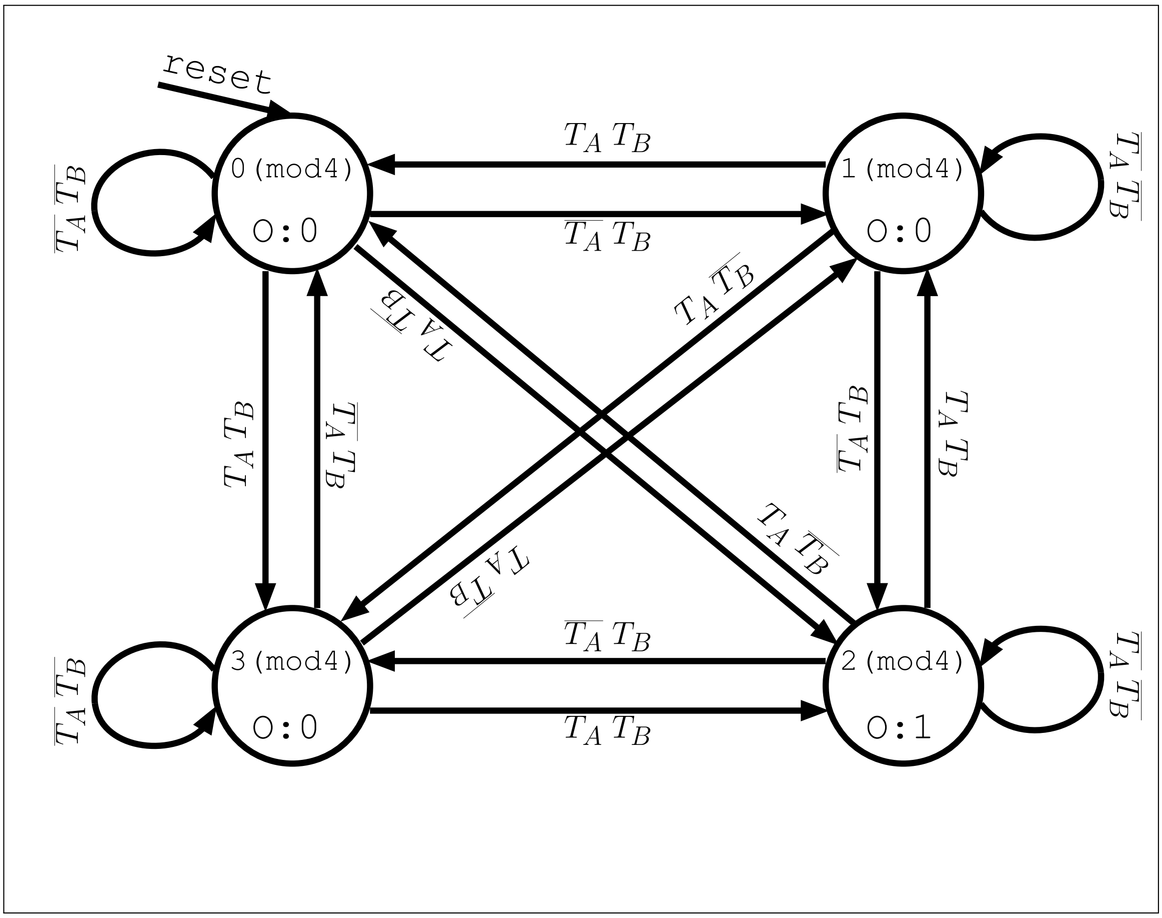
3 Finite State Machines (FSM) [30 points]

You are given two *one-bit* input signals (T_A and T_B) and one *one-bit* output signal (O) for the following modular equation: $2N(T_A)+N(T_B) \equiv 2 \pmod 4$. In this modular equation, $N(T_A)$ and $N(T_B)$ represent the **total number of times** the inputs T_A and T_B are high (i.e., logic 1) at each positive clock edge, respectively. The one-bit output signal, O, is set to 1 when the modular equation is satisfied (i.e., $2N(T_A) + N(T_B) \equiv 2 \pmod 4$), and 0 otherwise. An example that sets $O = 1$ at the end of the fourth cycle would be:

- (1^{st} cycle) $T_A = 0$ ($N(T_A) = 0$), $T_B = 0$ ($N(T_B) = 0$), $2N(T_A) + N(T_B) \equiv 0 \pmod 4 \Rightarrow O = 0$
- (2^{nd} cycle) $T_A = 1$ ($N(T_A) = 1$), $T_B = 1$ ($N(T_B) = 1$), $2N(T_A) + N(T_B) \equiv 3 \pmod 4 \Rightarrow O = 0$
- (3^{rd} cycle) $T_A = 1$ ($N(T_A) = 2$), $T_B = 0$ ($N(T_B) = 1$), $2N(T_A) + N(T_B) \equiv 1 \pmod 4 \Rightarrow O = 0$
- (4^{th} cycle) $T_A = 0$ ($N(T_A) = 2$), $T_B = 1$ ($N(T_B) = 2$), $2N(T_A) + N(T_B) \equiv 2 \pmod 4 \Rightarrow O = 1$

(a) [10 points] You are given a partial **Moore** machine state transition diagram that corresponds to the modular equation described above. However, the input labels of most of the transitions are still missing in this diagram. Please label the transitions with the correct inputs so that the FSM correctly implements the above specification.



(b) [10 points] Describe the FSM with Boolean equations assuming that the states are encoded with **one-hot encoding**. Assign state encodings while using the **minimum** possible number of bits to represent the states. Please indicate the values you assign to each state.

State assignments: 0 (mod 4): 0001, 1 (mod 4): 0010, 2 (mod 4): 0100, 3 (mod 4): 1000
CS denotes current states, and NS denotes next states.
$NS[0] = CS[0]\ \overline{T_A}\ \overline{T_B} + CS[1]\ T_A\ T_B + CS[2]\ T_A\ \overline{T_B} + CS[3]\ \overline{T_A}\ T_B$
$NS[1] = CS[1]\ \overline{T_A}\ \overline{T_B} + CS[2]\ T_A\ T_B + CS[3]\ T_A\ \overline{T_B} + CS[0]\ \overline{T_A}\ T_B$
$NS[2] = CS[2]\ \overline{T_A}\ \overline{T_B} + CS[3]\ T_A\ T_B + CS[0]\ T_A\ \overline{T_B} + CS[1]\ \overline{T_A}\ T_B$
$NS[3] = CS[3]\ \overline{T_A}\ \overline{T_B} + CS[0]\ T_A\ T_B + CS[1]\ T_A\ \overline{T_B} + CS[2]\ \overline{T_A}\ T_B$
$O[0] = CS[2]$

(c) [10 points] Describe the FSM with Boolean equations assuming that the states are encoded with **binary encoding** (i.e., fully encoding). Assign state encodings while using the **minimum** possible number of bits to represent the states. Please indicate the values you assign to each state.

State assignments: 0 (mod 4): 00, 1 (mod 4): 01, 2 (mod 4): 10, 3 (mod 4): 11
CS denotes current states, and NS denotes next states.
$NS[0] = \overline{CS[0]}\ T_B + CS[0]\ \overline{T_B}$
$NS[1] = CS[0]\ (CS[1] \text{ XOR } T_A \text{ XOR } T_B) + \overline{CS[0]}\ (T_A \text{ XOR } CS[1])$
$O[0] = CS[1]\ \overline{CS[0]}$