Math 381H Final Exam

Sara Patricia Huston

TOTAL POINTS

98.5 / 100

QUESTION 1

1 Problem 1 part 1: digraph **3 / 3**

✓ **- 0 pts**

`![1a.jpeg](/files/e59ae11c-8a25-480a-9b9d-5ffe46c48a53)Correct`

- 0.5 pts Click here to replace this description.

- 1 pts Click here to replace this description.

- 2 pts Click here to replace this description.

- 1 pts Click here to replace this description.

- 3 pts Click here to replace this description.

QUESTION 2

2 Problem 1 part 2: partition **3 / 3**

✓ **- 0 pts**

`![1b.jpeg](/files/e2064333-94a3-4003-9c30-6cf9ec4abb5d)Correct`

- 3 pts Click here to replace this description.

- 2 pts correct answer, wrong reason.

QUESTION 3

3 Problem 2: number of integer solutions **6.5 / 8**

- 0 pts

`![2.jpeg](/files/dacfa103-aa04-462d-834b-7e7655816257)Correct`

✓ **- 1.5 pts** *Click here to replace this description.*

- 8 pts Click here to replace this description.

- 5 pts Click here to replace this description.

- 4 pts Click here to replace this description.

- 6 pts Click here to replace this description.

- 3 pts Click here to replace this description.

- 2 pts Click here to replace this description.

QUESTION 4

4 Problem 3 part 1: True/false **2 / 2**

✓ **- 0 pts**

`![3a.jpeg](/files/c3d4ddc2-0836-4b62-b974-4406d77d0f80)Correct`

QUESTION 5

5 Problem 3 part 2: contrapositive **2 / 2**

✓ **- 0 pts**

`![3b.jpeg](/files/1b649100-2d2d-4cd3-bb85-e52a340e7e93)Correct`

- 1 pts Click here to replace this description.

- 2 pts Click here to replace this description.

QUESTION 6

6 Problem 3 part 3: negation **2 / 2**

✓ **- 0 pts**

`![3c.jpeg](/files/ce3e2b70-d21d-4fc5-9465-7408f7d080a4)`Correct

- **- 1 pts** Click here to replace this description.
- **- 2 pts** Click here to replace this description.

QUESTION 7

7 Problem 4: solve system of congruences **8 / 8**

✓ **- 0 pts**

`![4.jpeg](/files/161aabb2-c5d2-4c2a-a1ae-0ab5acb55a63)`Correct

- **- 5 pts** Click here to replace this description.
- **- 2 pts** Click here to replace this description.
- **- 4 pts** Click here to replace this description.
- **- 8 pts** Click here to replace this description.

QUESTION 8

8 Problem 5: are the statements equivalent? **8 / 8**

✓ **- 0 pts**

`![5.jpeg](/files/47a400a3-b62f-41f8-bc2f-3a9750518ae0)`Correct

- **- 8 pts** Click here to replace this description.
- **- 4 pts** Click here to replace this description.
- **- 3 pts** Click here to replace this description.
- **- 8 pts** this is only one specific case.
- **- 4 pts** what about the case where n is not equal to 1 mod 2?
- **- 4 pts** you need to show statement 1 implies statement 2, and statement 2 implies statement 1

QUESTION 9

9 Problem 6: number of 2 part partitions

8 / 8

✓ **- 0 pts**

`![6.jpeg](/files/f3bb444e-22df-4383-83fa-f43a0e9793db)`Correct

- **- 8 pts** Click here to replace this description.
- **- 1 pts** Click here to replace this description.
- **- 2.5 pts** Click here to replace this description.
- **- 6 pts** Click here to replace this description.
- **- 4 pts** Click here to replace this description.
- **- 0.5 pts** Click here to replace this description.
- **- 0 pts**

`![16.jpeg](/files/d30205d6-5df8-47f3-b386-3610d89c8e48)`

QUESTION 10

10 Problem 7: t-shirts **8 / 8**

✓ **- 0 pts**

`![7.jpeg](/files/62fa31dd-2d8c-4c0a-8a22-16809cdc88c8)`Correct

- **- 4 pts** Click here to replace this description.
- **- 2.5 pts** Click here to replace this description.
- **- 3 pts** Click here to replace this description.
- **- 1 pts** Click here to replace this description.
- **- 0 pts** Click here to replace this description.
- **- 6 pts** Click here to replace this description.
- **- 0.5 pts** Click here to replace this description.

