
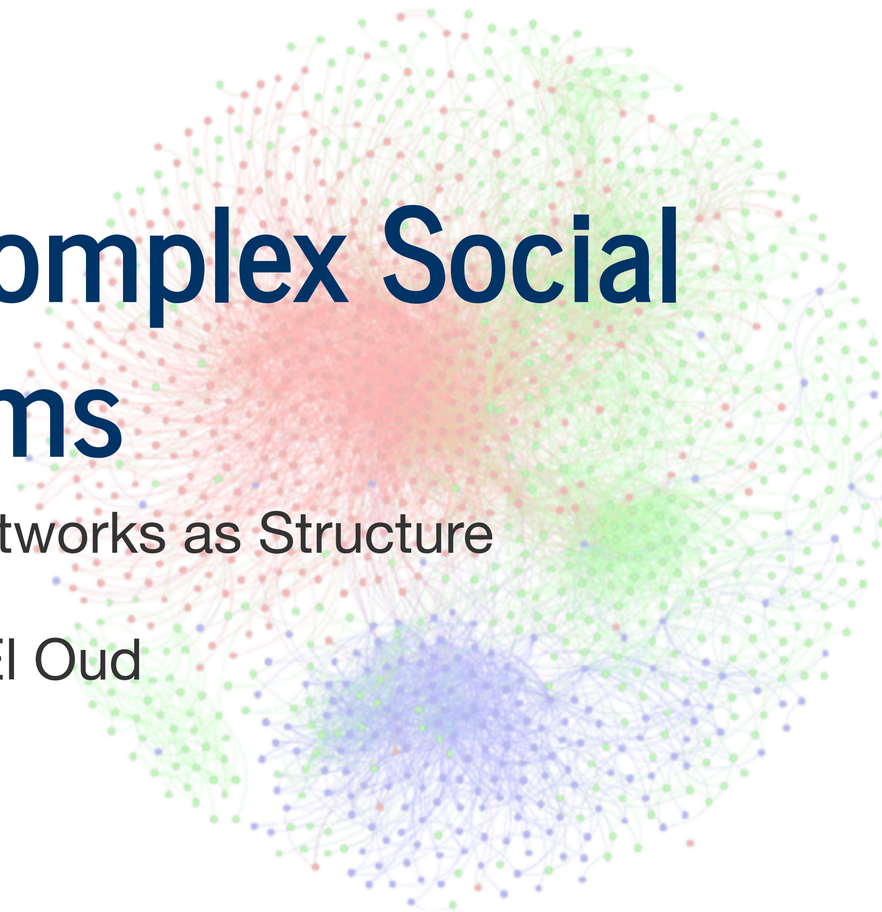

Introduction to Complex Social Systems

Empirical Patterns and Networks as Structure

Sara Maria El Oud

Introduction

The Universe Is Complex—and So Are We

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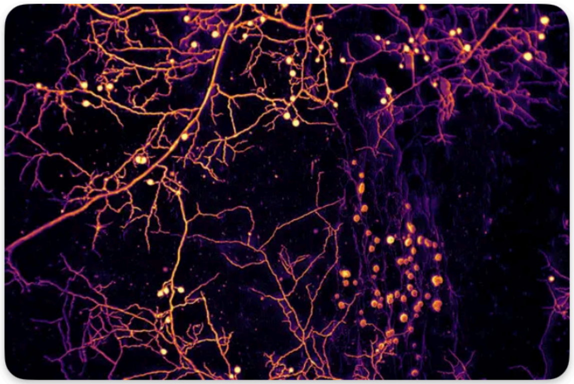
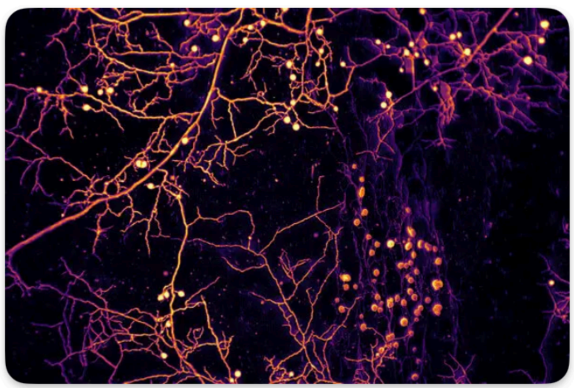


The Universe Is Complex—and So Are We



The Universe Is Complex—and So Are We

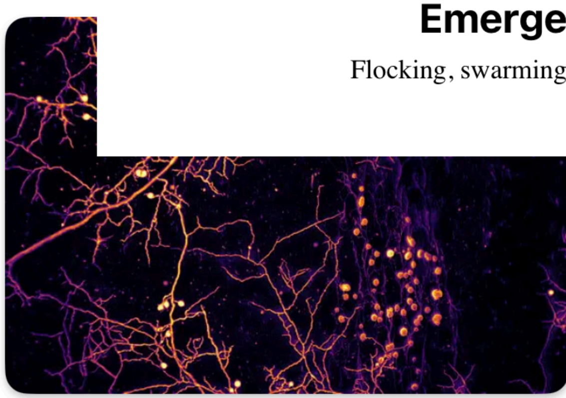
Simple Local Rules

local interactions, repeated interactions, no central control

↓

Emergent Behavior

Flocking, swarming, mobilizing, self-organization



What Are Social Systems?

- **Social systems** are made up of people or groups interacting with each other, often through communication, shared norms, and institutions.
- They are **decentralized**, often informal, and evolve over time.
- These interactions form **structures**—networks of influence, affiliation, or exchange.
- What matters is not just *who the individuals are*, but *how they interact*.

> *Social systems are **dynamic, adaptive, and shaped by feedback**—ideal candidates for modeling collective behavior.*

Collective Dynamics in Social Systems

- In social systems, **individual decisions and interactions** create **group-level outcomes**.
- These outcomes aren't centrally planned—but they **emerge**.
- Think protests, viral memes, social norms, or **entire economies**.

> ***emergent phenomena*** *from* ***decentralized, local interactions.***

Why These Systems Are Hard to Model (I)

- They are **not in equilibrium**—no static solution or fixed point.
- They involve **a large number of interacting components**—people, groups, topics, behaviors.
- These components are highly **heterogeneous**—differing in form, influence, and behavior.
- Many **different processes** unfold simultaneously, and operate at **multiple time-scales**.

> *Modeling these systems requires tools that account for both **diversity** and **dynamical evolution**.*

Why These Systems Are Hard to Model (II)

- Both the **structure** (who connects to whom)
  and the **interactions** (how they influence each other) are complex.

- These give rise to **non-linear dynamics**:
  - small inputs can trigger large cascades,
  - feedback, thresholds, and tipping points shape outcomes.

- In online systems, interactions often occur **at a distance**,
  asynchronously, and across multiple platforms.

> ***Complex structure + complex interactions → complex dynamics*** *Yet we still observe* ***patterns*** *—sharp transitions, scaling laws, and evolving communities.*

Why Now?

- We live in the **age of data**—we can observe social systems at scale, in real time.
- Online activity leaves **digital traces** of communication, coordination, and conflict.
- Advances in **Natural Language Processing (NLP)** allow us to quantify language, sentiment, and discourse.
- Computational tools make it possible to **model and simulate** complex systems that were once inaccessible.

> *We are now uniquely positioned to study human collective behavior—**empirically, rigorously, and at scale**.*

Why Study Online Social Systems?

- Our lives increasingly unfold **online**—but the dynamics are anything but virtual.
- Online platforms shape how we **communicate**, **coordinate**, and **mobilize**.
- To address today's challenges, we need to understand how **collective behavior emerges** online:
  - In **public health**: How misinformation spreads and shapes action
  - In **democracy**: How polarization and protest movements grow
  - In **markets**: How trust collapses and shocks cascade
  - In **online communities**: How norms, identities, and groups evolve
  - Across **platforms**: How narratives migrate, persist, and adapt
- Understanding complexity helps us **anticipate**, **intervene**, and **design** better systems.
- We now have the tools to **observe**, **quantify**, and **model** complex systems at scale.
- These insights generalize to **biological, ecological, and technological systems** as well.

