

2. A circuit has four inputs and two outputs. The inputs $A_{3:0}$ represent a number from 0 to 15. Output P should be TRUE if the number is prime (0 and 1 are not prime, but 2,3,5 and so on are prime). Output D should be TRUE if the number is divisible by 3. (*hint: 0 is not divisible by 3*).

(a) (2 points) Complete the following truth table

A_3	A_2	A_1	A_0	P	D
0	0	0	0	0	0
0	0	0	1	0	0
0	0	1	0	1	0
0	0	1	1	1	1
0	1	0	0	0	0
0	1	0	1	1	0
0	1	1	0	0	1
0	1	1	1	1	0
1	0	0	0	0	0
1	0	0	1	0	1
1	0	1	0	0	0
1	0	1	1	1	0
1	1	0	0	0	1
1	1	0	1	1	0
1	1	1	0	0	0
1	1	1	1	0	1

- (b) (4 points) Write Sums of Products (SOP) representation for P and D . No simplification required here.

Solution:

$$P = A_3 A_2 A_1 A_0 + A_3 A_2 A_1 \overline{A_0} + A_3 A_2 A_1 A_0 +$$

$$A_3 A_2 \overline{A_1} \overline{A_0} + A_3 A_2 \overline{A_1} A_0 + A_3 A_2 A_1 \overline{A_0}$$

$$D = \overline{A_3} \overline{A_2} A_1 A_0 + \overline{A_3} A_2 A_1 \overline{A_0} + A_3 \overline{A_2} \overline{A_1} A_0 + A_3 A_2 \overline{A_1} \overline{A_0} + A_3 A_2 A_1 A_0$$

- (c) (4 points) Write simplified equations for both P and D .
(*Hint: you can use Karnaugh maps to simplify equations*)

Solution:

$$P = \overline{A_3} A_2 A_0 + \overline{A_3} A_1 A_0 + \overline{A_3} \overline{A_2} A_1 + \overline{A_2} A_1 A_0 \text{ or}$$

$$P = \overline{A_3} A_1 A_0 + \overline{A_3} \overline{A_2} A_1 + \overline{A_2} A_1 A_0 + A_2 \overline{A_1} A_0$$

D can not be simplified further!

$$D = \overline{A_3} \overline{A_2} A_1 A_0 + \overline{A_3} A_2 A_1 \overline{A_0} + A_3 \overline{A_2} \overline{A_1} A_0 + A_3 A_2 \overline{A_1} \overline{A_0} + A_3 A_2 A_1 A_0$$