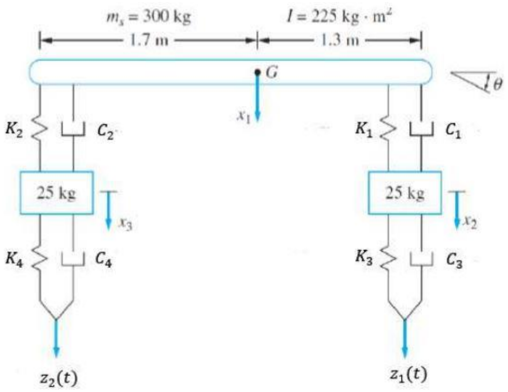

$$z_1(t) = 0.02[1 - \cos^2\left(\frac{10\pi vt}{6}\right)\left[1 - u\left(t - \frac{0.6}{v}\right)\right]$$

$$z_2(t) = z_1\left(t - \frac{L}{v}\right)$$

$$x_1(0) = 0.03, x_2(0) = 0, x_3(0) = 0, v = \frac{60m}{s}$$

Using Lagrange multipliers to derive the systems matrices.

$$T = \frac{1}{2}m_s\dot{x}_1^2 + \frac{1}{2}I\dot{\theta}^2 + \frac{1}{2}m_a\dot{x}_2^2 + \frac{1}{2}m_a\dot{x}_3^2$$

$$U = \frac{1}{2}k_3(x_2 - z_1)^2 + \frac{1}{2}k_1(x_2 - (x_1 + a\theta))^2 + \frac{1}{2}k_4(x_3 - z_2)^2 + \frac{1}{2}k_2(x_3 - (x_1 - (l - a)\theta))^2$$

$$W_{\theta_{n.c}} = [\mathrm{c}_1(\dot{x}_2 - \dot{x}_1 - \mathrm{a}\dot{\theta})\mathrm{a} - \mathrm{c}_2(\dot{x}_3 - \dot{x}_1 + (\mathrm{l} - \mathrm{a})\dot{\theta})(\mathrm{l} - \mathrm{a})]\delta\theta$$

$$W_{x_{1n.c}} = [\mathrm{c}_1(\dot{x}_2 - \dot{x}_1 - a\dot{\theta}) + c_2(\dot{x}_3 - \dot{x}_1 + (l - a)\dot{\theta})]\delta\mathrm{x}_1$$

$$W_{x_{2n.c}} = [-c_3(\dot{x}_2 - \dot{z}_1) - c_1(\dot{x}_2 - \dot{x}_1 - a\dot{\theta})]\delta x_2$$

$$W_{x_{3n.c}} = [-c_4(\dot{x}_3 - \dot{z}_2) - c_2(\dot{x}_3 - \dot{x}_1 + (l - a)\dot{\theta})]\delta x_3$$

$$Q_{\theta_{n.c}} = \mathrm{c}_1(\dot{x}_2 - \dot{x}_1 - \mathrm{a}\dot{\theta})\mathrm{a} - \mathrm{c}_2(\dot{x}_3 - \dot{x}_1 + (\mathrm{l} - \mathrm{a})\dot{\theta})(\mathrm{l} - \mathrm{a})$$

$$Q_{x_{1n.c}} = \mathrm{c}_1(\dot{x}_2 - \dot{x}_1 - a\dot{\theta}) + c_2(\dot{x}_3 - \dot{x}_1 + (l - a)\dot{\theta})$$

$$Q_{x_{2n.c}} = -c_3(\dot{x}_2 - \dot{z}_1) - c_1(\dot{x}_2 - \dot{x}_1 - a\dot{\theta})$$