What This Course Is About

We'll build a framework to model these systems using:

- **Networks** to represent structure
- **Stochastic processes** to capture dynamics
- **Simulation and theory** to explore how complex behaviors emerge

> *From local rules to global behavior:*
> *a path from* ***empirical patterns*** *to* ***explanatory models****.*

My Research Lens

Personal Trajectory

- **Undergraduate studies in Physics at AUB**, with wide-ranging interests in biology, anthropology, and social systems
  → First exposure to simulation as a way to explore complex behavior

- **Early research**: Collaborated with AUB's Computer Science department
  → Developed a software tool for detecting and quantifying corneal haze from medical imaging

- **PhD in Physics at GWU**, joining the **Dynamic Online Networks Lab** led by Dr. Neil Johnson
  → Shifted focus to modeling large-scale social systems using tools from statistical physics, network science, and simulation

Dynamic Online Networks Lab

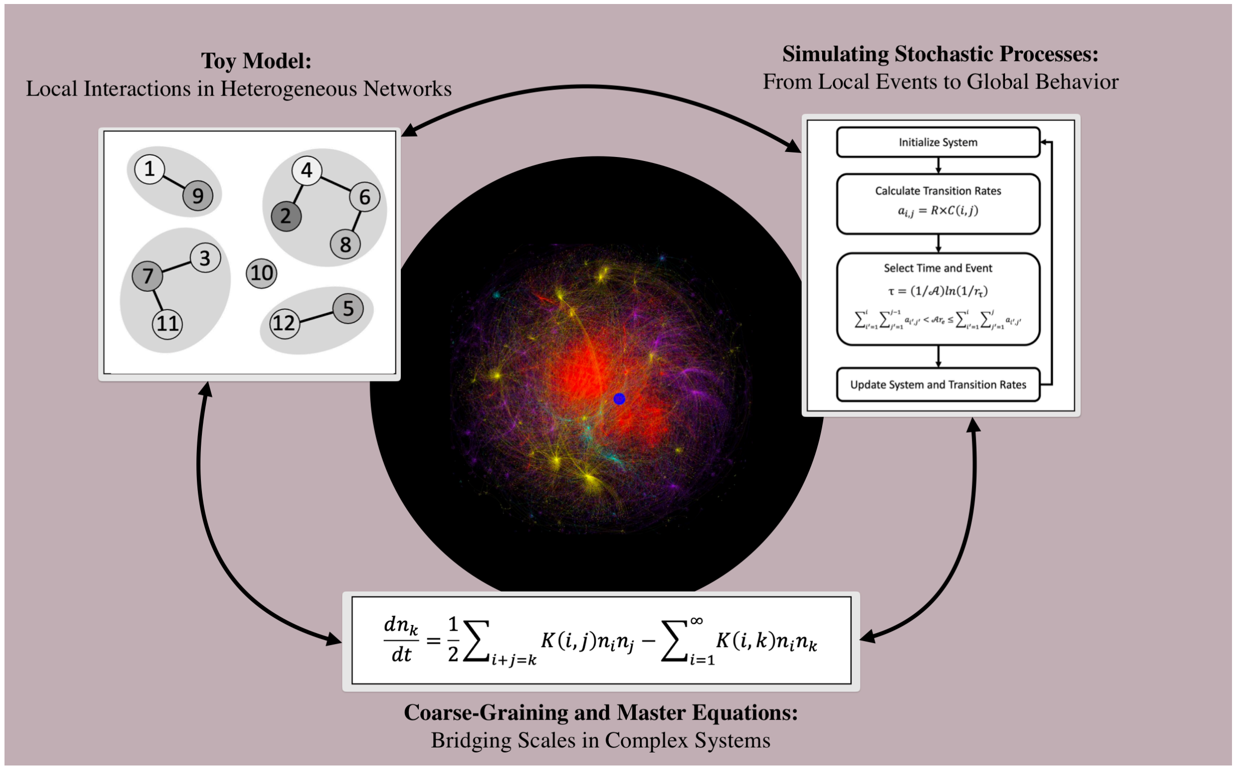
What We Do:

- **Data**: Collect online data across platforms, focusing on hate and extremist ecosystems
- **Networks**: Map structure and connectivity to analyze group dynamics
- **NLP**: Quantify and track evolving narratives over time
- **Theory**: Build **physics-based models** to capture dynamics and reproduce empirical patterns

Key Research Themes

- **Mapping the online landscape**
  Track how communities form, grow, and link across platforms using real-time data
- **Resilient ecosystems of extremism and hate**
  Study how groups adapt, fragment, and reconnect after moderation efforts
- **Narrative dynamics and misinformation**
  Analyze how competing narratives emerge, evolve, and influence behavior
- **Cross-platform flow**
  Understand how users, content, and strategies migrate across digital environments
- **Network-based interventions**
  Design structural nudges to disrupt harmful coordination without censoring speech

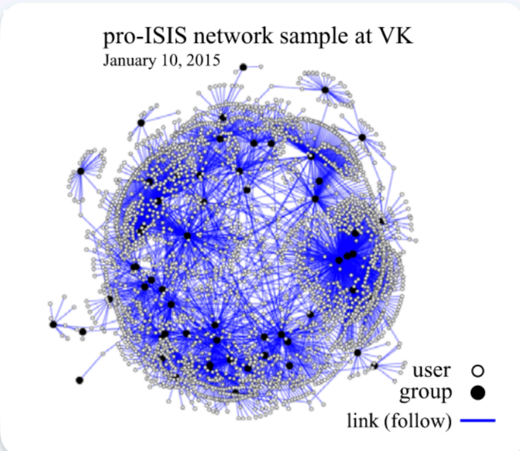
PHD: Aggregation Dynamics of Heterogenous Systems



Motivating Phenomena

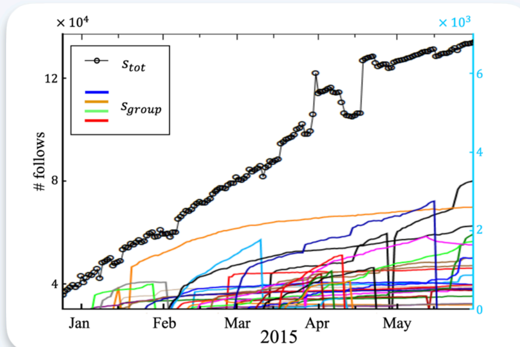
Pro-ISIS Online Mobilization on VK (2014–2015)

(a) Network Snapshot



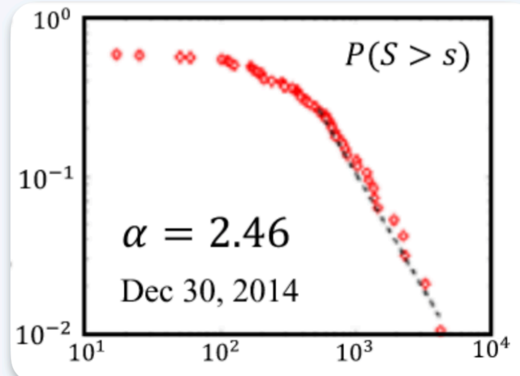
- **Many Interacting Components**
- **Complex Community Structure**

(b) Temporal Growth Patterns



- **Sudden Overall Onset (Dec 30 2014)**
- **Diverse Group Growth Patterns**
  - Sudden Starts
  - Varied Growth Rates
  - Disappearances or Takedowns

