Midterm #2 • Graded

Student

Sara Huston

Total Points

60 / 60 pts

Question 1

Question 1 10 / 10 pts

1.1 Question 1 Part a 5 / 5 pts

- ✓ 0 pts Correct
 - 1 pt small error
 - 2.5 pts incorrect roots but correct method
 - + 41 pts (make up/ars grading): total points out of 60
 - + 52.5 pts (make up/ars grading): total points out of 60
 - + 49 pts (make up/ars grading): total points out of 60
 - + 53.5 pts (make up/ars grading): total points out of 60
 - + 47.5 pts (make up/ars grading): total points out of 60

1.2 — Question 1 Part b **5** / 5 pts

- ✓ 0 pts Correct
 - 2 pts minor error
 - 5 pts minimal progress
 - + 1 pt small bonus modifier
 - 5 pts (make up/ars grading): total points out of 60 given with 1a
 - 3 pts major error

Question 2 20 / 20 pts

2.1 Question 2 Part a 10 / 10 pts

- ✓ 0 pts Correct
 - 0.5 pts small error
 - 3 pts minor error
 - 5 pts major error
 - **6 pts** limited progress or on the wrong track
 - + 1 pt small bonus modifier
 - 0.5 pts small error modifier
 - 10 pts (make up/ars grading): total points out of 60 given with 1a

2.2 Question 2 Part b 10 / 10 pts

- ✓ 0 pts Correct
 - 1 pt small error
 - 4 pts medium error
 - 6 pts major error
 - 8 pts minimal progress
 - + 1.5 pts small bonus modifier
 - 10 pts (make up/ars grading): total points out of 60 given with 1a

Question 3

Question 3 15 / 15 pts

- ✓ 0 pts Correct
 - 1 pt \lambda=0 is gives non-trivial solution
 - 1 pt eigenvalues for \lambda>0 should be n^2/4
 - 3 pts eigenvalue incorrect
 - **1 pt** eigenfunction incorrect
 - 2 pts all possible cases of \lambda not considered
 - **5 pts** incomplete and/or major error solution
 - **5 pts** did not find eigenvalues for non-trivial solution
 - **10 pts** no serious attempt
 - 2 pts incorrect final solution and unclear steps
 - 15 pts (make up/ars grading): total points out of 60 given with 1a

Question 4 15 / 15 pts

- ✓ 0 pts Correct
 - 0.5 pts needed to write 4-\alpha^2 inside square root if writing gen. sol. with Euler's identity
 - 1 pt small mistakes in general solution
 - **3 pts** general solution incorrect
 - 0.5 pts one of two constants in the general solution evaluated incorrectly
 - **1 pt** did not determine constants in general solution
 - **1 pt** constants in particular solution incorrect
 - 3 pts some steps correct but incomplete with insufficient work
 - **1 pt** seems to be correct method but steps very hard to follow
 - 1 pt decaying oscillation for 0<\alpha<2</p>
 - **2 pts** second part incorrect with little work shown
 - 3 pts second part incorrect with no work shown
 - **5 pts** insufficient work
 - 8 pts no serious attempt
 - 15 pts (make up/ars grading): total points out of 60 given with 1a
 - 1 pt no work shown in second part. how did you get it?
 - 15 pts no attempts

Math 383.004: Exam 2

Oct 28 2024

Instructor: Saiful Tamim

- Calculators are NOT allowed.
- Show as much work as possible for full and partial credit.
- All work must be clear and readable.
- Use proper mathematical notation.

Last Name, First Name: HUSTON

- Clearly indicate your answer, e.g., by boxing it.
- All differential equations are ordinary and standard notations are assumed.
- Sign the honor pledge below <u>after</u> completing the exam.

PID: 130459812		200	
Honor Pledge: I have neither given r	nor received unautho	rized help on t	this exam.
Signature:			

1. (10 pts) Find the homogeneous solution to the following ordinary differential equations in terms of real functions.