QUESTION 11

11 Problem 8 part 1: which ordered pairs

must be contained in R **4 / 4**

✓ **- 0 pts**

`![8a.jpeg](/files/c7056547-b362-4ad2-8e24-37685b3e37f3)`Correct

- **- 1 pts** Click here to replace this description.
- **- 0.5 pts** Click here to replace this description.
- **- 2 pts** Click here to replace this description.
- **- 3 pts** Click here to replace this description.

QUESTION 12

12 Problem 8 part 2: which ordered pairs cannot be contained in R **4 / 4**

✓ **- 0 pts**

`![8b.jpeg](/files/469f0395-ac43-4979-a806-546a955bc3e3)`Correct

- **- 3 pts** Click here to replace this description.
- **- 2 pts** Click here to replace this description.
- **- 4 pts** Click here to replace this description.
- **- 1 pts** Click here to replace this description.

QUESTION 13

13 Problem 9: proof in Zmod7 **10 / 10**

✓ **- 0 pts**

`![9.jpeg](/files/9bec7017-7be1-45ce-b77b-ceb1837a66a4)`Correct

- **- 5 pts** I don't understand your reasoning
- **- 1 pts** what if the numbers are not consecutive?
- **- 8 pts** Click here to replace this description.
- **- 6 pts** you've only considered one specific case

QUESTION 14

14 Problem 10: contrapositive proof **10 / 10**

✓ **- 0 pts**

`![10.jpeg](/files/1f8ad02d-be69-4872-912c-42df4d038372)`Correct

- **- 1.5 pts** Click here to replace this description.
- **- 3 pts** you haven't proved the contrapositive
- **- 2 pts** Click here to replace this description.
- **- 8 pts** Click here to replace this description.
- **- 10 pts** Click here to replace this description.
- **- 1 pts** Click here to replace this description.
- **- 0.5 pts** what is the formula?

`![14.jpeg](/files/55084e75-4485-43c6-92ed-e419c93dfb49)`

- **- 6 pts** you only considered specific cases

QUESTION 15

15 Problem 11: proof of divisibility by 12 **10 / 10**

✓ **- 0 pts**

`![11.jpeg](/files/5b1d4760-0a02-4fa8-a714-b8218354bc08)`Correct

- **- 1 pts** Click here to replace this description.
- **- 2 pts** Click here to replace this description.
- **- 3 pts** Click here to replace this description.
- **- 4 pts** Click here to replace this description.
- **- 5 pts** you can't assume m+n is divisible by 12
- **- 6 pts** Click here to replace this description.
- **- 7 pts** Click here to replace this description.
- **- 8 pts** Click here to replace this description.
- **- 9 pts** Click here to replace this description.

- 10 pts Click here to replace this description.

- 2 pts how do we know this is true?

`![2.jpeg](/files/2274e784-9587-485b-83c6-3f89a52ba209)`

- 5 pts you can't assume this is true

`![3.jpeg](/files/da50ca40-eb46-4dc5-998b-d5068d48a088)`

- 6 pts divisibility by 2 and 6 does not imply divisibility by 12

- 5 pts Click here to replace this description.

- 6 pts you can't use k in both expressions

`![6.jpeg](/files/bbd896b5-d28a-4690-a9e6-f8c9c3793899)`

- 2 pts how do we know this is true?

`![8.jpeg](/files/3f1f589e-dcac-4219-abac-b749612033d6)`

- 7 pts the contrapositive of the original statement does not allow you to assume this

`![9.jpeg](/files/0c3b409d-4296-478f-a304-4c0a3fadf2c4)`

- 3 pts where does this come from?

`![12.jpeg](/files/7cf0a7b2-c9e0-44a3-a8dd-09cc705f3499)`

- 5 pts where does this come from?

