

Midterm #1

● Graded

Student

Sara Huston

Total Points

50.5 / 60 pts

Question 1

Question #1

20 / 20 pts

✓ - 0 pts Correct

- 8 pts Major mistake
- 10 pts fundamental mistake
- 8 pts incomplete work and/or unclear steps
- 15 pts no serious attempt
- 2 pts small mistake
- 3 pts last part incorrect
- 2 pts solution not explicit
- 12 pts fundamental mistake. no correct steps.
- 2 pts seems correct but work was hard to follow
- 13 pts fundamental error (graded out of 15)
- 5 pts correct. (graded out of 15)
- 1 pt error in the last part

Question 2

Question #2

6 / 10 pts

- 0 pts Correct

- 2.5 pts very good progress, but minor error

✓ - 4 pts correct method and good setup, but major error with the rest of the solution

- 7.5 pts applied correct method but major issue
- 10 pts minimal progress or on the wrong track
- + 1 pt small bonus modifier
- 1 pt small error modifier
- + 5 pts correct. graded out of 15 (makeup)

Question 3

Question #3

4.5 / 10 pts

– 0 pts Correct

✓ – 2.5 pts error in critical points

– 4 pts major issue with critical points

– 2 pts condition for stability: minor issue

✓ – 4 pts condition for stability: medium issue

– 6 pts condition for stability: major issue

– 10 pts minimal progress

✓ + 1 pt small bonus modifier

– 1 pt small error modifier

– 5 pts significant mistakes (make up)

Question 4

Question #4

20 / 20 pts

✓ – 0 pts Correct

– 8 pts fundamental error. acceleration is not constant.