$$Q_{x_{3n.c}} = -c_4(\dot{x}_3 - \dot{z}_2) - c_2(\dot{x}_3 - \dot{x}_1 + (l - a)\dot{\theta})$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial\dot{\theta}}\right) - \frac{\partial T}{\partial\theta} + \frac{\partial U}{\partial\theta} = Q_{\theta_{n.c}}$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x_1}}\right)-\frac{\partial T}{\partial x_1}+\frac{\partial U}{\partial x_1}=Q_{x_1 n.c}$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x_2}}\right)-\frac{\partial T}{\partial x_2}+\frac{\partial U}{\partial x_2}=Q_{x_2 n.c}$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x_3}}\right)-\frac{\partial T}{\partial x_3}+\frac{\partial U}{\partial x_3}=Q_{x_3 n.c}$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right)=I\ddot{\theta},\frac{\partial T}{\partial \theta}=0,\frac{\partial U}{\partial \theta}=ak_1\big((x_1+a\theta)-x_2\big)+(l-a)k_2(-x_1+(l-a)\theta+x_3)$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x_1}}\right)=m_s\ddot{x}_1,\frac{\partial T}{\partial x_1}=0,\frac{\partial U}{\partial x_1}=k_1\big((x_1+a\theta)-x_2\big)+k_2\big((x_1-(l-a)\theta)-x_3\big)$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x_2}}\right)=m_a\ddot{x}_2,\frac{\partial T}{\partial x_2}=0,\frac{\partial U}{\partial x_2}=k_3(x_2-z_1)+k_1(x_2-(x_1+a\theta))$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x_3}}\right)=m_a\ddot{x}_3,\frac{\partial T}{\partial x_3}=0,\frac{\partial U}{\partial x_3}=k_4(x_3-z_2)+k_2(x_3-x_1+(l-a)\theta)$$

$$\begin{aligned}I\ddot{\theta}+ak_1\big((x_1+a\theta)-x_2\big)+(l-a)k_2(-x_1+(l-a)\theta+x_3)\\=\mathrm{c}_1\big(\dot{x_2}-\dot{x_1}-\mathrm{a}\dot{\theta}\big)\mathrm{a}-\mathrm{c}_2\big(\dot{x_3}-\dot{x_1}+(\mathrm{l}-\mathrm{a})\dot{\theta}\big)(\mathrm{l}-\mathrm{a})\end{aligned}$$

$$\begin{aligned}\rightarrow I\ddot{\theta}+(a^2c_1+(l-a)^2c_2)\dot{\theta}+(c_1a-c_2(l-a))\dot{x_1}-c_1a\dot{x_2}+c_2(l-a)\dot{x_3}\\+(a^2k_1+(l-a)^2k_2)\theta+(k_1a-k_2(l-a))x_1-k_1ax_2+k_2(l-a)x_3=0\end{aligned}$$

$$\begin{aligned}m_s\ddot{x}_1+k_1\big((x_1+a\theta)-x_2\big)+k_2\big((x_1-(l-a)\theta)-x_3\big)\\=\mathrm{c}_1\big(\dot{x}_2-\dot{x_1}-a\dot{\theta}\big)+c_2(\dot{x}_3-\dot{x_1}+(l-a)\dot{\theta})\end{aligned}$$

$$\begin{aligned}\rightarrow m_s\ddot{x}_1+(ac_1-(l-a)c_2)\dot{\theta}+(c_1+c_2)\dot{x_1}-c_1\dot{x_2}-c_2\dot{x_3}+(ak_1-(l-a)k_2)\theta+(k_1+k_2)x_1\\-k_1x_2-k_2x_3=0\end{aligned}$$

$$m_a\ddot{x}_2+k_3(x_2-z_1)+k_1\big(x_2-(x_1+a\theta)\big)=-c_3(\dot{x}_2-\dot{z}_1)-c_1\big(\dot{x}_2-\dot{x_1}-a\dot{\theta}\big)$$

$$m_a\ddot{x}_2-ac_1\dot{\theta}-c_1\dot{x_1}+(c_1+c_3)\dot{x_2}-ak_1\theta-\mathrm{k}_1x_1+(k_1+k_3)x_2=c_3\dot{z}_1+k_3z_1$$

$$m_a\ddot{x}_3+k_4(x_3-z_2)+k_2(x_3-x_1+(l-a)\theta)=-c_4(\dot{x}_3-\dot{z}_2)-c_2\big(\dot{x}_3-\dot{x_1}+(l-a)\dot{\theta}\big)$$

$$m_a\ddot{x}_3+c_2(l-a)\dot{\theta}-c_2\dot{x_1}+(k_2+k_4)\dot{x_3}+\mathrm{k}_2(l-a)\theta-\mathrm{k}_2x_1+(c_2+c_4)x_3=c_4\dot{z}_2+k_4z_2$$

