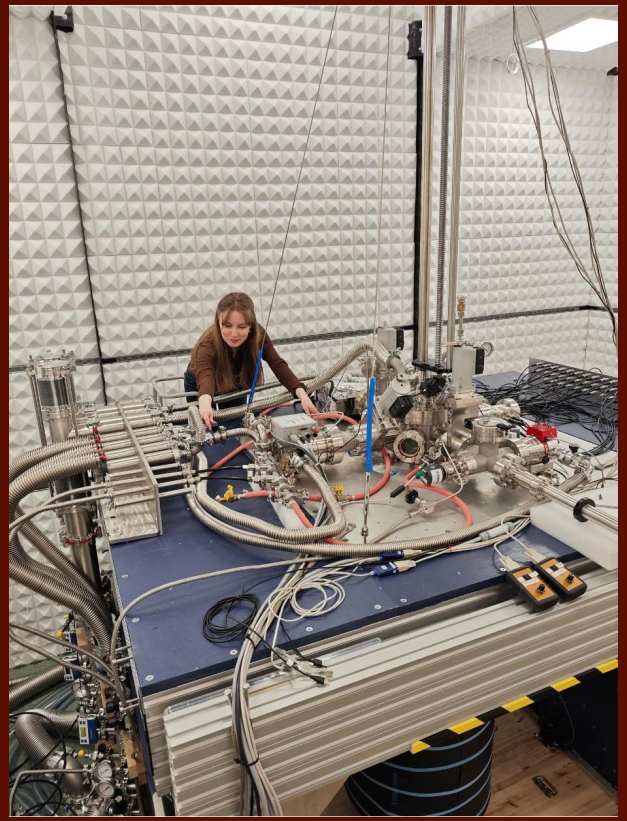
THE MAP OF MATHEMATICS

"All Ye Know On Earth"

Sarah Kate Sweeney
Lecture 1

WHO AM I, AND WHAT IS THIS?

- Sarah Kate Sweeney (just Sarah's fine)
- Graduate of UCC Astrophysics
- Currently in Research Masters in Macroscopic Quantum Matter
- Student Email (I monitor this one)- 120317406@umail.ucc.ie
- Feel free to reach out to me!
- Lecture 1 of CEIA Leaving Certificate Mathematics Tutorials
- "Map of Mathematics" (Introductory Lecture)
- Other Lectures given by me: Algebra 2, Complex Numbers



I have a question for you…

THE MAP OF MATHEMATICS

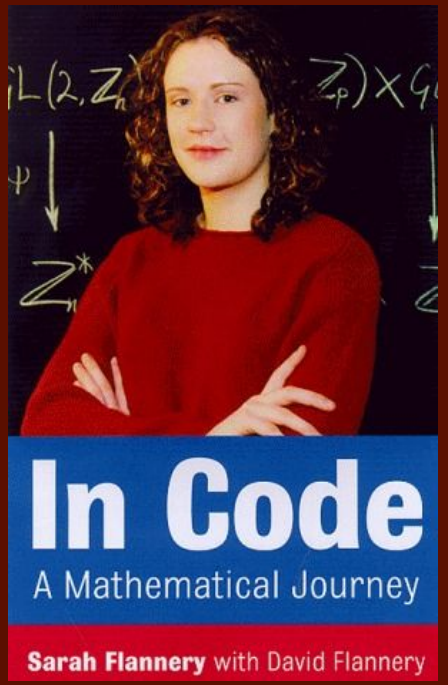
Concepts	**Properties of Models**	**Arithmetic**	**Algebra**	**Techniques**	**Applications**
Conceptual models of the world 'A framework in which we allow something to be true' At the end of 'why' is 'Eureka!'	*Numeric properties:* Natural, Integer, Rational, Real, Imaginary, Complex, Hyper Complex *Boolean Properties:* True, False *Set Properties:* Elements	*Numerical Operations:* +, -, x, /, log, exp, mod *Boolean Operations:* And, Or, Not *Set Operations:* Union, Intersection	Dealing with missing data Solving Equations: a = b + 2 b = 2a Dealing with functions: $y = f(x) = x^2 - 1$	Boolean Calculus Cryptography Geometry Linear Algebra Matrices Set Theory Statistics Trigonometry Vectors	AI Chemistry Economics Engineering Finance Genomics IT Medicine Physics Social Science

