Determining Authorship of the Disputed Federalist Papers using K-Means and HAC Clustering Algorithms

Homework 4 Applied Machine Learning IST 707

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Overview and Instructions

In this homework assignment, you are going to use clustering methods to solve a mystery in history: who wrote the disputed essays, Hamilton or Madison?

1. About the Federalist Papers Quote from the Library of Congress http://www.loc.gov/rr/program/bib/ourdocs/federalist.html The Federalist Papers were a series of eighty-five essays urging the citizens of New York to ratify the new United States Constitution. Written by Alexander Hamilton, James Madison, and John Jay, the essays originally appeared anonymously in New York newspapers in 1787 and 1788 under the pen name "Publius." A bound edition of the essays was first published in 1788, but it was not until the 1818 edition published by the printer Jacob Gideon that the authors of each essay were identified by name. The Federalist Papers are considered one of the most important sources for interpreting and understanding the original intent of the Constitution.

2. About the disputed authorship The original essays can be downloaded from the Library of Congress. http://thomas.loc.gov/home/histdox/fedpapers.html In the author column, you will find 74 essays with identified authors: 51 essays written by Hamilton, 15 by Madison, 3 by Hamilton and Madison, 5 by Jay. The remaining 11 essays, however, is authored by "Hamilton or Madison". These are the famous essays with disputed authorship. Hamilton wrote to claim the authorship before he was killed in a duel. Later Madison also claimed authorship. Historians were trying to find out which one was the real author.

3. Computational approach for authorship attribution In 1960s, statistician Mosteller and Wallace analyzed the frequency distributions of common function words in the Federalist Papers, and drew their conclusions. This is a pioneering work on using mathematical approaches for authorship attribution. http://www.stat.cmu.edu/~vlachos/courses/724/final/mosteller.pdf Nowadays, authorship attribution has become a classic problem in the data mining field, with applications in forensics (e.g. deception detection), and information organization.

In this homework you are provided with the Federalist Paper data set. The features are a set of "function words", for example, "upon". The feature value is the percentage of the

word occurrence in an essay. For example, for the essay “Hamilton_fed_31.txt”, if the function word “upon” appeared 3 times, and the total number of words in this essay is 1000, the feature value is 3/1000=0.3%

Now you are going to try solving this mystery using clustering algorithms k-Means and HAC. Document your analysis process and draw your conclusion on who wrote the disputed essays. Provide evidence for each method to demonstrate what patterns had been learned to predict the disputed papers, for example, visualize the clustering results and show where the disputed papers are located in relation to Hamilton and Madison’s papers. By the way, where are the papers with joint authorship located? For k-Means and EM, analyze the centroids to explain which attributes are most useful for clustering. Hint: the centroid values on these dimensions should be far apart from each other to be able to distinguish the clusters.

Read in Dataset

```
fedpapers_orig<-read.csv("~/Downloads/week4_resources/fedPapers85_fromClass.csv",
na.strings = c(""))

# check for missing values
sum(is.na(fedpapers_orig))

## [1] 0
```

Read in all Necessary Libraries

```
library(tidyverse)

## ── Attaching core tidyverse packages ──────────────────────── tidyverse 2.0.0 ──
## ✓ dplyr     1.1.3     ✓ readr     2.1.4
## ✓ forcats   1.0.0     ✓ stringr   1.5.1
## ✓ ggplot2   3.4.4     ✓ tibble    3.2.1
## ✓ lubridate 1.9.3     ✓ tidyr     1.3.0
## ✓ purrr     1.0.2
## ── Conflicts ────────────────────────────────────────── tidyverse_conflicts() ──
## ✖ dplyr::filter() masks stats::filter()
## ✖ dplyr::lag()    masks stats::lag()
## ℹ Use the conflicted package (<http://conflicted.r-lib.org/>) to force all conflicts to become errors
```

```
library(ggplot2)
#install.packages("factoextra")
library(factoextra)

## Welcome! Want to learn more? See two factoextra-related books at https://goo.gl/ve3WBa

library(stats)
```

Exploratory Data Analysis

Firstly, let's remove the papers written by John Jay, as we already know that the disputed papers must be written by either Madison or Hamilton:

```
fedpapers<-fedpapers_orig[fedpapers_orig$author != "Jay",]
head(fedpapers)

##   author       filename    a   all  also    an   and   any   are    as    at
## 1  dispt dispt_fed_49.txt 0.280 0.052 0.009 0.096 0.358 0.026 0.131 0.122 0.017
## 2  dispt dispt_fed_50.txt 0.177 0.063 0.013 0.038 0.393 0.063 0.051 0.139 0.114
## 3  dispt dispt_fed_51.txt 0.339 0.090 0.008 0.030 0.301 0.008 0.068 0.203 0.023
## 4  dispt dispt_fed_52.txt 0.270 0.024 0.016 0.024 0.262 0.056 0.064 0.111 0.056
## 5  dispt dispt_fed_53.txt 0.303 0.054 0.027 0.034 0.404 0.040 0.128 0.148 0.013
## 6  dispt dispt_fed_54.txt 0.245 0.059 0.007 0.067 0.282 0.052 0.111 0.252 0.015
##      be  been   but    by   can    do  down  even every   for.  from   had   has
## 1 0.411 0.026 0.009 0.140 0.035 0.026 0.000 0.009 0.044 0.096 0.044 0.035 0.017
## 2 0.393 0.165 0.000 0.139 0.000 0.013 0.000 0.025 0.000 0.076 0.101 0.101 0.013
## 3 0.474 0.015 0.038 0.173 0.023 0.000 0.008 0.015 0.023 0.098 0.053 0.008 0.015
## 4 0.365 0.127 0.032 0.167 0.056 0.000 0.000 0.024 0.040 0.103 0.079 0.016 0.024
## 5 0.344 0.047 0.061 0.209 0.088 0.000 0.000 0.020 0.027 0.141 0.074 0.000 0.054
## 6 0.297 0.030 0.037 0.186 0.000 0.000 0.007 0.007 0.007 0.067 0.096 0.022 0.015
##    have her   his   if.   in.  into    is    it   its   may  more  must my
## 1 0.044    0 0.017 0.000 0.262 0.009 0.157 0.175 0.070 0.035 0.026 0.026  0
## 2 0.152    0 0.000 0.025 0.291 0.025 0.038 0.127 0.038 0.038 0.000 0.013  0
## 3 0.023    0 0.000 0.023 0.308 0.038 0.150 0.173 0.030 0.120 0.038 0.083  0
## 4 0.143    0 0.024 0.040 0.238 0.008 0.151 0.222 0.048 0.056 0.056 0.071  0
## 5 0.047    0 0.020 0.034 0.263 0.013 0.189 0.108 0.013 0.047 0.067 0.013  0
## 6 0.119    0 0.067 0.030 0.401 0.037 0.260 0.156 0.015 0.074 0.045 0.015  0
```

```
##      no   not now    of    on   one  only    or   our shall should    so  some
## 1 0.035 0.114    0 0.900 0.140 0.026 0.035 0.096 0.017 0.017  0.017 0.035 0.009
## 2 0.000 0.127    0 0.747 0.139 0.025 0.000 0.114 0.000 0.000  0.013 0.013 0.063
## 3 0.030 0.068    0 0.858 0.150 0.030 0.023 0.060 0.000 0.008  0.068 0.038 0.030
## 4 0.032 0.087    0 0.802 0.143 0.032 0.048 0.064 0.016 0.016  0.032 0.040 0.024
## 5 0.047 0.128    0 0.869 0.054 0.047 0.027 0.081 0.027 0.000  0.000 0.027 0.067
## 6 0.059 0.134    0 0.876 0.141 0.052 0.022 0.074 0.030 0.015  0.030 0.007 0.045
##    such  than  that   the their  then there things  this    to up  upon   was
## 1 0.026 0.009 0.184 1.425 0.114 0.000 0.009  0.009 0.044 0.507  0 0.000 0.009
## 2 0.000 0.000 0.152 1.254 0.165 0.000 0.000  0.000 0.051 0.355  0 0.013 0.051
## 3 0.045 0.023 0.188 1.490 0.053 0.015 0.015  0.000 0.075 0.361  0 0.000 0.008
## 4 0.008 0.000 0.238 1.326 0.071 0.008 0.000  0.000 0.103 0.532  0 0.000 0.087
## 5 0.027 0.047 0.162 1.193 0.027 0.007 0.007  0.000 0.094 0.485  0 0.000 0.027
## 6 0.015 0.030 0.208 1.469 0.089 0.007 0.007  0.000 0.126 0.445  0 0.000 0.007
##    were  what  when which   who  will  with would your
## 1 0.017 0.000 0.009 0.175 0.044 0.009 0.087 0.192    0
## 2 0.000 0.000 0.000 0.114 0.038 0.089 0.063 0.139    0
## 3 0.015 0.008 0.000 0.105 0.008 0.173 0.045 0.068    0
## 4 0.079 0.008 0.024 0.167 0.000 0.079 0.079 0.064    0
## 5 0.020 0.020 0.007 0.155 0.027 0.168 0.074 0.040    0
## 6 0.030 0.015 0.037 0.186 0.045 0.111 0.089 0.037    0
```

