Approximating π using the Monte Carlo Method

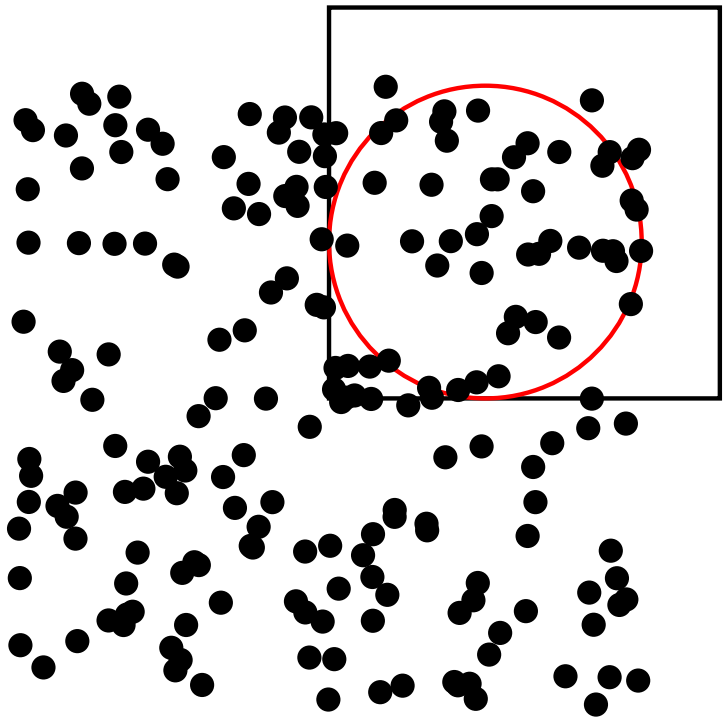
Introduction

The Monte Carlo method is a technique for approximating the value of π by randomly generating points within a square and counting the number of points that fall within a circle inscribed within the square.

The Algorithm

1. Start with a square of side length 2, with a circle inscribed within it of radius 1.
2. Generate a large number of random points (x, y) within the square.
3. Count the number of points that fall within the circle, which can be determined by the equation $x^2 + y^2 \leq 1$.
4. Use the ratio of the number of points within the circle to the total number of points to approximate π.

Visualizing the Monte Carlo Method



Explanation

The black dots represent the random points generated within the square. The red circle represents the circle inscribed within the square. The ratio of the number of black dots inside the circle to the total number of dots will approximate π.

Calculating the Approximation

Example

Let's say we generated 1000 random points and 800 of them fall within the circle. The approximation of π would be: $\frac{800}{1000} \times 4 = 3.2$

Accuracy

As the number of random points generated increases, the approximation of π becomes more accurate.

Calculating the Limits of Error

Formula

The formula to calculate the limit of error is: $E = \frac{\pi - \pi_{approx}}{\pi} \times 100\%$

Example

If the approximation of π is 3.2, the error limit would be:
$E = \frac{\pi - 3.2}{\pi} \times 100\% = 1.875\%$

Improving Accuracy

To improve the accuracy of the approximation, we can increase the number of random points generated or decrease the limit of error.

Conclusion

Summary

The Monte Carlo method is a useful technique for approximating the value of π by randomly generating points within a square and counting the number of points that fall within a circle inscribed within the square.

Advantages

Simple to implement Can be easily parallelized for faster computation Can be applied to other mathematical problems

Disadvantages

Requires a large number of random points for high accuracy May not be as accurate as other methods