Applied Statistical Analysis I

Hypothesis testing, experiments, difference in means

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Hypothesis Testing

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- What is a hypothesis?

Hypothesis Testing

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- What are the five steps of null-hypothesis significance testing?

Hypothesis Testing

- A **hypothesis** is a statement about a population. It is a prediction that a parameter takes a particular numerical value or falls in a certain range of values. Agresti and Finlay, 2009, p.143

Hypothesis Testing

- Usually, we want to learn about population parameters based on sample statistics $\rightarrow$ statistical inference.
- For example: is a population parameter different from zero? If you find a value different from zero in your sample, can you be sure it's not just random variation?
- Formulate competing hypotheses:
  H_0: The population parameter equals zero (null hypothesis).
  H_A: The population parameter differs from zero (alternative hypothesis).

The 5 Steps of Hypothesis Testing

TABLE 6.1: The Five Parts of a Statistical Significance Test

1. **Assumptions**
   Type of data, randomization, population distribution, sample size condition
2. **Hypotheses**
   Null hypothesis, H_0 (parameter value for "no effect")
   Alternative hypothesis, H_a (alternative parameter values)
3. **Test statistic**
   Compares point estimate to H_0 parameter value
4. ***P*-value**
   Weight of evidence against H_0; smaller P is stronger evidence
5. **Conclusion**
   Report P-value
   Formal decision (optional; see Section 6.4)

Figure 1: Source: Agresti and Finlay, 2009, p.147

Hypothesis Testing: Step 1

Assumptions Agresti and Finlay, 2009, p.144

- Type of data: continuous, categorical, etc.
- Sampling method: randomly obtained (e.g., random sample)
- Population distribution: assume a certain distribution (e.g., normal)
- Sample size: validity improves with larger samples

Hypothesis Testing: Step 2

State the Hypotheses

- Example: Someone claims the mean income of a country is 1400.
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  $H_0 : \mu = 1400 \quad \text{vs.} \quad H_A : \mu \neq 1400$
- In other words: H_0 = the population mean **is equal** to the sample mean and H_A = the population mean **is not equal** to the sample mean.

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- In other words: H_0 = the population mean **is equal** to the sample mean and H_A = the population mean **is not equal** to the sample mean.
- The hypothesis can be one-sided (<, >, ≥, ≤) or two-sided (=, $\neq$).

Hypothesis Testing: Step 3

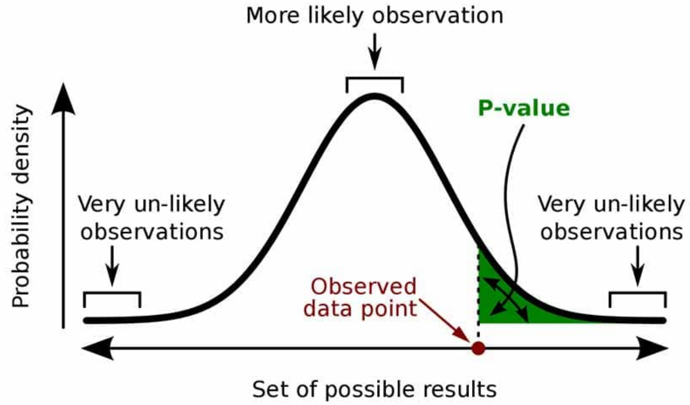
Test Statistic

- "The test statistic summarizes how far the estimate falls from the parameter value in H_0. Often this is expressed by the number of standard errors between the estimate and the H_0 value." Agresti and Finlay, 2009, p.145
- Depending on the distribution of the test statistic, it is often referred to as a z-statistic (when the population standard deviation is known and n is large) or a t-statistic (when the n is small and/or the population variance is unknown).

Step 4: p-value

- p-value: The p-value is the probability that the test statistic equals the observed value or a value even more extreme in the direction predicted by H_A. It is calculated by presuming that H_0 is true. The smaller the p-value, the stronger the evidence is against H_0 (Agresti and Finlay 2009, 145).

Step 4: p-value



A **p-value** (shaded green area) is the probability of an observed (or more extreme) result assuming that the null hypothesis is true.