`![13.jpeg](/files/1d18ddc7-be1b-49ec-8461-ef0d7ebdb3a2)`

- 4 pts you need to prove this

`![15.jpeg](/files/b1099d8a-7e26-4781-98e9-60f1b1115aec)`

QUESTION 16

16 Problem 12: proof bit strings **10 / 10**

✓ **- 0 pts**

`![12.jpeg](/files/2e102672-1b7a-48be-9229-d872a3027548)`Correct

- 5 pts Click here to replace this description.

- 10 pts Click here to replace this description.

- 7 pts I don't understand your reasoning

- 1 pts Click here to replace this description.

- 7 pts

`![5.jpeg](/files/18dd4d78-660e-4590-8f7d-abd8f666848c)`

- 7 pts I don't understand this

`![7.jpeg](/files/1a9558f0-af85-4748-8a42-28df2f578b11)`

- 10 pts I don't understand your reasoning

McCombs Math 381H Final Exam Spring 2023

Print Name Sara Huston UNC email: sarahust@ad.unc.edu

1. There are 12 questions on the exam, worth a total of 100 points.
2. No credit will be given for correct answers without supporting work and/or explanation.
3. Final answers must be clearly indicated. All supporting work must be legible and shown on these exam pages.
4. All proofs must be written using correct notation and complete sentences where appropriate.

Sign the Honor Pledge

I have neither given nor received any unauthorized help on this exam, and I have conducted myself within the guidelines of the University Honor Code.

Pledge: ______

McCombs Math 381H Final Exam Spring 2023

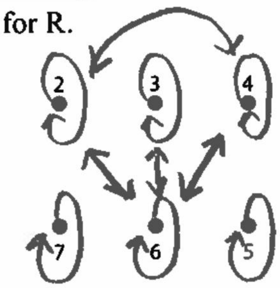
6 points

1. Let set $A=\{2,3,4,5,6,7\}$ with relation $R: A \to A$ defined as follows.

$$\boxed{\text{For every } x, y \in A,\ xRy \text{ means that } \gcd(x,y) \neq 1}$$

x & y are not relatively prime ⇒ share a common divisor

(i) Draw a digraph for R.



6,2 = 2
3,6 = 3
4,6 = 2
4,5 = 1
7,5 = 1

2,5 = 1
2,3 = 1
2,4 = 2
2,2 = 2
3,3 = 3
4,4 = 4

5,5 = 5
6,6 = 6
7,7 = 7

(ii) Determine whether R induces a partition of set A. You must explain your answer.

partition: must = A, no overlapping, no empty sets

$\underbrace{\{2,4,6\} \vee \{7\} \vee \{5\} \vee \{3,6\}}_{\text{double count } 6 \neq \text{ set } A}$

8 points

2. Let x, y, w, k, m, d, and n be non-negative integers.
Determine the total number of different solutions to the given equation such that exactly three of the variables are odd. You must show correct supporting work. No need to simplify your answer.

4 even 3 odd 7 total

$$x+y+w+k+m+d+n=965$$

$x = 2a$ $y = 2b$ $w = 2c$ $k = 2e$ for some integer

$m = 2f+1$ $d = 2t+1$ $n = 2s+1$ a, b, c, e, f, t, s ∈

$2a + 2b + 2c + 2e + 2t + 1 + 2f + 1 + 2s + 1 = 965$

$2a + 2b + 2c + 2e + 2t + 2f + 2s + 3 = 965$

$2a + 2b + 2c + 2e + 2t + 2f + 2s = 962$

$2(a+b+c+e+t+f+s) = 962$

$\underbrace{a+b+c+e+t+f+s}_{7 \text{ variables} \Rightarrow 7 \text{ flavors}} = 481 \Rightarrow 481$ donuts

★ donut problem

$\binom{k+n-1}{n} = \binom{481+7-1}{481} = \binom{487}{481} = \frac{487!}{481!(487-481)!}$

$\binom{487}{481} = \binom{487}{6} =$

6 points

3. Consider the following proposition. If p is a prime number, then $2^{p-1} \equiv 1 (\text{mod } p)$.

p = prime $\Rightarrow$ 19 is prime $T \rightarrow T$

(i) Determine whether the proposition is true or false when $p = 19$.
You must show correct supporting work.