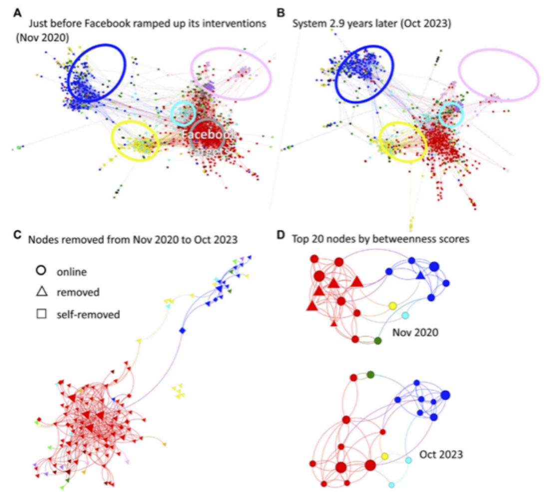
(c) Group Size Distribution (Onset)



- **Heavy Tailed Group Size Distribution**
- **Few Large, Many Small Groups**
- **Power-Law Pattern**

Competing Online Vaccination Communities

Resilience to Disruption



Source: Illari et al., *Frontiers in Complex Systems* (2024)

Complex Structure and Dynamics

- **Three-body problem:** distinct substructures
- **Complex inter- and intra- community structure**
- Communities follow **different strategies and growth patterns**
- The system is **competitive and dynamic**, not static
- Interactions span **geographies**

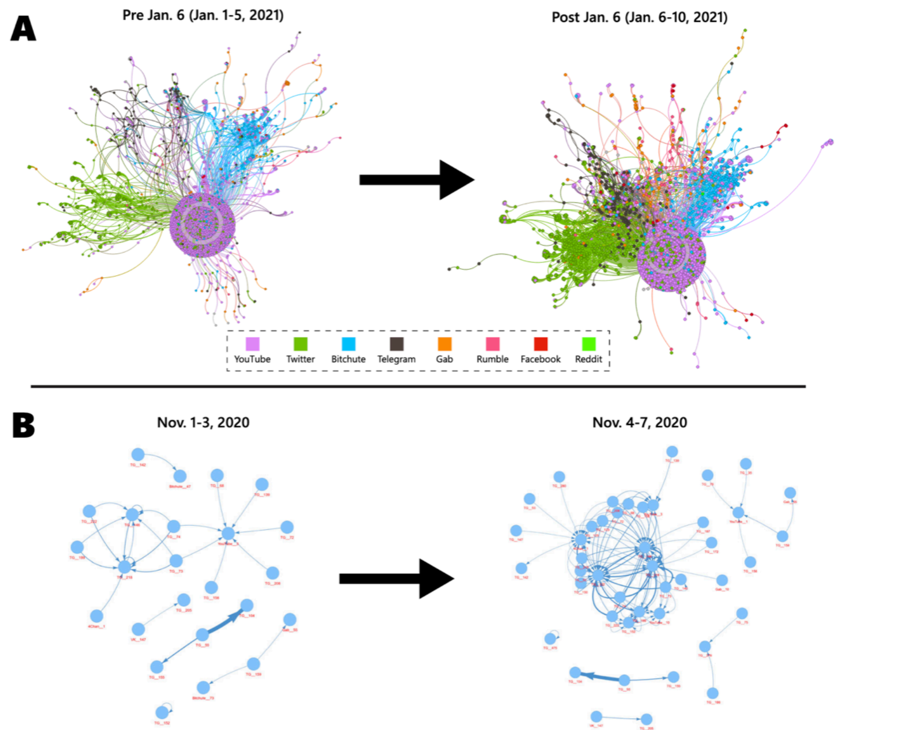
Heterogeneity and Narative Complexity

- Communities show diverse **internal structure and topical focus**
- Group narratives are **multi-topic and entangled across issues**
- Narratives **shift and adapt over time**

Adaptation and Resilience

- Community structure persists through **removal**
- **Robustness and reconfiguration** of network

The Global Hate Ecosystem: Complex, Interconnected, and Reactive



Source: Verma, A., Sear, R., & Johnson, N. F. (2024). *U.S. Election Hardens Hate Universe*. arXiv:2405.00459.

Offline-Event (January 6, 2021)

- Trump Supporters Stormed U.S. Capitol

Online Hate Communities

- Network **reorganizes and hardens**

online–offline feedback

What We've Seen So Far

Structure

- Many **heterogeneous nodes** (users, groups, topics) across platforms
- Communities form **modular, interlinked structures**
- Network connectivity spans **platforms and geographies**

Processes

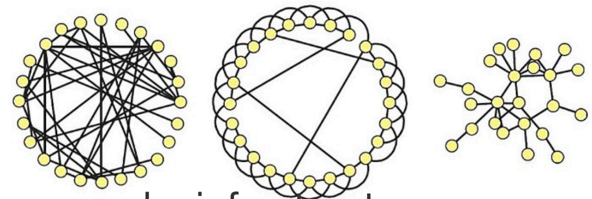
- **Links form dynamically** between nodes and groups
- Different nodes show **different growth and survival strategies**
- Groups can **grow rapidly**, **reposition**, **disappear**,or **shift identity and focus**

Emergent Patterns

- **Sudden onset** of activity in individual communities
- **Power-law scaling** in community sizes
- Systems are **adaptive and resilient** to disruption

Networks

Why Networks?



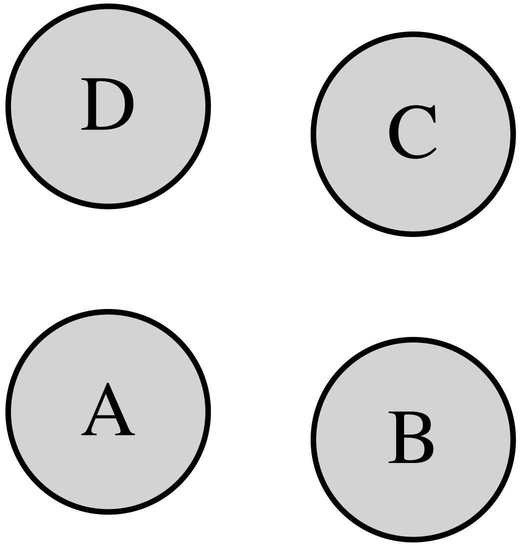
“Behind many complex systems is a network.”

- Networks emerge whenever you have **interactions**: people, genes, code, infrastructure
- Networks = **abstractions** to see relationships
- Found across domains (eg/ Social, Biological, Technological, Linguistic, …)
- Why this surge in network science?
  1. Explosion of **large-scale interaction data**
  2. Discovery of **universal patterns** — power laws, small-world structure

> ***Understanding a system’s structure is the first step to understanding its dynamics***

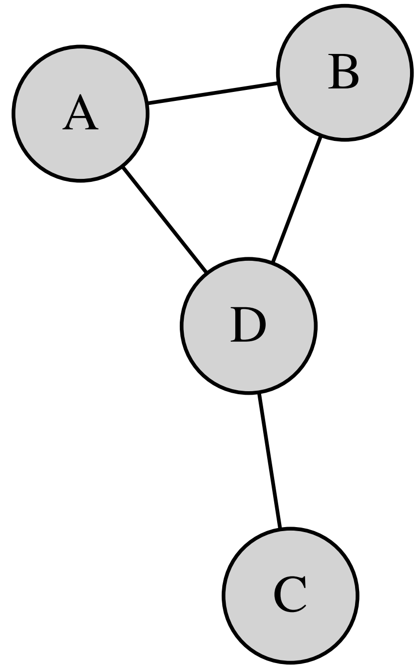
Representing Networks: Simple building blocks, many forms

