MATH347.003.FA23 Midterm 1

Sara Huston

TOTAL POINTS

31 / 35

QUESTION 1

Q1 10 pts

1.1 a **2 / 2**

✓ **+ 2 pts** *Computed the dot product to be 12*

1.2 b **3 / 3**

✓ **+ 1 pts** *Found a unit vector $$\vec{w}$$ perpendicular to $$\vec{u}$$ one possible choice is: $$(-2/ \sqrt{5}, 1/\sqrt{5})$$*

✓ **+ 1 pts** *Found a vector $$\vec{w}$$ perpendicular to $$\vec{u}$$ one possible choice is: $$(-2, 1)$$*

✓ **+ 1 pts** *Set up the dot product for $$\vec{w} = (w_1, w_2)$$ being $$\vec{w} \cdot \vec{u} = 0$$*

1.3 c **1 / 1**

✓ **+ 1 pts** *Found correct condition of: $$m=k$$*

1.4 d **2 / 2**

✓ **+ 2 pts** *Determined the matrix product to be:*
$$\begin{bmatrix}
21 & 5 & 35
17 & 5 & 29 \\ 18& 6 & 15
\end{bmatrix}$$

1.5 e **2 / 2**

✓ **+ 2 pts** *Compute correct solution of 0*

QUESTION 2

Q2 9 pts

2.1 a **4 / 4**

✓ **+ 1 pts** *Compatible $$A$$ Matrix*

✓ **+ 1 pts** *Compatible $$B$$ Matrix*

Showed $$AB \neq BA$$

✓ **+ 1 pts** *Computed $$AB$$ Correctly*

✓ **+ 1 pts** *Computed $$BA$$ Correctly*

2.2 b **3 / 3**

Using Matrix Inverse $$A = B^{-1}$$

✓ **+ 1 pts** *Identified that the desired matrix $$A$$ is the inverse of $$B$$ i.e. $$A = B^{-1}$$*

✓ **+ 1 pts** *Used the formula for finding the inverse of a $$2 \times 2$$ matrix: $$\frac{1}{det(B)}$$ $$\begin{bmatrix} 4 & -3\\ -1 & 1 \end{bmatrix}$$*

✓ **+ 1 pts** *Correct final answer: $$\begin{bmatrix} 4 & -3\\ -1 & 1\end{bmatrix}$$*

2.3 c **0 / 2**

✓ **+ 0 pts** *Incorrect*

QUESTION 3

3 Q3 **9 / 9**

✓ **+ 9 pts** *Computed correctly: $$A^{-1} =*
\begin{bmatrix}
*1/2 & -1/2 & -1/2 & 1/2 *
*-1/2 & 1/2 & -1/2 & 1/2 *

*-1/2 & -1/2 & 1/2 & 1/2 *

1/2 & 1/2 & 1/2 & -1/2

\end{bmatrix} $$

QUESTION 4

4 Q4 **4 / 4**

✓ **+ 1 pts** *Wrote the system of linear equations in matrix form*

✓ **+ 1 pts** *First Correct Row Operation*

✓ **+ 1 pts** *Second Correct Row Operation*

✓ **+ 1 pts** *Correct answer: $$q = 4$$*

QUESTION 5

Q5 3 pts

5.1 a **1 / 1**

✓ **+ 1 pts** *Correct*

5.2 b **0 / 1**

✓ **+ 0 pts** *Incorrect*

5.3 c **0 / 1**

✓ **+ 0 pts** *Incorrect*

MATH 347-003
Oct 2, 2023

Midterm 1

Time Limit: 50 minutes
Instructor: Shiying Li

Print your first and last name neatly in the box below, and print your PID neatly in the box at the bottom of every page .

Name: Sara Huston

Please read the following directions carefully:

- This exam contains 5 pages (including this cover page) and 5 questions. The point total is 35 points. You have 50 minutes to complete this exam. Manage your time carefully.
- Make sure the order of pages of your exam papers is correct when turning in.
- This is a closed-book closed-note, individual exam. No other aids are allowed, i.e. no calculators, no internet, no textbook etc.
- To receive full credit, please show all relevant work. Partial credit will be for any correct work that is presented.
- Only work written on the exam sheet in the **designated space** for each question will be graded.
- You are not allowed to use your own scratch paper. You may get additional scratch paper from the instructor, if you need it.
- If you have extraneous scratch work in the designated answer space, please make sure to mark out anything you don't want graded.
- Please sign and date the honor pledge below. **Your exam cannot be graded if you do not sign below.**

Honor Pledge:

I have read and understood the directions on the previous page, and I certify that I have neither given nor received any unauthorized assistance on this exam. Further, I pledge that my conduct on this exam is in full compliance with UNC's Honor Code.

Signature:

Date: 10/02/23

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1. Vectors and matrices.

(a) (2 points) Calculate the dot product $\mathbf{u} \cdot \mathbf{v}$, where $\mathbf{u} = (1, 2)$ and $\mathbf{v} = (2, 5)$.

$u_1 \circ v_1 + u_2 \circ v_2 =$

$1 \cdot 2 + 2 \circ 5$

$2 + 10 = 12$

(b) (3 points) Find a unit vector $\mathbf{w}$ that is perpendicular to $\mathbf{u}$.