Let's group by author and try to understand the variance in writing styles between them.

```
#deleting file name
fedpapers_eda<-fedpapers[,-2]
#Take mean frequency of each group grouped by author

group_fedpapers<- fedpapers_eda %>% group_by(author) %>%
  summarise(across(everything(), mean),
            .groups = 'drop')  %>%
  as.data.frame()
group_fedpapers
```

```
##    author         a       all      also        an       and       any
## 1      HM 0.2133333 0.04266667 0.006000000 0.04700000 0.5306667 0.01800000
## 2 Hamilton 0.3156078 0.05376471 0.004784314 0.08080392 0.3394902 0.04674510
## 3  Madison 0.2698000 0.05533333 0.011066667 0.05946667 0.4196667 0.02980000
## 4    dispt 0.3039091 0.05554545 0.011272727 0.04845455 0.3560000 0.04218182
##         are        as        at        be      been       but        by
## 1 0.08233333 0.0800000 0.04433333 0.06666667 0.02133333 0.02833333 0.1750000
## 2 0.07254902 0.1177255 0.04882353 0.30837255 0.06192157 0.03015686 0.1045882
## 3 0.07486667 0.1340667 0.02953333 0.28953333 0.07406667 0.03540000 0.1660667
## 4 0.09590909 0.1380000 0.04700000 0.35918182 0.05500000 0.03154545 0.1615455
##         can         do       down       even      every       for.
## 1 0.005333333 0.002333333 0.0000000000 0.002333333 0.007666667 0.08566667
## 2 0.038235294 0.006607843 0.0019803922 0.012294118 0.020803922 0.09245098
## 3 0.028333333 0.004000000 0.0009333333 0.009800000 0.031066667 0.09453333
## 4 0.042545455 0.007909091 0.0013636364 0.013636364 0.041090909 0.10000000
##        from        had        has       have        her        his
## 1 0.10866667 0.08133333 0.04600000 0.06666667 0.020333333 0.08700000
## 2 0.08021569 0.01743137 0.04811765 0.09894118 0.007058824 0.03192157
## 3 0.06853333 0.02246667 0.05106667 0.10046667 0.007800000 0.01886667
## 4 0.08009091 0.02245455 0.02490909 0.07872727 0.006909091 0.01963636
##         if.        in.       into         is         it        its        may
## 1 0.007666667 0.249000 0.02966667 0.1040000 0.1086667 0.05400000 0.01733333
## 2 0.029019608 0.343902 0.02131373 0.1594118 0.1585882 0.05254902 0.06219608
## 3 0.021466667 0.286400 0.02686667 0.1714000 0.1498000 0.04806667 0.06560000
## 4 0.021363636 0.276000 0.02236364 0.1638182 0.1489091 0.03463636 0.06927273
##        more       must         my         no        not        now
## 1 0.04200000 0.01366667 0.0053333333 0.02400000 0.03466667 0.012666667
## 2 0.03996078 0.03370588 0.0042549020 0.03188235 0.09029412 0.006549020
## 3 0.04953333 0.03380000 0.0018666667 0.04280000 0.09500000 0.005133333
## 4 0.04872727 0.03963636 0.0006363636 0.03054545 0.10790909 0.002818182
##         of         on        one       only         or        our      shall
## 1 0.8386667 0.09366667 0.04400000 0.01500000 0.05366667 0.00600000 0.004666667
## 2 0.9571765 0.04743137 0.03568627 0.02031373 0.09801961 0.02266667 0.021705882
```

```
## 3 0.8687333 0.10506667 0.04280000 0.02360000 0.08420000 0.01786667 0.019866667
## 4 0.8851818 0.11254545 0.04236364 0.02663636 0.09054545 0.01663636 0.008000000
##       should         so       some       such       than       that        the
## 1 0.002333333 0.01733333 0.02133333 0.03100000 0.03766667 0.1016667 1.267000
## 2 0.029392157 0.02896078 0.01601961 0.02894118 0.04454902 0.2211765 1.289941
## 3 0.023733333 0.02900000 0.02133333 0.02873333 0.04100000 0.2015333 1.375267
## 4 0.017181818 0.03163636 0.03481818 0.02072727 0.03845455 0.1998182 1.307000
##       their       then      there     things       this         to
## 1 0.13233333 0.009666667 0.004333333 0.000000000 0.06500000 0.3826667
## 2 0.07468627 0.005196078 0.037098039 0.003372549 0.09349020 0.5910784
## 3 0.08913333 0.006200000 0.007733333 0.001400000 0.08313333 0.4568667
## 4 0.09263636 0.008181818 0.013727273 0.002363636 0.08363636 0.4526364
##          up        upon         was        were        what        when
## 1 0.009000000 0.005333333 0.11400000 0.04733333 0.002333333 0.01566667
## 2 0.004568627 0.047313725 0.02060784 0.01745098 0.013627451 0.01225490
## 3 0.001266667 0.002200000 0.02573333 0.02086667 0.011600000 0.00600000
## 4 0.001545455 0.001181818 0.02663636 0.02090909 0.011363636 0.01190909
##       which         who        will        with       would        your
## 1 0.1373333 0.04733333 0.01666667 0.09700000 0.02566667 0.000000000
## 2 0.1604902 0.03292157 0.09239216 0.07896078 0.12272549 0.002078431
## 3 0.1638000 0.02653333 0.10666667 0.07706667 0.06066667 0.002266667
## 4 0.1692727 0.02618182 0.12663636 0.07490909 0.07036364 0.000000000
```

