HW3

1. $91_{10} + C6_{16}$

① Convert each operand to binary

$91_{10} = b001011011$ (U9Q0) $91-64=27$ $27-16=11$ $11-8=3$ $3-2=1$ $1-1=0$

$C6_{16} = b011000110$ (U9Q0) $C_{16} = b1100$ $6_{16} = b0110$

② Perform arithmetic in binary

$$b\ 0\ 0\ 1\ 0\ 1\ 1\ 0\ 1\ 1$$
$$+\ b\ 0\ 1\ 1\ 0\ 0\ 0\ 1\ 1\ 0$$
$$\overline{b\ 1\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 1}\quad (U9Q0)$$

③ Convert to decimal

$2^8 + 2^5 + 2^0 = 256 + 32 + 1 = \boxed{d289}$

④ Check:

$91_{10} \rightarrow$ already in base 10

$C6_{16} \rightarrow 12 \cdot 16^1 + 6 \cdot 16^0 = 192 + 6 = 198_{10}$

$$\begin{array}{r} d\ 198 \\ +\ d91 \\ \hline d\ 269 \end{array}$$

2. $11_8 - 11_{10}$

① Convert each operand to binary

$11_8 = b001001$

$11_{10} = b001011$

$-11_{10} = b110101$ 2's complement; 6-bit integer

② Perform arithmetic in binary

$$b001001$$
$$+\ b110101$$
$$\overline{b111110}$$ 2's complement; 6-bit integer

③ Convert to decimal

Answer: 111110 $\boxed{-2_{10}}$

$-$ Answer: 000010 $\rightarrow 2$ ✓

④ Check: $11_8 = d9$, $11_{10} = d11$ $9-11 = -2$ ✓

3. $12.3125_{10} + 0110_{I2Q2}$

① Convert each operand to binary

$12.3125_{10} = 12 \frac{5}{16} = 12 + \frac{1}{4} + \frac{1}{16}$

$12.3125_{10} = 1100 \, 0101_{I4Q4}$

0110_{I2Q2} is already in binary → convert to $0001 1000_{I4Q4}$

② Perform arithmetic in binary

$\quad 1100 0101_{I4Q4}$
$+ \, 0001 1000_{I4Q4}$
$\quad \overline{1101 1101}_{I4Q4}$

③ Convert to decimal

$2^3 + 2^2 + 2^0 + 2^{-1} + 2^{-2} + 2^{-4} = 8 + 4 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{16} = \boxed{13.8125_{10}}$

④ Check:

$0110_{I2Q2} = 2^0 + 2^{-1} = 1.5_{10}$

$12.3125_{10} + 1.5_{10} = 13.8125_{10} \; \checkmark$

4. $5.75_{10} - 7.125_{10}$

① Convert each operand to binary

$5.75_{10} = 0101 110_{I4Q3}$

$7.125_{10} = 0111 001_{I4Q3}$

$-7.125_{10} = 1000 111_{I4Q3}$

② Perform arithmetic in binary

$\quad 0101 110_{I4Q3}$
$+ \, 1000 111_{I4Q3}$
$\quad \overline{1110 101}_{I4Q3}$

④ Check

$\quad 7.125$
$\quad -5.750$
$\quad \overline{1.375} \; \checkmark$

③ Convert to decimal

Answer $= 1110 101_{I4Q3}$ $\boxed{-1.375_{10}}$

$-$Answer $= 0001 011_{I4Q3} \rightarrow 1.375$

5. $9_{10} \cdot 3_{10}$

① Convert each operand to binary

$9_{10} = 1001_{U4Q0}$

$3_{10} = 11_{U2Q0}$

② Perform arithmetic in binary

$$
\begin{array}{r}
1001_{U4Q0} \\
\times \quad 11_{U2Q0} \\
\hline
1001 \\
+ 10010 \\
\hline
011011_{U6Q0}
\end{array}
\qquad \rightarrow \quad 11011_{U5Q0}
$$

④ Check:

$9_{10} \cdot 3_{10} = 27_{10}$ ✓

③ Convert to decimal

$11011_{U5Q0} \rightarrow 2^4 + 2^3 + 2^1 + 2^0 = \boxed{27_{10}}$

6. $(-5)_{10} \cdot (-6)_{16}$

① Convert each operand to binary

$(-5)_{10} = 1011_{I4Q0}$

$5_{10} = 0101_{I4Q0}$

$(-6)_{16} = 1010_{I4Q0}$

$6_{16} = 0110_{I4Q0}$

③ Convert to decimal

$00011110_{I8Q6} = 16 + 8 + 4 + 2$

$= \boxed{30_{10}}$

④ Check: $(-5)_{10} \cdot (-6)_{16} = 30_{10}$ ✓

② Perform arithmetic in binary

$$
\begin{array}{r}
1011_{I4Q0} \\
\times 1010_{I4Q0} \\
\hline
\end{array}
$$

negative second operand

$$
\begin{array}{r}
1\ 1\ 1\ 1\ 1\ 1\ 1\ 0\ 1\ 1\ 0 \\
1\ 1\ 1\ 1\ 0\ 1\ 1\ 0\ 0\ 0 \\
1\ 1\ 1\ 0\ 1\ 1\ 0\ 0\ 0\ 0 \\
1\ 1\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0 \\
1\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0 \\
+ 1\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0 \\
\hline
0\ 0\ 0\ 0\ 1\ 1\ 1\ 1\ 0 \\
\end{array}
$$

$I8Q0$

7. $9.5_{10} \cdot 2.625_{10}$

① Convert each operand to binary

$9.5_{10} = 1001 1_{U4Q1}$

$2.625_{10} = 10101_{U2Q3}$

$8 + 1 + 0.5$

$2 + 0.5 + 0.125$

② Perform arithmetic in binary

$$
\begin{array}{r}
10011_{U4Q1} \\
\times\ 10101_{U2Q3} \\
\hline
10011 \\
1001100 \\
+\ 100110000 \\
\hline
0110001111_{U6Q4}
\end{array}
$$

④ Check:

$9.5_{10} \cdot 2.625_{10} = 24.9375_{10}$ ✓

③ Convert to decimal

$0110001111_{U6Q4} = 16 + 8 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = \boxed{24.9375_{10}}$

8. $(-1.25)_{10} \cdot 3.5_{10}$

① Convert each operand to binary

$(-1.25)_{10} = 1011_{I2Q2}$

$1.25_{10} = 0101_{I2Q2}$

$3.5_{10} = 0111_{I3Q1}$

② Perform arithmetic in binary

$$
\begin{array}{r}
1011_{I2Q2} \\
\times\ 0111_{I3Q1} \\
\hline
11111011 \\
1111011 0 \\
+\ 1110 1100 \\
\hline
11011101_{I5Q3}
\end{array}
$$

④ Check:

$-1.25_{10} \cdot 3.5_{10} = -4.375_{10}$ ✓

③ Convert to decimal

Answer: 11011101_{I5Q3}

$-$ Answer: $00100011_{I5Q3} \rightarrow 4 + \frac{1}{4} + \frac{1}{8} = 4.375$

$\boxed{-4.375_{10}}$

Challenge 1 | -5.6875_{10} in single-precision floating point format

To binary:

$5.6875 \rightarrow 101.1011_{U3Q4}$

$1.011011_{U1Q6} \times 2^2$

store 5 as $5 + 127 = 132$

Floating point: $(-1)^{sign} \cdot (1.\,fraction) \times 2^{exp - 127}$

Significand: $011011000\ldots$

Exponent: $10000100 \qquad \leftarrow 128_{10} + 4_{10} = 132_{10}$

Float: $1\ 10000100\ 01101100000000000000000$

sign   exponent   significand

$0xC2360000$

Challenge 2 | $0x2E4B68DF$ as single-precision floating point

$0\ 0101110\ 0100101101101000110111111$

sign   exp   significand

$(-1)^0 \cdot (1.10010110110100011011111_{U1Q23}) \cdot 2^{01011100_2 - 127_{10}}$

$\left(1 + \frac{1}{2} + \frac{1}{16} + \frac{1}{64} + \frac{1}{128} + \frac{1}{512} + \frac{1}{1024} + \frac{1}{2^{12}} + \frac{1}{2^{16}} + \frac{1}{2^{17}} + \frac{1}{2^{19}} + \frac{1}{2^{20}} + \frac{1}{2^{21}} + \frac{1}{2^{22}} + \frac{1}{2^{23}}\right) \cdot 2^{92 = 1}$

4.625×10^{-11}