Math 381H Test 1

Sara Patricia Huston

TOTAL POINTS

60.25 / 77

QUESTION 1

1 rewrite expression **7 / 7**

✓ **- 0 pts**

`![1.jpeg](/files/392ed0e3-c549-4c35-bd2a-e582bde78cfa)`*Correct*

QUESTION 2

2 valid proof method **5.25 / 7**

✓ **- 1.75 pts** *Click here to replace this description.*

QUESTION 3

3 truth table **7 / 7**

✓ **- 0 pts**

`![3.jpeg](/files/76611d28-530d-43c4-afa6-b2d6c9f6cb66)`*Correct*

QUESTION 4

4 true or false **10 / 10**

✓ **- 0 pts**

`![p4.jpeg](/files/470e142e-8637-4e02-986d-837746716e82)`*Correct*

QUESTION 5

5 truth value of p **10 / 10**

✓ **- 0 pts**

`![5.jpeg](/files/c7725f34-30c5-45f5-b683-8415c282a927)`*Correct*

QUESTION 6

6 logical equivalence **12 / 12**

✓ **- 0 pts**

`![6.jpeg](/files/e7c80a6c-e2f8-40be-9265-59685260a2bf)`*Correct*

QUESTION 7

7 Induction Proof **6 / 12**

✓ **- 6 pts** *see me for details*

QUESTION 8

8 If n is prime >3, then n + 2 and n + 4 cannot both be prime **3 / 12**

✓ **- 9 pts** *see me for details*

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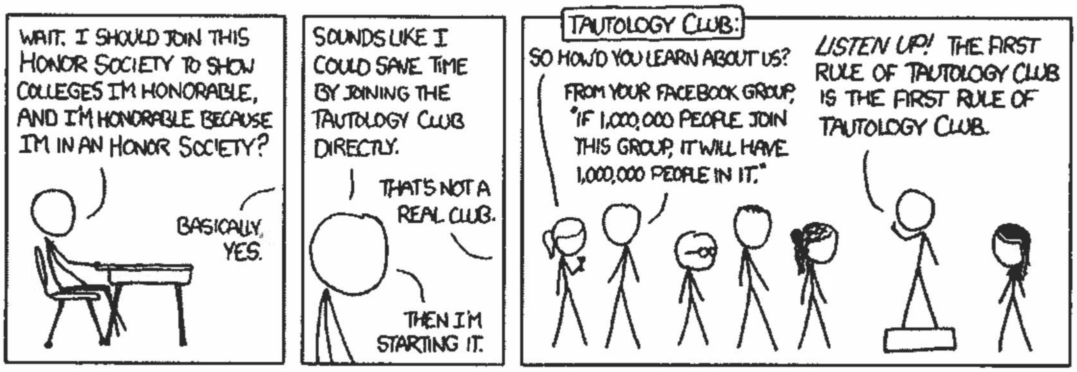

1. There are 8 questions on the test, worth a total of 77 points.
2. No credit will be given for correct answers without supporting work and/or explanation.
3. Final answers must be clearly indicated. All supporting work must be legible and shown on these test pages.
4. All proofs must be written using correct notation and complete sentences where appropriate.

Sign the Honor Pledge

I have followed the guidleines listed on the Exam Instructions Sheet.
I have neither given nor received any unauthorized help on this exam, and
I have conducted myself within the guidelines of the University Honor Code.

Pledge: ______

You may assume the following statements are true.

Every positive integer greater than 1 is either prime, or has at least one prime divisor.

Let m and n be integers.

m	n	$m \pm n$	$m \cdot n$
even	*even*	*even*	*even*
even	*odd*	*odd*	*even*
odd	*even*	*odd*	*even*
odd	*odd*	*even*	*odd*

Let x and y be real numbers.

x	y	$x \pm y$	$x \cdot y$	$\frac{x}{y}$
rational	*rational*	*rational*	*rational*	*rational*, provided $y \neq 0$
rational	*irrational*	*irrational*, provided $x \neq 0$	*irrational*, provided $x \neq 0$	*irrational*, provided $x \neq 0$
irrational	*irrational*	*irrational*, provided $x \neq -y$		

Let n be an integer.
n is even if and only if n^2 is even.
n is odd if and only if n^2 is odd.