- Nodes $N = \{A, B, C, D\}$



Representing Networks: Simple building blocks, many forms

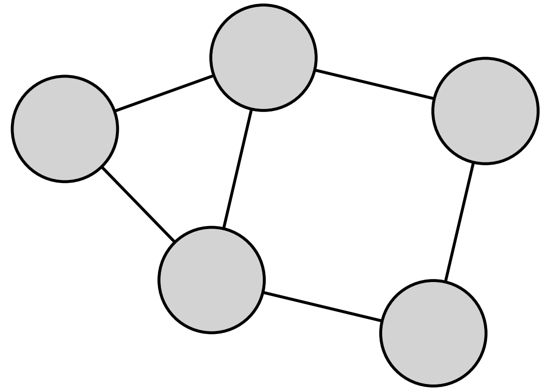
- Nodes $N = \{A, B, C, D\}$
- Edges $E = \{(A, B), (C, D)\}$



Representing Networks: Simple building blocks, many forms

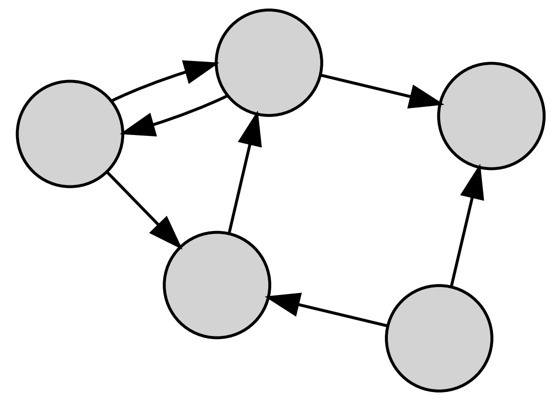
- $G = (N, E)$
- Adjacency Matrix A_{ij}

$$A = \begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$



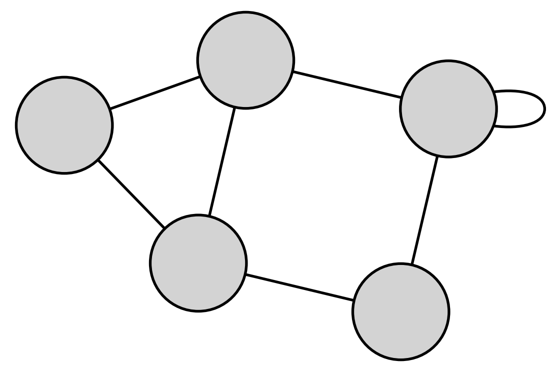
Representing Networks: Simple building blocks, many forms

- $G = (N, E)$
- Adjacency Matrix A_{ij}
- **Directed** vs **Undirected**:
  - **Undirected**: $A_{ij} = A_{ji}$ (*symmetric*)
  - **Directed** (digraph): $A_{ij} \neq A_{ji}$ (*asymmetric*)



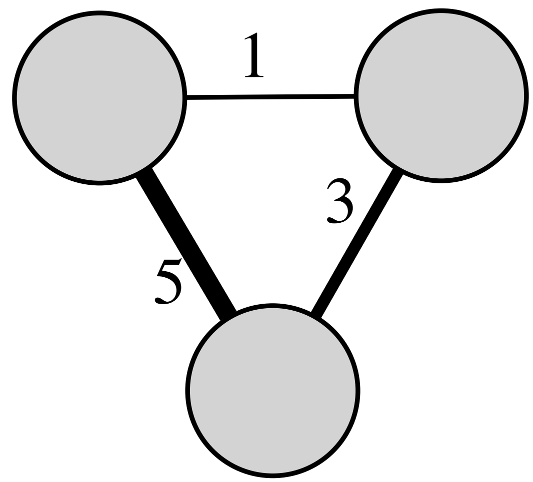
Representing Networks: Simple building blocks, many forms

- $G = (N, E)$
- Adjacency Matrix A_{ij}
- **Directed** vs **Undirected**
- **Self-loops**:
  - An edge from a node to itself
  - In adjacency matrix: $A_{ii} = 1$



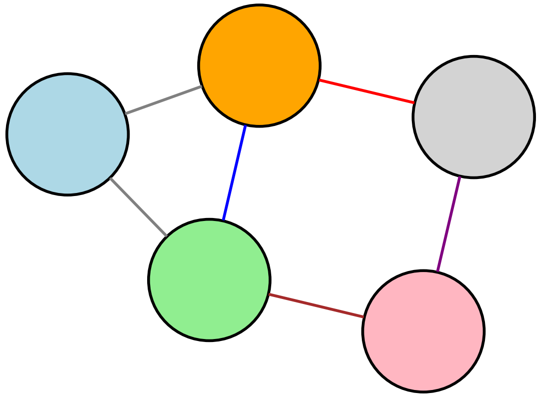
Representing Networks: Simple building blocks, many forms

- $G = (N, E)$
- Adjacency Matrix A_{ij}
- **Directed** vs **Undirected**
- **Self-loops**
- **Weighted** vs **Unweighted**:
  - **Unweighted**: $A_{ij} = 0$ or 1
  - **Weighted**: $A_{ij} \in \mathbb{R}_{\geq 0}$



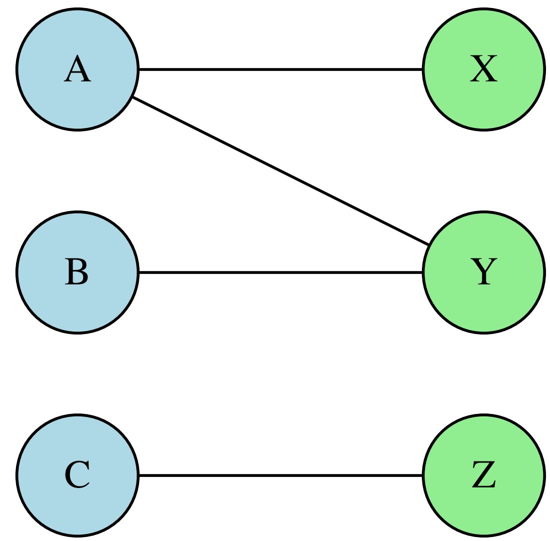
Representing Networks: Simple building blocks, many forms

- $G = (N, E)$
- Adjacency Matrix A_{ij}
- **Directed** vs **Undirected**
- **Self-loops**
- **Weighted** vs **Unweighted**
- **Node/Edge Attributes**:
  - Node attributes: assign a property to each node → x_i
  - Edge attributes: assign a property or weight to each edge → w_{ij}
  - Example:
    - x_A = "student", x_B = "professor"
    - w_{AB} = "collaborates", or $w_{AB} = 0.8$



Representing Networks: Simple building blocks, many forms

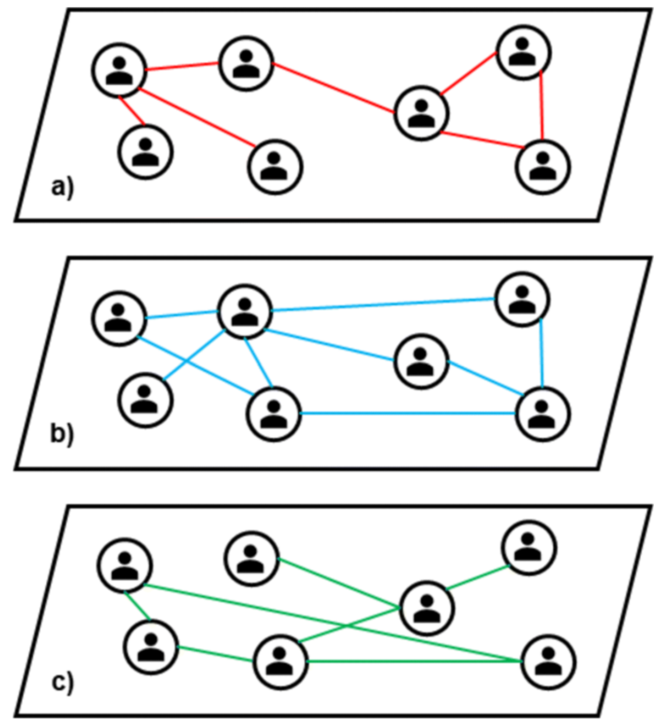
- $G = (N, E)$
- Adjacency Matrix A_{ij}
- **Directed** vs **Undirected**
- **Self-loops**
- **Weighted** vs **Unweighted**
- **Node/Edge Attributes**
- **Bipartite**:
  - two distinct types of nodes
  - edges only across types
  - e.g., users and items, authors and papers



$$\text{Nodes:} \quad N = N_1 \cup N_2, \quad N_1 \cap N_2 = \emptyset$$
$$\text{Edges:} \quad E \subseteq N_1 \times N_2$$

