Qno 25.7

Home Task

"18K-0318"

(1) $\frac{dy}{dt} = -2y + 5e^{-t} = f(t_0, y_0, z_0)$

$\frac{dz}{dt} = \frac{-yz^2}{2} = g(t_0, y_0, z_0)$

Range $t = 0$ to $t = 0.4$

using $h = 0.1$

with $y(0) = 2$

$z(0) = 4$

1st iteration:

$\frac{dy}{dt} = -2y + 5e^{-t}$

$K_1 = h f(t_0, y_0, z_0)$

$= 0.1(-2(2) + 5e^0)$

$K_1 = 0.1$

for L_1

$L_1 = hg(t_0, y_0, z_0)$

$= 0.1\left(\frac{-2(4)^2}{2}\right)$

$L_1 = -1.6$

for K_2

$$K_2 = hf\left(t_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}, z_0 + \frac{L_1}{2}\right)$$

$$= 0.1\left(0 + \frac{0.1}{2}, 2 + \frac{0.1}{2}, 4 - \frac{1.6}{2}\right)$$

$$= 0.1\left(0.05, 2.05, 3.2\right)$$

$$K_2 = 0.1\left(-2(2.05) + 5e^{-0.05}\right)$$

$$K_2 = 0.065614$$

for L_2

$$L_2 = hg\left(t_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}, z_0 + \frac{L_1}{2}\right)$$

$$= 0.1\, g\left(0 + \frac{0.1}{2}, 2 + \frac{0.1}{2}, 4 - \frac{1.6}{2}\right)$$

$$= 0.1\, g\left(0.05, 2.05, 3.2\right)$$

$$= 0.1\left(-\frac{2.05(3.2)^2}{2}\right)$$

$$L_2 = -1.0496$$

for K_3

$$K_3 = hf\left(t_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}, z_0 + \frac{L_2}{2}\right)$$

$$= 0.1 f\left(0 + \frac{0.1}{2}, 2 + \frac{0.065614}{2}, 4 - \frac{1.0496}{2}\right)$$

$$= 0.1 f(0.05, 2.032807, 3.4752)$$

$$= 0.1\left(-2(2.032807) + 5e^{-0.05}\right)$$

$$= 0.069053$$

for L_3

$$L_3 = hg\left(t_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}, z_0 + \frac{L_2}{2}\right)$$

$$= 0.1 g(0.05, 2.032807, 3.4752)$$

$$= 0.1\left(-\frac{2.032807(3.4752)^2}{2}\right)$$

$$L_3 = -1.227512$$

for K_4

$$K_4 = hf\left(t_0 + \frac{h}{2}, y_0 + \frac{K_3}{2}, z_0 + \frac{L_3}{2}\right)$$

$$= 0.1 f\left(0 + \frac{0.1}{2}, 2 + \frac{0.069053}{2}, 4 - \frac{1.227514}{2}\right)$$

$$= 0.1 f(0.05, 2.034527, 3.386244)$$

$$= 0.1\left(-2(2.034527) + 5e^{-0.05}\right)$$

$$= 0.068709$$

for L_4

$$L_4 = hg\left(t_0 + \frac{h}{2}, y_0 + \frac{K_3}{2}, z_0 + \frac{L_3}{2}\right)$$

$$L_4 = 0.1\, g(0.05, 2.034527, 3.386244$$

$$= 0.1\left(-\frac{2.034527(3.386244)^2}{2}\right)$$

$$\boxed{L_4 = -1.166460}$$

$t_1 = t_{01} + h$

$$y_1 = y_0 + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$$

$$y_1 = 2 + \frac{1}{6}(0.1 + 2(0.065614) + 2(0.069058) + 0.068709)$$

$$y_1 = 2.073007$$

$$z_1 = z_0 + \frac{1}{6}(K_1 + 2L_2 + 2L_3 + L_4)$$

$$= 4 + \frac{1}{6}(-1.6 + 2(-1.0496) + 2(-1.22512) - 1.166460)$$

$$z_1 = 2.779886$$

$$t_1 = t_0 + h$$

$$= 0 + 0.1$$

$$t_1 = 0.1$$

2nd iteration:

