TEST

Graded

Student

Sara Huston

Total Points

27 / 28 pts

Question 1

P1 — **6** / 7 pts

+ **0 pts** Correct

+ **6 pts** 2 minor miscalculations: there must be 8 and 7 instead of 7, and 6.

Question 2

P2 — **7** / 7 pts

+ **0 pts** Correct

+ **7 pts** 11*10/2=55. Otherwise, correct.

Question 3

P3 — **7** / 7 pts

+ **0 pts** Correct

+ **7 pts** Where is h_{11}?

Question 4

P4 — **7** / 7 pts

+ **0 pts** Correct

+ **7 pts** Point adjustment

Q1 P1

7 Points

Put the Pledge: your signature; the usage of pdf files is strongly advised.
You may have up to 7+7+7+7 exam points (from 20, i.e., up to 140%).

How many hands of 4 cards each (permuting cards doesn't change the hand) can be formed from the set of 10 cards with the denominations $0, 1, 2, \dots, 9$ if no 3 consecutive cards are to be in a hand.

Sara Muston

$\boxed{1.}$ 4 hand cards

set of 10 cards (0...9)

No 3 consecutive cards in a hand

Free − B_1 + B_{12}

$\underbrace{\binom{10}{4}}_{\text{10 cards, choose 4}}$ − three consecutive hands + 4 consecutive hands

$\binom{10}{4} = \frac{10\cdot 9\cdot 8\cdot 7}{\cancel{4}\cdot\cancel{3}\cdot\cancel{2}} = 10\cdot 3\cdot 7 = 210$

4 consecutive:

7 choices

0,1,2,3
1,2,3,4
⋮
6,7,8,9

$\Big\}$ 6

3 consecutive

7 $\Big\{$

0,1,2 → 3,4,5,6,7,8,9
1,2,3 → 0,4,5,6,7,8,9
2,3,4 → 0,1,5,6,7,8,9
3,4,5 → 0,1,2,6,7,8,9
4,5,6 → 0,1,2,3,7,8,9
5,6,7 → 0,1,2,3,4,8,9
6,7,8 → 0,1,2,3,4,5,9
7,8,9 → 0,1,2,3,4,5,6,

place triplet

$\binom{7}{1}\cdot \underline{\quad}$ remaining digit

$= 10-3=7$ choices

$\binom{10}{4} - \binom{7}{1}\binom{7}{1} + \binom{6}{1}$

add back 4 consecutive bc they are double counted

$210 - 49 + 6 = \boxed{167}$

Scanned with CamScanner

Q2 P2
7 Points

PUT YOUR NAME WITH AT LEAST ONE PROBLEM, PLEASE.

Determine the number of combinations with 9 objects (9-baskets) from a set containing 8 objects "A", 4 objects "B", 3 objects "C".
Use the inclusion-exclusion principle.

CamScanner 10-28-2024 10.41.pdf Download

Your browser does not support PDF previews. You can download the file instead.

