

1 Boolean Logic Circuits [45 points]

During your job interview, you are asked to design a combinational circuit with a four-bit input, $\{A, B, C, D\}$ (A is the most significant bit and D is the least significant bit), and two 1-bit outputs, Fib and $G3$. The value of each output is determined as follows:

- The output Fib is 1 only when the input 4-bit number is a Fibonacci number. You can calculate Fibonacci numbers as follows, $f(0) = 0$, $f(1) = 1$, and $f(n) = f(n-1) + f(n-2)$ for $n \geq 2$.
- The output $G3$ is 1 only when the input 4-bit number is greater than 3.
- Otherwise, the corresponding output is zero.

Please answer the following three questions.

- (a) [10 points] Fill in the missing entries in the truth table below for the combinational circuit you are designing and express the output Fib in the *sum of products* representation.

Inputs				Outputs	
A	B	C	D	Fib	$G3$
0	0	0	0	1	0
0	0	0	1	1	0
0	0	1	0	1	0
0	0	1	1	1	0
0	1	0	0	0	1
0	1	0	1	1	1
0	1	1	0	0	1
0	1	1	1	0	1
1	0	0	0	1	1
1	0	0	1	0	1
1	0	1	0	0	1
1	0	1	1	0	1
1	1	0	0	0	1
1	1	0	1	1	1
1	1	1	0	0	1
1	1	1	1	0	1

$$\begin{aligned}
 Fib = & (\overline{A} \cdot \overline{B} \cdot \overline{C} \cdot \overline{D}) + (\overline{A} \cdot \overline{B} \cdot \overline{C} \cdot D) + (\overline{A} \cdot \overline{B} \cdot C \cdot \overline{D}) + (\overline{A} \cdot \overline{B} \cdot C \cdot D) + (\overline{A} \cdot B \cdot \overline{C} \cdot D) + \\
 & (A \cdot \overline{B} \cdot \overline{C} \cdot \overline{D}) + (A \cdot B \cdot \overline{C} \cdot D)
 \end{aligned}$$

- (b) [15 points] Simplify the *Fib* expression using Boolean minimization rules. Show your work step-by-step.

$$\begin{aligned}
 Fib &= (\overline{A} \cdot \overline{B} \cdot \overline{C} \cdot \overline{D}) + (\overline{A} \cdot \overline{B} \cdot \overline{C} \cdot D) + (\overline{A} \cdot \overline{B} \cdot C \cdot \overline{D}) + (\overline{A} \cdot \overline{B} \cdot C \cdot D) + (\overline{A} \cdot B \cdot \overline{C} \cdot D) + \\
 &+ (A \cdot \overline{B} \cdot \overline{C} \cdot \overline{D}) + (A \cdot B \cdot \overline{C} \cdot D) \\
 Fib &= ((\overline{A} \cdot \overline{B}) \cdot ((\overline{C} \cdot \overline{D}) + (\overline{C} \cdot D) + (C \cdot \overline{D}) + (C \cdot D))) + (\overline{A} \cdot B \cdot \overline{C} \cdot D) + (A \cdot \overline{B} \cdot \overline{C} \cdot \overline{D}) + (A \cdot B \cdot \overline{C} \cdot D) \\
 Fib &= ((\overline{A} \cdot \overline{B}) \cdot (1)) + (\overline{A} \cdot B \cdot \overline{C} \cdot D) + (A \cdot \overline{B} \cdot \overline{C} \cdot \overline{D}) + (A \cdot B \cdot \overline{C} \cdot D) \\
 Fib &= (\overline{A} \cdot \overline{B}) + (\overline{A} \cdot B \cdot \overline{C} \cdot D) + (A \cdot \overline{B} \cdot \overline{C} \cdot \overline{D}) + (A \cdot B \cdot \overline{C} \cdot D) \\
 Fib &= (\overline{A} \cdot \overline{B}) + (\overline{C} \cdot ((\overline{A} \cdot B \cdot D) + (A \cdot \overline{B} \cdot \overline{D}) + (A \cdot B \cdot D))) \\
 Fib &= (\overline{A} \cdot \overline{B}) + (\overline{C} \cdot ((B \cdot D) + (A \cdot \overline{B} \cdot \overline{D}))) \\
 Fib &= (\overline{A} \cdot \overline{B}) + (B \cdot \overline{C} \cdot D) + (A \cdot \overline{B} \cdot \overline{C} \cdot \overline{D}) \\
 Fib &= (\overline{A} \cdot \overline{B}) + (A \cdot \overline{B} \cdot \overline{C} \cdot \overline{D}) + (B \cdot \overline{C} \cdot D) \\
 Fib &= (\overline{A} \cdot \overline{B}) + (\overline{B} \cdot \overline{C} \cdot \overline{D}) + (B \cdot \overline{C} \cdot D)
 \end{aligned}$$

- (c) [20 points] Find the simplest representation of the *G3* output by using *only* 2-input NAND gates. Show your work step-by-step.

$$\begin{aligned}
 G3 &= \overline{(\overline{A} \cdot A)} \cdot \overline{(\overline{B} \cdot B)} \\
 \textbf{Explanation:} \\
 G3 &= (\overline{A} \cdot B \cdot \overline{C} \cdot \overline{D}) + (\overline{A} \cdot B \cdot \overline{C} \cdot D) + (\overline{A} \cdot B \cdot C \cdot \overline{D}) + (\overline{A} \cdot B \cdot C \cdot D) + (A \cdot \overline{B} \cdot \overline{C} \cdot \overline{D}) + (A \cdot \overline{B} \cdot \overline{C} \cdot D) \\
 &+ (A \cdot \overline{B} \cdot C \cdot \overline{D}) + (A \cdot \overline{B} \cdot C \cdot D) + (A \cdot B \cdot \overline{C} \cdot \overline{D}) + (A \cdot B \cdot \overline{C} \cdot D) + (A \cdot B \cdot C \cdot \overline{D}) + (A \cdot B \cdot C \cdot D) \\
 G3 &= (\overline{A} \cdot B \cdot ((\overline{C} \cdot \overline{D}) + (\overline{C} \cdot D) + (C \cdot \overline{D}) + (C \cdot D))) + (A \cdot \overline{B} \cdot ((\overline{C} \cdot \overline{D}) + (\overline{C} \cdot D) + (C \cdot \overline{D}) + (C \cdot D))) \\
 &+ (A \cdot B \cdot ((\overline{C} \cdot \overline{D}) + (\overline{C} \cdot D) + (C \cdot \overline{D}) + (C \cdot D))) \\
 G3 &= (\overline{A} \cdot B \cdot (1)) + (A \cdot \overline{B} \cdot (1)) + (A \cdot B \cdot (1)) \\
 G3 &= (\overline{A} \cdot B) + (A \cdot \overline{B}) + (A \cdot B) \\
 G3 &= A + B \\
 G3 &= \overline{\overline{A + B}} \\
 G3 &= \overline{\overline{A} \cdot \overline{B}} \\
 G3 &= \overline{(\overline{A} \cdot A)} \cdot \overline{(\overline{B} \cdot B)}
 \end{aligned}$$