Choose “meaningful” words to analyze

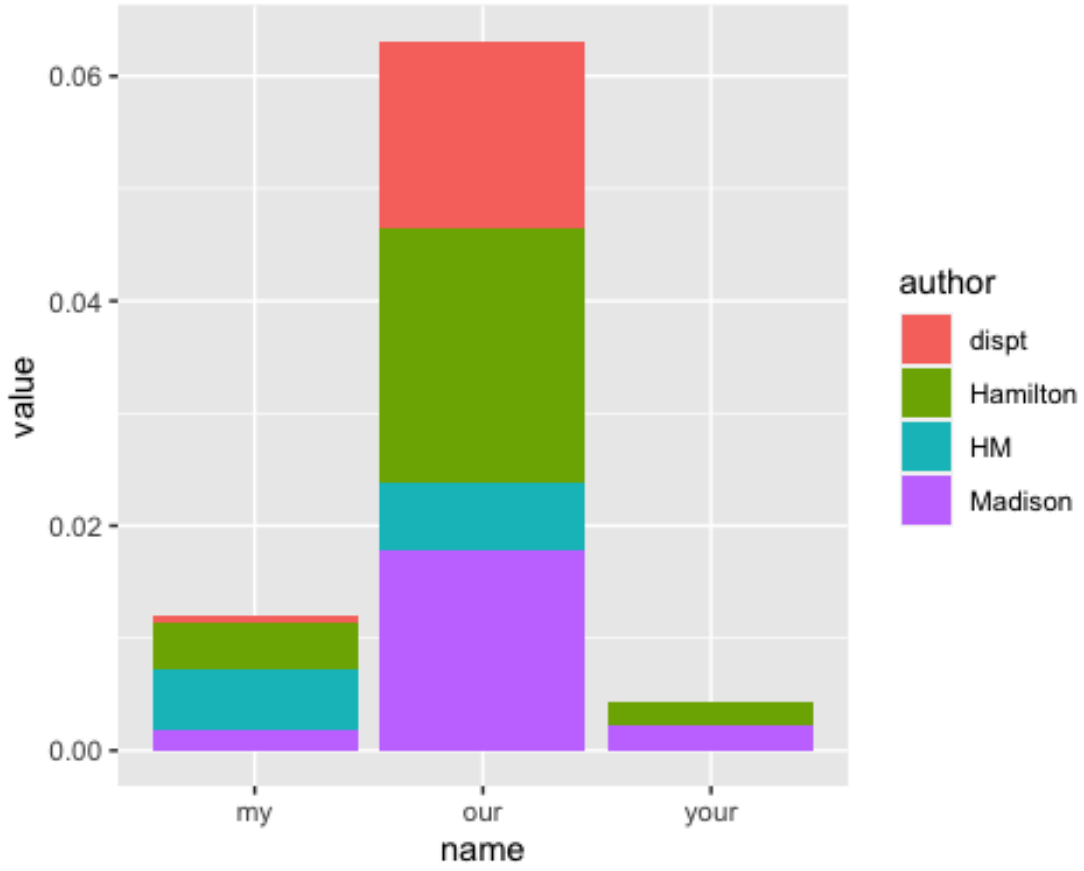
As we know, some words are more meaningful than others. Let’s see who writes most frequently in a first and second person tone (an informal tone, call to action!). Therefore, let’s look at the relative frequencies of “my”, “our”, and “your” based on the author.

```
informal<-select(group_fedpapers, c('author','my', 'our', 'your'))
informal_chart<-pivot_longer(informal, -author)
informal_chart
```

```
## # A tibble: 12 × 3
##    author   name     value
```

```
##    <chr>    <chr>    <dbl>
##  1 HM       my     0.00533
##  2 HM       our    0.006
##  3 HM       your   0
##  4 Hamilton my     0.00425
##  5 Hamilton our    0.0227
##  6 Hamilton your   0.00208
##  7 Madison  my     0.00187
##  8 Madison  our    0.0179
##  9 Madison  your   0.00227
## 10 dispt    my     0.000636
## 11 dispt    our    0.0166
## 12 dispt    your   0
```

```
ggplot(data=informal_chart, aes(x = name, y=value, fill = author)) +
  geom_col(position=position_stack())
```



Based on the above chart, Hamilton appears to write more frequently with the words “my” and “our”, which gives him a more personal tone and a more communal feel with his audience. Both Hamilton and Madison use the word “your” in relatively equal frequencies. Strangely, the disputed papers do not use the word “your” at all.

“Our” is used much more frequently than the words “my” and “your”, suggesting that both Madison and Hamilton used “us” and “we” verbiage as a way to relate to their audience as Americans and call them to action.

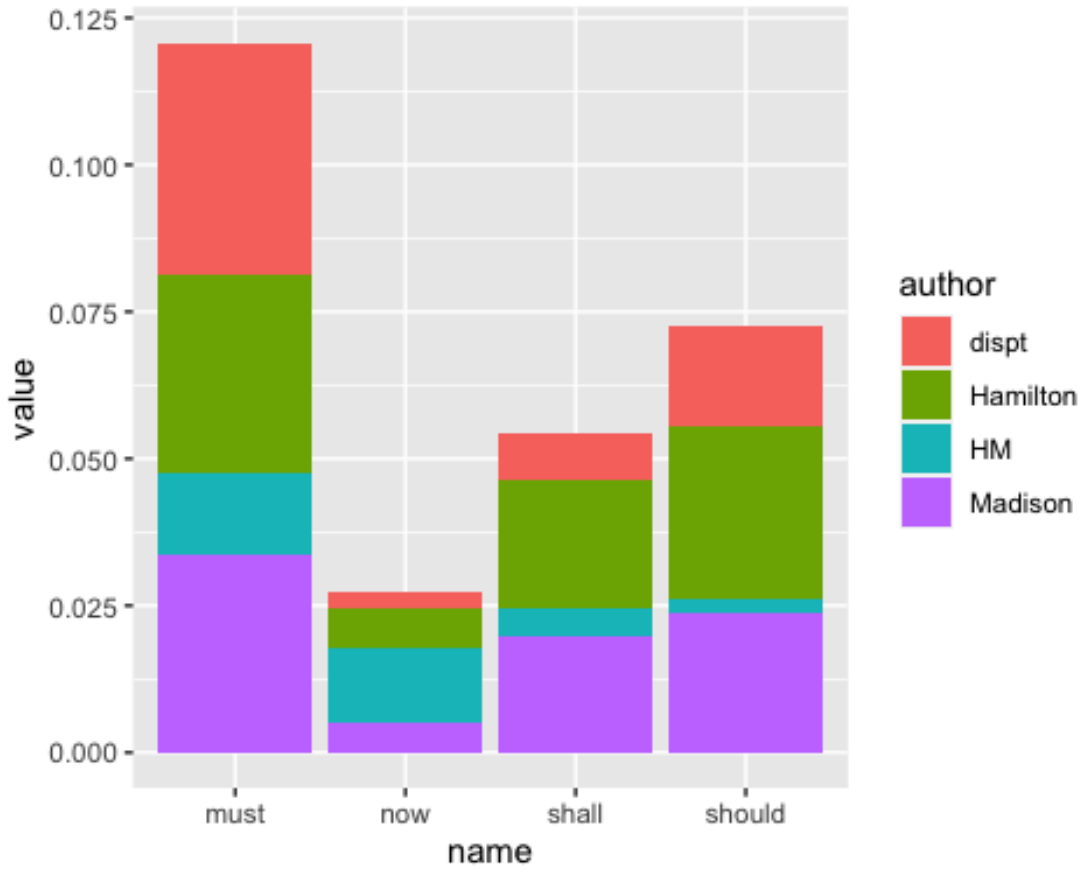
Let’s follow the idea of “call to action” words in our exploratory data analysis. It is intuitive that one writer may have a more powerful, or strong writing style. Certain words imply urgency, or action, such as the following: must, now, shall, and should. Let’s examine the relative frequency of each for each author:

```
action<-select(group_fedpapers, c('author','must', 'now', 'shall', 'should'))
action_chart<-pivot_longer(action, -author)
action_chart

## # A tibble: 16 × 3
##    author   name     value
##    <chr>    <chr>    <dbl>
```

```
##  1 HM       must   0.0137
##  2 HM       now    0.0127
##  3 HM       shall  0.00467
##  4 HM       should 0.00233
##  5 Hamilton must   0.0337
##  6 Hamilton now    0.00655
##  7 Hamilton shall  0.0217
##  8 Hamilton should 0.0294
##  9 Madison  must   0.0338
## 10 Madison  now    0.00513
## 11 Madison  shall  0.0199
## 12 Madison  should 0.0237
## 13 dispt    must   0.0396
## 14 dispt    now    0.00282
## 15 dispt    shall  0.008
## 16 dispt    should 0.0172
```

```
ggplot(data=action_chart, aes(x = name, y=value, fill = author)) +
  geom_col(position=position_stack())
```



Again, Hamilton seems to use these "call to action" words slightly more frequently. This provides a bit of preliminary context to our analysis regarding the tones of each of our writers, which may useful in identifying the author of the disputed papers.