Representing Networks: Simple building blocks, many forms

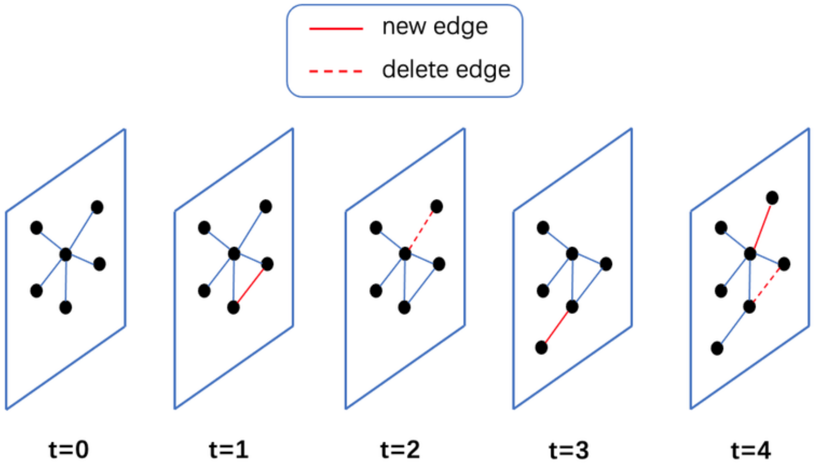
- $G = (N, E)$
- Adjacency Matrix A_{ij}
- **Directed** vs **Undirected**
- **Self-loops**
- **Weighted** vs **Unweighted**
- **Node/Edge Attributes**
- **Bipartite**
- **Multiplex**:
  - same nodes connected in multiple ways
  - different **layers** (communication, collaboration, etc.)



Representing Networks: Simple building blocks, many forms

- $G = (N, E)$
- Adjacency Matrix A_{ij}
- **Directed** vs **Undirected**
- **Self-loops**
- **Weighted** vs **Unweighted**
- **Node/Edge Attributes**
- **Bipartite**
- **Multiplex**
- **Temporal**:
  - edges and nodes vary over time
  - sequence of graphs

$$G(t) = (N(t), E(t))$$



Measuring Networks: Degree

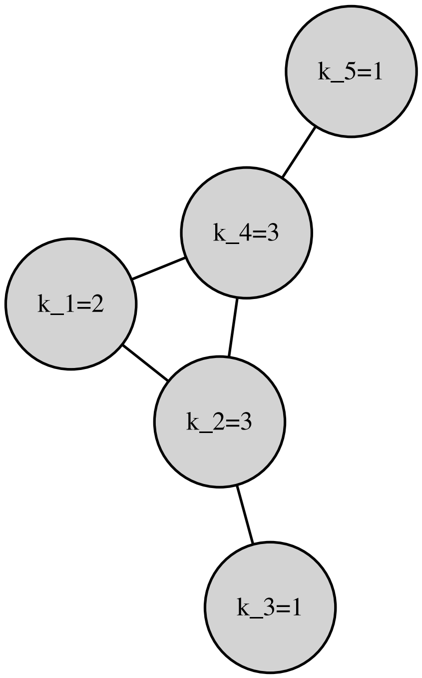
Node Degree

- Degree of a node = number of edges
- For undirected graphs:

$$k_i = \sum_j A_{ij}$$

- For directed graphs:

$$k_i^{\text{in}} = \sum_j A_{ji}, \quad k_i^{\text{out}} = \sum_j A_{ij}$$



Measuring Networks: Degree Distribution

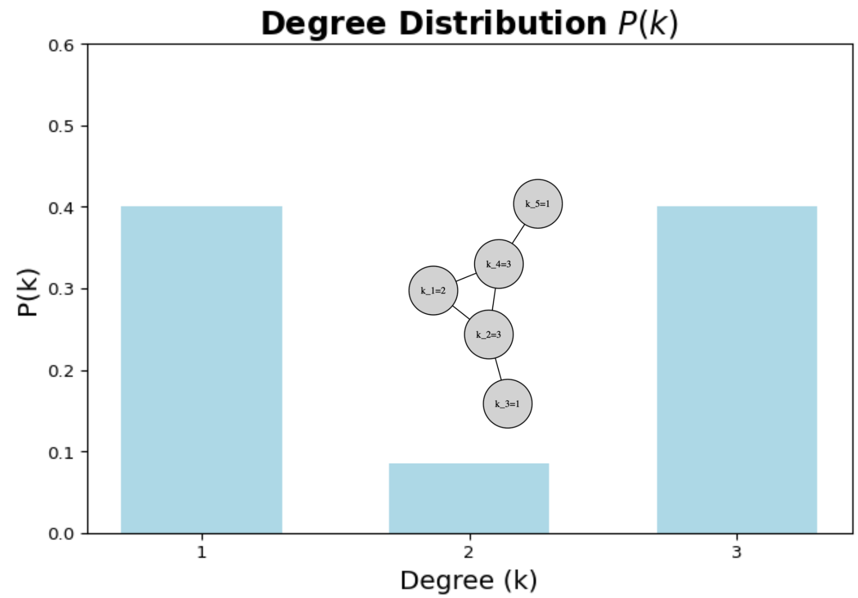
Node Degree $k_i = \sum_j A_{ij}$

Degree Distribution

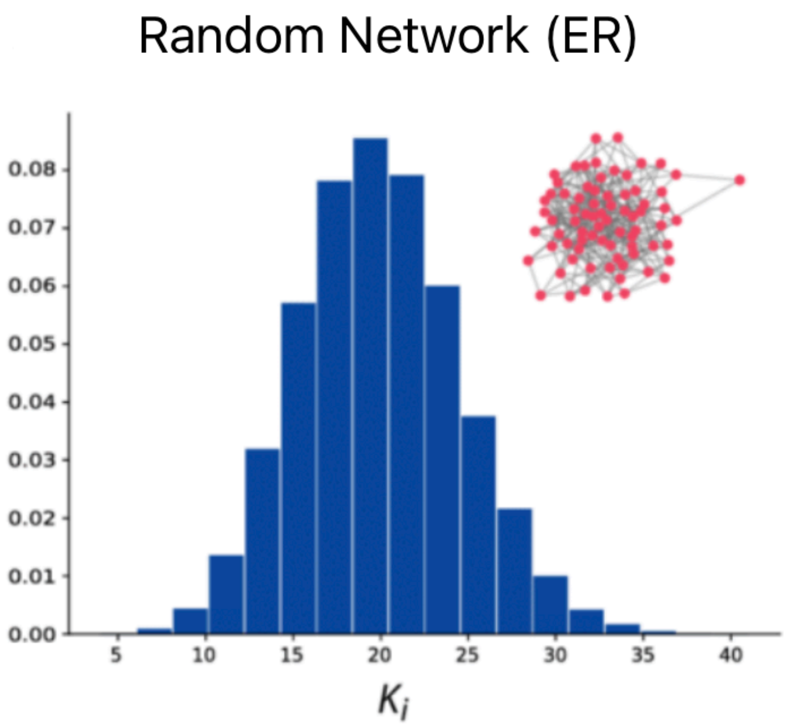
- $P(k)$ = probability a node has degree k

