8 Systolic Arrays [30 points]

A systolic array consists of 3x4 Processing Elements (PEs), interconnected as shown in Figure 1. The inputs of the systolic array are labeled as H0, H1, H2 and V0,V1,V2,V3. Figure 2 shows the PE logic, which performs a multiply and accumulate operation (MAC), and it saves the result in an internal register (reg). Figure 2 also shows how each PE propagates its inputs. We make the following assumptions:

- The latency of each MAC is one cycle.
- The propagation of the values from i_0 to o_0 , and from i_1 to o_1 , takes one cycle.
- The initial value of all registers is zero.
- You can input a value more than once in the systolic array.

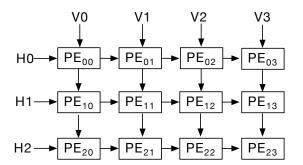


Figure 1: PE array

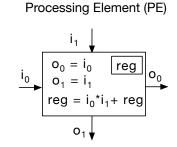


Figure 2: Processing Element (PE)

Your goal is to use this systolic array to perform the convolution of a 3x3 image (matrix I) with three 2x2 filters (matrices F, G, and H), to obtain three outputs (matrices O, U, and E):

As an example, the convolution of the matrix I with the filter F is computed as follows:

- $O_{00} = I_{00} * F_{00} + I_{01} * F_{01} + I_{10} * F_{10} + I_{11} * F_{11}$
- $O_{01} = I_{01} * F_{00} + I_{02} * F_{01} + I_{11} * F_{10} + I_{12} * F_{11}$
- $O_{10} = I_{10} * F_{00} + I_{11} * F_{01} + I_{20} * F_{10} + I_{21} * F_{11}$
- $O_{11} = I_{11} * F_{00} + I_{12} * F_{01} + I_{21} * F_{10} + I_{22} * F_{11}$

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You should compute the three convolutions in the minimum possible amount of cycles. Fill the following table with:

- 1. The input values (matrices I, F, G, and H) in the correct input ports of the systolic array (the values can be repeated).
- 2. The output values and the corresponding PE where the outputs (matrices O, U, and E) are generated.

Fill the gaps only with relevant information.

cycle	H 0	H1	H2	$\mathbf{V0}$	V1	V2	V3	PE_{00}	PE_{01}	PE_{02}	PE_{03}	PE_{10}	PE_{11}	PE_{12}	PE_{13}	PE_{20}	PE_{21}	PE_{22}	PE_{23}
0	$ F_{00} $			I_{00}															
1	F_{01}	G_{00}		I_{01}	I_{01}														
2	F_{10}	G_{01}	H_{00}	I_{10}	I_{02}	I_{10}													
3				I_{11}	I_{11}	I_{11}	I_{11}	O_{00}											
4		G_{11}	H_{10}		I_{12}	I_{20}	I_{12}		O_{01}			U_{00}							
5			H_{11}			I_{21}	I_{21}			O_{10}			U_{01}			E_{00}			
6							I_{22}				O_{11}			U_{10}			E_{01}		
7															U_{11}			E_{10}	
8																			E_{11}
9																			
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