– 8 pts fundamental error

– 4 pts error using IC leading to wrong solution

– 6 pts right direction but incomplete

– 6 pts solved the wrong ODE

– 15 pts no serious attempt

– 2 pts steps unclear

– 2 pts small error

– 5 pts (b) small error in ODE, incomplete solution.
(c) no explanation

Math 383: Exam 1
Sep 27, 2024
Instructor: Saiful Tamim

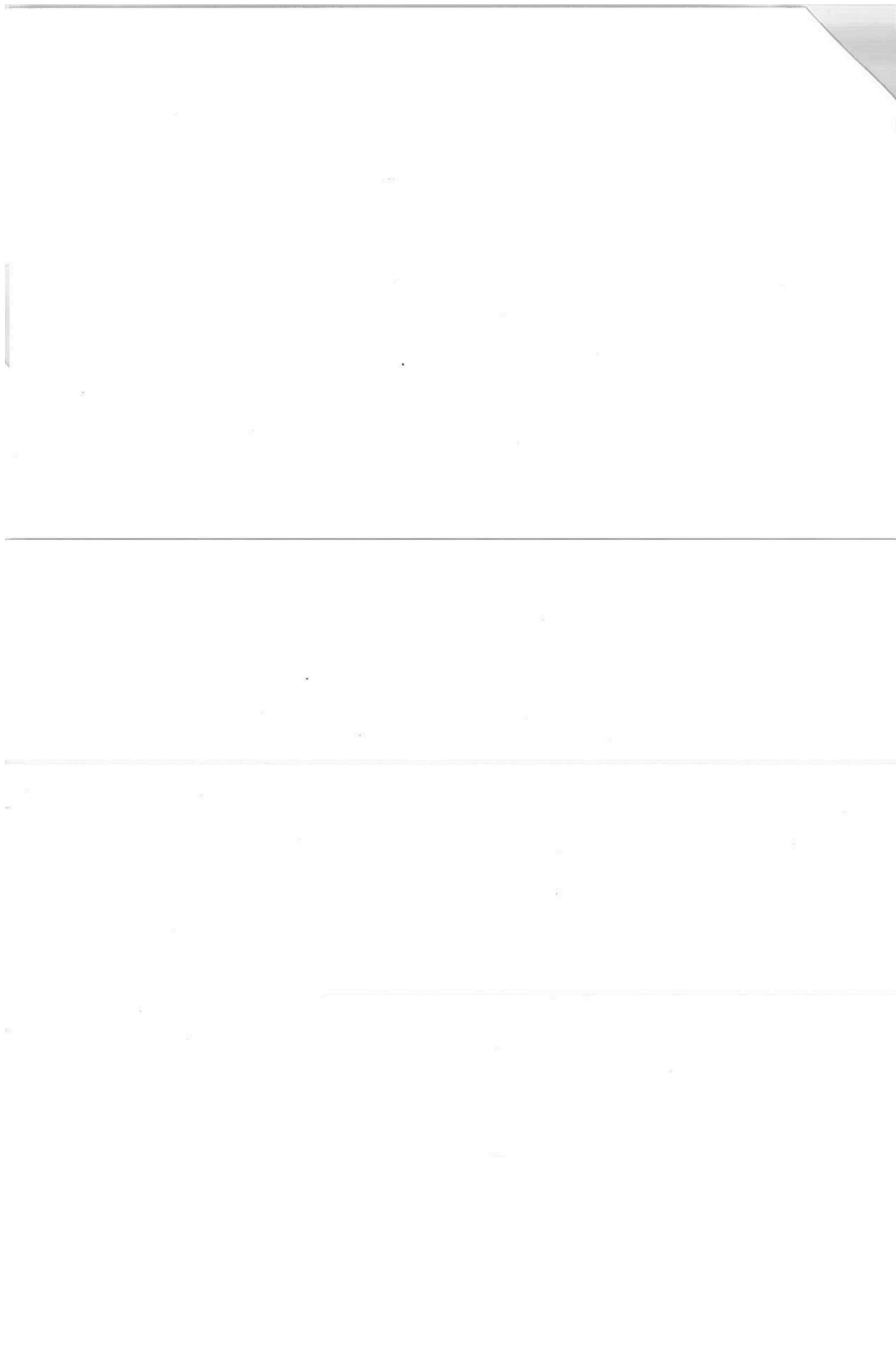
- Calculators are NOT allowed.
- Show work where possible for full and partial credit.
- **Work must be clear and readable for full credit.**
- Use proper mathematical notation.
- Clearly indicate your answer, e.g., by boxing it.
- Sign the honor pledge below after completing the exam.

Last Name, First Name: MUSTON, Sara

PID: 730459812

Honor Pledge: I have neither given nor received unauthorized help on this exam.

Signature: 



1. (20 points) Find the explicit solution to the following initial value problem, where k is a constant.

$$x' + x - kx^2 = 0, \quad x(0) = 1.$$

Determine the long-term solution $x(t \rightarrow \infty)$.

$$\frac{dx}{dt} + x - kx^2 = 0$$

$$\frac{dx}{dt} + x = kx^2$$

bernoulli:

$$v = x^{-1}$$

$$\frac{dv}{dt} = -x^{-2} \cdot \frac{dx}{dt}$$

$$\frac{dx}{dt} = -x^2 \cdot \frac{dv}{dt}$$

$$-x^2 \cdot \frac{dv}{dt} + x = kx^2$$

$$\frac{dv}{dt} - x^{-1} = k$$

$$\frac{dv}{dt} = (k + v)$$

$$\int \frac{dv}{k+v} = \int dt$$

$$\ln k - v = t + C$$

$$k - v = C \cdot e^t$$

$$v = k - C \cdot e^t$$

$$x^{-1} = k - C \cdot e^t$$

$$x = \frac{1}{k - C \cdot e^t}$$

$$1 = \frac{1}{k - C \cdot e^0}$$

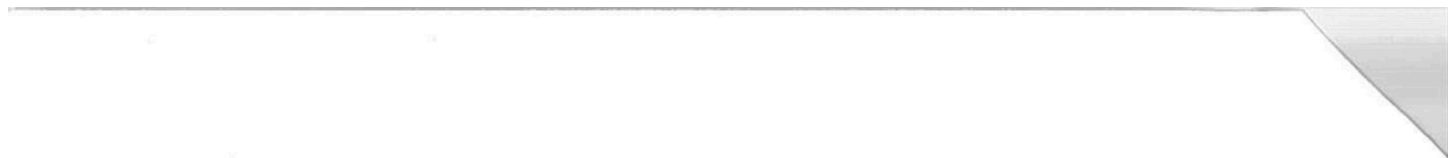
$$k - C = 1$$

$$C = k - 1$$

$$x(t) = \frac{1}{k + (1-k) \cdot e^t}$$

$$x(t \rightarrow \infty) = x(t)$$

$$\text{long term} = \boxed{0}$$



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2. (10 points) Find the explicit general solution of the following ODEs

$$y' = 2(2x + y)^2$$

$$\frac{dy}{dx} = 2(2x + y)^2$$

$$v = 2x + y$$

$$\frac{dv}{dx} = 2 + \frac{dy}{dx}$$

$$y = 2x - v$$

$$\frac{dy}{dx} = +2 \quad \frac{dy}{dx}$$

$$\int \frac{1}{4} \left(\frac{-1}{v+1} + \frac{1}{v-1} \right) dv = \int dx$$

$$+2 + \frac{dv}{dx} = 2 \cdot v^2$$

$$\frac{1}{4} (-\ln|v+1| + \ln|v-1|) = x + C$$

$$\ln\left(\frac{v-1}{v+1}\right) = 4x + C$$

$$\frac{dv}{dx} = 2v^2 + 2$$

$$= 2(v^2 + 1)$$

$$\frac{v-1}{v+1} = C \cdot e^{4x}$$

$$\frac{dv}{2v^2 + 2} = \frac{dx}{1} \quad \frac{dv}{2v^2 + 2} = dx$$

$$v-1 = C \cdot e^{4x} \cdot (v+1)$$

$$v - v \cdot C \cdot e^{4x} = 1 + C \cdot e^{4x}$$

$$\frac{1}{2} \frac{dv}{(v-1)(v+1)} = dx$$

$$v = \frac{1 + C \cdot e^{4x}}{1 - C \cdot e^{4x}}$$

$$1 = A(v-1) + B(v+1)$$

$$y = \frac{1 + C \cdot e^{4x}}{1 - C \cdot e^{4x}} - 2x$$

$$1 = Av - A + Bv + B$$

$$1 = v(A+B) + (B-A)$$

$$B = \frac{1}{2}$$

$$1 = B - A$$

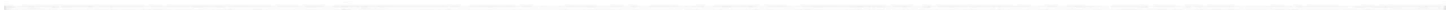
$$A = -\frac{1}{2}$$

$$A = -B$$

$$1 = B + B$$

$$1 = 2B$$

$$y = \frac{1}{2} \tan^{-1}\left(\frac{2x+y}{1}\right) + C$$



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3. (10 points) Consider the following ODE

$$x' = ax - x^3$$

By tuning the constant parameter a , it is possible to modify the stability of the system. What is the range of values of a for which there will be a non-zero stable equilibrium point in the system?

$$\frac{dx}{dt} = ax - x^3$$

Critical points:

$$0 = ax - x^3$$

$$0 = x(a - x^2)$$

$$x = 0$$

$$a - x^2 = 0$$

$$a = x^2$$

$$f'(x) = a - 3x^2$$

$$f' < 0 \Rightarrow \text{stable}$$

$$a - 3x^2 < 0$$

$$\begin{aligned} x^2 - 3x^2 &< 0 \\ &> 0 \\ -2x^2 &< 0 \end{aligned}$$

$$x^2 > 0 \text{ for all } x \text{ except } 0$$

$$\Rightarrow -2x^2 < 0 \text{ for all } x \text{ except } 0$$

so

$$a = x^2$$

$$a = 0^2$$

$$\Rightarrow a \neq 0$$

For all
 $a = x^2$
with $a \neq 0$



4. (20 point) The driver in a car traveling at 10 ms^{-1} begins applying the break which reduces the speed at a rate proportional to time t . If the speed drops to 5 ms^{-1} in 10 s, how far the car has traveled within that time?