for K_1

$$K_1 = hf(t_1, y_1, z_1)$$

$$= 0.1 f(0.1, 2.073007, 2.779886)$$

$$= 0.1\left(-2(2.073007) + 5e^{-2.779886}\right)$$

$$= -0.383579 \quad 0.037817$$

$$L_1 = hg(t_1, y_1, z_1)$$

$$= 0.1(0.1, 2.073007, 2.77886)$$

$$= 0.1\left(\frac{-2.073007(2.77886)^2}{2}\right)$$

$$= -0.800395$$

For K_2

$$K_2 = hf\left(t_1 + \frac{h}{2}, y_1 + \frac{K_1}{2}, z_1 + \frac{L_1}{2}\right)$$

$$= 0.1 f\left(0.1 + \frac{0.1}{2}, 2.073007 + \frac{0.383}{2}, 2.779886 - \frac{0.8}{2}\right)$$

$$= 0.1 f(0.15, 1.881218, 2.379689)$$

$$= 0.1\left(-2(1.881218) + 0.5e^{0.15}\right)$$

$$K_2 = 0.054110$$

for

$$L_2 = hg\left(t_1 + \frac{h}{2}, y_1 + \frac{K_1}{2}, z_1 + \frac{L_1}{2}\right)$$

$$= 0.1\, g(0.15, 1.881218, 2.379689)$$

$$= 0.1\left(-1.881218\,(2.379689) - \frac{2}{2}\right)$$

$$= -0.532659$$

For K_3

$$K_3 = hf\left(t_1 + \frac{h}{2}, y_1 + \frac{K_2}{2}, z_1 + \frac{L_2}{2}\right)$$

$$= 0.1\left(0.15, 2.073007 + \frac{0.054110}{2}, 2.779886 + \frac{0.532}{2}\right)$$

$$= 0.1(0.15, 2.100062, 2.1513557)$$

$$= 0.1\left(-2(2.100062) + 5e^{-0.15}\right)$$

$$= 0.010342$$

for L_3

$$L_3 = hf\left(t_1 + \frac{h}{2}, y_1 + \frac{K_2}{2}, z_1 + \frac{L_2}{2}\right)$$

$$= 0.1\, f(0.15, 2.073007, 2.1513557)$$

$$= 0.1\left(-2.073007\,\frac{(2.1513557)^2}{2}\right)$$

$$= -0.479728$$

for K_4

$$K_4 = hf\left(t_1 + \frac{h}{2}, y_1 + \frac{K_3}{2}, \beta_1 + \frac{L_3}{2}\right)$$

$$= 0.1 f\left(0.15, 2.078007 + \frac{0.010342}{2}, 2.779886 - \frac{0.47}{2}\right)$$

$$= 0.1 f(0.15, 2.078178, 2.540026)$$

$$= 0.1\left(-\frac{3.078178(2.540026)^2}{2}\right)$$

$$= -0.670392 \quad 0.014718$$

for L_4

$$L_4 = hf\left(t_1 + \frac{h}{2}, y_1 + \frac{K_3}{2}, \beta_1 + \frac{L_3}{2}\right)$$

$$= 0.1 f(0.15, 2.078178, 2.540026)$$

$$= 0.1\left(\frac{-2.078178(2.540026)^2}{2}\right)$$

$$= -0.670392$$

for y_2

$$y_2 = y_1 + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$$

$$= 2.073007 + \frac{1}{6}(0.037817 + 2(0.054110) + 2(0.0110342) + 0.014718)$$

$$= 2.103478$$

for z_2

$$Z_2 = Z_1 + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$$

$$= 2.779886 + \frac{1}{6}(-0.800395 + 2(-0.532659) + 2(-0.49928) + 0.67088)$$

$$= 2.191359.$$

$$t_2 = t_1 + h$$

$$= 0.1 + 0.1$$

$$t_2 = 0.2$$