# MATH347.003.FA23 Midterm 1

#### Sara Huston

TOTAL POINTS

### 31 / 35

**QUESTION 1** 

Q1 10 pts

1.1 a 2 / 2

 $\sqrt{+2}$  pts Computed the dot product to be 12

1.2 b 3 / 3

√ + 1 pts Found a unit vector \$\$\vec{w}\$\$
perpendicular to \$\$\vec{u}\$\$ one possible choice is:
\$\$(-2/ \sqrt{5}, 1/\sqrt{5})\$\$
√ + 1 pts Found a vector \$\$\vec{w}\$\$ perpendicular

to  $$$\vec{u}$$  one possible choice is: \$\$(-2, 1)\$\$  $\checkmark + 1$  pts Set up the dot product for  $$$\vec{w} = (w_1, w_2)$$  being  $$$\vec{w} \cdot \cdot \cdot \vec{u} = 0$$ \$

1.3 C 1 / 1

 $\sqrt{+1}$  pts Found correct condition of: \$\$m=k\$\$

1.4 d 2 / 2

√ + 2 pts Determined the matrix product to be:

\$\$\begin{bmatrix}

21 & 5 & 3511

17 & 5 & 29 \\ 18& 6 & 15

\end{bmatrix}\$\$

1.5 e 2 / 2

√ + 2 pts Compute correct solution of 0

**QUESTION 2** 

Q2 9 pts

2.1 a 4/4

√ + 1 pts Compatible \$\$A\$\$ Matrix

√ + 1 pts Compatible \$\$B\$\$ Matrix

Showed \$\$AB \neq BA\$\$

√ + 1 pts Computed \$\$AB\$\$ Correctly

√ + 1 pts Computed \$\$BA\$\$ Correctly

2.2 b 3 / 3

Using Matrix Inverse  $\$A = B^{-1}$ 

√ + 1 pts Identified that the desired matrix \$\$A\$\$ is

the inverse of \$B\$ i.e.  $$A = B^{-1}$ \$

√ + 1 pts Used the formula for finding the inverse of

a \$\$2 \times 2\$\$ matrix: \$\$\frac{1}{det(B)}\$\$

\$\$\begin{bmatrix} 4 & -3\\ -1 & 1 \end{bmatrix}\$\$

√ + 1 pts Correct final answer: \$\$\begin{bmatrix} 4

& -3\\ -1 & 1\end{bmatrix}\$\$

2.3 C 0 / 2

√ + 0 pts Incorrect

**OUESTION 3** 

3 O3 9 / 9

 $\checkmark$  + 9 pts Computed correctly: \$\$A^{-1} =

\begin{bmatrix}

1/2 & -1/2 & -1/2 & 1/2 \\

-1/2 & 1/2 & -1/2 & 1/2 \\

```
-1/2 & -1/2 & 1/2 & 1/2 \\
1/2 & 1/2 & 1/2 & -1/2 \\
\text{end{bmatrix} $$
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QUESTION 4

4 Q4 4 / 4

√ + 1 pts Wrote the system of linear equations in matrix form

√ + 1 pts First Correct Row Operation

√ + 1 pts Second Correct Row Operation

 $\sqrt{+1}$  pts Correct answer: \$\$q = 4\$\$

**QUESTION 5** 

Q5 3 pts

5.1 a 1 / 1

√ + 1 pts Correct

5.2 **b** 0 / 1

√ + 0 pts Incorrect

5.3 **C 0 / 1** 

√ + 0 pts Incorrect

MATH 347-003 Oct 2, 2023

# Midterm 1

Time Limit: 50 minutes Instructor: Shiying Li

Print your first and last name neatly in the box below, and print your PID neatly in the box at the bottom of every page.

Name:

Sara Huston

### Please read the following directions carefully:

- This exam contains 5 pages (including this cover page) and 5 questions. The point total is 35 points. You have 50 minutes to complete this exam. Manage your time carefully.
- Make sure the order of pages of your exam papers is correct when turning in.
- This is a closed-book closed-note, individual exam. No other aids are allowed, i.e. no calculators, no internet, no textbook etc.
- To receive full credit, please show all relevant work. Partial credit will be for any correct work that is presented.
- Only work written on the exam sheet in the designated space for each question will be graded.
- You are not allowed to use your own scratch paper. You may get additional scratch paper from the instructor, if you need it.
- If you have extraneous scratch work in the designated answer space, please make sure to mark out anything you don't want graded.
- Please sign and date the honor pledge below. Your exam cannot be graded if you do not sign below.

## Honor Pledge:

I have read and understood the directions on the previous page, and I certify that I have neither given nor received any unauthorized assistance on this exam. Further, I pledge that my conduct on this exam is in full compliance with UNC's Honor Code.