Hypothesis Testing: Step 5

Conclusion

- Validate whether the obtained test statistic is unlikely to occure, under the assumption that the null hypothesis (H_0) is true $\rightarrow$ p-value, if probability is low, we reject H_0.
- Select an α-level, which indicates acceptable probability of Type 1 error (usually 0.05, 0.01).
- If p-value < α, we reject H_0 $\rightarrow$ proof by contradiction.
- Be careful when you are drawing a conclusion - there is still a probability that we falsely reject the null, even if p-value < α.

		Null hypothesis (H_0) is	
		True	**False**
Decision about Null hypothesis (H_0)	**Don’t reject**	Correct inference (true negative) (probability = $1-\alpha$)	Type II Error (false negative) (probability = β)
	Reject	Type I Error (false positive) (probability = α)	Correct inference (true positive) (probability = $1-\beta$)

We are most concerned about Type I error: the probability of obtaining a false positive result.

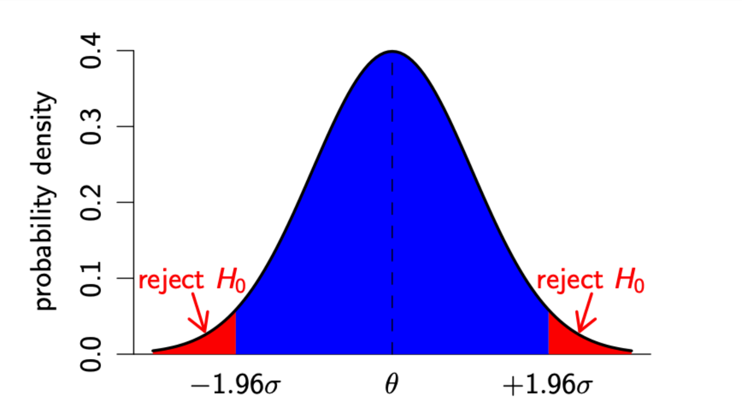
Calculating a Test Statistic

- Let's take our previous example: We take a random sample of size n = 1000 and obtain a mean of 1350 with a standard deviation of 750.
- At a level of 95% confidence, the critical z-values on a standard normal distribution are ±1.96
- How much is the sample mean (i.e., 1350) away from the hypothesized population mean (i.e., 1400) in repeated samples? Calculate the z-score of your realized sample:

$$z = \frac{\bar{x} - \mu_{pop}}{\sigma_{pop}/\sqrt{n}} = \frac{1350 - 1400}{\widehat{750}/\sqrt{1000}} = -2.1$$

- This is smaller than the critical value and you would reject H_0.

Calculating a Test Statistic



Decision Based on Confidence Intervals

- A $100(1-\alpha)\%$ **confidence interval (CI)** for a population mean is an interval that, in repeated random samples, would contain the true mean about $100(1-\alpha)\%$ of the time.
- For a two-sided test with significance level $\alpha = 0.05$, the CI is a **95% CI**.
- Our example:

$$\bar{x} \pm z_{1-\alpha/2} \cdot \frac{s}{\sqrt{n}} = 1350 \pm 1.96 \cdot \frac{750}{\sqrt{1000}} = [1303, 1396].$$

- The hypothesized mean 1400 is not in this 95% CI $\Rightarrow$ reject H_0 at $\alpha = 0.05$.
- **Key idea:** For a two-sided test at level α,

$$\mu_0 \notin 100(1-\alpha)\% \text{ CI} \quad \Longleftrightarrow \quad p < \alpha.$$

Example

Significance test for a mean (t-test)

TABLE 6.2: Responses of Subjects on a Scale of Political Ideology

	Race		
Response	Black	White	Other
1. Extremely liberal	10	36	1
2. Liberal	21	109	13
3. Slightly liberal	22	124	13
4. Moderate, middle of road	74	421	27
5. Slightly conservative	21	179	9
6. Conservative	27	176	7
7. Extremely conservative	11	28	2
	$n = 186$	$n = 1073$	$n = 72$

H_0: Mean political ideology is 'moderate' $\rightarrow \mu = 4$
H_A: Mean political ideology falls in liberal or conservative direction $\rightarrow \mu \neq 4$ ($\mu < 4$ or $\mu > 4$)

Significance test for a mean (t-test)

- With $\bar{x}$ = 4.075, s = 1.512, we can calculate the SE:

$$SE = \frac{s}{\sqrt{n}} = \frac{1.512}{\sqrt{186}} = 0.111$$

- And then the test statistic:

$$t = \frac{\bar{x} - \mu_0}{SE} = \frac{4.075 - 4}{0.111} = 0.68$$

- Degrees of freedom: $df = n - 1 = 185$.
- How to interpret this value?

