MATHS 2107 Statistics and Numerical Methods
Assignment 1
Due: Monday 9th August 2021 5pm - (Week 3)

When presenting your solutions to the assignment, please include some explanation in words to accompany your calculations. It is not necessary to write a lengthy description, just a few sentences to link the steps in your calculation. Messy, illegible or inadequately explained solutions may be penalised. The marks awarded for each part are indicated in boxes.

This assignment has 4 questions, for a total of 31 marks.

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1. A recent study found that 15.0% of passenger vehicles had defective tyres and 12.0% had defective brakes. Assume further that given that a passenger vehicle has a defective tyres, the probability that it has defective brakes is 0.3. In what follows, let T be the event that a *randomly chosen* vehicle has defective tyres and B be the event that it has defective brakes.

   $\boxed{3}$ (a) Express the given information in probability notation.

   $\boxed{2}$ (b) Are the events B and T independent? Explain your answer without performing any further calculations.

   $\boxed{2}$ (c) Find $P(B \cap T)$.

   $\boxed{2}$ (d) Find $P(B \cup T)$.

   $\boxed{2}$ (e) Given that a randomly chosen vehicle is found to have defective brakes, what is the probability that it has defective tyres. Express your answer using probability notation.

2. A rare disease affects 2% of the population. A test has a sensitivity of 97%, i.e., it will give a positive result 97% of the time that a person actually has the disease. The same test also has a specificity of 94%, i.e., it will give a negative result 94% of the time when a person does not have the disease. Denote the event that a random person has a disease by D, and the event that a randomly selected person has a positive blood test by T.

   $\boxed{3}$ (a) Express the given information in probability notation.

   $\boxed{2}$ (b) Calculate the probability that a randomly selected person has a positive blood test.

   $\boxed{2}$ (c) Calculate the probability that given a randomly selected person has a positive blood test that they have the disease.

3. A car manufacturer offers a 5yr/Unlimited km warranty on one of its models. Historically, 12% of cars sold will require repairs under the terms of this warranty. Suppose a dealer sells 175 cars of that model, and let X be the number of those cars that will require service under the terms of the warranty.

   $\boxed{4}$ (a) A suitable probability model for X is a binomial distribution. What are the parameters of the binomial model in this case. Justify the use of a binomial model in context.

   $\boxed{2}$ (b) Find the probability that exactly 18 cars require service under the warranty.

   $\boxed{2}$ (c) Find the probability that *at least* 18 cars require service under the warranty.

4. Consider two independent random variables X and Y, such that

$$\begin{aligned} \mathrm{E}[X] &= 4 \\ \mathrm{E}[Y] &= 6 \\ \mathrm{var}(X) &= 2 \\ \mathrm{var}(Y) &= 3 \end{aligned}$$

Let

$$Z = X - Y.$$

$\boxed{2}$ (a) Calculate $E[Z]$.

$\boxed{3}$ (b) Calculate $\mathrm{var}(Z)$.

1 a) let T be the event of having defective tyres

let B be the event of having defective brakes

we know:

$P(T) = 0.15$

$P(B) = 0.12$

$P(B|T) = 0.3$

b) These events are not independent. We know this because the $P(B) \neq P(B|T)$ this means that $P(B|T)$ is dependent on $P(B)$ & $P(T)$, therefore making these events dependent on eachother.

c) $P(B \cap T)$

we know that $P(B|T)$ is equivilent to $\frac{P(B \cap T)}{P(T)}$, therefore substitute $\frac{P(B \cap T)}{P(T)}$ for $P(B|T)$ in $P(B|T) = 0.3$