Every rational number is can be written in the form $r = \frac{m}{n}$, where m and n are not both even, i.e., r can be *reduced to lowest terms*.

$\sqrt{2}$ is irrational.

7 points

1. Let w, r, and m be meaningful statements. Rewrite the following **without using the negation operator**.

$$\neg w \to (\neg r \vee \neg m)$$

$\neg(\neg r \vee \neg m) \to w$

$(r \wedge m) \to w$

7 points

2. Suppose p and q are meaningful statements. Which of the following represent a valid method to prove that $p \to q$ is true.

I Show that	**II** Show that	**III** Assume that	**IV** Assume that
$\neg p \to \neg q$ is false.	$\neg(\neg p \vee q)$ is false.	$\neg q \to \neg p$ is true.	$\neg p \vee q$ is false.

maybe

inverse

contrapositive

$p \to q$

$p \to q$
$T \to F$

- ○ ~~I only~~
- ○ II only
- ○ III only
- ● IV only
- ○ I and II only
- ○ I and III only
- ○ ~~II and IV only~~
- ○ all of them
- ○ none of them

$q \to p$ $\quad$ $p \wedge \neg q$ $\quad$ $\neg(p \to q)$

assume $p \to q$ $\quad T \to F$ $\quad$ $\neg(p \to q)$ $\quad T \to F$

show that its true bc it can't be false

7 points

3. Complete the truth table to answer the given question.

M	N	$\neg M$	$\neg N$	$\neg(M \to (\neg N)) \equiv ((\neg M) \to N)$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

$\neg(\neg M \vee \neg N)$ $\quad M \vee N$

$(M \wedge N) \equiv$ $\quad M \vee N$

How many rows yield an outcome of FALSE for the statement $\neg(M \to (\neg N)) \equiv ((\neg M) \to N)$?

- ○ exactly one
- ● exactly two
- ○ exactly three
- ○ all of them
- ○ none of them

$F = T \to T$

$F \neq T$

$F \equiv T \to F$

$F \equiv F$

$\neg(T \to (\neg T)) \equiv ((\neg T) \to F$

$(T \to (F)) \equiv$ $\quad F \to F$

$\neg(F)$ $\quad T$

$T \equiv T$

$\neg(T \to (\neg F)) \equiv (\neg T \to F)$

$(T \to T)$ $\quad F \to F$

$\neg(T)$ $\quad T$

$F \not\equiv T$

10 points

4. Let m and n be positive integers. Determine whether the given statement is true or false. If it is true, give a proof. If it is false, give a specific counterexample.

If $n^3 + n$ is odd, then $\left(\frac{n}{m}\right)^2 = 2$

$n^3 + n = 2k + 1$

two cases:

$n =$ odd or $n =$ even

odd^3 + odd

odd + odd

even $\neq$ odd

even^3 + even

even + even

even $\neq$ odd

Thus, since n^3+n can never be odd, i.e. it is always False, the statement is vacuously True

$F \to$ anything

$\underbrace{\phantom{F \to \text{anything}}}_{\text{always True}}$

$n = 2k+1$

$(2k+1)^3 + 2k+1$

$(2k+1)(4k^2+4k+1) + 2k+1$

$8k^3 + 8k^2 + 4k^2 + 2k + 4k + 1 + 2k + 1$

$2(4k^3 + 6k^2 + 4k + 1)$ integer divisible by 2

$n = 2k$

$(2k)^3 + 2k$ $\quad 2(4k^3 + k)$

$8k^3 + 2k$

integer divisible by 2

10 points

5. Suppose m, k, g, and p are meaningful statements such that each of the following is true.

I. m II. $\neg k \to \neg m$ III. $k \to g$ IV. $p \vee g$

Determine the truth-value of statement p. You must show supporting work.

