

APPENDIX

Dynamic Modelling and Control Design of a Pediatric Robotic Lower-Limb Exoskeleton

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For the right leg of a coupled human-exoskeleton system with 5 DOFs, the inertia matrix $M(q)$, Coriolis and centripetal matrix $C(q, \dot{q})$, gravity vector $G(q)$, and torque vector τ are provided below:

A. INERTIA MATRIX ($M(q)$)

$$M(q) = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} & M_{15} \\ M_{21} & M_{22} & M_{23} & M_{24} & M_{25} \\ M_{31} & M_{32} & M_{33} & M_{34} & M_{35} \\ M_{41} & M_{42} & M_{43} & M_{44} & M_{45} \\ M_{51} & M_{52} & M_{53} & M_{54} & M_{55} \end{bmatrix} \quad (\text{A1})$$

where

$$\begin{aligned} M_{11} = & I_{yy,1} + I_{zz,2} + I_{zz,3} + I_{zz,4} + I_{zz,5} + 0.25 a_1^2 m_1 + a_1^2 m_2 + a_1^2 m_3 + 0.25 a_2^2 m_2 + a_1^2 m_4 + \\ & a_2^2 m_3 + a_1^2 m_5 + a_2^2 m_4 + 0.25 a_3^2 m_3 + a_2^2 m_5 + a_3^2 m_4 + a_3^2 m_5 + 0.25 a_4^2 m_4 + a_4^2 m_5 + 0.25 a_5^2 m_5 + \\ & lc_{2,1}^2 m_2 + lc_{2,2}^2 m_2 + lc_{3,1}^2 m_3 + lc_{3,2}^2 m_3 + lc_{4,1}^2 m_4 + lc_{4,2}^2 m_4 + lc_{5,1}^2 m_5 + lc_{5,2}^2 m_5 + lc_1^2 m_1 + \\ & lc_3^2 m_1 + I_{xx,2} \cos(\theta_2)^2 + I_{xx,3} \cos(\theta_2)^2 + I_{yy,4} \cos(\theta_2)^2 + I_{yy,5} \cos(\theta_2)^2 - I_{zz,2} \cos(\theta_2)^2 - I_{zz,3} \cos(\theta_2)^2 - \\ & I_{zz,4} \cos(\theta_2)^2 - I_{zz,5} \cos(\theta_2)^2 - I_{xx,3} \cos(\theta_2)^2 \cos(\theta_3)^2 + I_{xx,4} \cos(\theta_2)^2 \cos(\theta_3)^2 + I_{xx,4} \cos(\theta_2)^2 \cos(\theta_4)^2 + \\ & I_{xx,5} \cos(\theta_2)^2 \cos(\theta_3)^2 + I_{xx,5} \cos(\theta_2)^2 \cos(\theta_4)^2 + I_{xx,5} \cos(\theta_2)^2 \cos(\theta_5)^2 + I_{yy,3} \cos(\theta_2)^2 \cos(\theta_3)^2 - I_{yy,4} \cos(\theta_2)^2 \cos(\theta_3)^2 - \\ & I_{yy,4} \cos(\theta_2)^2 \cos(\theta_4)^2 - I_{yy,5} \cos(\theta_2)^2 \cos(\theta_3)^2 - I_{yy,5} \cos(\theta_2)^2 \cos(\theta_4)^2 - I_{yy,5} \cos(\theta_2)^2 \cos(\theta_5)^2 + 2 I_{xz,2} \cos(\theta_2) \sin(\theta_2) + \\ & a_2 lc_{2,1} m_2 + a_3 lc_{3,1} m_3 + a_4 lc_{4,1} m_4 + a_5 lc_{5,1} m_5 + a_1 lc_1 m_1 - 0.25 a_2^2 m_2 \cos(\theta_2)^2 - a_2^2 m_3 \cos(\theta_2)^2 - \\ & a_2^2 m_4 \cos(\theta_2)^2 - 0.25 a_3^2 m_3 \cos(\theta_2)^2 - a_2^2 m_5 \cos(\theta_2)^2 - a_3^2 m_4 \cos(\theta_2)^2 - a_3^2 m_5 \cos(\theta_2)^2 - lc_{2,1}^2 m_2 \cos(\theta_2)^2 + \\ & lc_{2,3}^2 m_2 \cos(\theta_2)^2 - lc_{3,1}^2 m_3 \cos(\theta_2)^2 + lc_{3,3}^2 m_3 \cos(\theta_2)^2 - lc_{4,2}^2 m_4 \cos(\theta_2)^2 + lc_{4,3}^2 m_4 \cos(\theta_2)^2 - \\ & lc_{5,2}^2 m_5 \cos(\theta_2)^2 + lc_{5,3}^2 m_5 \cos(\theta_2)^2 + lc_{3,1}^2 m_3 \cos(\theta_2)^2 \cos(\theta_3)^2 - lc_{3,2}^2 m_3 \cos(\theta_2)^2 \cos(\theta_3)^2 - lc_{4,1}^2 m_4 \cos(\theta_2)^2 \cos(\theta_3)^2 - \\ & lc_{4,1}^2 m_4 \cos(\theta_2)^2 \cos(\theta_4)^2 + lc_{4,2}^2 m_4 \cos(\theta_2)^2 \cos(\theta_3)^2 + lc_{4,2}^2 m_4 \cos(\theta_2)^2 \cos(\theta_4)^2 - lc_{5,1}^2 m_5 \cos(\theta_2)^2 \cos(\theta_3)^2 - \\ & lc_{5,1}^2 m_5 \cos(\theta_2)^2 \cos(\theta_4)^2 + lc_{5,2}^2 m_5 \cos(\theta_2)^2 \cos(\theta_3)^2 - lc_{5,1}^2 m_5 \cos(\theta_2)^2 \cos(\theta_5)^2 + lc_{5,2}^2 m_5 \cos(\theta_2)^2 \cos(\theta_4)^2 + \\ & lc_{5,2}^2 m_5 \cos(\theta_2)^2 \cos(\theta_5)^2 + 2 I_{yz,3} \cos(\theta_2) \cos(\theta_3) \sin(\theta_2) - 2 I_{xx,4} \cos(\theta_2)^2 \cos(\theta_3)^2 \cos(\theta_4)^2 - 2 I_{xx,5} \cos(\theta_2)^2 \cos(\theta_3)^2 \cos(\theta_4)^2 - \\ & 2 I_{xx,5} \cos(\theta_2)^2 \cos(\theta_3)^2 \cos(\theta_5)^2 - 2 I_{xx,5} \cos(\theta_2)^2 \cos(\theta_4)^2 \cos(\theta_5)^2 + 2 I_{yy,4} \cos(\theta_2)^2 \cos(\theta_3)^2 \cos(\theta_4)^2 + \\ & 2 I_{yy,5} \cos(\theta_2)^2 \cos(\theta_3)^2 \cos(\theta_4)^2 + 2 I_{yy,5} \cos(\theta_2)^2 \cos(\theta_3)^2 \cos(\theta_5)^2 + 2 I_{yy,5} \cos(\theta_2)^2 \cos(\theta_4)^2 \cos(\theta_5)^2 + \\ & 2 I_{xz,3} \cos(\theta_2) \sin(\theta_2) \sin(\theta_3) + a_3 a_4 m_4 \cos(\theta_4) + 2 a_3 a_4 m_5 \cos(\theta_4) + 2 a_1 lc_{2,3} m_2 \cos(\theta_2) + 2 a_1 lc_{3,3} m_3 \cos(\theta_2) + \\ & 2 a_2 lc_{3,2} m_3 \cos(\theta_3) + 2 a_1 lc_{4,3} m_4 \cos(\theta_2) + 2 a_3 lc_{4,1} m_4 \cos(\theta_4) + 2 a_1 lc_{5,3} m_5 \cos(\theta_2) - 2 a_4 lc_{5,2} m_5 \cos(\theta_5) - \\ & a_1 a_2 m_2 \sin(\theta_2) - 2 a_1 a_2 m_3 \sin(\theta_2) - 2 a_1 a_2 m_4 \sin(\theta_2) - 2 a_1 a_2 m_5 \sin(\theta_2) + a_2 a_3 m_3 \sin(\theta_3) + \\ & 2 a_2 a_3 m_4 \sin(\theta_3) + 2 a_2 a_3 m_5 \sin(\theta_3) - a_4 a_5 m_5 \sin(\theta_5) - 2 a_1 lc_{2,1} m_2 \sin(\theta_2) + 2 a_2 lc_{3,1} m_3 \sin(\theta_3) - \\ & 2 a_3 lc_{4,2} m_4 \sin(\theta_4) - 2 a_4 lc_{5,1} m_5 \sin(\theta_5) + 2 I_{xy,3} \cos(\theta_2)^2 \cos(\theta_3) \sin(\theta_3) - 2 I_{xy,4} \cos(\theta_2)^2 \cos(\theta_3) \sin(\theta_3) - \\ & 2 I_{xy,5} \cos(\theta_2)^2 \cos(\theta_3) \sin(\theta_3) - 2 I_{xy,4} \cos(\theta_2)^2 \cos(\theta_4) \sin(\theta_4) - 2 I_{xy,5} \cos(\theta_2)^2 \cos(\theta_4) \sin(\theta_4) - \\ & 2 I_{xy,5} \cos(\theta_2)^2 \cos(\theta_5) \sin(\theta_5) - a_2 lc_{2,1} m_2 \cos(\theta_2)^2 - a_3 lc_{3,1} m_3 \cos(\theta_2)^2 + 0.25 a_3^2 m_3 \cos(\theta_2)^2 \cos(\theta_3)^2 + \\ & a_3^2 m_4 \cos(\theta_2)^2 \cos(\theta_3)^2 + a_3^2 m_5 \cos(\theta_2)^2 \cos(\theta_3)^2 - 0.25 a_4^2 m_4 \cos(\theta_2)^2 \cos(\theta_3)^2 - 0.25 a_4^2 m_4 \cos(\theta_2)^2 \cos(\theta_4)^2 - \\ & a_4^2 m_5 \cos(\theta_2)^2 \cos(\theta_3)^2 - a_4^2 m_5 \cos(\theta_2)^2 \cos(\theta_4)^2 - 0.25 a_5^2 m_5 \cos(\theta_2)^2 \cos(\theta_3)^2 - 0.25 a_5^2 m_5 \cos(\theta_2)^2 \cos(\theta_4)^2 - \end{aligned}$$