Sara Huston

2. 9 baskets 8 objects A, 4 objects B, 3 objects C

$N_9 = \text{Free} - B_1 - B_2 - B_3 + B_{23}$

$\text{Free} = x_1 + x_2 + x_3 = 9$

$= R(3,9) = \binom{9+3-1}{3-1} = \binom{11}{2}$

$= \frac{11 \cdot 10}{2} = 66$

$B_1 = \text{Bad}_1 = x_1 \geq 9$

$B_2 = \text{Bad}_2 = x_2 \geq 5$ $B_{12} = B_{13} = 0$

$B_3 = \text{Bad}_3 = x_3 \geq 4$ $B_{23} = 1$

$B_1 = \binom{9+3-1-9}{3-1} = \binom{2}{2} = 1$

$B_2 = \binom{9+3-1-5}{3-1} = \binom{6}{2} = \frac{6 \cdot 5}{2} = 15$

$B_3 = \binom{9+3-1-4}{3-1} = \binom{7}{2} = \frac{7 \cdot 6}{2} = 21$

$B_{23} = \binom{9+3-1-9}{3-1} = \binom{2}{2} = 1$ $\boxed{N_9 = 30}$

$N_9 = 66 - 15 - 1 - 21 + 1$

Scanned with CamScanner

Q3 P3

7 Points

Assuming that a pair of rabbits gives birth to 6 pairs in half a year, which is (approximately) the maturity period for "real" rabbits, find the recurrence relation for the number of pairs of rabbits g_n at the (beginning of the) n-th half-a-year, solve it (i.e., find a formula in terms of λ) for 1 mature pair to begin with, i.e. when $g_1 = 1,\ g_2 = 7,\ g_3 = 13$ and so on. Thus, the unit of time is 6 months in this problem instead of 1 month for Fibonacci. What is g_{11} (the reproduction period for real rabbits lasts, indeed, about $11 * 0.5 = 5.5$ years)?

▾ **CamScanner 10-28-2024 10.41 2.pdf** Download

Your browser does not support PDF previews. You can download the file instead.

Sara Huston

$\boxed{3.}$ 6 pairs in half a year
half a year = maturity period

$$\boxed{g_n = g_{n-1} + 6g_{n-2}}$$

$g_1 = 1 \quad g_2 = 7 \quad g_3 = 13$

$13 = 7 + 6g_0$

$6 = 6g_0 \Rightarrow g_0 = 1$

$\lambda^n = \lambda^{n-1} + 6\lambda^{n-2}$

$\lambda^2 = \lambda + 6$

$$\boxed{g_n = \left(\frac{3}{5}\right)(3)^n + \left(\frac{2}{5}\right)(-2)^n}$$

$\lambda^2 - \lambda - 6 = 0$

$(\lambda - 3)(\lambda + 2) = 0$

$\boxed{\lambda_1 = 3} \quad \boxed{\lambda_2 = -2}$

$g_n = c_1(3)^n + c_2(-2)^n$

$1 = c_1 + c_2$

$1 = 3c_1 - 2c_2$ $\quad \boxed{c_2 = \frac{2}{5}}$

$c_1 = 1 - c_2$

$1 = 3(1 - c_2) - 2c_2$ $\quad \boxed{c_1 = \frac{3}{5}}$

$1 = 3 - 3c_2 - 2c_2$

$-2 = -5c_2$

Scanned with CamScanner

Q4 P4

7 Points

Find but do not solve algebraically the recurrence relation for the number x_n of perfect coverings of the shape $1 \times n$ by monominos (single boxes) and dominos such that no three monominos are
consecutive (two can be consecutive). Here $x_1 = 1, x_2 = 2, x_3 = 2$; continuing, find x_4, x_5.

▾ **CamScanner 10-28-2024 10.41 3.pdf** Download

Your browser does not support PDF previews. You can download the file instead.

Sara Huston

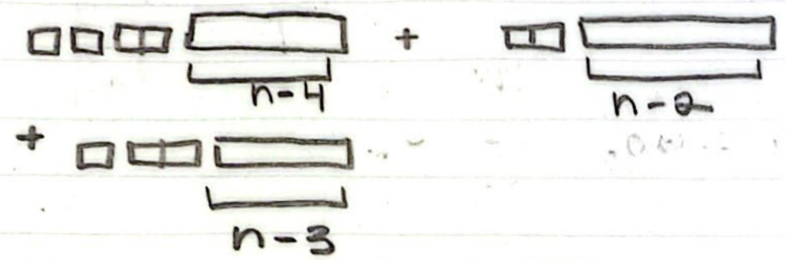
$\boxed{4.}$ $1 \times n$ monominos, dominos

No three monominos

$X_0 = 1$ $X_1 = 1$ $X_2 = 2$ $X_3 = 2$

$X_4 = 4$

$X_n =$



$$\boxed{X_n = X_{n-4} + X_{n-3} + X_{n-2}}$$

$X_5 = 5$

$X_4 = 1 + 1 + 2 = 4$ $\boxed{X_4 = 4}$

$X_5 = 2 + 2 + 1 = 5$ $\boxed{X_5 = 5}$

Scanned with CamScanner