

# On the (im)possibility of fairness

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presented by Sarah Dean

Fairness in ML, September 2017

# “Similar people should be treated similarly”

but similar in what sense?

On Monday, we considered the fairness constraint

$$D(f(x), f(x')) \leq d(x, x')$$

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 $f : \mathcal{X} \rightarrow \mathcal{D}$

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which amounts to a Lipschitz condition on the decision map  $f : \mathcal{X} \rightarrow \mathcal{D}$

- ▶ Sensitivity to definition of  $d$
- ▶ What is the space of individuals  $\mathcal{X}$ ? The feature space?

# Three spaces of the decision pipeline

We distinguish between the *construct space*  $\mathcal{C}$ , the *observed space*  $\mathcal{O}$ , and the *decision space*  $\mathcal{D}$ .

Decision space	Construct space	Observed space
College performance	intelligence	IQ
College performance	HS success	GPA
Recidivism	"criminality"	family history of crime
Recidivism	risk-averseness	age
Employee productivity	knowledge of job	years experience

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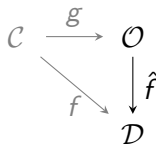
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- ▶ Imperfections in choice of construct vs. observed space?
- ▶ Should we distinguish between the decision space (label space) and the outcome space?

# Decision pipeline as maps between spaces

We consider transformations between spaces,

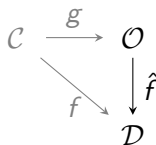
- ▶ observation processes  $g : \mathcal{C} \rightarrow \mathcal{O}$
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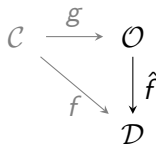
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The **distortion**  $\rho_h$  of map  $h : \mathcal{X} \rightarrow \mathcal{Y}$  is

$$\sup_{x, x' \in \mathcal{X}} |d_{\mathcal{X}}(x, x') - d_{\mathcal{Y}}(h(x), h(x'))|$$

and  $\rho(\mathcal{X}, \mathcal{Y}) = \min_h \rho_h$

# Individual fairness and what-you-see-is-what-you-get

A map  $f : \mathcal{C} \rightarrow \mathcal{D}$  is  $(\epsilon, \epsilon')$ -**fair** if for all  $x, x' \in \mathcal{C}$

$$d_{\mathcal{C}}(x, x') \leq \epsilon \implies d_{\mathcal{D}}(f(x), f(x')) \leq \epsilon'$$

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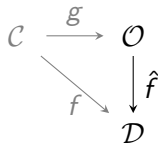
**Axiom** (WYSIWYG) The distortion between  $\mathcal{C}$  and  $\mathcal{O}$  is at most  $\delta$

An **individual fairness mechanism** ( $\text{IFM}_{\epsilon}$ ) is a nontrivial mapping  $\hat{f} : \mathcal{O} \rightarrow \mathcal{D}$  with  $\rho_{\hat{f}} \leq \epsilon$ .

# Fairness is possible!!!

## Theorem

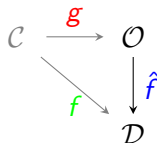
*Under WYSIWYG, an  $IFM_{\delta'}$  guarantees  $(\epsilon, \delta + \delta')$ -fairness.*



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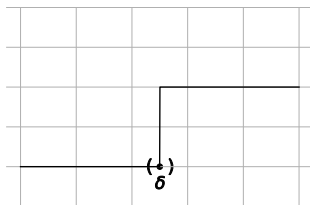
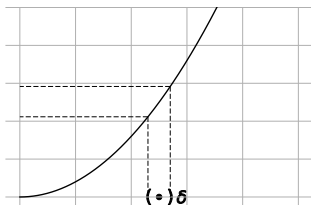
## Theorem

*Under WYSIWYG( $\epsilon$ ), any nontrivial map  $\hat{f} : \mathcal{O} \rightarrow \mathcal{D}$  is not  $(\delta - \epsilon, \delta')$ -fair for any  $\delta, \delta' < 1$  if  $\mathcal{D}$  is discrete*

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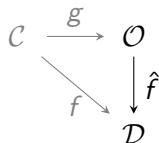
- What about randomization?



## Beyond individuals: structural bias

How might bias against certain groups manifest itself in this framework?