$$V_i = 10 \text{ m/s} \quad t = 0$$

$$V_f = 5 \text{ m/s} \quad \Delta t = 10 \text{ s}$$

$$a(t) = -t \cdot k \quad \leftarrow \text{constant}$$

$$v(t) \sim t$$

$$v(t) = \frac{-t^2}{2} \cdot k + V_0$$

$$v(t) = \frac{-t^2}{2} \cdot k + 10$$

$$x(t) = \frac{-t^3}{3} \cdot k + 10t + x_0 \quad \leftarrow 0$$

$$x(t) = \frac{-t^3}{3} \cdot k + 10t$$

$$5 = -\frac{10^2 \cdot k}{2} + 10$$

$$-5 = -50 \cdot k$$

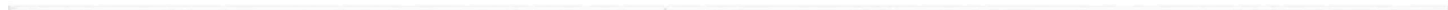
$$k = \frac{1}{10}$$

$$x(10) = 100 - \frac{100}{3}$$

$$x(10) = 66.7 \text{ m}$$

$$x(t) = \frac{-t^3}{3} \cdot \frac{1}{10} + 10t$$

$$x(10) = \frac{-1000}{3 \cdot 10} + 10 \cdot 10$$



Useful Integrals

$$\int u dv = uv - \int v du$$

$$\int u^n du = \frac{1}{n+1} u^{n+1} + C \text{ if } n \neq -1$$

$$\int \frac{du}{u} = \ln |u| + C$$

$$\int e^u du = e^u + C$$

$$\int a^u du = \frac{a^u}{\ln a} + C$$

$$\int \sin u du = -\cos u + C$$

$$\int \cos u du = \sin u + C$$

$$\int \sec^2 u du = \tan u + C$$

$$\int \csc^2 u du = -\cot u + C$$

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1} \frac{u}{a} + C$$

$$\int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1} \frac{u}{a} + C$$

$$\int \frac{du}{a^2 - u^2} = \frac{1}{2a} \ln \left| \frac{u+a}{u-a} \right| + C$$

$$\frac{dy}{dx} = 2(2x+y)^2$$

$$v = 2x + y$$

$$\frac{dv}{dx} = 2 + \frac{dy}{dx}$$

$$-2 + \frac{dy}{dx} = 2 \cdot 2v^2$$

$$\frac{dv}{dx} = 2 + 2v^2$$

$$\frac{dv}{dx} = 2 + \frac{dv}{dx}$$

$$dy$$

$$\frac{dv}{dx} = \frac{dv}{dx} - 2$$

$$\frac{dx}{dt} + x - kx^2 = 0$$

$$\frac{dx}{dt} + x = kx^2$$

$$\frac{dx}{dt} = kx^2 - x$$

$$\frac{dx}{kx^2 - x} = dt$$

$$\frac{1}{x(kx-1)} = dt$$

$$\int \frac{k}{kx-1} + \frac{-1}{x} = dt$$

$$1 = Ax + B(kx-1)$$

$$\ln t - 1 - \ln x = t + c$$

$$1 = Ax + Bkx - B$$

$$\ln \frac{kx-1}{x} = t + c$$

$$1 = x(A+Bk) + (-B)$$

$$\frac{kx-1}{x} = e^{t+c}$$

$$B = -1$$

$$x e^{t+c} = kx - 1$$

$$0 = A + Bk$$

$$1 = kx - x \cdot e^{t+c}$$

$$0 = A - k$$

$$1 = x(k - e^{t+c})$$

$$A = k$$

$$\frac{1}{k - e^{t+c}} = x$$