2. A circuit has four inputs and two outputs. The inputs $A_{3:0}$ represent a number from 0 to 15. Output P should be TRUE if the number is prime (0 and 1 are not prime, but 2,3,5 and so on are prime). Output D should be TRUE if the number is divisible by 3. (*hint: 0 is not divisible by 3*).

   (a) (2 points) Complete the following truth table

A_3	A_2	A_1	A_0	P	D
0	0	0	0	0	0
0	0	0	1	0	0
0	0	1	0	1	0
0	0	1	1	1	1
0	1	0	0	0	0
0	1	0	1	1	0
0	1	1	0	0	1
0	1	1	1	1	0
1	0	0	0	0	0
1	0	0	1	0	1
1	0	1	0	0	0
1	0	1	1	1	0
1	1	0	0	0	1
1	1	0	1	1	0
1	1	1	0	0	0
1	1	1	1	0	1

   (b) (4 points) Write Sums of Products (SOP) representation for P and D. No simplification required here.

Solution:

$P = A_3\,A_2\,A_1\,A_0 + A_3\,A_2\,A_1\,A_0 + A_3\,A_2\,A_1\,A_0 +$
$A_3\,A_2\,A_1\,A_0 + A_3\,A_2\,A_1\,A_0 + A_3\,A_2\,A_1\,A_0$
$D = \overline{A_3}\,\overline{A_2}\,A_1\,A_0 + \overline{A_3}\,A_2\,A_1\,\overline{A_0} + A_3\,\overline{A_2}\,\overline{A_1}\,A_0 + A_3\,A_2\,\overline{A_1}\,\overline{A_0} + A_3\,A_2\,A_1\,A_0$

   (c) (4 points) Write simplified equations for both P and D.
(*Hint: you can use Karnaugh maps to simplify equations*)

Solution:

$P = \overline{A_3}\,A_2\,A_0 + \overline{A_3}\,A_1\,A_0 + \overline{A_3}\,\overline{A_2}\,A_1 + \overline{A_2}\,A_1\,A_0$ **or**
$P = \overline{A_3}\,A_1\,A_0 + \overline{A_3}\,\overline{A_2}\,A_1 + \overline{A_2}\,A_1\,A_0 + A_2\,\overline{A_1}\,A_0$
D can not be simplified further!
$D = \overline{A_3}\,\overline{A_2}\,A_1\,A_0 + \overline{A_3}\,A_2\,A_1\,\overline{A_0} + A_3\,\overline{A_2}\,\overline{A_1}\,A_0 + A_3\,A_2\,\overline{A_1}\,\overline{A_0} + A_3\,A_2\,A_1\,A_0$