Significance test for a mean (t-test)

- For a **two-sided** $\alpha = 0.05$ and df = 185, the critical value is : $t_{0.975,185} \approx 1.97$
- Our $t = 0.68$ is far inside $[-1.97, 1.97] \Rightarrow$ fail to reject H_0.
- p-value: $P(|T| > 0.68) \approx 0.50$ (from the t distribution).

Significance test for a mean (t-test)

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- p-value: $P(|T| > 0.68) \approx 0.50$ (from the t distribution).

Checking the CIs too:

$$4.075 \pm 1.97 \cdot 0.111 = [3.86,\ 4.29]$$

Since 4 is inside the 95% CI, result agrees: fail to reject H_0.

Significance test for a mean (t-test)

What is the conclusion?

p-value $\approx 0.50 > 0.05$, thus **we cannot reject the null (H_0)**. It is plausible that the population mean is 4, and therefore moderate.

Introduction to causality

Causal effect

What is a causal effect?

What is the fundamental problem of causal inference?

Key idea

A **causal effect** is the change in an outcome Y that would occur if we switched a unit from one condition to another.

Variables

- **Treatment** T**:**
  - $T = 1$ = treated
  - $T = 0$ = control
- **Outcome** Y**:** what we measure

Potential outcomes

$$Y_i(1) = \text{outcome if treated}$$

$$Y_i(0) = \text{outcome if control}$$

Individual causal effect

$$\tau_i = Y_i(1) - Y_i(0)$$

The difference between what *did* happen and what *would have happened* in the counterfactual world.

Causal effect

The fundamental problem with causal inference: “we can only observe, at most, one of the two quantities, for any individual at a particular point in time”
$\rightarrow$ **The causal effect is unobservable.**

(Bueno de Mesquita & Fowler, 2021, p.164)

SATE

What is the sample average treatment effect (SATE)?

What can we actually observe?

SATE

$$\frac{1}{n}\sum_{i=1}^{n}\{Y_i(1) - Y_i(0)\}$$

- It is the average of individual-level treatment effects in the sample.
- SATE is unobservable due to the fundamental problem of causal inference $\rightarrow$ we only observe **sample difference in means**.

SATE: Difference in Means

Naïve estimator (difference in means)

$$\hat{\Delta} = \bar{Y}_{T=1} - \bar{Y}_{T=0}$$

Difference in average outcome between treated and control groups.

Key point

- $\hat{\Delta}$ is an **unbiased estimate of SATE** only if treatment is as-if random.
- Otherwise it may be **biased by selection** (treated and control differ in unobserved ways).
- Correlation does not imply causation.

Bias

What are the sources of bias? And how can we overcome bias?

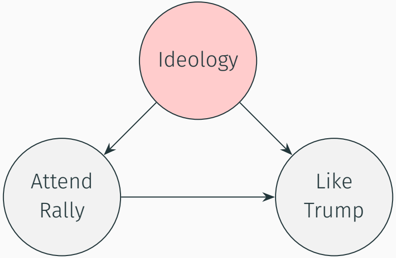
Bias

- **Baseline differences**: "...difference in the average potential outcome between two groups (e.g., the treated and untreated groups), even when those two groups have the same treatment status"
- **Confounders** may cause baseline differences, which may cause bias (omitted variable bias).

Bias

Confounders

- It has an effect on the treatment status.
- It also affects the potential outcome *beyond* its effect through treatment.



Other sources of bias

Bias

Other sources of bias

- Reverse causality
- Unobserved unit heterogeneity (special type of OVB)
- Post-treatment bias

Bias

How can we overcome bias?

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   Randomly assigning units to treatment and control groups eliminates baseline differences.
   "But, the ideal experiment that we'd like to run is often impractical, infeasible, or unethical".

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   Randomly assigning units to treatment and control groups eliminates baseline differences.
   "But, the ideal experiment that we'd like to run is often impractical, infeasible, or unethical".
3. **Causal inference methods**: Difference-in-Differences design, Instrumental Variables etc.

(BdM and Fowler, 2021, 215-239)