- ○ Statement p must be true.
- ○ Statement p must be false.
- ● Not enough information to determine the truth-value of statement p.

$m = T$ $\quad \neg k \to \neg m = T$ $\quad k \to g = T$ $\quad p \vee g = T$

1. Since $m \equiv T$, for $\neg k \to \neg m \equiv T$, $\neg k \to \neg T \equiv \neg k \to F$

k must be T

2. Since $k \equiv T$, for $k \to g \equiv T$, $T \to g$

g must be T

3. Since $g \equiv T$, for $p \vee g \equiv T$, $p \vee T \equiv T$

thus, the p value is unknown

12 points

6. Let p, q, r, and w be meaningful statements.

Determine which of the following are logically equivalent to the statement $p \to (q \wedge r \wedge w)$.

You must show supporting work.

I $p \vee (\neg q \wedge \neg r \wedge \neg w)$ II $((q \wedge r) \to \neg w) \to \neg p$ III $(p \wedge \neg r) \to (w \vee q)$

- ○ I and III only
- ○ I only
- ○ II and III only
- ○ III only
- ○ I and II only
- ● II only
- ○ ~~all of them~~
- ○ none of them

I. $p \vee (\neg q \wedge \neg r \wedge \neg w)$

$\neg p \to (\neg q \wedge \neg r \wedge \neg w)$

Not equivalent

$p = T$
$Q = F$
$R = F$
$W = F$

$\neg T \to (\neg F \wedge \neg F \wedge \neg F)$

$F \to T$

T

$T \to (F \wedge F \wedge F)$

$T \to F$

F

II. $((q \wedge r) \to \neg w) \to \neg p$

$(\neg(q \wedge r) \vee \neg w) \to \neg p$

$p \to \neg(\neg(q \wedge r) \vee \neg w)$

$p \to (q \wedge r \wedge w)$

equivalent

III. $(p \wedge \neg r) \to (w \vee q)$

$p = T$ $w = T$
$r = F$ $q = T$

$(T \to \neg F) \to (T \vee T)$

$(T \to T) \to T$

$T \to T$

T

Not equivalent

$T \to (T \wedge T \wedge F)$

$T \to F$

F

12 points

7. Use Mathematical Induction to prove the given statement.

If n is a positive integer, then $n^3 - n$ is divisible by 6.

$n > 0$

1. Basis step

$$\frac{n^3-n}{6} = \frac{1^3-1}{6} = \frac{0}{6} = 0$$

integer thus, divisible by 6

2. Induction hypothesis

For some specific integer $k \geq 1$,

$k^3 - k = 6L \qquad k^3 = 6L + k \qquad k = k^3 - 6L$

3. Induction step

$(k+1)^3 - (k+1)$

$(k+1)(k^2+2k+1) - k - 1$

$k^3 + 2k^2 + k + k^2 + 2k + 1 - k - 1$

$k^3 + 3k^2 + 2k$

$k(k^2 + 3k + 2)$

$k(k+2)(k+1)$

$3!$

$3 \cdot 2 \cdot 1$

$= 6$

divisible by 6

Thus, by induction on n, If n is a positive integer, then $n^3 - 6 = 6L$

12 points

8. Let n be an integer, with $n > 3$. Prove the given statement.

If n is prime, then $n+2$ and $n+4$ cannot both be prime.

$n = 2k+1$

$2k+1+2$ $2k+1+4$

$2k+3$ $2k+5 = 2L+1$

$2k+3 = 2T+1$

$n = 2k$

$2k+2 =$ even

proof by contradiction $T \to F$

T $n = 2k+1$ $n+2 =$ $n+4$

T $2k+1+2$ $2k+1+4$

must be odd to be prime or else it can be divided by 2, not 2 since $n>3$

$2k+3$ $2k+5$

example of both being prime is

both can be prime

$k = 7$ $14+5$

$14+3$ 19

17

$((\neg q \lor \neg r) \lor \neg w)$

$q \land r \land w$

$T \to F$

$T \longrightarrow F$

$T \to F$

$p = T$

$T \to F$

F

F