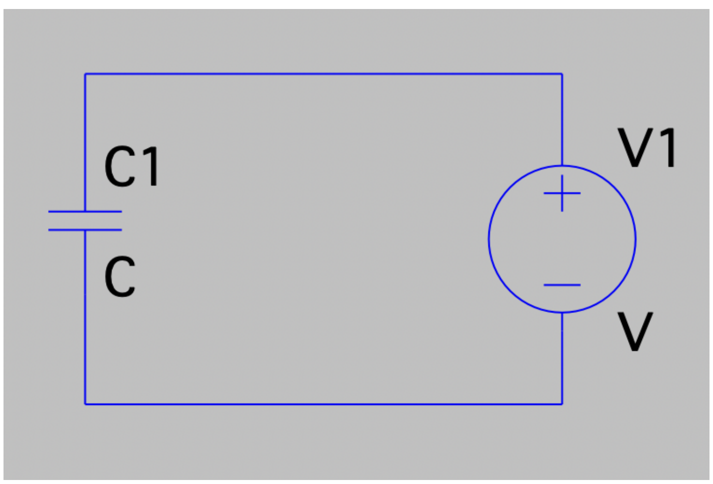
Capacitors & Capacitance

1 What's a capacitor?

Imagine we have two metal plates, with a thin sheet of insulating material (called a **dielectric**) in the middle. Let's now connect each plate to a wire, and form a series circuit with a voltage source.



If we turn on the voltage source, that will cause a current to flow from the positive terminal, to the negative terminal. But, the dielectric will not allow a current through it! Instead, positive charge builds up on the plate nearest the positive terminal of the voltage source. This charge creates an electric field, which is felt by the other plate. The positive charges in one plate attract negative charges in the other plate.

Now, you have one plate that is positively charged opposite one plate that is negatively charged. They can feel each others electric fields - there is voltage between the plates! You are now storing a little bit of electrical energy in those plates.

This device is called a **parallel plate capacitor** and is a device used in circuits to store a small amount of electric charge.

As you might imagine, the more voltage we initially apply, the more charge we can build up on those plates. However, there is a particular amount of charge a particular capacitor can hold, and we call this amount it's **capacitance**.

To define the capacitance of a capacitor, we find the ratio of charge it can hold for a certain applied voltage. That is to say:

$C = \frac{Q}{V}$
where C is capacitance, Q is charge and V is voltage.

The unit of capacitance is the **Farad**, named after Michael Faraday (remember him?).

Try Q1!

2 What physical features affect capacitance?

Okay, so now we know how to build a capacitor: two metal plates with a dielectric in the middle. But how can we build capacitors of different capacitances? What physical features of a capacitor affect it's capacitance?

First, let's think about the **common area of the plates**. Your initial instinct might be that the more area you have, the more charge can build up on the plates, and so, larger capacitance. You'd be right! You can check this by experimenting with plate capacitors with different areas, and you'll find that capacitance is directly proportional to area. That is to say:

$C \propto A$

Next, let's look at the distance between the plates. As you might imagine, the farther apart the plates are, the less charge will be induced in the negative plate, and so the weaker the induced voltage across the plates will be ($d \propto V$). Experimenting with capacitors confirms this intuition. Therefore, we can say that:

$C = \frac{1}{d}$

Finally, let's look at the dielectric. Different substances will have different 'permittivities'; that is, the substances ability to store electric energy and allow the electric field to permeate through it. Since the voltage across the plates is caused by the electric field, the permittivity of the dielectric affects the capacitance. If you experiment with capacitors of different dielectrics, you will find that capacitance is directly proportional to permittivity.

$C \propto \epsilon$

We now have three relationships between physical features of a capacitor and it's capacitance. Putting them all together, we get this equation:

$C = \frac{A\epsilon}{d}$

Try Q2!

3 Energy in a Capacitor

We have established that a capacitor stores electrical energy. What is we need to know exactly how much energy is inside a capacitor? The energy comes from the work being done by the battery to charge the capacitor. We use the following equation to find the energy in the capacitor:

$W = \frac{1}{2}CV^2$

The derivation of this expression is outside the scope of the LC, since it requires an understanding of integration (it is taught in HL Maths, but not Physics). If you would like some intuition, look at this YT video:
https://www.youtube.com/watch?v=8F969kSlsAI

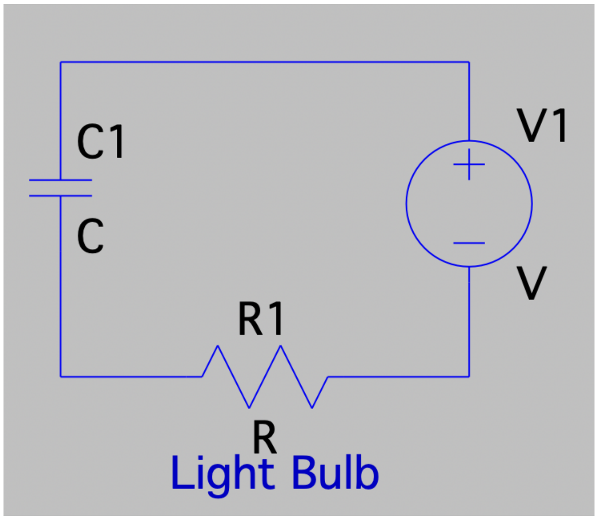
4 Current Flow in a Capacitor Circuit

4.1 DC

When you connect a capacitor to a DC source, electrons will flow off one plate and onto another. This sets up a potential difference across the plates. Once the voltage across the capacitor matches the voltage from the source, there is no longer a difference in potential between them, and so, no more current flows.

4.2 AC

Consider a capacitor connected to an AC source. As with DC, electrons flow from one plate to the other. But, since the current is AC, the polarity (direction) keeps changing; the current continues to flow. We can do experiments with capacitors in AC to see how they behave.



Consider two capacitors, one $100\mu F$ and another $200\mu F$. Connect each one to a power supply and a light bulb in series (like the above circuit diagram). What you'll find is that the bulb connected to the $200\mu F$ capacitor will light up brighter than the $100\mu F$ capacitor. Ergo, we can say that the higher the capacitance, the higher the current.

Let's repeat this experiment, but hold the capacitance constant, say at $100\mu F$. This time, we will vary the frequency of the AC. What you'll find this time, is that the higher the frequency of the AC, the brighter the light bulb will be.

5 Capacitors in Practice

We have established that capacitors store energy, but we don't use them in the same way we use batteries. They are used as a part of more complex circuits. Let's look at some of them.

5.1 RC Filters in radios

Imagine you have an AC signal that contains not just one frequency, but many. A situation where you might have this is in a radio: different stations emit radio waves (electromagnetic waves) of different frequencies. With a variable capacitor in paralell with a resistor (an RC circuit), it is possible to filter out all frequencies apart from one narrow range. This is how you 'tune in' to a station.

5.2 Rectifiers

'Rectification' is the process of converting AC to DC. We do this diodes. However, the output of a diode bridge is usually unsteady, and features what we call 'ripple'; a little bit of AC that leaked into the output signal. We can smooth this out using an RC circuit.

6 Practice Problems

1. A parallel plate capacitor is charged to a voltage of 50V. The charge on the capacitor is $2x10^{-5}$ C. What is the capacitance of the capacitor? (Ans: $0.4\mu F$)

2. A pair of metal plates $0.5M^2$ in area are 0.1mm apart. The dielectric is air, with a permittivity of $8.9x10^{-12}Fm^{-1}$. What is the capacitance? (Ans: $4.445x10^{-9}F$).