K-Means Algorithm

Now we will use our K-means algorithm to sort the documents into clusters.

```
#remove author name
fedpapers_km<-fedpapers[,-1]
rownames(fedpapers_km)<-fedpapers_km[,1]
#view new dataframe with each row labeled by file name.
fedpapers_km[,1]<-NULL
```

Run K-Means Clustering

```
#install.packages("stats")
library(stats)
library(factoextra)
```

```r
#set seed
set.seed(22)
#drop author names for cluster
#fedpapers_km2<-fedpapers[,-1]
cluster<-kmeans(fedpapers_km, 5, nstart = 25)
fedpapers_km$clusters<-as.factor(cluster$cluster)
fedpapers$clusters<-as.factor(cluster$cluster)
#view results
print(cluster)
```

```
## K-means clustering with 5 clusters of sizes 14, 13, 22, 16, 15
##
## Cluster means:
##          a        all       also         an        and        any        are
## 1 0.3465000 0.04557143 0.008142857 0.09221429 0.3001429 0.05385714 0.05207143
## 2 0.2590000 0.05815385 0.012076923 0.05046154 0.3574615 0.03469231 0.08646154
## 3 0.3323636 0.05227273 0.004181818 0.07077273 0.3445909 0.04422727 0.07872727
## 4 0.2750625 0.06356250 0.003187500 0.08168750 0.3610000 0.04700000 0.08212500
## 5 0.2796667 0.05000000 0.009200000 0.05840000 0.4607333 0.02793333 0.08173333
##          as        at        be      been       but        by       can
## 1 0.1197143 0.04242857 0.3321429 0.06492857 0.02914286 0.1065714 0.03400000
## 2 0.1513077 0.03253846 0.3343077 0.05592308 0.03169231 0.1576923 0.03200000
## 3 0.1081818 0.06186364 0.3152273 0.05745455 0.02945455 0.1191818 0.03422727
## 4 0.1242500 0.04000000 0.2983125 0.06062500 0.03200000 0.0886875 0.05131250
## 5 0.1174667 0.03766667 0.2344667 0.07120000 0.03473333 0.1696000 0.02620000
##          do        down      even       every      for.       from
## 1 0.006785714 0.0010000000 0.00800000 0.01964286 0.09342857 0.07971429
## 2 0.004846154 0.0019230769 0.01076923 0.03638462 0.09115385 0.06476923
## 3 0.006909091 0.0015000000 0.01504545 0.02840909 0.09736364 0.08245455
## 4 0.006562500 0.0033750000 0.01231250 0.01818750 0.08731250 0.07918750
## 5 0.005066667 0.0002666667 0.01006667 0.02253333 0.09720000 0.08580000
##         had        has       have        her        his        if.        in.
## 1 0.01757143 0.04292857 0.09335714 0.000500000 0.02307143 0.03057143 0.3887143
## 2 0.01507692 0.03784615 0.09115385 0.004538462 0.01615385 0.02084615 0.2868462
```

```
## 3 0.01840909 0.05427273 0.09436364 0.015636364 0.03804545 0.02981818 0.3210000
## 4 0.01406250 0.04156250 0.09887500 0.001687500 0.03106250 0.02681250 0.3203750
## 5 0.04300000 0.04533333 0.09793333 0.011800000 0.03473333 0.01840000 0.2839333
##        into        is        it       its       may      more      must
## 1 0.01814286 0.1677143 0.1842143 0.04907143 0.06692857 0.03557143 0.02578571
## 2 0.02138462 0.1710000 0.1536154 0.04661538 0.06346154 0.05323077 0.03423077
## 3 0.02509091 0.1542727 0.1370909 0.04695455 0.05850000 0.04781818 0.03722727
## 4 0.01681250 0.1723125 0.1655625 0.06118750 0.06956250 0.03706250 0.04025000
## 5 0.03146667 0.1395333 0.1372000 0.04260000 0.05386667 0.04053333 0.02893333
##          my         no        not        now         of         on        one
## 1 0.003214286 0.03900000 0.09550000 0.010142857 1.0425000 0.04671429 0.03228571
## 2 0.002153846 0.04423077 0.09523077 0.004000000 0.8984615 0.12669231 0.04269231
## 3 0.004909091 0.02990909 0.09359091 0.005727273 0.9251818 0.05968182 0.04177273
## 4 0.004437500 0.03050000 0.08156250 0.004875000 0.9122500 0.03793750 0.03325000
## 5 0.001066667 0.02726667 0.09213333 0.005466667 0.8583333 0.08620000 0.04013333
##        only        or       our     shall    should        so      some
## 1 0.02364286 0.09650000 0.005428571 0.02564286 0.02742857 0.02885714 0.01757143
## 2 0.02623077 0.08953846 0.015000000 0.01838462 0.02353846 0.02507692 0.02192308
## 3 0.01622727 0.09686364 0.040363636 0.01986364 0.02618182 0.02845455 0.01754545
## 4 0.02250000 0.10112500 0.015000000 0.01943750 0.03400000 0.03293750 0.01181250
## 5 0.02260000 0.07700000 0.015066667 0.01073333 0.01606667 0.02860000 0.03186667
##        such        than        that         the       their        then       there
## 1 0.02850000 0.04628571 0.2338571 1.394857 0.05842857 0.004285714 0.037428571
## 2 0.02369231 0.03646154 0.1969231 1.481538 0.08430769 0.006000000 0.007769231
## 3 0.02854545 0.05200000 0.2208636 1.147364 0.08272727 0.005954545 0.034227273
## 4 0.03006250 0.03693750 0.2151875 1.390937 0.07937500 0.006312500 0.038500000
## 5 0.02746667 0.03773333 0.1780000 1.220600 0.10386667 0.007133333 0.011866667
##       things        this          to          up        upon         was
## 1 0.003071429 0.10928571 0.5442857 0.0011428571 0.04771429 0.01642857
## 2 0.002000000 0.08276923 0.4475385 0.0004615385 0.00000000 0.02607692
## 3 0.003863636 0.09559091 0.5812273 0.0084090909 0.04372727 0.01727273
## 4 0.003500000 0.07737500 0.6515000 0.0024375000 0.04718750 0.02412500
## 5 0.000600000 0.07886667 0.4317333 0.0033333333 0.00600000 0.04913333
```

```
##        were        what        when       which         who        will        with
## 1 0.01700000 0.016500000 0.017357143 0.1444286 0.02971429 0.08950000 0.07550000
## 2 0.01946154 0.011230769 0.010230769 0.1620000 0.02123077 0.12584615 0.07546154
## 3 0.01754545 0.010000000 0.013818182 0.1611364 0.03054545 0.09254545 0.08077273
## 4 0.01625000 0.016875000 0.006062500 0.1800000 0.03718750 0.10200000 0.07937500
## 5 0.02920000 0.008933333 0.007733333 0.1575333 0.03653333 0.07986667 0.08086667
##        would        your
## 1 0.15028571 0.000000000
## 2 0.07838462 0.002615385
## 3 0.13818182 0.003818182
## 4 0.07356250 0.001375000
## 5 0.04533333 0.000000000
##
## Clustering vector:
##    dispt_fed_49.txt    dispt_fed_50.txt    dispt_fed_51.txt    dispt_fed_52.txt
##                   2                   5                   2                   2
##    dispt_fed_53.txt    dispt_fed_54.txt    dispt_fed_55.txt    dispt_fed_56.txt
##                   5                   2                   3                   5
##    dispt_fed_57.txt    dispt_fed_62.txt    dispt_fed_63.txt  Hamilton_fed_1.txt
##                   2                   3                   5                   3
## Hamilton_fed_11.txt Hamilton_fed_12.txt Hamilton_fed_13.txt Hamilton_fed_15.txt
##                   3                   3                   3                   3
## Hamilton_fed_16.txt Hamilton_fed_17.txt Hamilton_fed_21.txt Hamilton_fed_22.txt
##                   1                   4                   1                   3
## Hamilton_fed_23.txt Hamilton_fed_24.txt Hamilton_fed_25.txt Hamilton_fed_26.txt
##                   4                   3                   3                   3
## Hamilton_fed_27.txt Hamilton_fed_28.txt Hamilton_fed_29.txt Hamilton_fed_30.txt
##                   4                   4                   4                   3
## Hamilton_fed_31.txt Hamilton_fed_32.txt Hamilton_fed_33.txt Hamilton_fed_34.txt
##                   4                   1                   4                   3
## Hamilton_fed_35.txt Hamilton_fed_36.txt Hamilton_fed_59.txt  Hamilton_fed_6.txt
##                   3                   4                   1                   5
## Hamilton_fed_60.txt Hamilton_fed_61.txt Hamilton_fed_65.txt Hamilton_fed_66.txt
```

