1 Boolean Circuit Minimization [20 points]

(a) [10 points] Convert the following Boolean equation so that it only contains NAND operations. Show your work step-by-step.

$$F = (\overline{A \cdot B} + C) + A \cdot C$$

$$F = \overline{\overline{\overline{(A \cdot B)} \cdot \overline{(A \cdot B)}} \cdot \overline{(A \cdot C)}}$$

Explanation:

$F = \overline{\overline{(\overline{A \cdot B} + C)} \cdot \overline{(A \cdot C)}}$
$F = \overline{((A \cdot B) \cdot \overline{C}) \cdot \overline{(A \cdot C)}}$
$F = \overline{(A \cdot B) \cdot \overline{(\overline{C} \cdot \overline{(A \cdot C)})}}$
$F = \overline{(A \cdot B)\overline{(A \cdot C)}}$
$F = \overline{\overline{\overline{(A \cdot B)} \cdot \overline{(A \cdot B)}} \cdot \overline{(A \cdot C)}}$

(b) [10 points] Using Boolean algebra, find the simplest Boolean algebra equation for the following min-terms. Show your work step-by-step. You may label the order of variables as ABCD (e.g., $\overline{A} \cdot B \cdot \overline{C} \cdot \overline{D}$ denotes 0100).

$$\sum(0000, 0100, 0101, 1000, 1100, 1101)$$

$$F = \overline{C} \cdot (B + \overline{D})$$

Explanation:

$F = (\overline{A} \cdot \overline{B} \cdot \overline{C} \cdot \overline{D}) + (\overline{A} \cdot B \cdot \overline{C} \cdot \overline{D}) + (\overline{A} \cdot B \cdot \overline{C} \cdot D) + (A \cdot \overline{B} \cdot \overline{C} \cdot \overline{D}) + (A \cdot B \cdot \overline{C} \cdot \overline{D}) + (A \cdot B \cdot \overline{C} \cdot D)$
$F = (B \cdot \overline{C} \cdot ((\overline{A} \cdot \overline{D}) + (\overline{A} \cdot D) + (A \cdot \overline{D}) + (A \cdot D))) + (\overline{C} \cdot \overline{D} \cdot ((\overline{A} \cdot \overline{B}) + (\overline{A} \cdot B) + (A \cdot \overline{B}) + (A \cdot B)))$
$F = B \cdot \overline{C} + \overline{C} \cdot \overline{D}$
$F = \overline{C} \cdot (B + \overline{D})$