$\therefore \frac{P(B \cap T)}{P(T)} = 0.3$

put value of $P(T)$ into equation

$\frac{P(B \cap T)}{0.15} = 0.3$

multiply both sides by 0.15 to find $P(B \cap T)$

$\frac{P(B \cap T)}{0.15}^{\times 0.15} = 0.3^{\times 0.15}$

$\Rightarrow P(B \cap T) = 0.045$

$\therefore$ The probability of both the tyres and breaks being defective is 0.045

d) $P(B \cup T)$

we know

$P(B \cup T) = P(B) + P(T) - P(B \cap T)$

Substitute in values of $P(B)$, $P(T)$ & $P(B \cap T)$

$P(B \cup T) = 0.12 + 0.15 - 0.045$

$= 0.225$

$\therefore$ The probability of either the tyres or breaks being defective is 0.225

e) want $P(T/B)$

we know

$P(T|B) = \frac{P(T \cap B)}{P(B)}$

Substitute in values for $P(T \cap B)$ and $P(B)$

$P(T|B) = \frac{0.045}{0.12}$

$= 0.375$

$\therefore$ The probability of a randomly selected vehicle with defective brakes has defective tyres is 0.375.

2 a) Let D be the rare disease event.
Let T be a postive result event

$P(D) = 0.02$
$P(T|D) = 0.97$
$P(T^c|D^c) = 0.94$
$P(D^c) = 1 - 0.02 = 0.98$

b) we want
$P(T)$ where $P(T) = P(T|D)\,P(D) + P(T|D^c)\,P(D^c)$ (law of total probability)

we know:
$P(T|D) = 0.97$
$P(D) = 0.02$
$P(D^c) = 0.98$

Find:
$P(T|D^c)$ where
$P(T|D^c) = 1 - P(T^c|D^c)$
$= 1 - 0.94$
$= 0.06$

Substitute into $P(T) = P(T|D)\,P(D) + P(T|D^c)\,P(D^c)$ to find $P(T)$
$P(T) = (0.97 \times 0.02) + (0.06 \times 0.98)$
$= 0.0194 + 0.0588$
$= 0.0782$
$\therefore$ The probability of a randomly selected person having a positive blood test is 0.0782

c) We want
$P(D|T)$

From Bayes Rule we know

$$P(D|T) = \frac{P(T|D)P(D)}{P(T|D)P(D) + P(T|D^c)P(D^c)}$$

Substitute in known values.

$$P(D|T) = \frac{0.97 \times 0.02}{(0.97 \times 0.02) + (0.06 \times 0.98)}$$

$$= \frac{0.00194}{0.00194 + 0.0588}$$

$$= \frac{0.00194}{0.0782}$$

≈ 0.2480 (4 decimal places)
$\therefore$ The probability of a randomly selected person having a positive blood test and having the disease is approx 0.2480

3 a) The parameters of a binomial distribution are the number of trials (denoted by n) and the probability of the event occouring (denoted by P).

in this case we know

$n = 175$

$p = 12\%$

$= 0.12$

since each trial is independent to eachother and each trial has only two possible outcomes

1. service required under werrenty
2. Service not required whilst under warrenty

a binomial distribution approach is appropriate.

b) using the MATLAB command

binopdf(18, 175, 0.12)

It is found that there is a 0.0766 probability that exactly 18 cars will be serviced under warrenty

c) using the MATLAB command

1 - binocdf(18, 175, 0.12)

It is found that there is a 0.7128 probability that at least 18 cars will be serviced under warrenty

4 $E[X] = 4$

$E[Y] = 6$

$var(X) = 2$

$var(Y) = 3$

$Z = X - Y$

a) Since $Z = X - Y$ we know

$E[Z] = E[X - Y]$

$E[Z] = E[X] - E[Y]$

Substitute in $E[X]$ & $E[Y]$

$E[Z] = 4 - 6$

$= -2$

$\therefore$ the value of $E[Z]$ is -2

b) since $Z = X - Y$ we know

$var(Z) = var(X - Y)$

$= var(X) + var(Y)$

Substitute in $var(X)$ & $var(Y)$

$var(Z) = 2 + 3$

$= 5$

$\therefore$ the value of $var(Z)$ is 5