```
##                 1                 1                 1                 1
## Hamilton_fed_67.txt Hamilton_fed_68.txt Hamilton_fed_69.txt  Hamilton_fed_7.txt
##                 4                 4                 1                 3
## Hamilton_fed_70.txt Hamilton_fed_71.txt Hamilton_fed_72.txt Hamilton_fed_73.txt
##                 4                 4                 3                 3
## Hamilton_fed_74.txt Hamilton_fed_75.txt Hamilton_fed_76.txt Hamilton_fed_77.txt
##                 1                 1                 3                 1
## Hamilton_fed_78.txt Hamilton_fed_79.txt  Hamilton_fed_8.txt Hamilton_fed_80.txt
##                 4                 3                 3                 4
## Hamilton_fed_81.txt Hamilton_fed_82.txt Hamilton_fed_83.txt Hamilton_fed_84.txt
##                 1                 4                 3                 1
## Hamilton_fed_85.txt  Hamilton_fed_9.txt       HM_fed_18.txt       HM_fed_19.txt
##                 4                 3                 5                 5
##       HM_fed_20.txt  Madison_fed_10.txt  Madison_fed_14.txt  Madison_fed_37.txt
##                 5                 5                 2                 5
##  Madison_fed_38.txt  Madison_fed_39.txt  Madison_fed_40.txt  Madison_fed_41.txt
##                 5                 2                 5                 5
##  Madison_fed_42.txt  Madison_fed_43.txt  Madison_fed_44.txt  Madison_fed_45.txt
##                 5                 2                 2                 2
##  Madison_fed_46.txt  Madison_fed_47.txt  Madison_fed_48.txt  Madison_fed_58.txt
##                 2                 2                 5                 2
##
## Within cluster sum of squares by cluster:
## [1] 1.182484 1.047539 1.782588 1.191858 1.325325
##  (between_SS / total_SS =  32.8 %)
##
## Available components:
##
## [1] "cluster"      "centers"      "totss"        "withinss"     "tot.withinss"
## [6] "betweenss"    "size"         "iter"         "ifault"

head(fedpapers_km)
```

```
##                      a   all  also    an   and   any   are    as    at    be
## dispt_fed_49.txt 0.280 0.052 0.009 0.096 0.358 0.026 0.131 0.122 0.017 0.411
## dispt_fed_50.txt 0.177 0.063 0.013 0.038 0.393 0.063 0.051 0.139 0.114 0.393
## dispt_fed_51.txt 0.339 0.090 0.008 0.030 0.301 0.008 0.068 0.203 0.023 0.474
## dispt_fed_52.txt 0.270 0.024 0.016 0.024 0.262 0.056 0.064 0.111 0.056 0.365
## dispt_fed_53.txt 0.303 0.054 0.027 0.034 0.404 0.040 0.128 0.148 0.013 0.344
## dispt_fed_54.txt 0.245 0.059 0.007 0.067 0.282 0.052 0.111 0.252 0.015 0.297
##                    been   but    by   can    do  down  even every   for.  from
## dispt_fed_49.txt 0.026 0.009 0.140 0.035 0.026 0.000 0.009 0.044 0.096 0.044
## dispt_fed_50.txt 0.165 0.000 0.139 0.000 0.013 0.000 0.025 0.000 0.076 0.101
## dispt_fed_51.txt 0.015 0.038 0.173 0.023 0.000 0.008 0.015 0.023 0.098 0.053
## dispt_fed_52.txt 0.127 0.032 0.167 0.056 0.000 0.000 0.024 0.040 0.103 0.079
## dispt_fed_53.txt 0.047 0.061 0.209 0.088 0.000 0.000 0.020 0.027 0.141 0.074
## dispt_fed_54.txt 0.030 0.037 0.186 0.000 0.000 0.007 0.007 0.007 0.067 0.096
##                     had   has  have her   his    if.   in.  into    is    it
## dispt_fed_49.txt 0.035 0.017 0.044    0 0.017 0.000 0.262 0.009 0.157 0.175
## dispt_fed_50.txt 0.101 0.013 0.152    0 0.000 0.025 0.291 0.025 0.038 0.127
## dispt_fed_51.txt 0.008 0.015 0.023    0 0.000 0.023 0.308 0.038 0.150 0.173
## dispt_fed_52.txt 0.016 0.024 0.143    0 0.024 0.040 0.238 0.008 0.151 0.222
## dispt_fed_53.txt 0.000 0.054 0.047    0 0.020 0.034 0.263 0.013 0.189 0.108
## dispt_fed_54.txt 0.022 0.015 0.119    0 0.067 0.030 0.401 0.037 0.260 0.156
##                     its   may  more  must my    no   not now    of    on   one
## dispt_fed_49.txt 0.070 0.035 0.026 0.026  0 0.035 0.114   0 0.900 0.140 0.026
## dispt_fed_50.txt 0.038 0.038 0.000 0.013  0 0.000 0.127   0 0.747 0.139 0.025
## dispt_fed_51.txt 0.030 0.120 0.038 0.083  0 0.030 0.068   0 0.858 0.150 0.030
## dispt_fed_52.txt 0.048 0.056 0.056 0.071  0 0.032 0.087   0 0.802 0.143 0.032
## dispt_fed_53.txt 0.013 0.047 0.067 0.013  0 0.047 0.128   0 0.869 0.054 0.047
## dispt_fed_54.txt 0.015 0.074 0.045 0.015  0 0.059 0.134   0 0.876 0.141 0.052
##                    only    or   our shall should    so  some  such  than  that
## dispt_fed_49.txt 0.035 0.096 0.017 0.017  0.017 0.035 0.009 0.026 0.009 0.184
## dispt_fed_50.txt 0.000 0.114 0.000 0.000  0.013 0.013 0.063 0.000 0.000 0.152
## dispt_fed_51.txt 0.023 0.060 0.000 0.008  0.068 0.038 0.030 0.045 0.023 0.188
## dispt_fed_52.txt 0.048 0.064 0.016 0.016  0.032 0.040 0.024 0.008 0.000 0.238
```

```
## dispt_fed_53.txt 0.027 0.081 0.027 0.000  0.000 0.027 0.067 0.027 0.047 0.162
## dispt_fed_54.txt 0.022 0.074 0.030 0.015  0.030 0.007 0.045 0.015 0.030 0.208
##                  the their  then there things  this    to up  upon    was
## dispt_fed_49.txt 1.425 0.114 0.000 0.009  0.009 0.044 0.507  0 0.000 0.009
## dispt_fed_50.txt 1.254 0.165 0.000 0.000  0.000 0.051 0.355  0 0.013 0.051
## dispt_fed_51.txt 1.490 0.053 0.015 0.015  0.000 0.075 0.361  0 0.000 0.008
## dispt_fed_52.txt 1.326 0.071 0.008 0.000  0.000 0.103 0.532  0 0.000 0.087
## dispt_fed_53.txt 1.193 0.027 0.007 0.007  0.000 0.094 0.485  0 0.000 0.027
## dispt_fed_54.txt 1.469 0.089 0.007 0.007  0.000 0.126 0.445  0 0.000 0.007
##                  were  what  when which   who  will  with would your clusters
## dispt_fed_49.txt 0.017 0.000 0.009 0.175 0.044 0.009 0.087 0.192    0       2
## dispt_fed_50.txt 0.000 0.000 0.000 0.114 0.038 0.089 0.063 0.139    0       5
## dispt_fed_51.txt 0.015 0.008 0.000 0.105 0.008 0.173 0.045 0.068    0       2
## dispt_fed_52.txt 0.079 0.008 0.024 0.167 0.000 0.079 0.079 0.064    0       2
## dispt_fed_53.txt 0.020 0.020 0.007 0.155 0.027 0.168 0.074 0.040    0       5
## dispt_fed_54.txt 0.030 0.015 0.037 0.186 0.045 0.111 0.089 0.037    0       2
```