Signature: 10/02/23

> 730459812 PID:

Date:

- 1. Vectors and matrices.
  - (a) (2 points) Calculate the dot product  $\mathbf{u} \cdot \mathbf{v}$ , where  $\mathbf{u} = (1, 2)$  and  $\mathbf{v} = (2, 5)$ .

$$U_1 \circ V_1 + W_2 \circ V_2 =$$

$$1 \cdot 2 + 2 \cdot 5 = 12$$

$$2 + 10 = 12$$

(b) (3 points) Find a unit vector w that is perpendicular to u.

$$||u|| = \sqrt{||z||^2 + 2^2} = \sqrt{5}$$
 change sign  
 $W = \left(\frac{2}{\sqrt{5}}, \frac{-1}{\sqrt{5}}\right)$  3 Plip

(c) (1 point) Matrix A has n rows and m columns, and matrix B has k rows and l columns. What is the condition on n, m, l, k for which the matrix product AB is well-defined (i.e., is legal and can be calculated).

$$m = k$$

(d) (2 points) Calculate the matrix product  $\begin{bmatrix} 1 & 3 & 6 \\ 2 & 1 & 5 \\ 3 & 0 & 0 \end{bmatrix} \begin{bmatrix} 6 & 2 & 5 \\ 5 & 1 & 4 \\ 0 & 0 & 3 \end{bmatrix}$ 

(e) (2 points) Calculate  $\begin{bmatrix} 2 \\ 3 \end{bmatrix}^T \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ .  $\begin{bmatrix} 2 & 3 \end{bmatrix}$  2 1 by 2 2 by 2

$$\begin{bmatrix} 23 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} -3 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \xrightarrow{+b=0}$$

#### 2. Matrix rules.

(a) (4 points) Provide an example that demonstrates that matrix multiplication is not commutative. That is, write two matrices A and B for which  $AB \neq BA$ . Then calculate AB and BA.

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} B = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 4 & 2 \end{bmatrix}$$

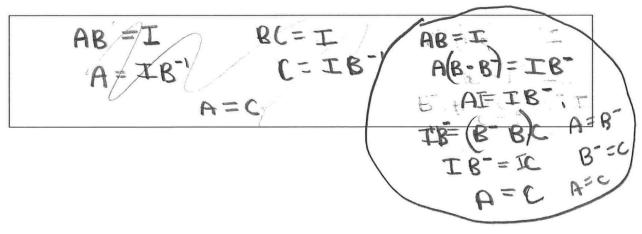
$$BA = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix}$$

(b) (3 points) If  $B = \begin{bmatrix} 1 & 3 \\ 1 & 4 \end{bmatrix}$ , find a  $2 \times 2$  matrix A that satisfies AB = I.

$$\frac{1}{4-3} \begin{bmatrix} 1 & 3 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 1 & 0 \\ 0 & 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 1 & 0 \\ 0 & 1 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 1 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 1 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 1 & -1 & 1 \end{bmatrix}$$

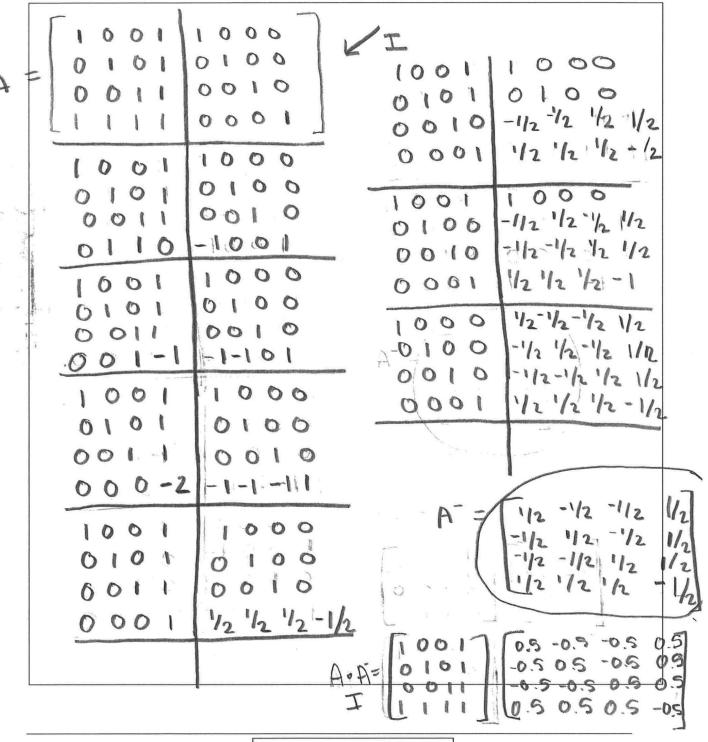
(c) (2 points) If AB = I = BC, use the associative law to prove that A = C.



3. (9 points) Calculate the inverse matrix  $A^{-1}$  for the  $4 \times 4$  matrix.

$$A = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}.$$

Show also the elimination steps of the augmented matrix.



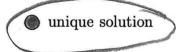
4. (4 points) Consider the equations

$$x+4y-2z = 1$$
$$x+7y-6z = 6$$
$$3y-qz = 5$$

Which number q makes this system singular?

- 5. Consider a system of 2 equations in 3 unknowns. Answer the following multiple choice problems. (Completely fill in the bubbles beside the correct choices.)
  - (a) (1 point) Which of the following describes the row picture
    - O the solution lies in the intersection of three lines
- the solution lies in the intersection of two planes
- (b) (1 point) Which of the following describes the column picture: the solution is the weights in a linear combination of column vectors of the coefficient matrix that produces
  - $\bigcirc$  a two-dimensional RHS vector
- a three-dimensional RHS vector
- (c) (1 point) What are the possible cases for solutions of this linear system? Check all that apply.

no solution



more than one solution

Don't forget to sign the Honor Pledge!

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