

# 1 Boolean Logic Circuits [45 points]

During your job interview, you are asked to design a combinational circuit with a four-bit input,  $\{A, B, C, D\}$  ( $A$  is the most significant bit and  $D$  is the least significant bit), and two 1-bit outputs,  $Fib$  and  $G3$ . The value of each output is determined as follows:

- The output  $Fib$  is 1 only when the input 4-bit number is a Fibonacci number. You can calculate Fibonacci numbers as follows,  $f(0) = 0$ ,  $f(1) = 1$ , and  $f(n) = f(n - 1) + f(n - 2)$  for  $n \geq 2$ .
- The output  $G3$  is 1 only when the input 4-bit number is greater than 3.
- Otherwise, the corresponding output is zero.

Please answer the following three questions.

- (a) [10 points] Fill in the missing entries in the truth table below for the combinational circuit you are designing and express the output  $Fib$  in the *sum of products* representation.

Inputs				Outputs	
$A$	$B$	$C$	$D$	$Fib$	$G3$
0	0	0	0	1	0
0	0	0	1	1	0
0	0	1	0	1	0
0	0	1	1	1	0
0	1	0	0	0	1
0	1	0	1	1	1
0	1	1	0	0	1
0	1	1	1	0	1
1	0	0	0	1	1
1	0	0	1	0	1
1	0	1	0	0	1
1	0	1	1	0	1
1	1	0	0	0	1
1	1	0	1	1	1
1	1	1	0	0	1
1	1	1	1	0	1

$$\begin{aligned}
 Fib = & (\overline{A} \cdot \overline{B} \cdot \overline{C} \cdot \overline{D}) + (\overline{A} \cdot \overline{B} \cdot \overline{C} \cdot D) + (\overline{A} \cdot \overline{B} \cdot C \cdot \overline{D}) + (\overline{A} \cdot \overline{B} \cdot C \cdot D) + (\overline{A} \cdot B \cdot \overline{C} \cdot D) + \\
 & (A \cdot \overline{B} \cdot \overline{C} \cdot \overline{D}) + (A \cdot B \cdot \overline{C} \cdot D)
 \end{aligned}$$

- (b) [15 points] Simplify the *Fib* expression using Boolean minimization rules. Show your work step-by-step.

$$\begin{aligned}
 Fib &= (\overline{A} \cdot \overline{B} \cdot \overline{C} \cdot \overline{D}) + (\overline{A} \cdot \overline{B} \cdot \overline{C} \cdot D) + (\overline{A} \cdot \overline{B} \cdot C \cdot \overline{D}) + (\overline{A} \cdot \overline{B} \cdot C \cdot D) + (\overline{A} \cdot B \cdot \overline{C} \cdot D) + \\
 &+ (A \cdot \overline{B} \cdot \overline{C} \cdot \overline{D}) + (A \cdot B \cdot \overline{C} \cdot D) \\
 Fib &= ((\overline{A} \cdot \overline{B}) \cdot ((\overline{C} \cdot \overline{D}) + (\overline{C} \cdot D) + (C \cdot \overline{D}) + (C \cdot D))) + (\overline{A} \cdot B \cdot \overline{C} \cdot D) + (A \cdot \overline{B} \cdot \overline{C} \cdot \overline{D}) + (A \cdot B \cdot \overline{C} \cdot D) \\
 Fib &= ((\overline{A} \cdot \overline{B}) \cdot (1)) + (\overline{A} \cdot B \cdot \overline{C} \cdot D) + (A \cdot \overline{B} \cdot \overline{C} \cdot \overline{D}) + (A \cdot B \cdot \overline{C} \cdot D) \\
 Fib &= (\overline{A} \cdot \overline{B}) + (\overline{A} \cdot B \cdot \overline{C} \cdot D) + (A \cdot \overline{B} \cdot \overline{C} \cdot \overline{D}) + (A \cdot B \cdot \overline{C} \cdot D) \\
 Fib &= (\overline{A} \cdot \overline{B}) + (\overline{C} \cdot ((\overline{A} \cdot B \cdot D) + (A \cdot \overline{B} \cdot \overline{D}) + (A \cdot B \cdot D))) \\
 Fib &= (\overline{A} \cdot \overline{B}) + (\overline{C} \cdot ((B \cdot D) + (A \cdot \overline{B} \cdot \overline{D}))) \\
 Fib &= (\overline{A} \cdot \overline{B}) + (B \cdot \overline{C} \cdot D) + (A \cdot \overline{B} \cdot \overline{C} \cdot \overline{D}) \\
 Fib &= (\overline{A} \cdot \overline{B}) + (A \cdot \overline{B} \cdot \overline{C} \cdot \overline{D}) + (B \cdot \overline{C} \cdot D) \\
 Fib &= (\overline{A} \cdot \overline{B}) + (\overline{B} \cdot \overline{C} \cdot \overline{D}) + (B \cdot \overline{C} \cdot D)
 \end{aligned}$$

- (c) [20 points] Find the simplest representation of the *G3* output by using *only* 2-input NAND gates. Show your work step-by-step.

$$\begin{aligned}
 G3 &= \overline{(\overline{A} \cdot A)} \cdot \overline{(\overline{B} \cdot B)} \\
 \textbf{Explanation:} \\
 G3 &= (\overline{A} \cdot B \cdot \overline{C} \cdot \overline{D}) + (\overline{A} \cdot B \cdot \overline{C} \cdot D) + (\overline{A} \cdot B \cdot C \cdot \overline{D}) + (\overline{A} \cdot B \cdot C \cdot D) + (A \cdot \overline{B} \cdot \overline{C} \cdot \overline{D}) + (A \cdot \overline{B} \cdot \overline{C} \cdot D) \\
 &+ (A \cdot \overline{B} \cdot C \cdot \overline{D}) + (A \cdot \overline{B} \cdot C \cdot D) + (A \cdot B \cdot \overline{C} \cdot \overline{D}) + (A \cdot B \cdot \overline{C} \cdot D) + (A \cdot B \cdot C \cdot \overline{D}) + (A \cdot B \cdot C \cdot D) \\
 G3 &= (\overline{A} \cdot B \cdot ((\overline{C} \cdot \overline{D}) + (\overline{C} \cdot D) + (C \cdot \overline{D}) + (C \cdot D))) + (A \cdot \overline{B} \cdot ((\overline{C} \cdot \overline{D}) + (\overline{C} \cdot D) + (C \cdot \overline{D}) + (C \cdot D))) \\
 &+ (A \cdot B \cdot ((\overline{C} \cdot \overline{D}) + (\overline{C} \cdot D) + (C \cdot \overline{D}) + (C \cdot D))) \\
 G3 &= (\overline{A} \cdot B \cdot (1)) + (A \cdot \overline{B} \cdot (1)) + (A \cdot B \cdot (1)) \\
 G3 &= (\overline{A} \cdot B) + (A \cdot \overline{B}) + (A \cdot B) \\
 G3 &= A + B \\
 G3 &= \overline{\overline{A} + \overline{B}} \\
 G3 &= \overline{\overline{A} \cdot \overline{B}} \\
 G3 &= \overline{(\overline{A} \cdot A)} \cdot \overline{(\overline{B} \cdot B)}
 \end{aligned}$$