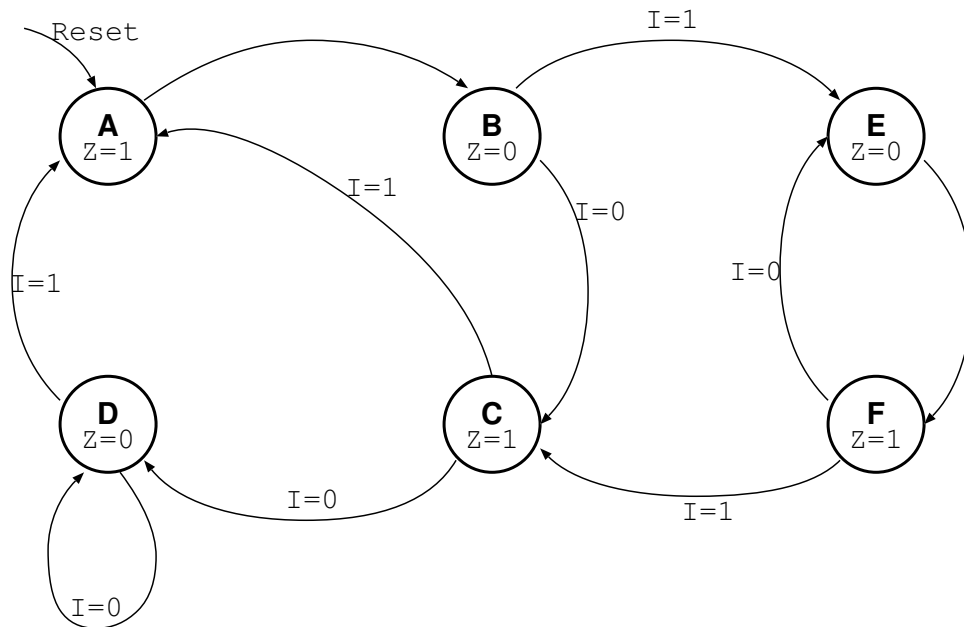


2. Consider the following state diagram of an FSM with 1-bit input (I) and 1-bit output (Z).



The state has been coded using a 3-bit vector $S = S_2 S_1 S_0$ according to the following table:

State			
name	S_2	S_1	S_0
A	0	0	0
B	0	0	1
C	0	1	0
D	0	1	1
E	1	0	0
F	1	0	1

- (a) (1 point) Is this a Moore or Mealy type FSM? Briefly explain.

Solution: Moore, outputs depend only on the present state and nothing else.

- (b) (6 points) Fill in the following state transition table that determines the next state vector $N = N_2 N_1 N_0$ based on the current state S and the input I .

State				Input	Next State			
name	S_2	S_1	S_0	I	name	N_2	N_1	N_0
A	0	0	0	X	B	0	0	1
B	0	0	1	0	C	0	1	0
B	0	0	1	1	E	1	0	0
C	0	1	0	0	D	0	1	1
C	0	1	0	1	A	0	0	0
D	0	1	1	0	D	0	1	1
D	0	1	1	1	A	0	0	0
E	1	0	0	X	F	1	0	1
F	1	0	1	0	E	1	0	0
F	1	0	1	1	C	0	1	0

Note that there are different ways of writing this table to represent the same result.

- (c) (3 points) Write the Next State Equations from the table you have filled above using either *Product of Sums (POS)* or *Sum of Products (SOP)* form. **Do not spend time minimizing the equations, this will be next question.**

Solution:

$$N_0 = \overline{S_2} \overline{S_1} \overline{S_0} + \overline{S_2} S_1 \overline{S_0} \overline{I} + \overline{S_2} S_1 S_0 \overline{I} + S_2 \overline{S_1} \overline{S_0}$$

$$N_1 = \overline{S_2} \overline{S_1} S_0 \overline{I} + \overline{S_2} S_1 \overline{S_0} \overline{I} + \overline{S_2} S_1 S_0 \overline{I} + S_2 \overline{S_1} S_0 I$$

$$N_2 = \overline{S_2} \overline{S_1} S_0 I + S_2 \overline{S_1} \overline{S_0} + S_2 \overline{S_1} S_0 \overline{I}$$

- (d) (6 points) Minimize the Next State Equations from the previous part. Note that the FSM requires only six states. This means that there are several State (S) / Input(I) combinations for which the outputs can be treated as *Don't Care*, which should help minimizing the boolean equations.

Hint: Consider using Karnaugh diagrams to solve this problem.

Solution:

It is important to note that for $S_2 S_1 = 11$ the next state N can be taken as $N_2 N_1 N_0 = XXX$. This can simplify the Boolean equations significantly. It is best to use a Karnaugh map to find the simplifications.

		$S_0 I$			
		00	01	11	10
$S_2 S_1$	00	1	1	0	0
	01	1	0	0	1
	11	X	X	X	X
	10	1	1	0	0

N_0

		$S_0 I$			
		00	01	11	10
$S_2 S_1$	00	0	0	0	1
	01	1	0	0	1
	11	X	X	X	X
	10	0	0	1	0

N_1

		$S_0 I$			
		00	01	11	10
$S_2 S_1$	00	0	0	1	0
	01	0	0	0	0
	11	X	X	X	X
	10	1	1	0	1

N_2

$$N_0 = \overline{S_1} \overline{S_0} + S_1 \overline{I}$$

$$N_1 = S_1 \overline{I} + S_2 S_0 I + \overline{S_2} S_0 \overline{I}$$

$$N_2 = S_2 \overline{S_0} + S_2 \overline{I} + \overline{S_2} \overline{S_1} S_0 I$$