3 Boolean Algebra [15 points]

(a) [5 points] Find the simplest sum-of-products representation of the following Boolean equation. Show your work step-by-step.

$F = B + (A + \overline{C}).(\overline{A} + \overline{B} + \overline{C})$

Answer: $F = A + B + \overline{C}$

Explanation:

$F = B + (A.\overline{A} + A.\overline{B} + A.\overline{C} + \overline{A}.\overline{C} + \overline{B}.\overline{C} + \overline{C}.\overline{C})$
$F = B + 0 + A.\overline{B} + \overline{C}.(A + \overline{A}) + \overline{B}.\overline{C} + \overline{C}$
$F = (B + A.\overline{B}) + \overline{C}.(A + \overline{A}) + (\overline{B}.\overline{C} + \overline{C})$
$F = (B + A) + \overline{C} + \overline{C}.(\overline{B} + 1)$
$F = A + B + \overline{C}$

(b) [5 points] Convert the following Boolean equation so that it only contains NAND operations. Show your work step-by-step.

$F = \overline{(A + B.C)} + \overline{C}$

Answer: $F = \overline{(\overline{(\overline{(A.A)}.\overline{(B.C)})}.C)}$

Explanation:

$F = \overline{(\overline{(\overline{(A + B.C)} + \overline{C})})}$
$F = \overline{((A + B.C).C)}$
$F = \overline{(\overline{\overline{(A + B.C)}}.C)}$
$F = \overline{(\overline{(\overline{A}.\overline{(B.C)})}.C)}$
$F = \overline{(\overline{(\overline{(A.A)}.\overline{(B.C)})}.C)}$

(c) [5 points] Using Boolean algebra, simplify the following min-terms: $\sum(3, 5, 7, 11, 13, 15)$

Show your work step-by-step.

Answer: $F = D.(B + C)$

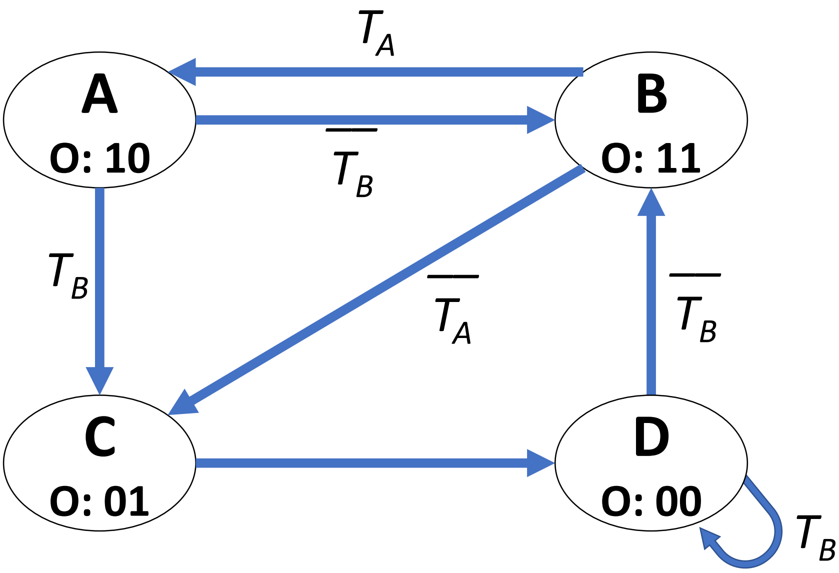
Explanation:

$\{3, 5, 7, 11, 13, 15\} = \{0011, 0101, 0111, 1011, 1101, 1111\}$

$F = (\overline{A}.\overline{B}.C.D) + (\overline{A}.B.\overline{C}.D) + (\overline{A}.B.C.D) + (A.\overline{B}.C.D) + (A.B.\overline{C}.D) + (A.B.C.D)$
$F = (C.D.((\overline{A}.\overline{B}) + (\overline{A}.B) + (A.\overline{B}) + (A.B))) + (B.D.((\overline{A}.\overline{C}) + (A.\overline{C})))$
$F = (C.D) + (B.\overline{C}.D)$
$F = D.(C + (B.\overline{C}))$
$F = D.(B + C)$

4 Finite State Machine [50 points]

You are given the following FSM with two one-bit input signals (T_A and T_B) and one two-bit output signal (O). You need to implement this FSM, but you are unsure about how you should encode the states. Answer the following questions to get a better sense of the FSM and how the three different types of state encoding we dicussed in the lecture (i.e., one-hot, binary, output) will affect the implementation.



(a) [3 points] There is one critical component of an FSM that is *missing* in this diagram. Please write what is missing in the answer box below.

The reset line or indication for initial state.

(b) [2 points] Of the two FSM types, what type of an FSM is this?

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(c) [5 points] List one major advantage of each type of state encoding below.

One-hot encoding	reduces next-state logic
Binary encoding	reduces FFs to hold state
Output encoding	reduces the output logic

(d) [10 points] Fully describe the FSM with equations given that the states are encoded with **one-hot** encoding. Assign state encodings such that numerical values of states increase monotonically for states A through D while using the **minimum** possible number of bits to represent the states with one-hot encoding. Indicate the values you assign to each state *and* simplify all equations:

State assignments: A: 0001, B: 0010, C: 0100, D: 1000
NS[3] = TB * TS[3] + TS[2]
NS[2] = TB * TS[0] + $\overline{TA}$ * TS[1]
NS[1] = $\overline{TB}$ * (TS[0] + TS[3])
NS[0] = TS[1] * TA
O[1] = TS[0] + TS[1]
O[0] = TS[1] + TS[2]

(e) [10 points] Fully describe the FSM with equations given that the states are encoded with **binary** encoding. Assign state encodings such that numerical values of states increase monotonically for states A through D while using the **minimum** possible number of bits to represent the states with binary encoding. Indicate the values you assign to each state *and* simplify all equations:

State assignments: A: 00, B: 01, C: 10, D: 11

$NS[1] = \overline{TS[1]} * (\overline{TS[0]} * TB + TS[0]\ \overline{TA}) + TS[1] * (\overline{TS[0]} + TS[0] * TB)$

$NS[0] = \overline{TS[1]} * \overline{TS[0]} * \overline{TB} + TS[1]$

$O[1] = TS[1]$

$O[0] = TS[1]\ \mathrm{XOR}\ TS[0]$

(f) [10 points] Fully describe the FSM with equations given that the states are encoded with **output** encoding. Use the **minimum** possible number of bits to represent the states with output encoding. Indicate the values you assign to each state *and* simplify all equations:

State assignments: A: 10, B: 11, C: 01, D: 00

$NS[1] = TS[1] * \overline{TS[0]} * \overline{TB} + TS[1] * TS[0] * TA + \overline{TS[1]} * \overline{TS[0]} * \overline{TB}$

$= \overline{TS[0]} * \overline{TB} + TS[1] * TS[0] * TA$

$NS[0] = TS[1] * \overline{TS[0]} + TS[1] * TS[0] * \overline{TA} + \overline{TS[1]} * \overline{TS[0]} * \overline{TB}$

$= TS[1] * (\overline{TS[0]} + TS[0] * \overline{TA}) + \overline{TS[1]} * \overline{TS[0]} * \overline{TB}$

$O[1] = TS[1]$

$O[0] = TS[0]$