True

$2^{19-1} \equiv 1(\text{mod } 19)$ $\quad 2^{18} \equiv 1(\text{mod } 19)$

$262144 - 1 \equiv 0(\text{mod } 19)$

$\frac{262143}{19} = 13797$, ★ see scrap if you need more $\Rightarrow$ divisible by 19 = true

(ii) Write the contrapositive of the original proposition.

$p \rightarrow q \qquad \neg q \rightarrow \neg p$

also: If $2^{p-1} \not\equiv 1(\text{mod } p)$, then p is not prime

If $2^{p-1} - 1$ is not divisible by p, then p is a composite number.

(iii) Write the negation of the original proposition.

$\neg(p \rightarrow q) \Rightarrow \neg(\neg p \vee q) \Rightarrow p \wedge \neg q$

p is a prime number and $2^{p-1} - 1$ is not divisible by p

also: p is a prime # and $2^{p-1} \not\equiv 1 (\text{mod } p)$

8 points

4. Find the smallest positive integer solution for the given congruence.

$$x \equiv 37(\text{mod } 52)$$
$$x \equiv 14(\text{mod } 41)$$

You must show correct supporting work.

gcd(52, 41) = 1
↓
prime

$x = 52k + 37$ for some $k \in \mathbb{Z}$

$52k + 37 \equiv 14(\text{mod } 41)$
$\equiv 11(\text{mod } 41)$

$11k + 37 \equiv 14(\text{mod } 41)$

$11k \equiv -23(\text{mod } 41)$

$11k \equiv 18(\text{mod } 41)$

$15 \cdot 11k \equiv 18 \cdot 15(\text{mod } 41)$
$\equiv 1(\text{mod } 41)$

$k \equiv 270(\text{mod } 41)$

$k \equiv 24(\text{mod } 41)$

$k = 41a + 24$ for some integer $a \in \mathbb{Z}$

$x = 52(41a + 24) + 37$

$x = 2132a + 1285$

1285

x	y	d	
1	0	41	
0	1	11	3
1	-3	8	1
-1	1+3 = 4	3	2
1+2 = 3	-3-4·2 = -11	2	1
-1-3 = -4	4+11 = 15	1	

$41 = 11 \cdot 3 + 8$
$11 = 8 \cdot 1 + 3$
$8 = 3 \cdot 2 + 2$
$3 = 2 \cdot 1 + 1$

$-4 \cdot 41 + 15 \cdot 11 = 1$

8 points

5. Let n be an integer. Determine whether the following statements are logically equivalent. You must explain your answer.

Statement 1:	n^2+3 is divisible by 4
Statement 2:	n^4-3 is divisible by 2

see scrap if you need more

$n^2+3 \equiv 0 \pmod 4$ $\quad$ $n^4-3 \equiv 0 \pmod 2$

$n^2+3 = 4k$ for some $k \in \mathbb{Z}$ $\quad$ $n^4-3 = 2L$ for some $L \in \mathbb{Z}$

$n^2 = 4k-3$ $\quad$ $n^4 = 2L-3$

$(n^2)^2 = (4k-3)^2$

$n^4 = 16k^2-24k+9$

$n^4 = 2(\underbrace{8k^2-12k+6}_{\text{integer}})-3$ $\quad$ $+12-3$

i.e. $n^4 = 2L-3$

thus, they are equivalent

8 points

6. Suppose the set $A=\{1,2,3,\ldots,50\}$ is divided into exactly two disjoint nonempty subsets.

For example, one possibility is $\{1\}$ $\{2,3,\ldots,50\}$.

see scrap work

Determine the total number of possibilities.

No need to simplify your answer. You must show correct supporting work.

2 is for empty set

$$\frac{2^{50}-2}{2}$$

$\square$ 50 different elements

$\square$ similar to bit things

possible # of subsets

order doesn't matter $\Rightarrow$ combination

~~repetition~~ not allowed $\Rightarrow \binom{n}{k}$

$2^{50}-\binom{50}{0}-\binom{50}{50}$

$= 2^{50}$ total possible

$$2^{50}-1+\binom{50}{25} \quad \binom{50}{1}\cdot\binom{49}{49}+\binom{50}{2}\cdot\binom{48}{48} \ldots + \binom{50}{25}\binom{25}{25}$$

place elements into size 1 subsets; place rest of elements; place elements into size 2 subset; place rest of elements; place elements into size 25 subsets; place rest of elements

possible w/out factoring out $\frac{1}{4}$

2

$\therefore$ Stop at 25 bc then it repeats $\rightarrow$ dont double count

repetition i.e. same ...

