Q1)

(a) $\int_{0.5}^{1} x^4 \, dx$

*) Trapezoidal

$\int_{0.1}$ $x_0 = 0.5$, $x_1 = 1$, $h = \frac{1-0.5}{2} = \frac{1}{4}$

$\int_{0.1}^{1} f(x)\,dx = \frac{1/2}{2}\left[(0.5)^4 + (1)^4\right]$

$= \boxed{0.~~132~~ ~~8125~~}$ $\boxed{0.265625}$

*) Simpson:-

1/3 :- $x_0 = 0.5$, $x_2 = 1$, $h = 0.25$

$x_1 = a + h = 0.5 + 0.25 = \boxed{0.75}$

$\int_{0.5}^{1} f(x) = \frac{0.25}{3}\left[(0.5)^4 + 4(0.75)^4 + (1)^4\right]$

$= \boxed{0.1942104}$

3/8 :- $x_0 = 0.5$, $x_3 = 1$, ~~$x_1 = 0.5 + 0.25 = 0.75$~~

$h = \frac{1 - 0.5}{2} = 0.25$, ~~$x_2 = \frac{0.75 + 0.25}{2}$~~

$x_m = \frac{1 + 0.5}{2} = 0.75$

$x_1 = \frac{0.5 + 0.75}{2} = \boxed{0.625}$

$x_2 = \frac{1 + 0.75}{2} = \boxed{0.875}$

$$\int_{0.5}^{1} f(x)\,dx = \frac{3}{8}(0.25)\left[(0.5)^4 + 3(0.625)^4 + 3(0.875)^4 + (1)^4\right]$$

~~$= \boxed{0.3725280762}$~~

$= \boxed{0.3073483057}$

(b)

b) Trapezoidal

~~$x_0 = -2$, $x_1 = 2$~~

~~$h = 2 + 2 = 4$~~

~~$\int_{-2}^{2} f(x) = \frac{4}{2}\left[(-2)^3 e^{-3} + (2)^3 e^{3}\right]$~~

$x_0 = 0, \quad x_1 = 0.5$

$h = 0.5$

$$\int_0^{0.5} f(x) = \frac{0.5}{2}\left[\left(\frac{2}{0-4}\right) + \left(\frac{2}{0.5-4}\right)\right]$$

$$= \boxed{-0.267}$$

Simpson's $(1/3)$:-

$x_0 = 0, \quad x_2 = 0.5,$

$h = \frac{0.5}{2} = 0.25$

$x_1 = 0.25 + 0 = 0.25$

$$\int_0^{0.5} f(x) = \frac{0.25}{3}\left[\left(\frac{2}{0-4}\right) + 4\left(\frac{2}{0.25-4}\right) + \left(\frac{2}{0.5-4}\right)\right]$$

$$= \boxed{-0.2670634}$$

Simpson's $(3/8)$

$x_0 = 0, \quad x_3 = 0.5$

$x_m = 0.25, \quad h = 0.25$

$x_1 = \frac{0 + 0.25}{2} = 0.125$

$x_2 = \frac{0.5 + 0.25}{2}$ $= 0.375$

$$\int_0^{0.5} f(x)\,dx = \frac{3}{8}(0.25)\left[\frac{2}{(0-4)}\right.$$

$$+3\left[\frac{2}{0.125-4}\right]+3\left[\frac{2}{0.375-4}\right]+\left.\frac{2}{0.5-4}\right]$$

$= -0.4000740$

(c)

b) Trapezoidal:-

~~$x_0 = 0, \; x_1 = 2, \; h = 2$~~

$$\int_0^2 f(x)\,dx = \frac{2}{2}\left[\frac{2}{0+4}+\frac{2}{2^2+4}\right.$$

$x_0 = 1, \; x_1 = 1.5, \; h = 0.5$

$$\int_0^{2} f(x)\,dx = \frac{0.5}{2}\left[(1)^2\ln(1)+(1.5)^2\ln(1.5)\right.$$

$$= 0.22807411$$

Simpson's (1/3)

$x_0 = 1,\ x_2 = 1.5,\ h = \frac{1.5-1}{2} = 0.25$

$x_1 = 1 + 0.25 = 1.25$

$$\int_1^{1.5} f(x) = \frac{0.25}{3}\left[(1)^2 \ln(1) + (1.25)^2 \ln(1.25) + (1.5)^2 \ln(1.5)\right]$$

$$= 0.10506079$$

Simpson's (3/8)

$x_0 = 1,\ x_3 = 1.5,\ h = \frac{1.5-1}{3} = 0.25$

$x_{m_2} = 1.25,\ x_{12} = 1.125,\ x_2 = 1.375$

$$\int_1^{1.5} f(x)\,dx = \frac{3}{8}(0.25)\left[(1)^2 \ln(1) + 3(1.125)^2 \ln(1.125) + 3(1.375)^2 \ln(1.375) + (1.5)^2 \ln(1.5)\right]$$

$= \boxed{0.29137855356}$

(d)

*) Trapezoidal

$n_0 = 0,\ n_1 = 1,\ h = 1$

$$\int_0^1 f(n)\,dn = \frac{1}{2}\left[(0)^2(e^{-0}) + (1)^2 e^{-1}\right]$$

$= \boxed{0.183937206}$

*) Simpson's (1/3)

$n_0 = 0,\ n_2 = 1,\ h = 0.5$

$n_1 = 0.5 + 0 = 0.5$

$$\int_0^1 f(n)\,dn = \frac{0.5}{3}\left[(0)^2 e^{-0} + 4(0.5)^2 e^{-0.5} + (1)^2 e^{-1}\right]$$

$= \boxed{0.162401}$

Simpson's (3/8)

$x_0 = 0$, $x_3 = 1$, $x_{m2} = 0.5$, $h = 0.5$

$x_1 = \frac{0.5+0}{2} = 0.25$, $x_2 = \frac{0.5+1}{2} = \boxed{0.75}$

$$\int_0^1 f(x)\,dx = \frac{3}{8}(0.5)\left[(0)^2 e^{-0} + 3(0.25)^2 e^{-0.25} + 3(0.75)^2 e^{-0.75} + (1)^2 e^{-1}\right]$$

$$= \boxed{0.24581683966}$$