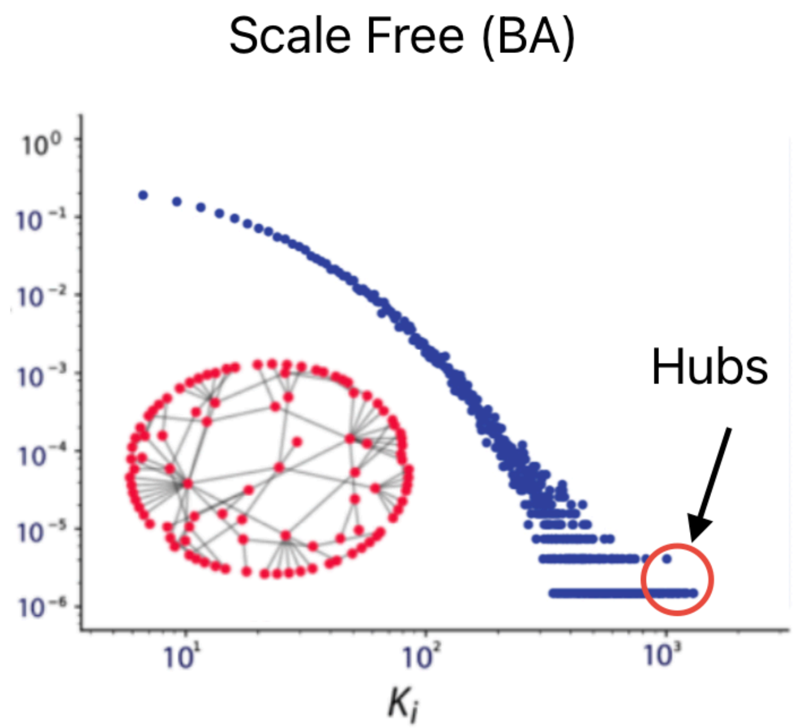
$$P(k) = \frac{N_k}{N}$$

where N_k is the number of nodes with degree k



Measuring Networks: Example Degree Distributions

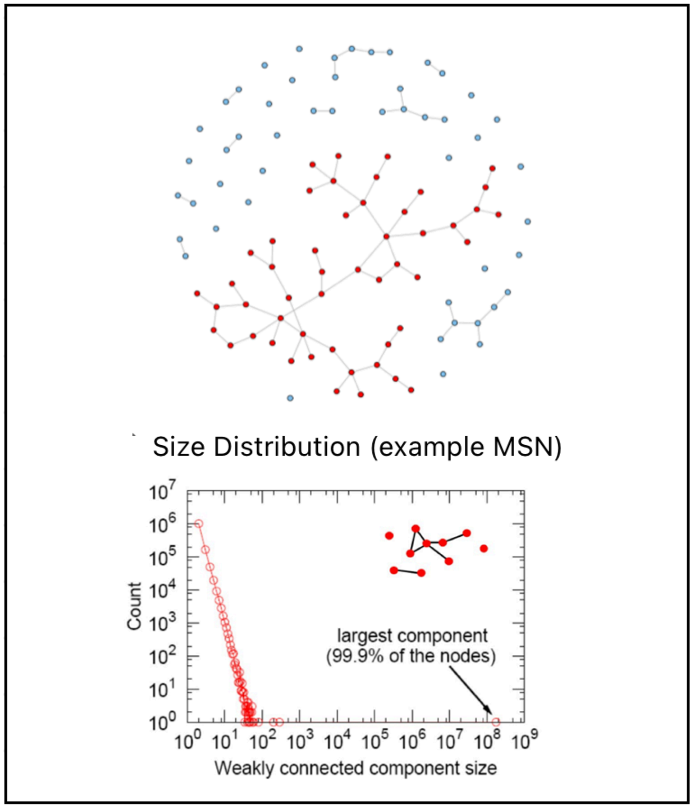




Measuring Networks: Connected Components

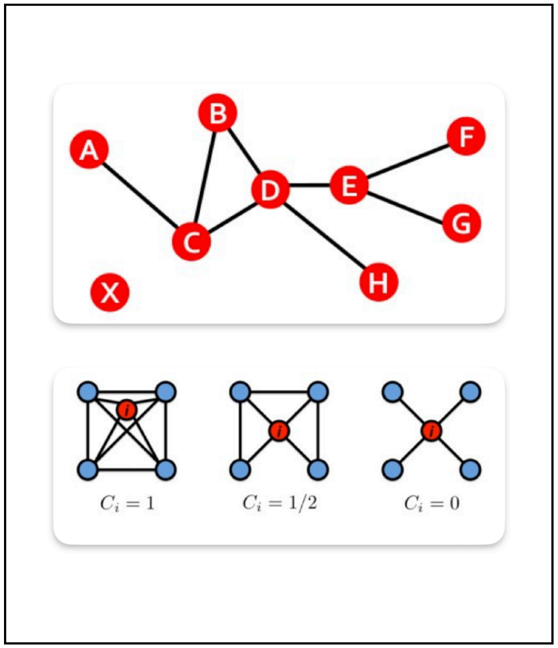
- A **connected component** is a group of nodes where each node is reachable from every other by a path
- Components are **disjoint** — each node belongs to exactly one component
  - **giant component** (red) — the largest, connected cluster
  - **small or isolated components** (blue) scattered across the network
- The **component size distribution** shows how fragmented or cohesive the system is
- Connected components help us understand:
  - Whether the system is **cohesive or fragmented**
  - How far **information, influence, or contagion** can spread
  - The system's **resilience** to failure or targeted removal

In our models, component growth and merging are central to understanding system-wide dynamics



Measuring Networks: Other Measures

- **Path Length**
  - Average shortest distance between nodes
  - Captures **efficiency** of flow or communication
- **Diameter**
  - Longest shortest path in the network
  - Indicates **network scale**
- **Clustering Coefficient**
  - Measures how interconnected neighbors are
  - Highlights **local cohesion** or triadic closure
- **Centrality Measures**
  - **Degree centrality**: direct influence
  - **Betweenness**: control over information flow
- *Many more …*

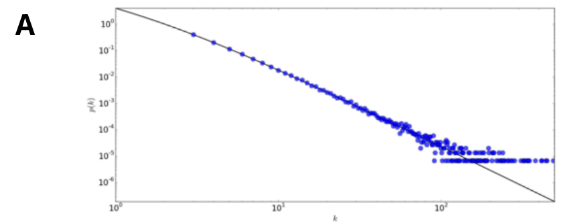


Observed Patterns in Real-World Networks

- **Scale-Free Networks**
  - A few nodes (hubs) have many connections; most have very few
  - Degree distribution is **heavy-tailed**
  - No typical node degree — the network is **scale-free**
  - Found in web, citation, and social systems
  - Follows a power law:

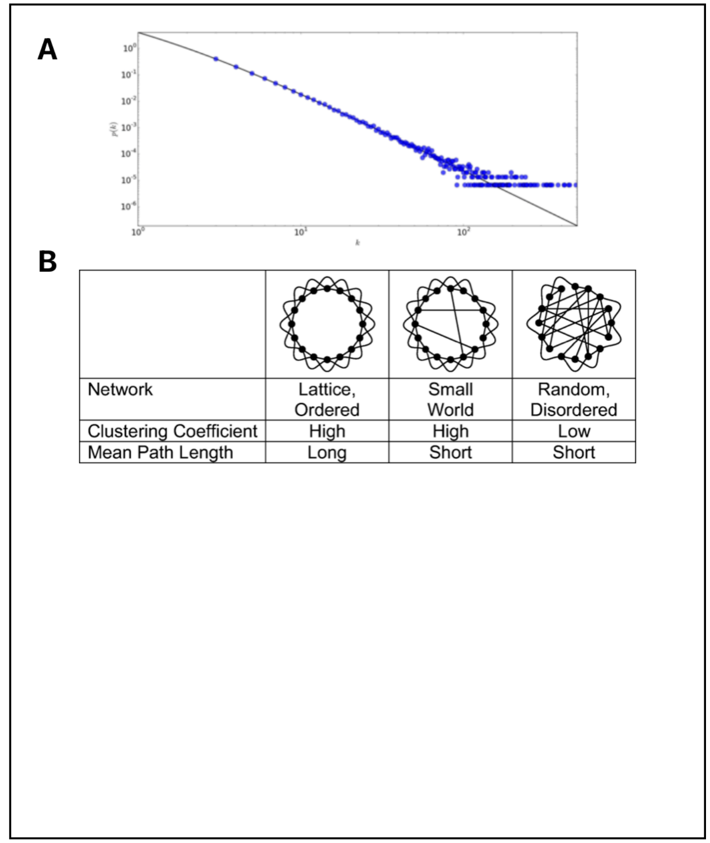
$$P(k) \sim k^{-\gamma}$$

  - Hubs shape **connectivity**, **resilience**, and **influence**



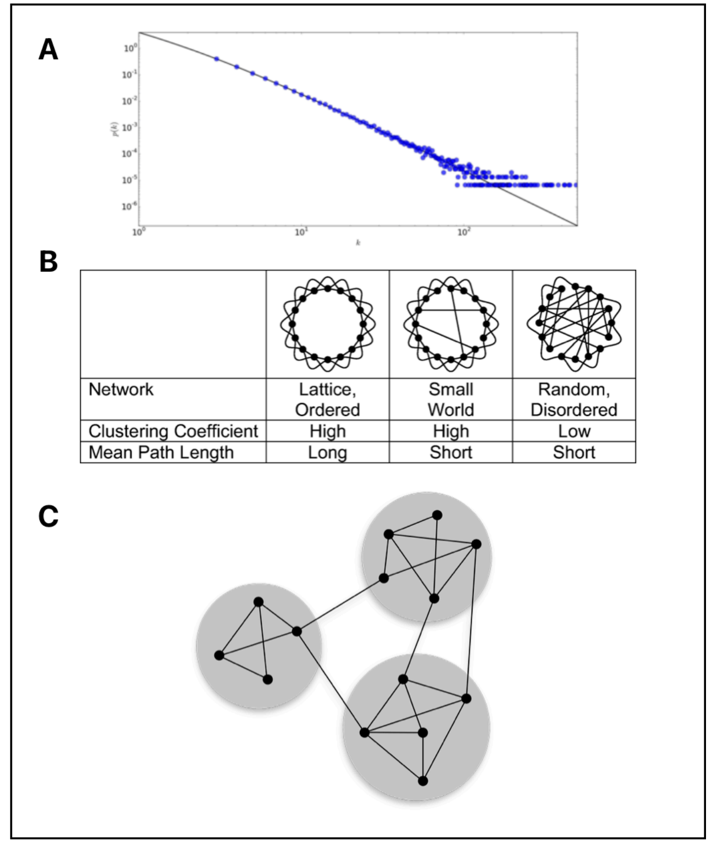
Observed Patterns in Real-World Networks

- **Scale-Free Networks**
  - $P(k) \sim k^{-\gamma}$
- **Small-World Networks**
  - High clustering and short average path lengths
  - "Six degrees of separation" in real-world networks
  - Most nodes are only a few steps apart — short path lengths
  - Found in social, biological, and neural systems
  - Supports fast spread of information with strong local structure



Observed Patterns in Real-World Networks

- **Scale-Free Networks**
  - $P(k) \sim k^{-\gamma}$
- **Small-World Networks**
  - High Clustering + Short Paths
- **Modularity and community structure**
  - Nodes form tightly connected clusters with sparse links between
  - Communities reflect roles, functions, or social groups
  - Important for understanding influence and containment
  - Found in social networks, biological systems, topic graphs



Random Networks: Modeling Networks from the Bottom Up

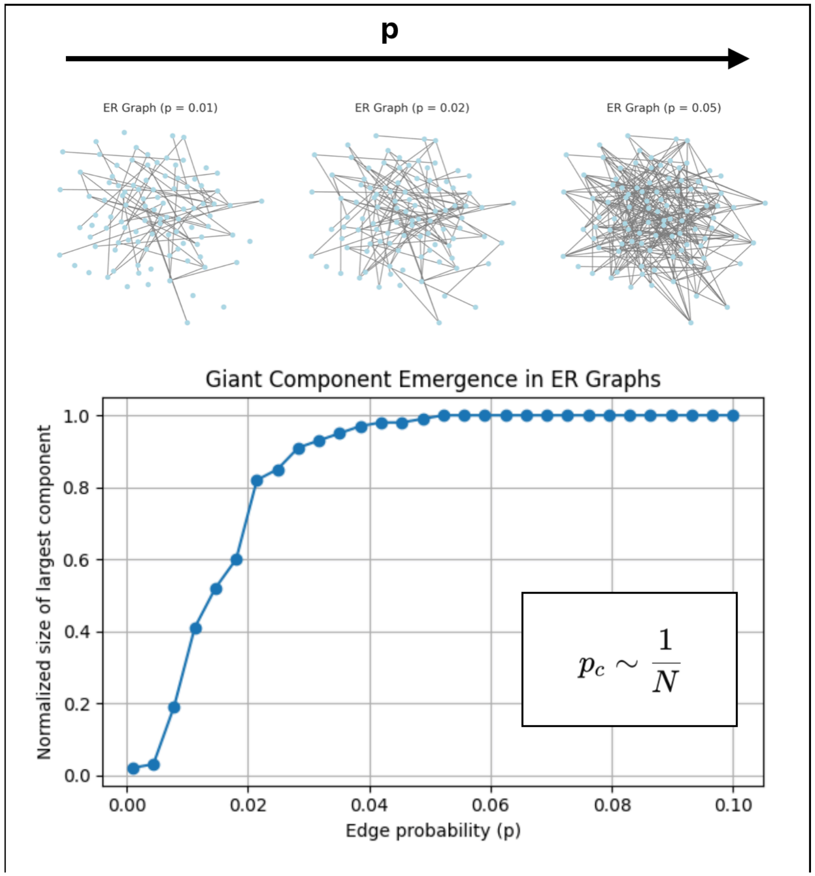
- **Generative models** explain where network structure comes from
- By defining **simple local rules**, we can generate networks with:
  - Different degree distributions
  - Different clustering and connectivity patterns
- **Why this matters**:
  - Provide **null models** — baselines to detect meaningful structure
  - Create **clean substrates** to test ideas about spreading, resilience, aggregation
  - Reveal **mechanisms** that give rise to real-world network features

> ***Simple rules → complex structure*** *From randomness to recognizable patterns*