8 points

7. The UNC Student Store holds a contest to hand out souvenir shirts featuring four designs from the Mathematics Hall of Fame. Each shirt is size XL.

700 different people enter the contest.
6 different people win a shirt.
Shirts with the same design are indistinguishable from one another.
The order in which the shirts are handed out does not matter.

700 people
6 winners
$\Rightarrow$ no repeats
order doesn't matter

Boy Genius   Air McCombs   GQ Cover   South Mark

1) order doesn't matter for ppl $\Rightarrow$ comb.
2) repitition of shirts allowed

How many different ways can the shirts be handed out?

No need to simplify your answer. You must show correct supporting work.

$$\underbrace{\binom{700}{6}}_{\text{choose 6 winners}} \cdot \underbrace{\binom{4}{1}}_{w1} \cdot \underbrace{\binom{4}{1}}_{w2} \cdot \underbrace{\binom{4}{1}}_{w3} \cdot \underbrace{\binom{4}{1}}_{w4} \cdot \underbrace{\binom{4}{1}}_{w5} \cdot \underbrace{\binom{4}{1}}_{w6}$$

choose shirt

$$\boxed{\binom{700}{6} \cdot 4^6}$$

8 points

8. Let $A = \{w, x, y\}$ with relation $R : A \to A$, such that R is transitive but NOT reflexive. Suppose R contains the ordered pairs (x,w), (w,x), and (y,w).

(i) What other ordered pair(s) **MUST** also be contained in R? You must explain your answer.

Transitive:
$xRw \wedge wRx \Rightarrow xRx$ and wRw

(x,w) and $(w,x) \Rightarrow \boxed{(x,x)}$ and $\boxed{(w,w)}$

(y,w) and $(w,x) \Rightarrow \boxed{(y,x)}$

(ii) What ordered pair(s) **CANNOT** be contained in R? You must explain your answer.

Since it is not reflexive and (x,x) and (w,w) are alredy in there, $\boxed{(y,y)}$ cannot be in there. Otherewise, the relation would be reflexive

10 points

9. Prove the given statement.

Every set containing exactly eight different integers contains at least two elements that are congruent in $\mathbb{Z}_7$.

7 types to choose from
choose 8

$\equiv 1$ $\equiv 2$ $\equiv 3$ $\equiv 4$ $\equiv 5$ $\equiv 6$ $\equiv 7$ $\equiv$

1 2 3 4 5 6 7 8

you run out of choices $\Rightarrow$ must pick any of 7 $\Rightarrow$ at least 2 are congruent

mod 7 has 7 choices for integers

$[0]_{\mathbb{Z}_7} = \{7k\}$ for some $k \in \mathbb{Z}$

$[1]_{\mathbb{Z}_7} = \{7k+1\}$

$[2]_{\mathbb{Z}_7} = \{7k+2\}$

$[3]_{\mathbb{Z}_7} = \{7k+3\}$

$[4]_{\mathbb{Z}_7} = \{7k+4\}$

$[5]_{\mathbb{Z}_7} = \{7k+5\}$

$[6]_{\mathbb{Z}_7} = \{7k+6\}$

Therefore, if you have to choose 8 integers, only 7 could not be congruent, then the other would have to be congruent to at least 1 of ~~the~~ others. choosing 8 from a set of 7 needs repitition of at least 1 ~~element~~

10 points

10. Let m and n be positive integers. Use the method of **proof by contrapositive** to prove the given statement.

If $m+n+2\sqrt{mn}$ is not a perfect square, then m is not a perfect square or n is not a perfect square

assume $q = F$

$p \to q$ $\quad \neg q \to \neg p$ show $p = F$

$\neg(\neg a \vee \neg b)$

$a \wedge b$

If m is a perfect square and n is a perfect square, then $m+n+2\sqrt{mn}$ is a perfect square.

