## 4. Amdahl's Law [40 points]

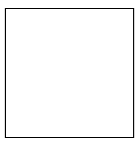
Consider Processor X with area A. You will analyze the performance of different processors with respect to Processor X. All processors discussed in this problem will be built on a die of area **16A**. Assume that the single-thread performance of a core increases with the **square root of its area**. You are given a workload where $S$ fraction of its work is serial and $P = 1 - S$ fraction of its work is **perfectly** parallelizable.

In this problem, we define, for a given program:

$$\text{Speedup of } Processor A = \frac{Runtime\ on\ Processor X}{Runtime\ on\ Processor A}$$

(a) Given Processor Z is made of a single core with area $16A$, what is the speedup of this processor when running a program with $S = 1$? Show your work.

**Processor Z**
1 *large* core of area $16A$



4
One core with 4 times the speedup of Processor X runs the whole program.

(b) Given the same Processor Z from part (a), what is the speed up of this processor when running a program with $S = 0$? Show your work.

4
One core with 4 times the speedup of Processor X runs the whole program.

(c) What is the speedup equation for a heterogeneous processor that has one large core with area $N \times A$ and the remaining die area filled with small cores, each with area $A$? Write in terms of S and N.
Assume that (i) only the large core will be running the serial portion, and (ii) all cores, including the large core, will be running the parallel portion of the program.

$$\frac{1}{\frac{S}{\sqrt{N}} + \frac{1-S}{(16-N)+\sqrt{N}}}$$

(d) Now, you are given a chance to design a heterogeneous processor of area $16A$ to run some workload. The large core on the processor has an area size of $M^2 \times A$, where $M$ is a natural number. What is $M$ such that the processor can maximize the speedup when running a program with $P = 0.8$? Show your work.

M = 3.
We have 1,2,3,4 as options for M. When plugged into the equation from the previous part, M=3 gives the best speedup.