

1 Boolean Logic Circuits [40 points]

During your job interview, you are asked to design a combinational circuit with a four-bit input, $\{A, B, C, D\}$ (A is the most significant bit and D is the least significant bit), and two 1-bit outputs, *Factorial* and *Div4*. The value of each output is determined as follows:

- The output *Factorial* is 1 only when the input 4-bit number is a product of ALL positive integers that are less than or equal to the input number.
- The output *Div4* is 1 only when the input 4-bit number is divisible by 4.
- Otherwise, the corresponding outputs are zero.

Please answer the following four questions.

- (a) [10 points] Fill in the missing entries in the truth table below for the combinational circuit you are designing.

Inputs				Outputs	
<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>Factorial</i>	<i>Div4</i>
0	0	0	0	1	1
0	0	0	1	1	0
0	0	1	0	1	0
0	0	1	1	0	0
0	1	0	0	0	1
0	1	0	1	0	0
0	1	1	0	0	0
0	1	1	1	0	0
1	0	0	0	0	1
1	0	0	1	0	0
1	0	1	0	0	0
1	0	1	1	0	0
1	1	0	0	0	1
1	1	0	1	0	0
1	1	1	0	0	0
1	1	1	1	0	0

- (b) [10 points] Express the output *Div4* as the simplest *sum of products* representation. Show your work step-by-step.

$$Div4 = (\overline{C} \cdot \overline{D})$$

Explanation:

$$Div4 = (\overline{A} \cdot \overline{B} \cdot \overline{C} \cdot \overline{D}) + (\overline{A} \cdot B \cdot \overline{C} \cdot \overline{D}) + (A \cdot \overline{B} \cdot \overline{C} \cdot \overline{D}) + (A \cdot B \cdot \overline{C} \cdot \overline{D})$$

$$Div4 = (\overline{A} \cdot \overline{C} \cdot \overline{D})(\overline{B} + B) + (A \cdot \overline{B} \cdot \overline{C} \cdot \overline{D}) + (A \cdot B \cdot \overline{C} \cdot \overline{D})$$

$$Div4 = (\overline{A} \cdot \overline{C} \cdot \overline{D}) + (A \cdot \overline{B} \cdot \overline{C} \cdot \overline{D}) + (A \cdot B \cdot \overline{C} \cdot \overline{D})$$

$$Div4 = (\overline{C} \cdot \overline{D})(A \cdot \overline{B} + A) + (A \cdot B \cdot \overline{C} \cdot \overline{D})$$

$$Div4 = (\overline{C} \cdot \overline{D} \cdot B) + (\overline{C} \cdot \overline{D} \cdot A) + (A \cdot B \cdot \overline{C} \cdot \overline{D})$$

$$Div4 = (\overline{C} \cdot \overline{D} \cdot A) + (\overline{C} \cdot \overline{D})(A \cdot B + \overline{B})$$

$$Div4 = (\overline{C} \cdot \overline{D})(A + A) + (\overline{C} \cdot \overline{D} \cdot B)$$

$$Div4 = (\overline{C} \cdot \overline{D}) + (\overline{C} \cdot \overline{D} \cdot B)$$

$$Div4 = (\overline{C} \cdot \overline{D})$$

- (c) [20 points] Find the simplest representation of the *Factorial* output by using *only* NOR gates. Show your work step-by-step.

$$Factorial = \overline{B + A + \overline{\overline{\overline{C + C + D + D}}}}$$

Explanation:

$$\overline{Factorial} = (A + B + C + D) \cdot (A + B + C + \overline{D}) \cdot (A + B + \overline{C} + D)$$

$$\overline{Factorial} = (A \cdot (A + B + C + \overline{D}) + B \cdot (A + B + C + \overline{D}) + C \cdot (A + B + C + \overline{D}) + D \cdot (A + B + C + \overline{D})) \cdot (A + B + \overline{C} + D)$$

$$\overline{Factorial} = (A + AB + AC + A\overline{D} + AB + B + BC + B\overline{D} + AC + CB + C + C\overline{D} + AD + BD + CD + D\overline{D}) \cdot (A + B + \overline{C} + D)$$

$$\overline{Factorial} = (A + B + C) \cdot (A + B + \overline{C} + D)$$

$$\overline{Factorial} = (A + AB + A\overline{C} + AD) + (AB + B + B\overline{C} + BD) + (AC + BC + CD)$$

$$Factorial = \overline{B + A + C\overline{D}}$$

$$Factorial = \overline{B + A + \overline{\overline{CD}}}$$

$$Factorial = \overline{B + A + \overline{\overline{C + D}}}$$

$$Factorial = \overline{B + A + \overline{\overline{\overline{C + C + D + D}}}}$$