$n = s^2$ for some $s, t \in \mathbb{Z}$

$m = t^2$

show $m+n+2\sqrt{mn} = k^2$ for some $k \in \mathbb{Z}$

$m+n+2\sqrt{mn} = m+n+2\sqrt{mn}$

$t^2+s^2+2\sqrt{t^2 \cdot s^2} = m+n+2\sqrt{mn}$

$t^2+s^2+2\sqrt{(t \cdot s)^2} = m+n+2\sqrt{mn}$

$t^2+s^2+2ts = m+n+2\sqrt{mn}$

$(\underbrace{t+s}_{\text{integer}})^2 = m+n+2\sqrt{mn}$

i.e,

$k^2 = m+n+2\sqrt{mn}$

---

thus, If $m+n+2\sqrt{mn}$ is not a perfect square, then m is not a perfect square or n is not a perfect square is true by contrapositive.

☆ see scrap work

10 points

11. Let m,n be prime such that $m>3$ and $n>3$.

Prove the given statement.

If $m-n=2$, then $m+n$ is divisible by 12.

show:
m+n = 12k for some k ∈ Z
m+n ≡ 0 (mod 12)

since m and n are prime, in mod 12, there are 4 possibilites for m and n

≡ 1 (mod 12) i.e, 12k+1

≡ 5 (mod 12) i.e, 12k+5

≡ 7 (mod 12) i.e, 12k+7

≡ 11 (mod 12) i.e, 12k+9

can't be ≡ 0 (mod 12)
i.e n or m ≠ 12k bc divisible by 12 ≠ prime k ∈ Z

can't be ≡ 2 (mod 12) i.e,
n or m ≠ 12k+2 k ∈ Z
≠ $2(\underbrace{6k+1}_{})$
≠ 2s integer s ∈ Z
bc divisible by 2 ≠ prime

can't be ≡ 3 (mod 12)
n or m ≠ 12k+3 s,k ∈ Z
≠ $3(\underbrace{4k+1}_{\text{integer}})$
≠ 3s,
bc divisible by 3 means not prime

can't be ≡ 4 (mod 12) k,s ∈ Z
n or m ≠ 12k+4 ⇒ $4(\underbrace{3k+1}_{\text{integer}})$
≠ 4s
bc divisible by 4 ≠ prime

can't be ≡ 6 (mod 12) k,s ∈ Z
m or n ≠ 12k+6 ⇒ $6(\underbrace{2k+1}_{\text{integer}})$
≠ 6s
bc divisible by 6 ≠ prime

can't be ≡ 8 (mod 12) s,k ∈ Z
m or n ≠ 8+12k ⇒ $4(\underbrace{2+3k}_{\text{integer}})$
≠ 4s
bc divisible by 4 ≠ prime

since m and n are 2 apart, there are possibilities:

m ≡ 11 (mod 12) n ≡ 1 (mod 12)
or vice versa bc of symmetry of solution

m ≡ 5 (mod 12) n ≡ 7 (mod 12)

m − n = 2 ⇒ m = n+2
⇒ n = m − 2
i.e, 2 apart

$\overbrace{11 (\text{mod } 12)}^{m} + \overbrace{1 (\text{mod } 12)}^{n} \equiv 0 (\text{mod } 12)$

$\overbrace{5 (\text{mod } 12)}^{m} + \overbrace{7 (\text{mod } 12)}^{n} \equiv 0 (\text{mod } 12)$

i.e, divisible by 12

(same holds if n ≡ 5 (mod 12) or 11 (mod 12) bc of symmetry

10 points
12. Let n be an integer such that $n \geq 2$. Prove the given statement.

There are exactly $2^n - n - 1$ length-n bit strings that contain at least two 0s.