Random Networks: Erdős–Rényi (ER) Random Graphs

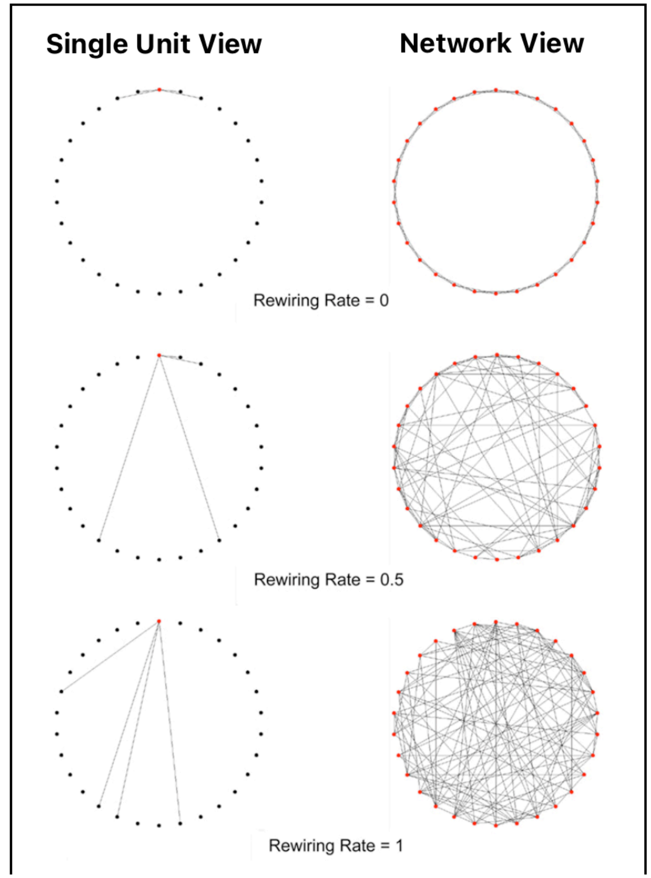
- Start with (N) isolated nodes
- For each pair of nodes, add an edge with probability (p)
- Result: A random graph (G(N, p))
- Degree distribution: Binomial (→ Poisson for large (N))
- Low clustering, no hubs
- Sudden appearance of a **giant component** when (p)
- Simple, powerful **null model** — baseline for randomness

Random Networks: Erdős–Rényi (ER) Random Graphs

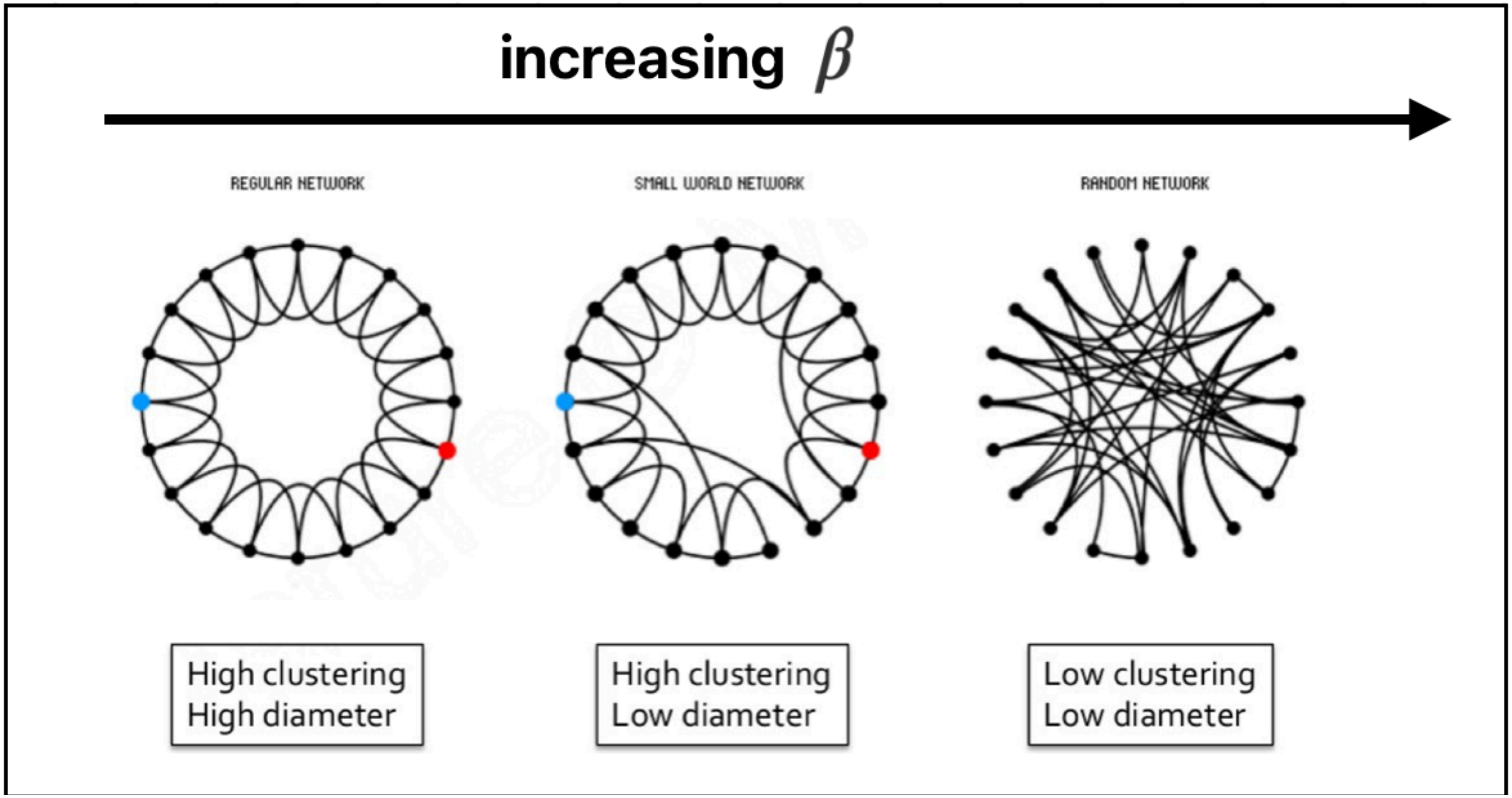


Small-World Networks: The Watts–Strogatz Model

- Start with a **regular ring lattice** each node connected to its k nearest neighbors
- Introduce randomness: **rewire** each edge with probability β
- For small β:
  - **High clustering** remains
  - **Path lengths drop sharply**
- Captures **small-world** behavior:
  - Short average path length $\sim \log N$
  - High clustering coefficient
- Seen in social, neural, and infrastructure networks



Small-World Networks: The Watts–Strogatz Model



Random Models That Yield Power-Laws

1. Barabási–Albert (BA) Model

- **Preferential attachment**: new nodes link to high-degree nodes
- “Rich Get Richer”

2. Copying Model *(directed networks)*

- Pick a node and **copy its link targets**
- With small mutations, still yields power-law

3. Node Duplication Model

- Duplicate a node and inherit edges
- Randomly prune or rewire some links

4. Edge-Based Attachment

- Pick an **edge**, connect to both its nodes
- Encourages growth near **already-connected areas**
- Reinforces dense substructures

5. Walk Attachment / Local Search

- New nodes walk the network to find where to attach
- Degree bias arises through **network visibility**
- Realistic for decentralized growth (e.g., peer-to-peer)

From Structure to Dynamics

Summary of Today

- We explored **online social systems** — dynamic, adaptive, and structurally complex
- Introduced **networks** as flexible models of structure and interaction
- Learned how to **quantify network structure**
  - Degree, clustering, components, modularity
- Simple generative rules → **emergent real-world patterns**
  - Scale-free distributions, small-world effects, community structure

> *Networks give us the **structure** — now we turn to **how systems evolve***

From Network to Events: Modeling System Evolution

- We now shift from **structure** to **dynamics**:
  - How local interactions give rise to collective behavior
  - How to model randomness and event-driven evolution

- Using a general framework:

$$(\Sigma, \mathcal{E}, \omega)$$

  - Σ — state space
  - $\mathcal{E}$ — events (discrete transitions)
  - ω — rates (how likely they are to happen)

From Network to Events: Modeling System Evolution

- We'll develop tools like:
  - The **Stochastic Master Equation**
  - **Mean-field approximations**
  - **Gillespie simulations**

- Then work through **key examples**, step by step
  - e.g., Birth–Death Process, Network Aggregation

- We'll build models **inspired by online social systems**, showing how local rules can reproduce global patterns

- Finally, we'll conclude with a **hands-on coding session** to simulate and visualize these dynamics in practice

From Network to Events: Modeling System Evolution

> *These are the **core ingredients** for understanding complex dynamics from simple rules*

- This framework works across systems:
  - Population models
  - Network aggregation
  - Epidemic dynamics
  - Cluster assembly