(a)
$$x'' - 5x' + 6x = 0$$

$$y'' = 2x' + 6x = 0$$

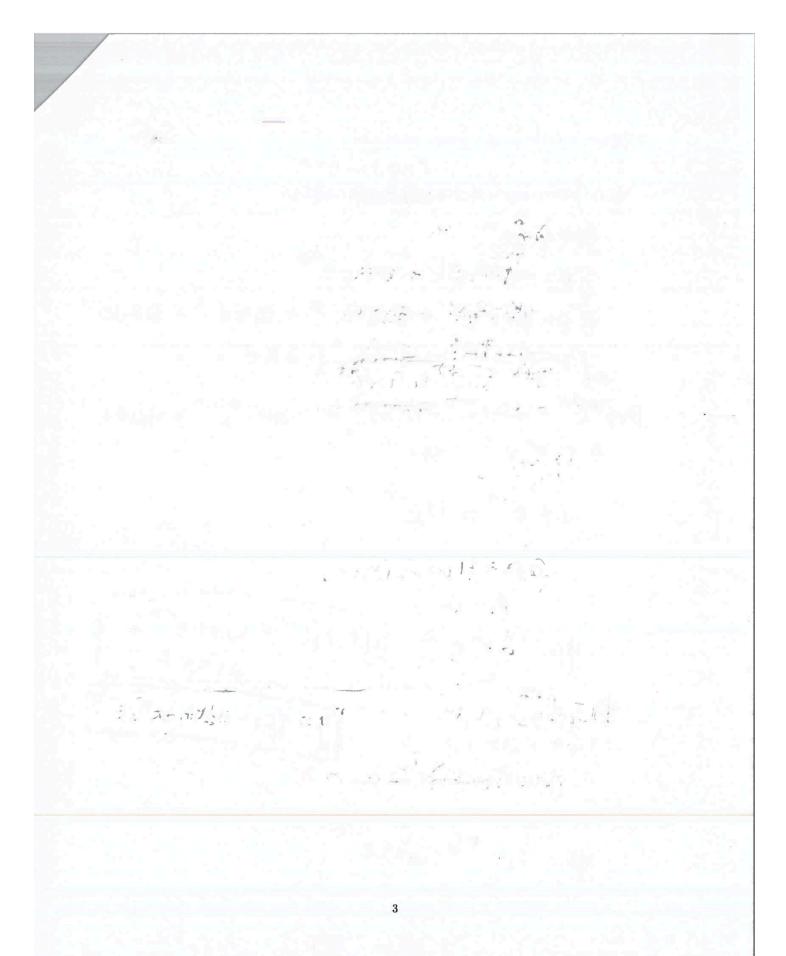
$$(x - 3)(x - 2) = 0$$

$$(x$$

(b)
$$y^{(4)} + 2y^{(2)} + y = 0$$
 $y = e^{rx}$
 $y + 2x^{2} + 1 = 0$
 $(r^{2} + 1)(r^{2} + 1) = 0$
 $r^{2} = -1$
 $r = \pm 1$

