

Solution: Not a full m-ary tree. All internal vertices do not have same no. of children.

(b) Determine whether it is a Balanced m-ary tree or not? Give reason.

Solution: A Balanced m-ary tree. Height h or h-1

(iv) Show the Preorder and Postorder traversal of the tree obtained in part (i).

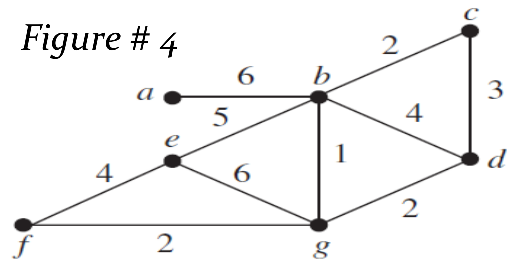
Solution:

PreOrder: N F G W B R 7 W B R 8 W B R 9 W B R 8 W B 9 W B 10 W B 11 W B 12 W B

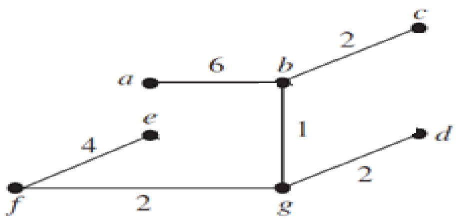
Post-Order: W B R 6 W B R 7 W B R 8 W B R 9 F W B 8 W B 9 W B 10 W B 11 W B 12 M N

(v) Find a minimum spanning tree for the graph given in figure 4 where the degree of each vertex in the spanning tree does not exceed 3.

Figure # 4



Solution:



QUESTION # 9: Combinatorics **[2 x 5 = 10 points]**

After Australia white-washed Pakistan in Test series and T20I. Head coach decides to have few changes in the team for the Srilankan team tour to Pakistan. Cricket team squad consists of 16 players. It includes 2 wicketkeepers and 5 bowlers.

(i) In how many ways can the head coach select a cricket team of eleven players if he has to select 1 wicket-keeper and at least 4 bowlers?

Solution:

We have to select a team of eleven players from a roster of 16 players. If that was the scenario then the total number of ways would be $^{16}C_{11}$. But here we have to select eleven players including 1 wicket keeper and 4 bowlers or, 1 wicket keeper and 5 bowlers.

There are a total number of 2 wicketkeepers to choose from and a total number of 5 bowlers to choose from.

So the number of ways of selecting 1 wicket keeper, 4 bowlers and 6 other players =

$^2C_1 \times {}^5C_4 \times {}^9C_6 = 840$.

Now let us find the number of ways of selecting 1 wicket keeper, 5 bowlers and 5 other players.

$^2C_1 \times {}^5C_5 \times {}^9C_5 = 252$

Therefore, the total number of ways of selecting the team = 840 + 252 = 1092. Hence the correct answer is 1092.

(ii) In how many ways can a cricket squad choose a captain, a vice-captain and a wicket-keeper from among themselves?

Solution:

Squad is consisting of 16 players so, $^{16}P_1 \times {}^{15}P_1 \times 2 = 480$.

(iii) A professor teaching discrete structures is making up a final exam. He has a pile of 24 questions on probability, 16 questions on combinatorics, and 10 questions on logic. He wishes to put five questions on each topic on the exam. Give an expression describing the number of different ways the above things can happen.

Solution:

Expression = $({}^{24}C_5)\,({}^{16}C_5)\,({}^{10}C_5) = 4.678 \times 10^{10}$.

(iv) What is the co-efficient of x^7y^{10} in the expansion of $(3x + 2y)^{17}$.

Solution:

Coefficient = $({}^{17}C_{10})\,(3)^7(2)^{10} = 4.355 \times 10^{10}$.

(v) Suppose that all license plates have three uppercase letters followed by three digits. How many license plates are possible in which all the letters and digits are distinct?

Solution:

Distinct License plates= 26*25*24*10*9*8 = 11,232,000.

QUESTION # 10: Discrete Probability and Pigeon Hole principle [2 x 5 = 10 points]

A bowl contains 7 red balls and 7 blue balls. You select balls at random without looking at them. Answer the following questions:

(i) How many balls must you select to be sure of having at least 3 balls of the same color?

Solution: 5 balls to be sure that at least 3 balls of the same color.

(ii) How many balls must you select to be sure of having at least 2 blue balls?

Solution: 9 balls to be sure that at least 2 blue balls.

(iii) Suppose the grades given out for a certain math class are A, A-, B+, B, B-, C, D, and E, and that there are 30 students in the class. At least how many students will earn the exact same grade?

Solution: Let N be 30 students and K be 8 different grades.

$\lceil N/K \rceil = \lceil 30/8 \rceil = 4$. So four students will earn the exact same grade.

(iv) A die is rolled and a coin is tossed, find the probability that the die shows an odd number and the coin shows a head.

Solution:

The sample space S of the experiment described in question 5 is as follows

S = { (1,H),(2,H),(3,H),(4,H),(5,H),(6,H),
(1,T),(2,T),(3,T),(4,T),(5,T),(6,T)}

Let E be the event "the die shows an odd number and the coin shows a head".

Event E may be described as follows

E={(1,H),(3,H),(5,H)}

The probability P(E) is given by

P(E) = n(E) / n(S) = 3 / 12 = 1 / 4

(v) Let a pair of dice be tossed. If the sum is 7, find the probability that one of the dice is 2.

Solution:

Let E be the event that a 2 appears on at least one of the two dice, and F be the event that the sum is 7.

Then

E = {(1, 2), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6) ,(3, 2) (4, 2), (5, 2), ,(6, 2)}

F = {(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)}

$E\cap F = \{(2, 5), (5, 2)\}$

$P(F) = 6/36$ and $P(E\cap F) = 2/36$.

Hence,

P(Probability that one of the dice is 2, given that the sum is 7)

$$= P(E|F)$$

$$= \frac{P(E\cap F)}{P(E)} = \frac{2/36}{6/36} = \frac{1}{3}$$

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BEST OF LUCK 😊