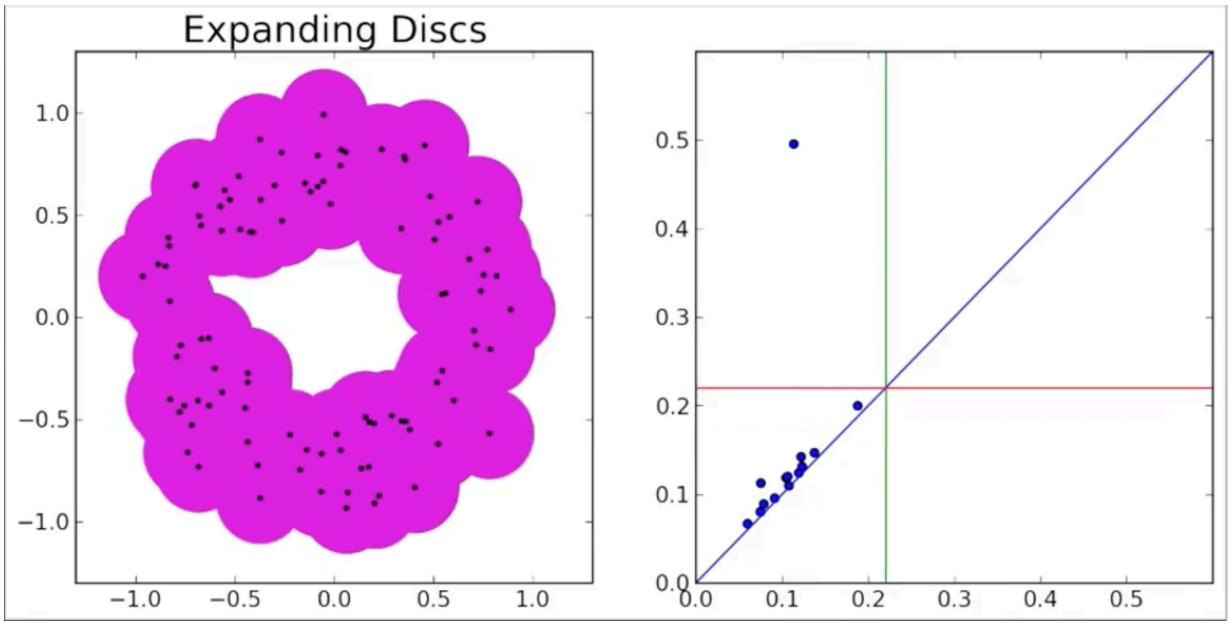
Applications of Persistence to Time Series Analysis (Liz Munch)

https://www.pathlms.com/siam/courses/4812/sections/6819/video_presentations/62605
time series classification (how to determine which dynamical system the time series is from)

- changing collections of topological space
  - Doing persistent homology on point cloud
  - I want to quantify what it means for the point cloud to look like a circle
  - If we just perceive the point cloud as it is then it is just a bunch of disconnected points scattered around, so a way to get around that is instead thinking about sticking a ball of fixed radius around each of my points and letting the ball expand. Then what i can do is to study the union of those balls/ discs expanding. I can study the expanding disc as a proxy for studying the topological space of that point cloud. The idea being that when the radius is very small we ware not doing a great job because all we see is a bunch of disconnected components, when the radius is too big then we are also not doing a great job because all we see is just one giant blob and I haven't actually seen the circular structure
  - Closer to the diagonal means a short life time -> because u r birth-ed and dead almost instantaneously,

There is a range of choices in the middle that gives me some sort of representation for the topology. The union of those discs looks something circular ish kind of

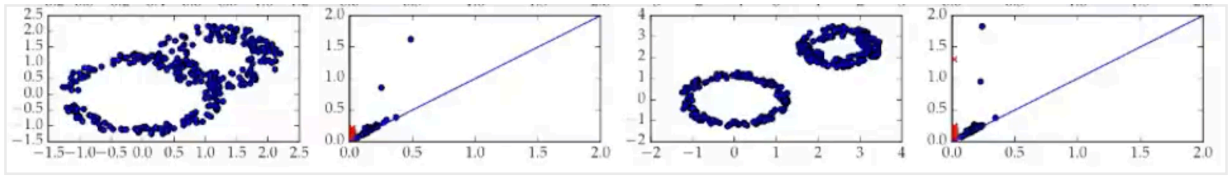


How to actually measure this?

- Persistence diagram (x-axis birth time of the feature, y-axis death of feature)
- notice that for each torus (one hole) corresponds to a significantly/ prominent off-diagonal point (similar persistence diagram)
- For spheres, it does not have any interesting first dimensional homology
- As a proxy for understand a embedded point clouds coming from manifolds or topological spaces I can use the persistence diagram.

What we can do with the persistence diagram

- We will have similar persistence diagrams for similar objects: e.g., two torus - at least in one dimensional structures



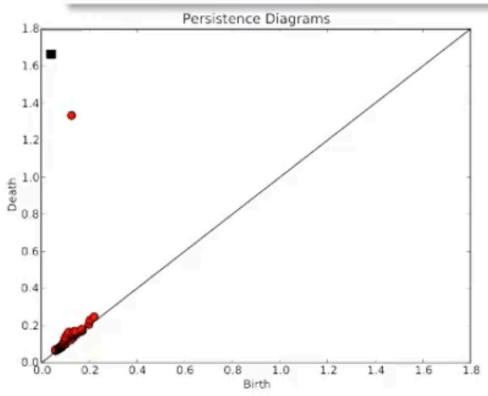
- How do we determine how they are different? = how to determine how far apart they are. E.g., when we are comparing a time series to our model, by looking at the distance between two off-diagonal points we can see how similar the two time series functions are

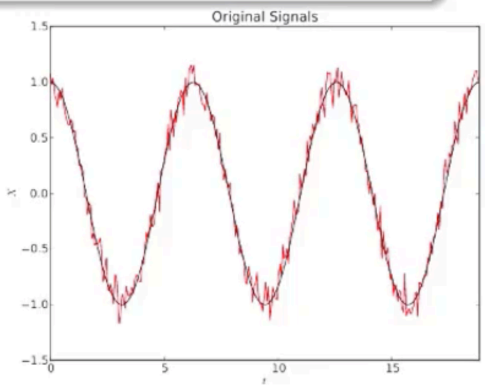
Noise resilience

Theorem (Cor of Cohen-Steiner et al., 2007)

Given two time series $f, g : \mathbb{R} \to \mathbb{R}$.
Let $\mathrm{dgm}(\mathrm{Takens}(f))$ be the persistence diagram of the Takens embedding of f, likewise for g, embedded with same η, m. Then

$$d_B(\mathrm{dgm}(\mathrm{Takens}(f)), \mathrm{dgm}(\mathrm{Takens}(g))) \leq \|f - g\|_\infty.$$





Application

- process
  - Time series
  - Transform to Takens Embedding
  - Generate persistence diagram
- Add stockastiticy (add noise — why would you want to add noise? What benefit??)