We will now add the results to the dataframe and visualize the clusters:

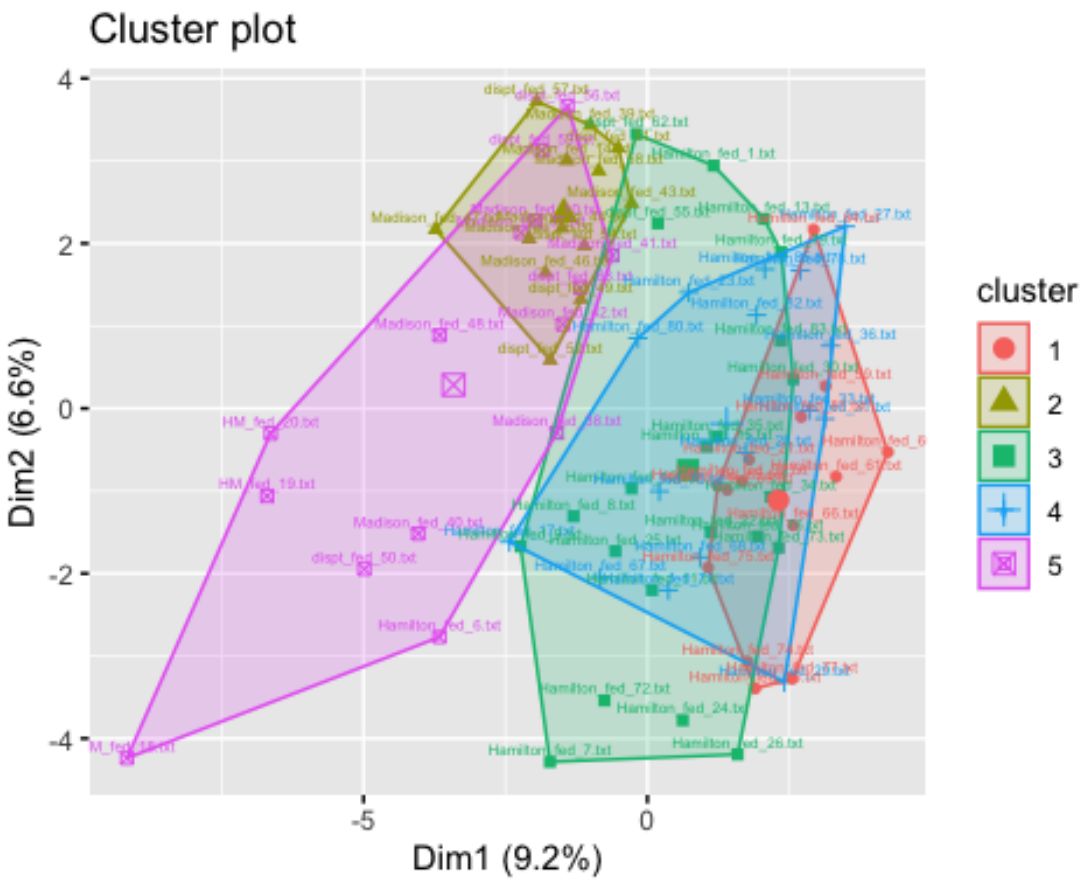
```
set.seed(22)
#first visualizing dataframe
head(fedpapers_km)

##                    a   all  also    an   and   any   are    as    at    be
## dispt_fed_49.txt 0.280 0.052 0.009 0.096 0.358 0.026 0.131 0.122 0.017 0.411
## dispt_fed_50.txt 0.177 0.063 0.013 0.038 0.393 0.063 0.051 0.139 0.114 0.393
## dispt_fed_51.txt 0.339 0.090 0.008 0.030 0.301 0.008 0.068 0.203 0.023 0.474
## dispt_fed_52.txt 0.270 0.024 0.016 0.024 0.262 0.056 0.064 0.111 0.056 0.365
## dispt_fed_53.txt 0.303 0.054 0.027 0.034 0.404 0.040 0.128 0.148 0.013 0.344
## dispt_fed_54.txt 0.245 0.059 0.007 0.067 0.282 0.052 0.111 0.252 0.015 0.297
##                  been   but    by   can    do  down  even every  for.  from
## dispt_fed_49.txt 0.026 0.009 0.140 0.035 0.026 0.000 0.009 0.044 0.096 0.044
## dispt_fed_50.txt 0.165 0.000 0.139 0.000 0.013 0.000 0.025 0.000 0.076 0.101
## dispt_fed_51.txt 0.015 0.038 0.173 0.023 0.000 0.008 0.015 0.023 0.098 0.053
## dispt_fed_52.txt 0.127 0.032 0.167 0.056 0.000 0.000 0.024 0.040 0.103 0.079
```

```
## dispt_fed_53.txt 0.047 0.061 0.209 0.088 0.000 0.000 0.020 0.027 0.141 0.074
## dispt_fed_54.txt 0.030 0.037 0.186 0.000 0.000 0.007 0.007 0.007 0.067 0.096
##                had   has  have her   his   if.   in.  into    is    it
## dispt_fed_49.txt 0.035 0.017 0.044    0 0.017 0.000 0.262 0.009 0.157 0.175
## dispt_fed_50.txt 0.101 0.013 0.152    0 0.000 0.025 0.291 0.025 0.038 0.127
## dispt_fed_51.txt 0.008 0.015 0.023    0 0.000 0.023 0.308 0.038 0.150 0.173
## dispt_fed_52.txt 0.016 0.024 0.143    0 0.024 0.040 0.238 0.008 0.151 0.222
## dispt_fed_53.txt 0.000 0.054 0.047    0 0.020 0.034 0.263 0.013 0.189 0.108
## dispt_fed_54.txt 0.022 0.015 0.119    0 0.067 0.030 0.401 0.037 0.260 0.156
##                 its   may  more  must my    no   not now    of    on   one
## dispt_fed_49.txt 0.070 0.035 0.026 0.026  0 0.035 0.114    0 0.900 0.140 0.026
## dispt_fed_50.txt 0.038 0.038 0.000 0.013  0 0.000 0.127    0 0.747 0.139 0.025
## dispt_fed_51.txt 0.030 0.120 0.038 0.083  0 0.030 0.068    0 0.858 0.150 0.030
## dispt_fed_52.txt 0.048 0.056 0.056 0.071  0 0.032 0.087    0 0.802 0.143 0.032
## dispt_fed_53.txt 0.013 0.047 0.067 0.013  0 0.047 0.128    0 0.869 0.054 0.047
## dispt_fed_54.txt 0.015 0.074 0.045 0.015  0 0.059 0.134    0 0.876 0.141 0.052
##                only    or   our shall should    so  some  such  than  that
## dispt_fed_49.txt 0.035 0.096 0.017 0.017  0.017 0.035 0.009 0.026 0.009 0.184
## dispt_fed_50.txt 0.000 0.114 0.000 0.000  0.013 0.013 0.063 0.000 0.000 0.152
## dispt_fed_51.txt 0.023 0.060 0.000 0.008  0.068 0.038 0.030 0.045 0.023 0.188
## dispt_fed_52.txt 0.048 0.064 0.016 0.016  0.032 0.040 0.024 0.008 0.000 0.238
## dispt_fed_53.txt 0.027 0.081 0.027 0.000  0.000 0.027 0.067 0.027 0.047 0.162
## dispt_fed_54.txt 0.022 0.074 0.030 0.015  0.030 0.007 0.045 0.015 0.030 0.208
##                 the their  then there things  this    to up  upon   was
## dispt_fed_49.txt 1.425 0.114 0.000 0.009  0.009 0.044 0.507  0 0.000 0.009
## dispt_fed_50.txt 1.254 0.165 0.000 0.000  0.000 0.051 0.355  0 0.013 0.051
## dispt_fed_51.txt 1.490 0.053 0.015 0.015  0.000 0.075 0.361  0 0.000 0.008
## dispt_fed_52.txt 1.326 0.071 0.008 0.000  0.000 0.103 0.532  0 0.000 0.087
## dispt_fed_53.txt 1.193 0.027 0.007 0.007  0.000 0.094 0.485  0 0.000 0.027
## dispt_fed_54.txt 1.469 0.089 0.007 0.007  0.000 0.126 0.445  0 0.000 0.007
##                 were  what  when which   who  will  with would your clusters
## dispt_fed_49.txt 0.017 0.000 0.009 0.175 0.044 0.009 0.087 0.192    0        2
## dispt_fed_50.txt 0.000 0.000 0.000 0.114 0.038 0.089 0.063 0.139    0        5
```

```
## dispt_fed_51.txt 0.015 0.008 0.000 0.105 0.008 0.173 0.045 0.068   0      2
## dispt_fed_52.txt 0.079 0.008 0.024 0.167 0.000 0.079 0.079 0.064   0      2
## dispt_fed_53.txt 0.020 0.020 0.007 0.155 0.027 0.168 0.074 0.040   0      5
## dispt_fed_54.txt 0.030 0.015 0.037 0.186 0.045 0.111 0.089 0.037   0      2
```

```
#drop final column and visualize clusters
fedpapers_km_fviz<-fedpapers_km[,-71]
fviz_cluster(cluster, data=fedpapers_km_fviz, labelsize = 5)
```