Formula for total # of bit strings

$= 2^n$

1. order matters ⇒ permutation
2. Repitition allowed ⇒ L^k

L = size of set

$|S| = |\{1,0\}| = 2 \Rightarrow L = 2$ i.e, 2^n

$$\underbrace{2^n}_{\text{total bit strings length-}n} - \overset{\text{\# that contain exactly 1 zero}}{n} - \underbrace{1}_{\text{\# that contains 0 zeros i.e, all 1}}$$

partition:

total − # that contains 1 0 − # that contains 1 0s

In any bit string length

$\binom{n}{n}$ ← # of spaces ← choose all w/ 1

i.e, choose n spaces w 1 ↓ choose n spaces w/ n # of 1's

$\binom{n}{n} = 1$

$\frac{\cancel{n!}}{\cancel{n!}(n-n)!} = 1$

$\binom{n}{1}\binom{n-1}{n-1}$ ← remaining # of spots

$= \frac{(n-1)!}{(n-1)!(n-1)(n-1)!} = 1$

place 1 zero ↓

$\frac{n!}{(n-1)!} \cdot 1$

$\frac{n\cancel{(n-1)!}}{\cancel{(n-1)!}} = n$

place the remaining spots w/ all 1's i.e, choose n-1 spaces w/ n-1

Sara Huston

$$\frac{2^{50}-2}{2}$$

20^5

$(t+s)(t+s)$

$t^2 + st + s^2 + st$

5. $n \equiv 1 \pmod 4$

$n \equiv 2 \pmod 4$

$n \equiv 3 \pmod 4$

$n \equiv 0 \pmod 4$

$n^2 + 3 \equiv 0 \pmod 4$

$n^2 + 3 = 4k$ $\quad n^4 - 3 = 2L$

$(n^2)^2 = (4k-3)^2$

$(n^4 = 16k^2 - 24k - 9$

$16k^2 - 24k - 9 - 3 = 2L$

$16k^2 - 24k - 6 = 2L$

$2(\underbrace{8k^2 - 12k - 3}_{\text{integer}}) = 2L$

see front & back

6. 2 choices:

$\frac{1}{1} \frac{0}{2} \cdots \frac{1}{50}$

$= 2^{50}$

each element has one place to go: either 1 or 2

$\underline{2} \cdot \underline{2} \cdot \underline{2} \cdot 2$

$= 2^{50}$

element 1 $\underbrace{\binom{2}{1}}$ $\cdot$ element 2 $\underbrace{\binom{2}{1}}$ $\cdot \underbrace{\binom{2}{1}} \cdots$

choose one subset 2^{50}

distinct into distinct

n^k

$$\frac{2^{50} \text{ total}}{2}$$

11. can't be $\equiv 9 \pmod{12}$ $\quad k, s \in \mathbb{Z}$

n or $m \neq 12k + 9 \Rightarrow 3\underbrace{(4k+3)}_{\text{integer}}$

$\neq 3s$

bc divisible 3 ≠ prime

can't be $\equiv 10 \pmod{12}$ $\quad k, s \in \mathbb{Z}$

n or $m \neq 12k + 10 \Rightarrow 2\underbrace{(6k+5)}_{\text{integer}}$

$\neq 2s$

bc divisible by 2 ≠ prime

$m = 11 \pmod{12}$ $\quad n \equiv 5 \pmod{12}$

or $7 \pmod{12}$

can't be either because:

$12k + 11 = 12f + 5$ $\quad R, f, k \in \mathbb{Z}$

or

$12f + 7$

$m - n \equiv 2 \pmod{12}$

$12k - 12f = 5 - 11$

$12\underbrace{(k-f)}_{\text{integer}} = -6$

$12R + 6 \not\equiv 2$

$12k - 12f = -11 + 7$

$12\underbrace{(k-f)}_{\text{integer}} = -5$

$12R + 5 \not\equiv 2 \pmod{12}$

See front & back

Sara Huston

3(i) $2^{19-1} =$

$2^{18} = (2^6)^3 \pmod{19}$

$= (64)^3 \pmod{19}$

$\equiv 7^3 \pmod{19}$

$= 343 \equiv 1 \pmod{19}$

$343 - 1 \equiv 0 \pmod{19}$

$\frac{342}{19} = 18$

i.e divisible by 19

checks for

4. $\frac{1285 - 37}{52} = k$

$k = 24$

$\frac{1285 - 14}{41} = l$

$l = 31$

$\frac{15 \cdot 11 - 1}{41} = 4$