

### 3 Boolean Algebra [15 points]

- (a) [5 points] Find the simplest sum-of-products representation of the following Boolean equation. Show your work step-by-step.

$$F = B + (A + \overline{C}).(\overline{A} + \overline{B} + \overline{C})$$

**Answer:**  $F = A + B + \overline{C}$

**Explanation:**

$$F = B + (A.\overline{A} + A.\overline{B} + A.\overline{C} + \overline{A}.\overline{C} + \overline{B}.\overline{C} + \overline{C}.\overline{C})$$

$$F = B + 0 + A.\overline{B} + \overline{C}.(A + \overline{A}) + \overline{B}.\overline{C} + \overline{C}$$

$$F = (B + A.\overline{B}) + \overline{C}.(A + \overline{A}) + (\overline{B}.\overline{C} + \overline{C})$$

$$F = (B + A) + \overline{C} + \overline{C}.(B + 1)$$

$$F = A + B + \overline{C}$$

- (b) [5 points] Convert the following Boolean equation so that it only contains NAND operations. Show your work step-by-step.

$$F = \overline{(A + B.C)} + \overline{C}$$

**Answer:**  $F = \overline{\overline{\overline{(A.A)}.(B.C))}.C$

**Explanation:**

$$F = \overline{\overline{\overline{(A + B.C)} + \overline{C}}}$$

$$F = \overline{\overline{(A + B.C).C}}$$

$$F = \overline{\overline{(A + \overline{B.C}).C}}$$

$$F = \overline{\overline{(\overline{A}.\overline{B.C}).C}}$$

$$F = \overline{\overline{\overline{\overline{(A.A)}.(B.C))}.C}}$$

- (c) [5 points] Using Boolean algebra, simplify the following min-terms:  $\sum(3, 5, 7, 11, 13, 15)$   
Show your work step-by-step.

**Answer:**  $F = D.(B + C)$

**Explanation:**

$$\{3, 5, 7, 11, 13, 15\} = \{0011, 0101, 0111, 1011, 1101, 1111\}$$

$$F = (\overline{A}.\overline{B}.C.D) + (\overline{A}.B.\overline{C}.D) + (\overline{A}.B.C.D) + (A.\overline{B}.C.D) + (A.B.\overline{C}.D) + (A.B.C.D)$$

$$F = (C.D.(\overline{A}.\overline{B}) + (\overline{A}.B) + (A.\overline{B}) + (A.B))) + (B.D.(\overline{A}.\overline{C}) + (A.\overline{C}))$$

$$F = (C.D) + (B.\overline{C}.D)$$

$$F = D.(C + (B.\overline{C}))$$

$$F = D.(B + C)$$