##

Visualizing bar chart of various clusters

```
ggplot(data=fedpapers, aes(x=author, fill=clusters)) + geom_bar(stat="count") + labs(title =
"Cluster Results from trial 1")
```

These cluster results tell us that the disputed papers are much more correlated with Madison's writing styles, which were shown in clusters 1 and 4. The disputed papers were predominantly shown in clusters 1 and 4 as well.

Hamilton's writing falls predominantly in clusters 3, 5, and slightly overlaps with cluster 1. Hamilton and Madison both overlap slightly in cluster 1. Based on the analysis of the clustering results, we chose a good initial number of clusters that showed a clear distinction between authors.

Based on these initial results, it seems much more likely that Madison is the author of these disputed papers.

Hierarchical Clustering Algorithms (HAC)

First, make a new df to pipe to the algorithm.

```
fedpapers_hac<-fedpapers[,2:72]
#making file name row names
rownames(fedpapers_hac)<-fedpapers_hac[,1]
fedpapers_hac[,1]<-NULL
```

```
#view df
head(fedpapers_hac)

##                    a   all  also    an   and   any   are    as    at    be
## dispt_fed_49.txt 0.280 0.052 0.009 0.096 0.358 0.026 0.131 0.122 0.017 0.411
## dispt_fed_50.txt 0.177 0.063 0.013 0.038 0.393 0.063 0.051 0.139 0.114 0.393
## dispt_fed_51.txt 0.339 0.090 0.008 0.030 0.301 0.008 0.068 0.203 0.023 0.474
## dispt_fed_52.txt 0.270 0.024 0.016 0.024 0.262 0.056 0.064 0.111 0.056 0.365
## dispt_fed_53.txt 0.303 0.054 0.027 0.034 0.404 0.040 0.128 0.148 0.013 0.344
## dispt_fed_54.txt 0.245 0.059 0.007 0.067 0.282 0.052 0.111 0.252 0.015 0.297
##                   been   but    by   can    do  down  even every  for.  from
## dispt_fed_49.txt 0.026 0.009 0.140 0.035 0.026 0.000 0.009 0.044 0.096 0.044
## dispt_fed_50.txt 0.165 0.000 0.139 0.000 0.013 0.000 0.025 0.000 0.076 0.101
## dispt_fed_51.txt 0.015 0.038 0.173 0.023 0.000 0.008 0.015 0.023 0.098 0.053
## dispt_fed_52.txt 0.127 0.032 0.167 0.056 0.000 0.000 0.024 0.040 0.103 0.079
## dispt_fed_53.txt 0.047 0.061 0.209 0.088 0.000 0.000 0.020 0.027 0.141 0.074
## dispt_fed_54.txt 0.030 0.037 0.186 0.000 0.000 0.007 0.007 0.007 0.067 0.096
##                    had   has  have her   his   if.   in.  into    is    it
## dispt_fed_49.txt 0.035 0.017 0.044   0 0.017 0.000 0.262 0.009 0.157 0.175
## dispt_fed_50.txt 0.101 0.013 0.152   0 0.000 0.025 0.291 0.025 0.038 0.127
## dispt_fed_51.txt 0.008 0.015 0.023   0 0.000 0.023 0.308 0.038 0.150 0.173
## dispt_fed_52.txt 0.016 0.024 0.143   0 0.024 0.040 0.238 0.008 0.151 0.222
## dispt_fed_53.txt 0.000 0.054 0.047   0 0.020 0.034 0.263 0.013 0.189 0.108
## dispt_fed_54.txt 0.022 0.015 0.119   0 0.067 0.030 0.401 0.037 0.260 0.156
##                    its   may  more  must my    no   not now    of    on   one
## dispt_fed_49.txt 0.070 0.035 0.026 0.026  0 0.035 0.114   0 0.900 0.140 0.026
## dispt_fed_50.txt 0.038 0.038 0.000 0.013  0 0.000 0.127   0 0.747 0.139 0.025
## dispt_fed_51.txt 0.030 0.120 0.038 0.083  0 0.030 0.068   0 0.858 0.150 0.030
## dispt_fed_52.txt 0.048 0.056 0.056 0.071  0 0.032 0.087   0 0.802 0.143 0.032
## dispt_fed_53.txt 0.013 0.047 0.067 0.013  0 0.047 0.128   0 0.869 0.054 0.047
## dispt_fed_54.txt 0.015 0.074 0.045 0.015  0 0.059 0.134   0 0.876 0.141 0.052
##                   only    or   our shall should    so  some  such  than  that
## dispt_fed_49.txt 0.035 0.096 0.017 0.017  0.017 0.035 0.009 0.026 0.009 0.184
## dispt_fed_50.txt 0.000 0.114 0.000 0.000  0.013 0.013 0.063 0.000 0.000 0.152
```