WELCOME TO THE PLATONIC ACADEMY

CONCEPTS AND MODELS

- **A framework in which we allow something to be true.**
- Example: coins
- Concepts/models have **properties**

THE MONK'S JOURNEY



I got this puzzle from Sarah Flannery's autobiography "In Code", where she gets it from Arthur Koestler's "Art of Creation"

THE MAP OF ~~MATHEMATICS~~ THINKING

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PROPERTIES OF CONCEPTS

What properties does this image have?

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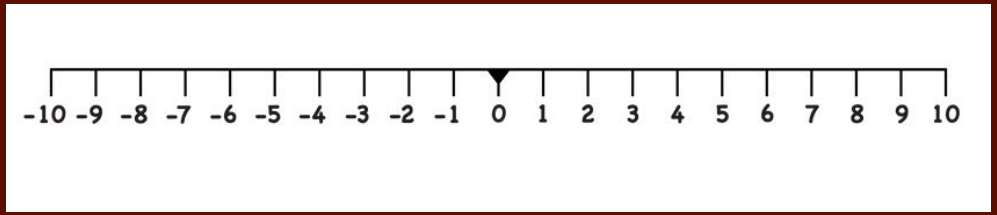
MESOPOTAMIAN MATHEMATICS

1	11	21	31	41	51
2	12	22	32	42	52
3	13	23	33	43	53
4	14	24	34	44	54
5	15	25	35	45	55
6	16	26	36	46	56
7	17	27	37	47	57
8	18	28	38	48	58
9	19	29	39	49	59
10	20	30	40	50	

BRAHMAGUPTA & THE EXPANSION OF THE NUMBER LINE

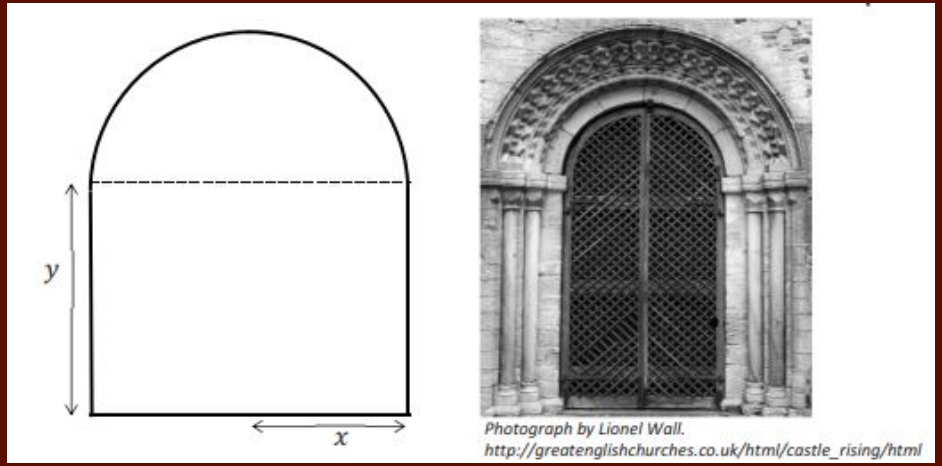


ALGEBRA

‘Al-jabr’ - broken parts/ bone setting

How can we deal with missing information?