$||u|| = \sqrt{1^2 + 2^2} = \sqrt{5}$

$w = \left\langle \frac{2}{\sqrt{5}}, \frac{-1}{\sqrt{5}} \right\rangle$

change sign & flip

(c) (1 point) Matrix A has n rows and m columns, and matrix B has k rows and l columns. What is the condition on n, m, l, k for which the matrix product AB is well-defined (i.e., is legal and can be calculated).

n by m k by l

$m = k$

(d) (2 points) Calculate the matrix product $\begin{bmatrix} 1 & 3 & 6 \\ 2 & 1 & 5 \\ 3 & 0 & 0 \end{bmatrix} \begin{bmatrix} 6 & 2 & 5 \\ 5 & 1 & 4 \\ 0 & 0 & 3 \end{bmatrix}$.

3 by 3 3 by 3

$\begin{bmatrix} 21 & 5 & 35 \\ 17 & 5 & 29 \\ 18 & 6 & 15 \end{bmatrix}$

$6+15 = 21$ $4+1=5$

$5+12+18$ $10+4+15$

$12+5=17$ 35 29

(e) (2 points) Calculate $\begin{bmatrix} 2 \\ 3 \end{bmatrix}^T \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix}$.

$[2\ 3]$ 1by2 2by2

1 by 2

$[2\,3] \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = [-3\ 2] \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix} = [0]$

-6
$+6 = 0$

1 by 2 2 by 1

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2. Matrix rules.

(a) (4 points) Provide an example that demonstrates that matrix multiplication is not commutative. That is, write two matrices A and B for which $AB \neq BA$. Then calculate AB and BA.

$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$

$AB \neq BA$

$AB = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 4 & 2 \end{bmatrix}$

$BA = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix}$

(b) (3 points) If $B = \begin{bmatrix} 1 & 3 \\ 1 & 4 \end{bmatrix}$, find a 2×2 matrix A that satisfies $AB = I$.

$\begin{bmatrix} 4 & -3 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

det $\frac{1}{4-3} = \frac{1}{1}$

$$\left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 1 & 4 & 0 & 1 \end{array}\right]$$
$$\left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & 1 & -1 & 1 \end{array}\right]$$
$$\left[\begin{array}{cc|cc} 1 & 0 & 4 & -3 \\ 0 & 1 & -1 & 1 \end{array}\right]$$

$\begin{bmatrix} 4 & -3 \\ -1 & 1 \end{bmatrix}$

(c) (2 points) If $AB = I = BC$, use the associative law to prove that $A = C$.

~~$AB = I$~~
~~$A = IB^{-1}$~~

$BC = I$
$C = IB^{-1}$

$A = C$

$AB = I$
$A(B \cdot B^{-}) = IB^{-}$
$A = IB^{-}$
~~$IB^{-} = (B^{-}B)C$~~
$IB^{-} = IC$
$A = C$

$A = B^{-}$
$B^{-} = C$
$A = C$

3. (9 points) Calculate the inverse matrix A^{-1} for the 4×4 matrix.

$$A = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}.$$

Show also the elimination steps of the augmented matrix.

$$A = \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \swarrow I$$

$$\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & -1 & 0 & 0 & 1 \end{array}$$

$$\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & -1 & -1 & 0 & 1 \end{array}$$

$$\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -2 & -1 & -1 & -1 & 1 \end{array}$$

$$\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1/2 & 1/2 & 1/2 & -1/2 \end{array}$$

$$\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1/2 & -1/2 & 1/2 & 1/2 \\ 0 & 0 & 0 & 1 & 1/2 & 1/2 & 1/2 & -1/2 \end{array}$$

$$\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1/2 & 1/2 & -1/2 & 1/2 \\ 0 & 0 & 1 & 0 & -1/2 & -1/2 & 1/2 & 1/2 \\ 0 & 0 & 0 & 1 & 1/2 & 1/2 & 1/2 & -1 \end{array}$$

$$\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1/2 & -1/2 & -1/2 & 1/2 \\ 0 & 1 & 0 & 0 & -1/2 & 1/2 & -1/2 & 1/2 \\ 0 & 0 & 1 & 0 & -1/2 & -1/2 & 1/2 & 1/2 \\ 0 & 0 & 0 & 1 & 1/2 & 1/2 & 1/2 & -1/2 \end{array}$$

$$A^{-} = \begin{bmatrix} 1/2 & -1/2 & -1/2 & 1/2 \\ -1/2 & 1/2 & -1/2 & 1/2 \\ -1/2 & -1/2 & 1/2 & 1/2 \\ 1/2 & 1/2 & 1/2 & -1/2 \end{bmatrix}$$

$$A \cdot A^{-} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0.5 & -0.5 & -0.5 & 0.5 \\ -0.5 & 0.5 & -0.5 & 0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & -0.5 \end{bmatrix}$$

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4. (4 points) Consider the equations

$$\begin{aligned} x+4y-2z &= 1 \\ x+7y-6z &= 6 \\ 3y-qz &= 5, \end{aligned}$$

Which number q makes this system singular?

1 4 -2 1
1 7 -6 6
0 3 -q 5

1 4 -2 1
0 3 -4 5
0 3 -q = 5

4

5. Consider a system of 2 equations in 3 unknowns. Answer the following multiple choice problems. (Completely fill in the bubbles beside the correct choices.)

(a) (1 point) Which of the following describes the row picture

- ○ the solution lies in the intersection of three lines
- ● the solution lies in the intersection of two planes

(b) (1 point) Which of the following describes the column picture : the solution is the weights in a linear combination of column vectors of the coefficient matrix that produces

- ○ a two-dimensional RHS vector
- ● a three-dimensional RHS vector

(c) (1 point) What are the possible cases for solutions of this linear system? Check all that apply.

- ● no solution
- ● unique solution
- ● more than one solution

Don't forget to sign the Honor Pledge!

ax + by + cz =
dx + fy + gz =

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