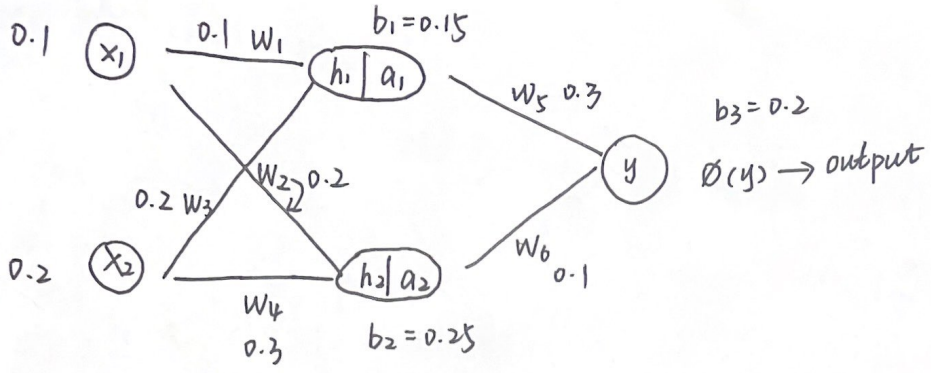

Error = $\sum \frac{1}{2}(\text{target} - \text{output})^2$

target output = 0.1 , learning rate η = 0.1

The Forward Pass

$h_1 = w_1 \cdot x_1 + w_2 \cdot x_2 + b_1 = 0.1 \times 0.1 + 0.2 \times 0.2 + 0.15 = 0.2$

$a_1 = \frac{1}{1+e^{-h_1}} = 0.549834$

$h_2 = w_3 \cdot x_1 + w_4 \cdot x_2 + b_2 = 0.2 \times 0.1 + 0.3 \times 0.2 + 0.25 = 0.33$

$a_2 = \frac{1}{1+e^{-h_2}} = 0.581759$

$y = w_5 \cdot a_1 + w_6 \cdot a_2 + b_3 = 0.3 \times 0.549834 + 0.1 \times 0.581759 + 0.2 = 0.4231261$

Output $= \frac{1}{1+e^{-y}} = 0.604231$

Error $= \frac{1}{2} \times (0.1 - 0.604231)^2 = 0.127124$

The Backwards Pass

$$\frac{\partial Error}{\partial W_5} = \frac{\partial Error}{\partial Output} \cdot \frac{\partial Output}{\partial y} \cdot \frac{\partial y}{\partial W_5}$$

$$\Rightarrow \begin{cases} 2 \cdot \frac{1}{2}(target - Output) \cdot (-1) = \frac{\partial Error}{\partial Output} = 0.60423 - 0.1 = 0.504231 \\ Output \cdot (1 - Output) = 0.604231 \times (1 - 0.60423) = 0.239136 = \frac{\partial Output}{\partial y} \\ a_1 = \frac{\partial y}{\partial W_5} = 0.549834 \end{cases}$$

$$\frac{\partial Error}{\partial W_5} = 0.504231 \times 0.239136 \times 0.549834 = 0.066299$$

$$new\_W_5 = W_5 - \eta \times \frac{\partial Error}{\partial W_5} = 0.3 - 0.1 \times 0.066299$$

(updated) $= 0.293370$

$$\frac{\partial Error}{\partial W_6} = \frac{\partial Error}{\partial Output} \cdot \frac{\partial Output}{\partial y} \cdot \frac{\partial y}{\partial W_6} \Rightarrow \frac{\partial y}{\partial W_6} = 0 + a_2 + 0 = a_2 = 0.581759$$

$$= 0.504231 \times 0.239136 \times 0.581759 = 0.070148$$

$$new\_W_6 = W_6 - \eta \times \frac{\partial Error}{\partial W_6} = 0.1 - 0.1 \times 0.070148 = 0.092985$$