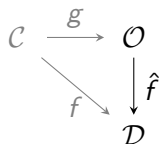
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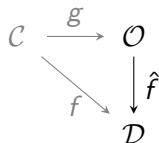
Measure distance between subsets  $X, X'$  in the same space  $\mathcal{X}$  with **Wasserstein distance**

$$\mathcal{W}_d(X, X') = \min_{\nu \in \mathcal{U}(X, X')} \int d_{\mathcal{X}}(x, x') \nu(x, x')$$

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Measure distance between subsets  $X, Y$  in the different spaces with **Gromov-Wasserstein distance**

$$\mathcal{G}(X, Y) = \frac{1}{2} \inf_{\mu} \int |d_{\mathcal{X}}(x, x') - d_{\mathcal{Y}}(y, y')| d\mu_X \times d\mu_X d\mu_Y \times d\mu_Y$$

# Structual bias and discrimination

The **between-groups** and **within-group distances** of  $\mathcal{X} = \bigcup_{i=1}^k X_i$  and  $\mathcal{Y} = \bigcup_{i=1}^k Y_i$  are respectively

$$\rho_b = \frac{1}{\binom{k}{2}} \mathcal{G}(\mathcal{X}, \mathcal{Y}), \quad \rho_w = \frac{1}{k} \sum_{i=1}^k \mathcal{G}(X_i, Y_i),$$

and the **group skew** is

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We have

- ▶  **$t$ -structural bias:**  $\sigma(\mathcal{C}, \mathcal{O}) > t$
- ▶  **$t$ -direct discrimination:**  $\sigma(\mathcal{O}, \mathcal{D}) > t$
- ▶ a  **$t$ -nondiscriminatory** mapping  $f : \mathcal{C} \rightarrow \mathcal{D}$  if  $\sigma(\mathcal{C}, \mathcal{D}) \leq t$

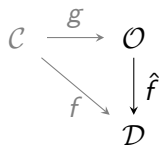
# Structural bias and discrimination

How does this notion of structural bias compare with an intuitive one?

What does direct discrimination look like? Can affirmative action be direct discrimination?

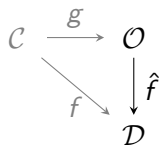
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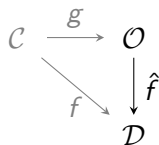
**Axiom** (WAE) For  $\mathcal{C} = X_1 \cup \dots \cup X_k$ ,

$$\mathcal{W}_{d_C}(X_i, X_j) < \epsilon \text{ for all } 1 \leq i, j \leq k$$



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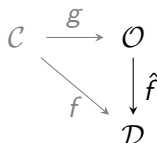
$$\mathcal{W}_{d_{\mathcal{C}}}(X_i, X_j) < \epsilon \text{ for all } 1 \leq i, j \leq k$$

A **group fairness mechanism** ( $\text{GFM}_{\epsilon}$ )  $f : \mathcal{O} \rightarrow \mathcal{D}$  with  $\mathcal{O} = Y_1 \cup \dots \cup Y_k$  satisfies  $\mathcal{W}_{d_{\mathcal{O}}}(f(Y_i), f(Y_j)) \leq \epsilon$

# Nondiscrimination is possible!

## Theorem

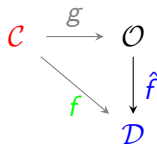
*Under WAE, a  $GFM_{\epsilon'}$  guarantees  $\frac{\max(\epsilon, \epsilon')}{\delta}$ -nondiscrimination*



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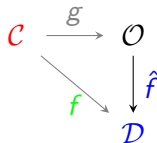
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Could achieving nondiscrimination in this setting require direct discrimination?

# Worldview comparison

## WYSIWYG vs. WAE

Each axiom induces fairness mechanism (group v. individual) to achieve fairness or nondiscrimination. Are they always incompatible?

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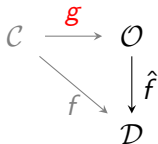
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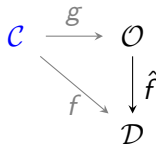
How can we understand the following as part of this framework?

- ▶ Observational measures:
  - ▶ demographic parity (equalized odds)
  - ▶ accuracy parity
  - ▶ true positive parity (equal opportunity)
  - ▶ predictive value parity
- ▶ Credit scores?
- ▶ COMPAS: Kristian Lum's approach

# Beyond conceptual design?



Assume good observations



Assume inherent equality

Framework allows for conceptual exploration and justification of a type of fairness mechanism.

- ▶ How do we model these spaces? How to explicitly encode structural bias at the modeling level?
- ▶ More sophisticated axioms?