Printed Last Name, First Name: HUSton, Sava PID: 730459812

Math 528, Section 2, Mid-Term 2

March 4, 2025

Problem 1 (15 pts). Find ranges for the real, constant coefficient c in the linear ODE y'' + 2y' + cy = 0 (where ' = d/dx) so that the basis of solutions consists of the given functions in i), ii), iii) below. In each case, give the parameters λ , a, ω in terms of c.

1,2=-2 ± 122-4c = -1 ± 14-c let 4=exx

i) two independent exponential functions of the form $e^{\lambda x}$

discriminant 70
$$1-C$$
 70 $\lambda_1=-1+\sqrt{1-C}$ $\lambda_2=-1-\sqrt{1-C}$

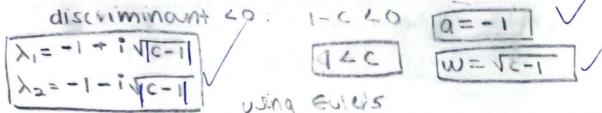
ii) independent functions of the form $x^0 e^{\lambda x}$ and $x e^{\lambda x}$

discreminant = 0:
$$1-c=0$$

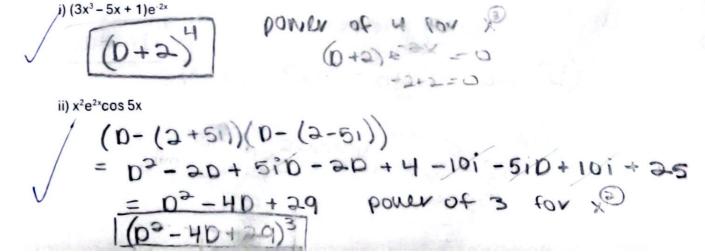
$$C=1$$

$$X=-1$$

iii) independent functions of the form of coses and $e^{ax}sin\omega x$ where a, ω are real.



Problem 2 (10 pts). Give real, constant-coefficient, linear operators for which the following functions are in the null space:



Problem 3 (20 pts) D = d/dx

i) Give the general homogeneous solution to the linear ODE (D 2 - 2D - 8) $y_H = 0$

$$(D-4)(D+2)y_{H}=0$$

 $D=4$
 $D=-2$
 $y_{H}=c_{1}e^{4x}+c_{2}e^{-2x}$

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ii)Use the results of Problem 2 i), convert the non-homogeneous ODE

(*) $(D^2 - 2D - 8)y_p = (3x^3 - 5x + 1)e^{-2x}$ into a higher order, homogeneous ODE for y_p .

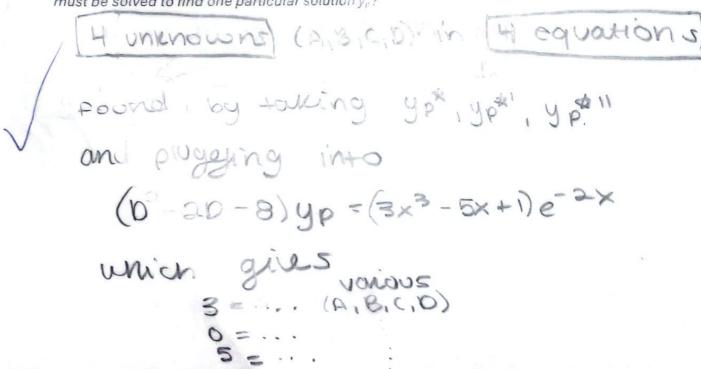
ODE in part ii). Make sure your answer includes the correct number of functions and

unknown constants

$$yp = Ae^{3x} + Bxe^{3x} + Cx^3e^{3x} + Dx^3e^{3x}$$

$$A_1B_1C_1D \in \mathbb{R} \qquad (-3)$$

iv) How many linear equations in how many unknowns will the above guess for y, give that must be solved to find one particular solution y,?



Problem 4 (15 pts). Give a basis of solutions of x'y" + -x y' + by = 0 when: cauchy let y= x": y'= xx" y"=(v)(v-1)x" X = X = (V)(V-1) + (-X) V X - + PX = 0 xr (r2-r-+b)=0 case 1 2 real sol *((+ = - = + + p) = 0 case = 1 real sol, 13-31=0 case1 4= (ci+ (21nx) x1 v(v-2)=0 y=c,x+c2x2 case 3: 2 imaginary V=0,2 = C1+C2 X2 SOIN: V=0,2 Y=C1+C2X2 SOIN: ci)(2 E IR 12-21+1=0 case 2 (r-1)(v-1)=0 y= (c1+(21nx)x' CIICE EIR V, 2 = 1 = ? (C, COSINX + C2 SIN INX) CIICA E TR

Problem 5 (20 pts). i. Use the method of variation of parameters to find the general solution for the 2^{nd} order non-homogeneous ODE $y'' + y = \csc x = r(x)$. (You are allowed to use the general variation of parameters formula.

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let y,=cosx y==sinx

$$W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = \cos^2 x + \sin^2 x = 1$$

$$y_p = u_1(x)y_1 + u_2(x)y_2$$

 $u_1(x) = (-) \begin{cases} v(x)y_2 & dx = (-) \\ v(y_1,y_2) \end{cases} = (-) \begin{cases} cscx & sinx dx = (-) \\ cscx & dx \end{cases}$

Ift U= COSX

 $\frac{du}{dx} = -\sin x dx \implies (-) \begin{cases} (-) \frac{1}{14} du = ||n||u|| = -||n||\cos x||$

uz(x)= S r(x) y, dx = Scocx · cosx dx = S 1 dx = x

YP=Grown In COSX + X SINX

general soin =

correct if NOI was sec X

particular

(3)
$$y = c_1 \cos x + c_2 \sin x + \cos x \ln |\cos x| + x \sin x$$

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Bonus (5 pts). Find the unique solution to this NH ODE give initial data $y(\pi/2)=0$, $y'((\pi/2)=0$. 4= C, COSX + C2SINX + COSXIN/COSX + XSINX 41 = - CISINX + C2COSX - SINX | n | COSX + COSX . () SINX + SINX + XCOSX Int mistake 0 = c, cos(=) + c2 sin(=) + cos(=)in/cos=1 cartier made + = SIN(=) the initial date = ca + II => ca = II il-posed quer fre 0 = - CISIN(\$) + C2(95(\$) - SIN(\$) In (05\$) Solution you had, this was the right conclusion.

+ # cos(#)

+ Underlined underined In 101 = CI=O+ underlined (+5) underine of (12 C1=0) DONOSINU would be

y= Isinx + cosx In 1cosx + xsinx

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Problem 6 (20 pts) The 2nd-order ODE y" + y'=0 does not depend on x. Solve the equation two ways: a. using the fact that the ODE is linear, constant coefficient, and homogeneous. B. using the substitution p(y) = y'. You will need to use the integral formula $\int dy/(a^2-y^2) = arc \sin(y/a) = \sin^{-1}(y/a)$ and the trig formula $\sin(a+b) = \sin a \cos b + \cos a \sin b$

$$\frac{dP}{dy} = 0$$

$$\frac{dP}{dy} =$$