$$y(x) = c_1 \cos x + c_2 \sin x + c_3(x) \cos x$$

$$+ c_4(x) \sin x$$

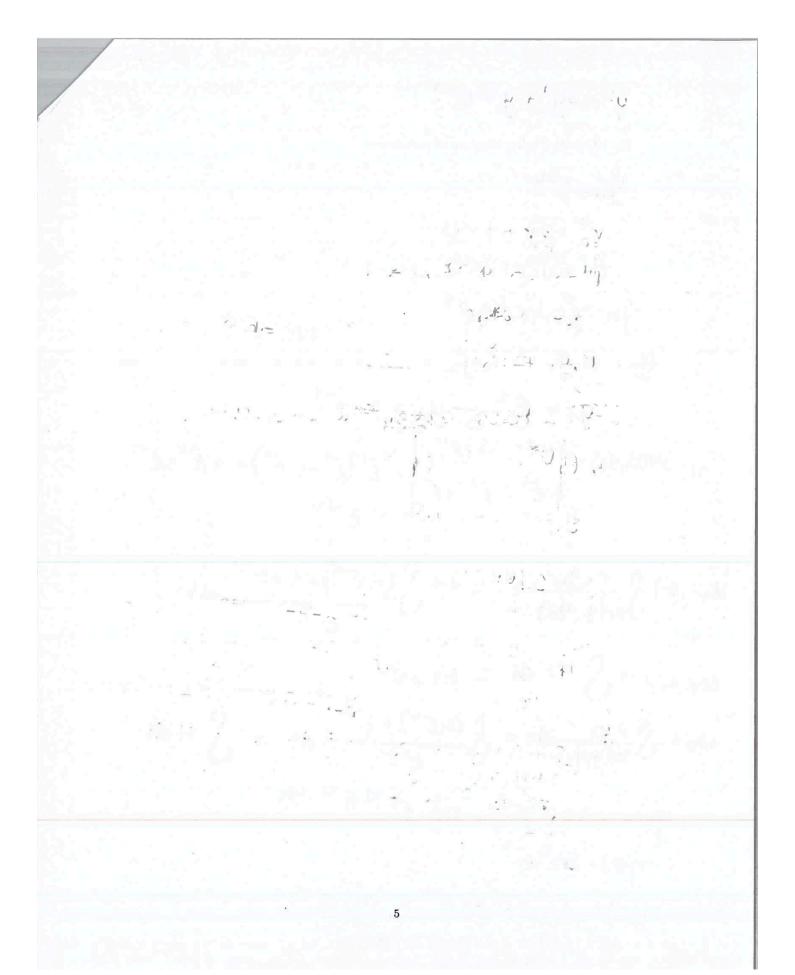


2. (20 pts) Find the complete general solution for

$$y'' + 2y' + y = 4e^{-t}$$

(a) (10 points) using method of undertermined coefficients

$$y_p = Ate^{-t}$$
 $y'p = -At^ae^{-t} + aAte^{-t}$
 $y''p = At^ae^{-t} - 2Ate^{-t} + aAe^{-t}$
 $y''p = At^ae^{-t} - 4Ate^{-t} + aAe^{-t}$
 $At^ae^{-t} = 4e^{-t}$
 $At^ae^{-t} = 4e^{-t}$



(b) (10 points) using variation of parameters

$$y_{h} = e^{vt}$$

$$y_{h} = e^{vt}$$

$$y_{h} = (c_{1} + c_{2} + t) = 0$$

$$y_{h} = (c_{1} + c_{2} + t) = 0$$

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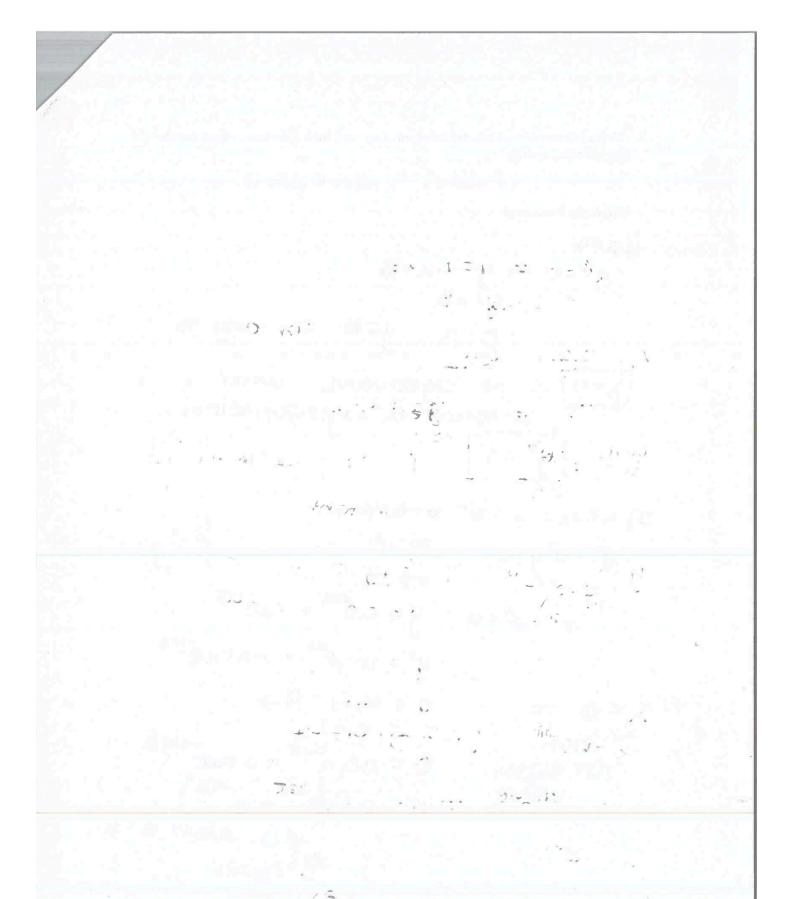
$$y_{h} = (c_{1} + c_{2} + t) = 0$$

