Persistence Theory: From quiver representation to data analysis

https://www.ams.org/journals/bull/2020-57-01/S0273-0979-2018-01647-9/S0273-0979-2018-01647-9.pdf

Euler's powerful perspective:
"The problem of the Seven Bridges of Konigsberg asked whether or not it was possible to devise a walk" around the city — so that the travelers cross each bridge exactly once. In order to answer this question, Euler showed that "discarding unimportant details (e.g., distances and the shapes of the land masses) leads to a topological problem, whose solution reduces to evaluating a computable invariant." [1] This is a fascinating point of view. Instead of meticulously trying to specify all the hyperparameters of the problem, isolating most important features (underlying features that do not change and are most essential to solving the problem) can provide the edge-cutting solutions.

Jose provides an intuitive definition of *the shape of a topological space:* "those properties which remain unchanged under deformation that do not tear holes or glue portions of the space together" [155] A central question in topology is how to determine if two spaces are topologically equivalent. The main idea for the solution to the problem posed is to associate algebraic objects — such as numbers, vector sapaces, and other structures—to a space.

Persistence homology is also effective in modeling, classification, and prediction tasks where the shape of data is relevant. One way of using persistence homology in TDA is by computing barcode-like invariants. By calculating persistent homology, or barcodes, we can understand the shape of data by examining different dimensional holes. The single long interval in the barcode for 09-dimensional persistence is indicative that the underlying data space is connected. (0-dimensional hole is connectivity) The two long intervals for 1-persistence homology corresponds to two 1-dimensional holes. And the single long interval corresponds to a significant void in the data. We then can infer from this barcodes that the shape of the data is topologically equivalent to that of a torus. [156] — take the image.

Another big theme in the persistence homology theory is stability what happens to the barcodes as the input data is changed slightly.

- others:

Eulerian walk: a path which traverses all edges in a graph exactly once; such walk exists if and only if the graph is connected, and the number of noes with odd degree is either zero or two.