Hidden Layer (W_1, W_2, W_3, W_4)

$$\frac{\partial Error}{\partial W_1} = \frac{\partial Error}{\partial Output} \cdot \frac{\partial Output}{\partial y} \cdot \frac{\partial y}{\partial a_1} \cdot \frac{\partial a_1}{\partial h_1} \cdot \frac{\partial h_1}{\partial W_1}$$

$$\Rightarrow \begin{cases} \frac{\partial Error}{\partial y} = \frac{\partial Error}{\partial Output} \cdot \frac{\partial Output}{\partial y} = 0.504231 \times 0.239136 = 0.120580 \\ \frac{\partial y}{\partial a_1} = W_5 + 0 + 0 = W_5 = 0.3 \\ \frac{\partial a_1}{\partial h_1} = a_1(1 - a_1) = 0.549834 \times (1 - 0.549834) = 0.247517 \\ \frac{\partial h_1}{\partial W_1} = X_1 = 0.1 \end{cases}$$

$$= 0.120580 \times 0.3 \times 0.247517 \times 0.1 = 0.000895$$

$$new\_W_1 = W_1 - \eta \times \frac{\partial Error}{\partial W_1} = 0.1 - 0.1 \times 0.000895 = 0.099910$$

$$\frac{\partial Error}{\partial W_2} = \frac{\partial Error}{\partial y} \cdot \frac{\partial y}{\partial a_1} \cdot \frac{\partial a_1}{\partial h_1} \cdot \frac{\partial h_1}{\partial W_2} \Rightarrow \frac{\partial h_1}{\partial W_2} = X_2 = 0.2$$

$$= 0.120580 \times 0.3 \times 0.247517 \times 0.2 = 0.001790$$

$$new\_W_2 = W_2 - \eta \cdot \frac{\partial Error}{\partial W_2} = 0.2 - 0.1 \times 0.00179 = 0.199821$$

$$\frac{\partial Error}{\partial W_3} = \frac{\partial Error}{\partial y} \cdot \frac{\partial y}{\partial a_2} \cdot \frac{\partial a_2}{\partial h_2} \cdot \frac{\partial h_2}{\partial W_3} \Rightarrow \begin{cases} \frac{\partial y}{\partial a_2} = W_6 = 0.1, \frac{\partial h_2}{\partial W_3} = X_1 = 0.1 \\ \frac{\partial a_2}{\partial h_2} = a_2(1 - a_2) = 0.581759 \times (1 - 0.581759) = 0.243315 \end{cases}$$

$$= 0.120580 \times 0.1 \times 0.243315 \times 0.1 = 0.000293$$

$$new\_W_3 = W_3 - \eta \cdot \frac{\partial Error}{\partial W_3} = 0.2 - 0.1 \times 0.000293 = 0.199971$$

$$\frac{\partial Error}{\partial W_4} = \frac{\partial Error}{\partial y} \cdot \frac{\partial y}{\partial a_2} \cdot \frac{\partial a_2}{\partial h_2} \cdot \frac{\partial h_2}{\partial W_4} = 0.120580 \times 0.1 \times 0.243315 \times 0.2 = 0.000587$$

$$new\_W_4 = W_4 - \eta \cdot \frac{\partial Error}{\partial W_4} = 0.3 - 0.1 \times 0.000587 = 0.299941$$

Update the Output $h_1 = new\_w_1 \cdot x_1 + new\_w_2 \cdot x_2 + b_1$

$h_1' = 0.0999105 \times 0.1 + 0.199821 \times 0.2 + 0.15 = 0.199956$

$a_1' = \frac{1}{1+e^{-h_1'}} = 0.549823$

$h_2' = new\_w_3 \cdot x_1 + new\_w_4 x_2 + b_2 = 0.199971 \times 0.1 + 0.299941 \times 0.2 + 0.25$

$= 0.329985$

$a_2' = \frac{1}{1+e^{-h_2'}} = 0.581756$

$y' = new\_w_5 \cdot a_1' + new\_w_6 \cdot a_2' + b_3$

$= 0.293370 \times 0.549823 + 0.092985 \times 0.581756 + 0.2 = 0.415396$

$new\_Output = \frac{1}{1+e^{-y'}} = 0.602381$

$new\_Error = \frac{1}{2}(0.1 - 0.602381)^2 = 0.126193$

Compared with original Error, it decreases