5. Caches [30 points]

A byte-addressable system with 16-bit addresses ships with a three-way set associative, write-back cache. The cache implements a true LRU replacement policy using the minimum number of replacement policy bits necessary to implement it. The tag store requires a total of 264 bits of storage. What is the block size of the cache? (Hint: $264 = 2^8 + 2^3$)

Answer:

2^5 bytes

Show all your work.

Assume t tag bits, n index bits and b block bits.
$t + n + b = 16$
LRU = $\lceil \log_2 3! \rceil = 3$ bits per set with 3-way associativity
Valid Bit = 1 bit per block
Dirty Bit = 1 bit per block
Tag bits = t bits per block
Number of sets = 2^n
Number of blocks = $3 * 2^n$ (3-way associative)
Tag store size = $3 * 2^n + 3 * 2^n * (2 + t) = 2^n * (9 + 3t)$
We get $2^n * (9 + 3t) = 2^8 + 2^3$
$2^n * (9 + 3t) = 2^3 * (2^5 + 1)$
$2^n * (9 + 3t) = 2^3 * 33$
So, $n = 3$ and $t = 8$. As a result, $b = 5$
Therefore, the block size is 2^5.