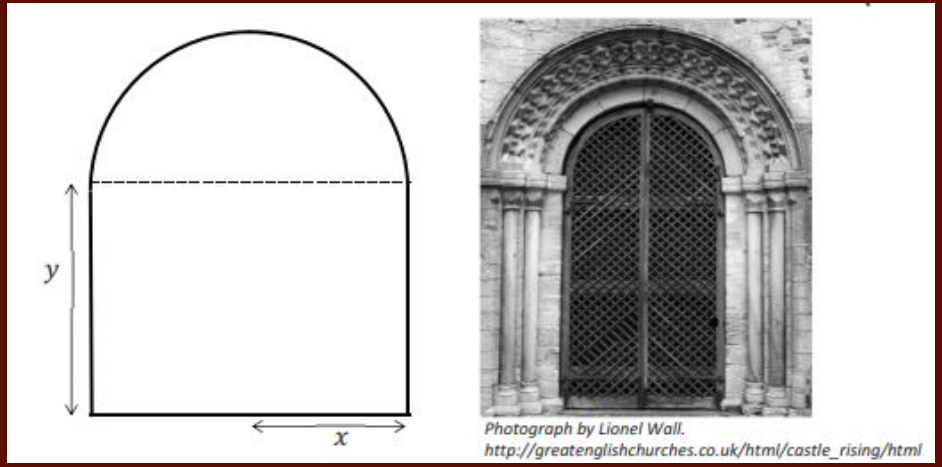
PAST PAPER EXAMPLE: 2019 PAPER 1, Q9



Photograph by Lionel Wall.
http://greatenglishchurches.co.uk/html/castle_rising/html

(a)(i) What is the perimeter P of the window, in terms of x & y?

PAST PAPER EXAMPLE: 2019 PAPER 1, Q9



Photograph by Lionel Wall.
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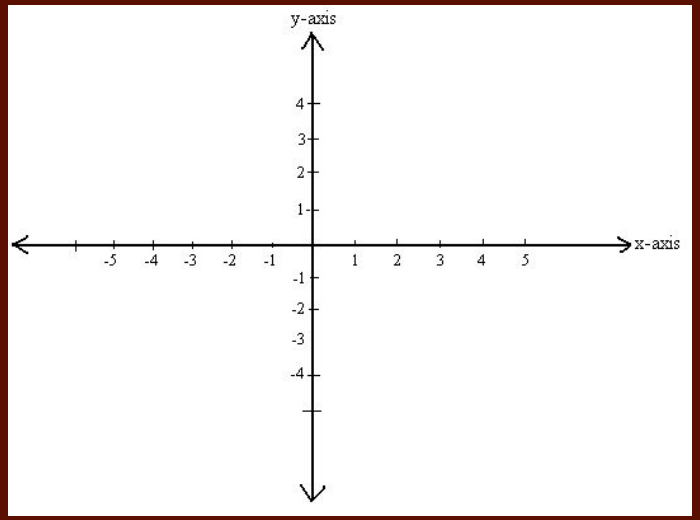
(ii) What if we know the perimeter is 12m?

Show that in this case, y = (12 - (2+pi)x)/2 for $0 < x < 12/(2+\pi)$.