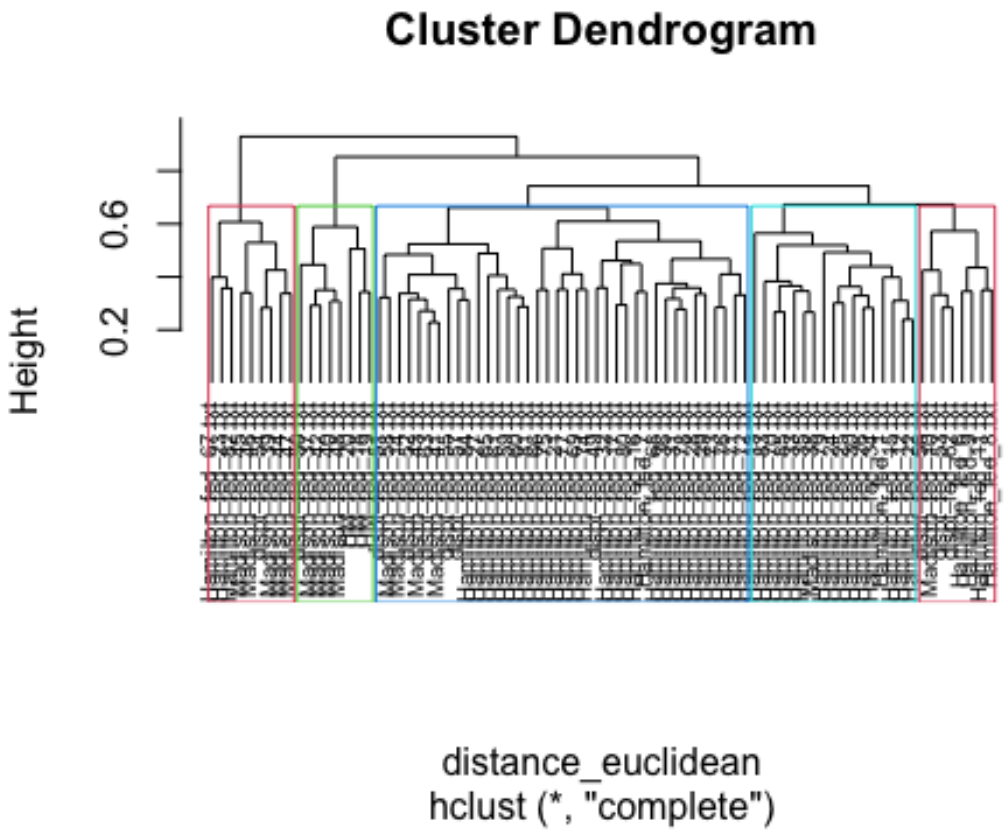
```
## dispt_fed_51.txt 0.023 0.060 0.000 0.008  0.068 0.038 0.030 0.045 0.023 0.188
## dispt_fed_52.txt 0.048 0.064 0.016 0.016  0.032 0.040 0.024 0.008 0.000 0.238
## dispt_fed_53.txt 0.027 0.081 0.027 0.000  0.000 0.027 0.067 0.027 0.047 0.162
## dispt_fed_54.txt 0.022 0.074 0.030 0.015  0.030 0.007 0.045 0.015 0.030 0.208
##                  the their  then there things  this    to up  upon   was
## dispt_fed_49.txt 1.425 0.114 0.000 0.009  0.009 0.044 0.507  0 0.000 0.009
## dispt_fed_50.txt 1.254 0.165 0.000 0.000  0.000 0.051 0.355  0 0.013 0.051
## dispt_fed_51.txt 1.490 0.053 0.015 0.015  0.000 0.075 0.361  0 0.000 0.008
## dispt_fed_52.txt 1.326 0.071 0.008 0.000  0.000 0.103 0.532  0 0.000 0.087
## dispt_fed_53.txt 1.193 0.027 0.007 0.007  0.000 0.094 0.485  0 0.000 0.027
## dispt_fed_54.txt 1.469 0.089 0.007 0.007  0.000 0.126 0.445  0 0.000 0.007
##                  were  what  when which   who  will  with would your
## dispt_fed_49.txt 0.017 0.000 0.009 0.175 0.044 0.009 0.087 0.192    0
## dispt_fed_50.txt 0.000 0.000 0.000 0.114 0.038 0.089 0.063 0.139    0
## dispt_fed_51.txt 0.015 0.008 0.000 0.105 0.008 0.173 0.045 0.068    0
## dispt_fed_52.txt 0.079 0.008 0.024 0.167 0.000 0.079 0.079 0.064    0
## dispt_fed_53.txt 0.020 0.020 0.007 0.155 0.027 0.168 0.074 0.040    0
## dispt_fed_54.txt 0.030 0.015 0.037 0.186 0.045 0.111 0.089 0.037    0
```

Now, we will calculate distance.

```
distance_euclidean<-dist(fedpapers_hac, method = "euclidean")
distance_maximum<-dist(fedpapers_hac, method = "maximum")
distance_manhattan<-dist(fedpapers_hac, method = "manhattan")
distance_canberra<-dist(fedpapers_hac, method = "canberra")
distance_binary<-dist(fedpapers_hac, method = "binary")
distance_minkowski<-dist(fedpapers_hac, method = "minkowski")
```

Cluster Dendrogram of Euclidean Distance

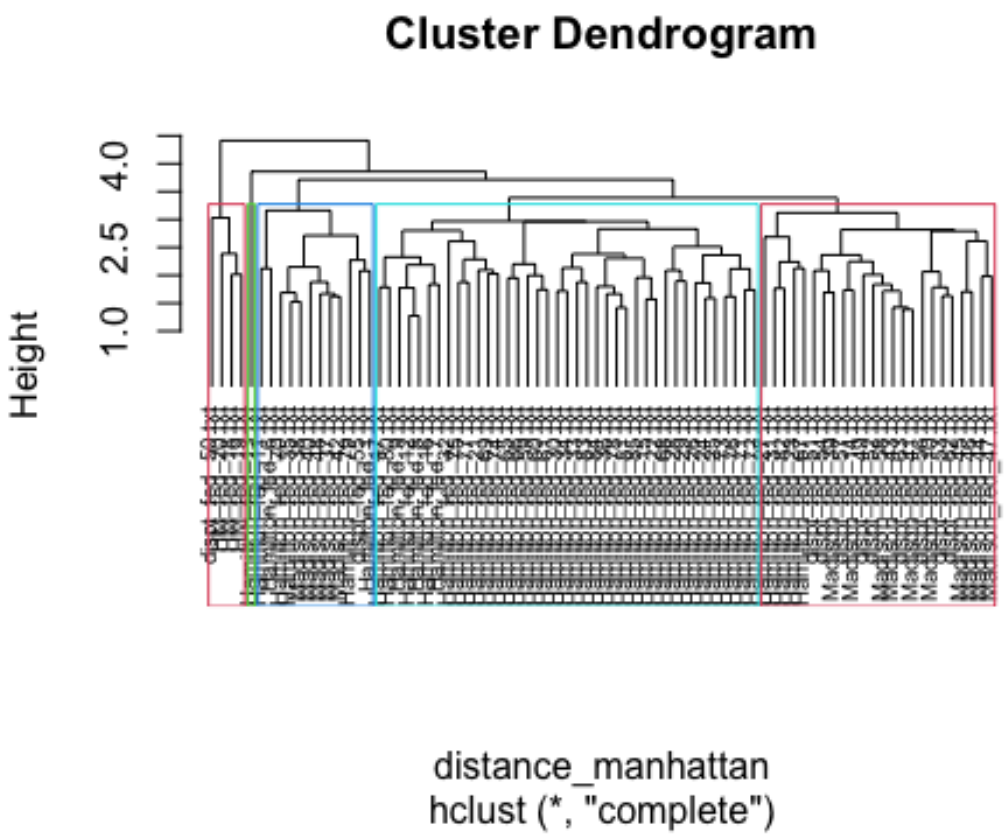
```
hac_euc<-hclust(distance_euclidean, method="complete")
plot(hac_euc, cex=0.6, hang=-1)
rect.hclust(hac_euc, k=5, border=2:5)
```



The above diagram puts disputed papers in the leafs closest to Madison's papers as you read the diagram from the bottom up. This again suggests that the disputed papers are closest in similarity to Madison, and are most likely written by him.

Visualizing Dendrogram with Manhattan Distance

```
hac_man<-hclust(distance_manhattan, method="complete")
plot(hac_man, cex=0.6, hang=-1)
rect.hclust(hac_man, k=5, border=2:5)
```



The Manhattan Distance Dendrogram is even more illustrative than the previous diagram, as it shows the large majority of disputed papers on the right handside. Here, we can clearly see that these disputed papers are closely connected to Madison.

Summary and Conclusions

To begin tackling the question of who wrote the disputed papers, we first began with some exploratory data analysis to see if there are clear differences in tone between the two authors. We found that Hamilton and Madison did in fact have a slight variance in tone (analyzing first/second person tone, as well as strong action words), with Hamilton being slightly more prone to use the first person and to use strong "call to action". While this initial analysis did not help much with answering our final question, it did prove that the two writers had somewhat distinguishable writing styles before jumping into our algorithms.

First, we used the K-Means clustering algorithm to group the papers. We found that 5 clusters was a good K value, and provided clear categories that separated Madison and Hamilton- with Madison falling predominantly into clusters 1 and 4, while Hamilton fell into clusters 3, 5, and also in 1. Ultimately, Madison's writing matched the disputed papers much more closely, supporting the conclusion that Madison wrote the papers.

When we conducted the HAC algorithm, we also found a strong correlation between Madison's writing and the disputed papers. The Manhattan distance was more useful to

conclude this correlation, as it showed the disputed papers grouped together on the right handside which were closely related to Madison. Ultimately, this also supported the conclusion that Madison wrote the papers.

Finally, based on our K-Means algorithm, we were able to examine the papers written jointly by both Hamilton and Madison. These appeared to actually be in a cluster of their own (2), which only overlapped slightly with Hamilton's writing.