$$M\begin{bmatrix}\ddot{\theta}\\ \ddot{x}_1\\ \ddot{x}_2\\ \ddot{x}_3\end{bmatrix}+C\begin{bmatrix}\dot{\theta}\\ \dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}+K\begin{bmatrix}\theta\\ x_1\\ x_2\\ x_3\end{bmatrix}=\begin{bmatrix}0\\ 0\\ c_3\dot{z}_1+k_3z_1\\ c_4\dot{z}_2+k_4z_2\end{bmatrix}$$

$$M=[\begin{matrix}I & 0 & 0 & 0\\ 0 & m_s & 0 & 0\\ 0 & 0 & m_a & 0\\ 0 & 0 & 0 & m_a\end{matrix}]$$

$$C=[\begin{matrix}a^2c_1+(l-a)^2c_2 & c_1a-(l-a)c_2 & -c_1a & c_2(l-a)\\ c_1a-c_2(l-a) & c_1+c_2 & -c_1 & -c_2\\ -c_1a & -c_1 & c_1+c_3 & 0\\ c_2(l-a) & -c_2 & 0 & c_2+c_4\end{matrix}]$$

$$K=[\begin{matrix}a^2k_1+(l-a)^2k_2 & ak_1-(l-a)k_2 & -ak_1 & (l-a)k_2\\ ak_1-(l-a)k_2 & k_1+k_2 & -k_1 & -k_2\\ -ak_1 & -k_1 & k_1+k_3 & 0\\ (l-a)k_2 & -k_2 & 0 & k_2+k_4\end{matrix}]$$

$$y_1=\theta$$

$$y_2=x_1$$

$$y_3=x_2$$

$$y_4=x_3$$

$$y_5=\dot{\theta}$$

$$y_6=\dot{x}_1$$

$$y_7=\dot{x}_2$$

$$y_8=\dot{x}_3$$

$$\dot{y}_1=y_5$$

$$\dot{y}_2=y_6$$

$$\dot{y}_3=y_7$$

$$\dot{y}_4=y_8$$

$$\dot{y_5} = \left(-\frac{1}{I}\right)\left[(a^2k_1 + (l-a)^2k_2)y_1 + \left(k_1a - k_2(l-a)\right)y_2 - k_1ay_3 + k_2(l-a)y_4 + (a^2c_1 + (l-a)^2c_2)y_5 + \left(c_1a - c_2(l-a)\right)y_6 - c_1ay_7 + c_2(l-a)y_8\right]$$

$$\dot{y_6} = \left(-\frac{1}{\mathrm{m_s}}\right)[(ak_1 - (l-a)k_2)\mathrm{y}_1 + (k_1 + k_2)y_2 - k_1y_3 - k_2y_4 + (ac_1 - (l-a)c_2)y_5 + (c_1 + c_2)y_6 - c_1y_7 - c_2y_8]$$

$$\dot{y_7} = \left(-\frac{1}{\mathrm{m_a}}\right)[-ak_1\mathrm{y}_1 - \mathrm{k}_1y_2 + (k_1 + k_3)y_3 - ac_1y_5 - c_1y_6 + (c_1 + c_3)y_7 - c_3\dot{z}_1 - k_3z_1]$$

$$\dot{y_8} = \left(-\frac{1}{m_a}\right)[\mathrm{k}_2(l-a)\mathrm{y}_1 - \mathrm{k}_2y_2 + (c_2 + c_4)y_4 + c_2(l-a)y_5 - c_2y_6 + (k_2 + k_4)y_8 - c_4\dot{z}_2 - k_4z_2]$$

$$z_1(t) = 0.02[1 - \cos^2(100\pi t)[1 - u(t-0.01)][u(t)]$$

$$\dot{z}_1(t) = -4\pi\cos(100\pi t)\sin(100\pi t)(u(t-0.01) - 1)$$

$$z_2(t) = 0.02[1 - \cos^2\left(100\pi(t-0.05)\right)[1 - u(t-0.06)][u(t-0.05)]$$

$$\dot{z}_2(t) = -4\pi u(t-0.05)\cos(100\pi(t-0.05))\sin(100\pi(t-0.05))(u(t-0.06) - 1)$$

PS: I have neglected the Dirac delta function in the derivative of z1 and z2; because they don't affect the results to a great extent. If we consider these functions, the calculations will become more complex.