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$$y_{h} = (c_{1} + c_{2} + t) = 0$$

$$y_{h} = (c_{1}$$

$$yp = (-)at^{2}e^{-t} + 4t^{2}e^{-t}u_{2} = 4t$$
 $yp = at^{2}e^{-t}$
 $ylt) = (c_{1} + c_{2}t)e^{-t} + at^{2}e^{-t}$
 $ylt) = (c_{1} + c_{2}t + at^{2})e^{-t}$



3. (15 pts) Determine all non-trivial eigenvalues and their associated eigenfunctions for the following system,

$$y'' + \lambda y = 0$$
 $y'(0) = 0, \quad y'(2\pi) = 0$

where λ is a constant.

$$y''=0 \Rightarrow y=Ax+B$$

 $y'=0 \Rightarrow y=Ax+B$
 $0=A$ $y=B$ for any B

[\ = 0] is an eigenvalue with associated eigenfunction

2)
$$\lambda < 0$$
: $\lambda = -\alpha^3$ $\alpha = (constant)$
 $y'''' - \alpha^2 y = 0$
 $y = e^{rx}$
 $y'' = \alpha^2 + cae^{\alpha x}$
 $y'' = \alpha c_1 e^{\alpha x} + -\alpha c_2 e^{\alpha x}$
 $\lambda < 0$ is

 $0 = \alpha_1 c_1 - \alpha c_2$
 $0 = \alpha c_1 e^{\alpha x} - \alpha c_2 e^{\alpha x}$
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$$= \alpha \, c_{3}(e^{\alpha 3\pi} - e^{-\alpha \pi})$$

$$\neq 0 \quad \forall n \in S \quad \alpha = 0$$

$$\Rightarrow c_{2} = 0$$

$$8 \quad \exists \quad c_{1} = 0$$

3)
$$\lambda 70: \lambda = a^2$$
 $a = constant$

FOV non thiriou solution:

$$a = \frac{n}{2} \left| \lambda = \frac{n^2}{4} \right|$$

$$y^* = \cos \frac{n}{2} x$$

eigen Function

4. (15 pts) Consider the following IVP

$$y'' + 2\alpha y' + 4y = 0$$
 $y(0) = y_0, y'(0) = 0$

where $\alpha > 0$ is a real constant.

(10 pts) Write down the solution for the IVP using Euler's identity.

$$y=e^{\nu x}$$
 $r^{a}+a\alpha r+4=0$

$$y = e^{-\alpha x} \left(c_1 \cos (H - \alpha^2) x + c_2 \sin (H - \alpha^2) x \right)$$

$$y' = -e^{-\alpha x} (y_0 \cos(H - \alpha^2)x + casin(H - \alpha^2)x)$$

(5 pts) Determine the range of values for α for which the above system exhibits exponentially decaying oscillation.

For valves C2-4 40 02 40 C3 L4 040042 decay oscillation (y10005 (4-02)x + y0 SIN(4-02)x keep oscillation V(2007-4.4 must be LO V402-16 decay HQ2-16 40 00° 2 4 H 02062