1 Boolean Logic Circuits [45 points]

During your job interview, you are asked to design a combinational circuit with a four-bit input, $\{A, B, C, D\}$ (A is the most significant bit and D is the least significant bit), and two 1-bit outputs, Fib and $G3$. The value of each output is determined as follows:

- The output Fib is 1 only when the input 4-bit number is a Fibonacci number. You can calculate Fibonacci numbers as follows, $f(0) = 0$, $f(1) = 1$, and $f(n) = f(n-1) + f(n-2)$ for $n \geq 2$.
- The output $G3$ is 1 only when the input 4-bit number is greater than 3.
- Otherwise, the corresponding output is zero.

Please answer the following three questions.

(a) [10 points] Fill in the missing entries in the truth table below for the combinational circuit you are designing and express the output Fib in the *sum of products* representation.

Inputs				**Outputs**	
A	B	C	D	Fib	$G3$
0	0	0	0	1	0
0	0	0	1	1	0
0	0	1	0	1	0
0	0	1	1	1	0
0	1	0	0	0	1
0	1	0	1	1	1
0	1	1	0	0	1
0	1	1	1	0	1
1	0	0	0	1	1
1	0	0	1	0	1
1	0	1	0	0	1
1	0	1	1	0	1
1	1	0	0	0	1
1	1	0	1	1	1
1	1	1	0	0	1
1	1	1	1	0	1

$Fib = (\overline{A} \cdot \overline{B} \cdot \overline{C} \cdot \overline{D}) + (\overline{A} \cdot \overline{B} \cdot \overline{C} \cdot D) + (\overline{A} \cdot \overline{B} \cdot C \cdot \overline{D}) + (\overline{A} \cdot \overline{B} \cdot C \cdot D) + (\overline{A} \cdot B \cdot \overline{C} \cdot D) +$
$(A \cdot \overline{B} \cdot \overline{C} \cdot \overline{D}) + (A \cdot B \cdot \overline{C} \cdot D)$

(b) [15 points] Simplify the Fib expression using Boolean minimization rules. Show your work step-by-step.

$Fib = (\overline{A} \cdot \overline{B} \cdot \overline{C} \cdot \overline{D}) + (\overline{A} \cdot \overline{B} \cdot \overline{C} \cdot D) + (\overline{A} \cdot \overline{B} \cdot C \cdot \overline{D}) + (\overline{A} \cdot \overline{B} \cdot C \cdot D) + (\overline{A} \cdot B \cdot \overline{C} \cdot D) + (A \cdot \overline{B} \cdot \overline{C} \cdot \overline{D}) + (A \cdot B \cdot \overline{C} \cdot D)$

$Fib = ((\overline{A} \cdot \overline{B}) \cdot ((\overline{C} \cdot \overline{D}) + (\overline{C} \cdot D) + (C \cdot \overline{D}) + (C \cdot D))) + (\overline{A} \cdot B \cdot \overline{C} \cdot D) + (A \cdot \overline{B} \cdot \overline{C} \cdot \overline{D}) + (A \cdot B \cdot \overline{C} \cdot D)$

$Fib = ((\overline{A} \cdot \overline{B}) \cdot (1)) + (\overline{A} \cdot B \cdot \overline{C} \cdot D) + (A \cdot \overline{B} \cdot \overline{C} \cdot \overline{D}) + (A \cdot B \cdot \overline{C} \cdot D)$

$Fib = (\overline{A} \cdot \overline{B}) + (\overline{A} \cdot B \cdot \overline{C} \cdot D) + (A \cdot \overline{B} \cdot \overline{C} \cdot \overline{D}) + (A \cdot B \cdot \overline{C} \cdot D)$

$Fib = (\overline{A} \cdot \overline{B}) + (\overline{C} \cdot ((\overline{A} \cdot B \cdot D) + (A \cdot \overline{B} \cdot \overline{D}) + (A \cdot B \cdot D)))$

$Fib = (\overline{A} \cdot \overline{B}) + (\overline{C} \cdot ((B \cdot D) + (A \cdot \overline{B} \cdot \overline{D})))$

$Fib = (\overline{A} \cdot \overline{B}) + (B \cdot \overline{C} \cdot D) + (A \cdot \overline{B} \cdot \overline{C} \cdot \overline{D})$

$Fib = (\overline{A} \cdot \overline{B}) + (A \cdot \overline{B} \cdot \overline{C} \cdot \overline{D}) + (B \cdot \overline{C} \cdot D)$

$Fib = (\overline{A} \cdot \overline{B}) + (\overline{B} \cdot \overline{C} \cdot \overline{D}) + (B \cdot \overline{C} \cdot D)$

(c) [20 points] Find the simplest representation of the $G3$ output by using *only* 2-input NAND gates. Show your work step-by-step.

$G3 = \overline{(\overline{A \cdot A}) \cdot (\overline{B \cdot B})}$

Explanation:

$G3 = (\overline{A} \cdot B \cdot \overline{C} \cdot \overline{D}) + (\overline{A} \cdot B \cdot \overline{C} \cdot D) + (\overline{A} \cdot B \cdot C \cdot \overline{D}) + (\overline{A} \cdot B \cdot C \cdot D) + (A \cdot \overline{B} \cdot \overline{C} \cdot \overline{D}) + (A \cdot \overline{B} \cdot \overline{C} \cdot D) + (A \cdot \overline{B} \cdot C \cdot \overline{D}) + (A \cdot \overline{B} \cdot C \cdot D) + (A \cdot B \cdot \overline{C} \cdot \overline{D}) + (A \cdot B \cdot \overline{C} \cdot D) + (A \cdot B \cdot C \cdot \overline{D}) + (A \cdot B \cdot C \cdot D)$

$G3 = (\overline{A} \cdot B \cdot ((\overline{C} \cdot \overline{D}) + (\overline{C} \cdot D) + (C \cdot \overline{D}) + (C \cdot D))) + (A \cdot \overline{B} \cdot ((\overline{C} \cdot \overline{D}) + (\overline{C} \cdot D) + (C \cdot \overline{D}) + (C \cdot D))) + (A \cdot B \cdot ((\overline{C} \cdot \overline{D}) + (\overline{C} \cdot D) + (C \cdot \overline{D}) + (C \cdot D)))$

$G3 = (\overline{A} \cdot B \cdot (1)) + (A \cdot \overline{B} \cdot (1)) + (A \cdot B \cdot (1))$

$G3 = (\overline{A} \cdot B) + (A \cdot \overline{B}) + (A \cdot B)$

$G3 = A + B$

$G3 = \overline{\overline{A + B}}$

$G3 = \overline{\overline{A} \cdot \overline{B}}$

$G3 = \overline{(\overline{A \cdot A}) \cdot (\overline{B \cdot B})}$