2. For this question, use the following truth table for a 4-input logic function called Z.

Input				**Output**
A	B	C	D	Z
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	X
0	1	0	0	1
0	1	0	1	X
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	X
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	0
1	1	1	1	0

(a) (1 point) What is the meaning of X in this truth table?

> **Solution:** The output value is not important for the functionality of the circuit. It can be taken as '0' or '1' to simplify the equations

(b) (6 points) A friend of yours has determined the following Boolean equation for Z:

$$Z = (B + D) \cdot (\overline{B} + \overline{C}) \cdot (A + \overline{C}) \cdot (\overline{A} + B + D) \cdot (A + B + \overline{C}) \cdot (A + C + \overline{D})$$

But he is not sure if this is correct. Verify whether or not the given equation matches the truth table given above. Is there something that your friend could have done better?

> **Solution:**
> The equation is not correct. You can see this if you mark the minterms on the truth table for each equation. The following are the problems:
>
> - $(A + \overline{C})$ is redundant if the X there was chosen as '1'
> - $(\overline{A} + B + D)$ is redundant.
> - $(A + B + \overline{C})$ is redundant, but $(\overline{A} + \overline{B} + C)$ is missing
> - $(A + C + \overline{D})$ is plain wrong. It covers 1 and X. Should not be there
>
> The X values have not been optimally used, this results in a more complex equation, there are more 0s than 1s, so a SOP form would probably be better, in addition there are redundant terms, the equation is not simplified

(c) (5 points) Derive your own *optimized* boolean equation corresponding to the same truth table using *sums-of-products* form. Try to take advantage of the 'X' values to minimize the equation as much as possible. (*Hint: use a Karnaugh map*)

> **Solution:** If you take all the Xs as '1', you can derive:
> $Z = (\overline{B} \cdot D) + (\overline{C} \cdot D) + (\overline{A} \cdot B \cdot \overline{C})$