COGITO, ERGO SUM - CARTESIAN COORDINATES

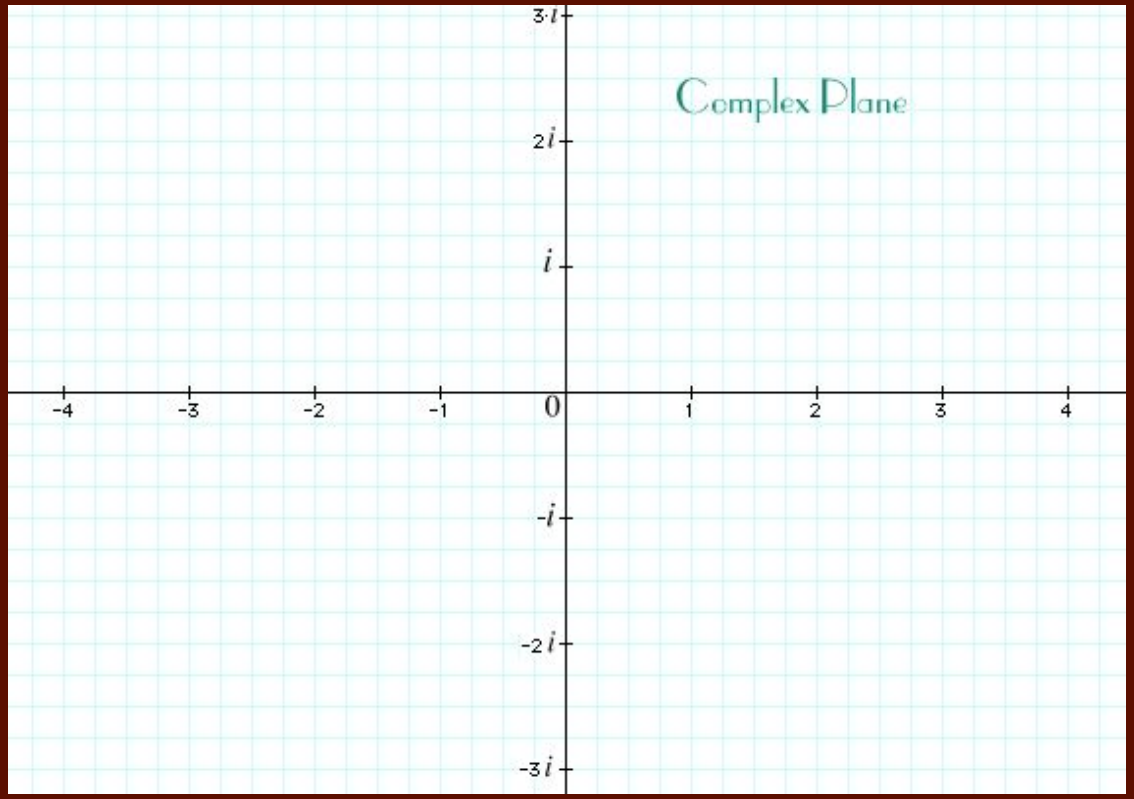
Plotting functions as graphs - the bridge between

Algebra and **Geometry**



GEROLAMO CARDANO & COMPLEX NUMBERS

THE NUMBER PLANE
(Numbers are now 2D)



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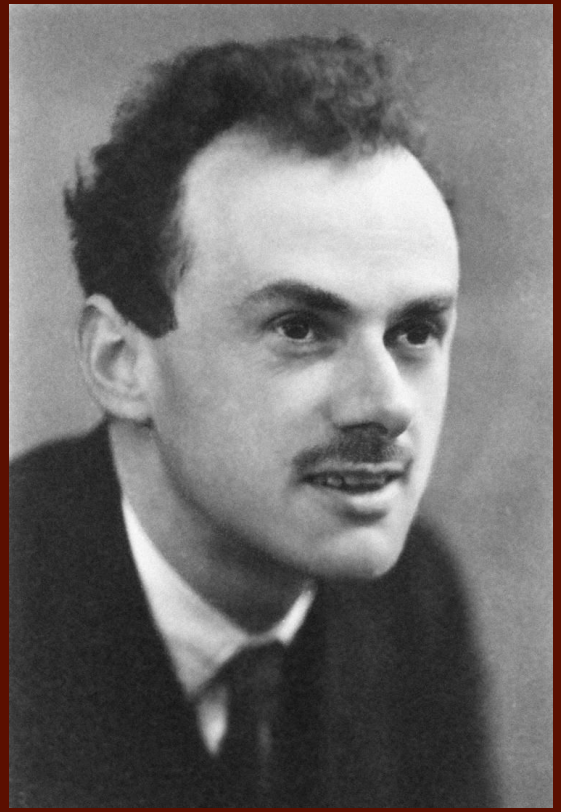
“Algebra is like sheet music. The important thing isn’t can you read the music, but can you hear it? Can you hear the music? “

- Niels Bohr’s Character in Christopher Nolan’s *Oppenheimer*

STORYTIME: PAUL DIRAC & ANTIMATTER

‘A theory with mathematical beauty is far more likely to be correct than an ugly one that fits some experimental data.’



ANTIMATTER FACTORY, CERN, GENEVA

"Beauty is truth, truth beauty,—that is all
Ye know on earth, and all ye need to know."

- *John Keats, "Ode on a Grecian Urn"*

I have a question for you (again)…