$$\begin{aligned}
& 0.25 a_5^2 m_5 \cos(\theta_2)^2 \cos(\theta_5)^2 + a_3 \text{lc}_{3,1} m_3 \cos(\theta_2)^2 \cos(\theta_3)^2 - a_4 \text{lc}_{4,1} m_4 \cos(\theta_2)^2 \cos(\theta_3)^2 - a_4 \text{lc}_{4,1} m_4 \cos(\theta_2)^2 \cos(\theta_4)^2 - \\
& a_5 \text{lc}_{5,1} m_5 \cos(\theta_2)^2 \cos(\theta_3)^2 - a_5 \text{lc}_{5,1} m_5 \cos(\theta_2)^2 \cos(\theta_4)^2 - a_5 \text{lc}_{5,1} m_5 \cos(\theta_2)^2 \cos(\theta_5)^2 + 2 I_{yz,4} \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) \sin(\theta_2) + \\
& 2 I_{xz,4} \cos(\theta_2) \cos(\theta_3) \sin(\theta_2) \sin(\theta_4) + 2 I_{xz,4} \cos(\theta_2) \cos(\theta_4) \sin(\theta_2) \sin(\theta_3) + 0.5 a_4^2 m_4 \cos(\theta_2)^2 \cos(\theta_3)^2 \cos(\theta_4)^2 + \\
& 2 a_4^2 m_5 \cos(\theta_2)^2 \cos(\theta_3)^2 \cos(\theta_4)^2 + 0.5 a_5^2 m_5 \cos(\theta_2)^2 \cos(\theta_3)^2 \cos(\theta_4)^2 + 0.5 a_5^2 m_5 \cos(\theta_2)^2 \cos(\theta_3)^2 \cos(\theta_5)^2 + \\
& 0.5 a_5^2 m_5 \cos(\theta_2)^2 \cos(\theta_4)^2 \cos(\theta_5)^2 - 2 I_{yz,4} \cos(\theta_2) \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) + 2 \text{lc}_{4,1}^2 m_4 \cos(\theta_2)^2 \cos(\theta_3)^2 \cos(\theta_4)^2 - \\
& 2 \text{lc}_{4,2}^2 m_4 \cos(\theta_2)^2 \cos(\theta_3)^2 \cos(\theta_4)^2 + 2 \text{lc}_{5,1}^2 m_5 \cos(\theta_2)^2 \cos(\theta_3)^2 \cos(\theta_4)^2 + 2 \text{lc}_{5,1}^2 m_5 \cos(\theta_2)^2 \cos(\theta_3)^2 \cos(\theta_5)^2 - \\
& 2 \text{lc}_{5,2}^2 m_5 \cos(\theta_2)^2 \cos(\theta_3)^2 \cos(\theta_4)^2 + 2 \text{lc}_{5,1}^2 m_5 \cos(\theta_2)^2 \cos(\theta_4)^2 \cos(\theta_5)^2 - 2 \text{lc}_{5,2}^2 m_5 \cos(\theta_2)^2 \cos(\theta_3)^2 \cos(\theta_5)^2 - \\
& 2 \text{lc}_{5,2}^2 m_5 \cos(\theta_2)^2 \cos(\theta_4)^2 \cos(\theta_5)^2 + 2 a_2 \text{lc}_{4,2} m_4 \cos(\theta_3) \cos(\theta_4) - 2 a_3 \text{lc}_{5,2} m_5 \cos(\theta_4) \cos(\theta_5) + \\
& a_2 a_4 m_4 \cos(\theta_3) \sin(\theta_4) + a_2 a_4 m_4 \cos(\theta_4) \sin(\theta_3) + 2 a_2 a_4 m_5 \cos(\theta_3) \sin(\theta_4) + 2 a_2 a_4 m_5 \cos(\theta_4) \sin(\theta_3) - \\
& a_3 a_5 m_5 \cos(\theta_4) \sin(\theta_5) - a_3 a_5 m_5 \cos(\theta_5) \sin(\theta_4) - a_2 \text{lc}_{2,3} m_2 \cos(\theta_2) \sin(\theta_2) - 2 a_1 \text{lc}_{3,2} m_3 \cos(\theta_3) \sin(\theta_2) - \\
& 2 a_2 \text{lc}_{3,3} m_3 \cos(\theta_2) \sin(\theta_2) - 2 a_2 \text{lc}_{4,3} m_4 \cos(\theta_2) \sin(\theta_2) + 2 a_2 \text{lc}_{4,1} m_4 \cos(\theta_3) \sin(\theta_4) + 2 a_2 \text{lc}_{4,1} m_4 \cos(\theta_4) \sin(\theta_3) - \\
& 2 a_2 \text{lc}_{5,3} m_5 \cos(\theta_2) \sin(\theta_2) - 2 a_3 \text{lc}_{5,1} m_5 \cos(\theta_4) \sin(\theta_5) - 2 a_3 \text{lc}_{5,1} m_5 \cos(\theta_5) \sin(\theta_4) - a_1 a_3 m_3 \sin(\theta_2) \sin(\theta_3) - \\
& 2 a_1 a_3 m_4 \sin(\theta_2) \sin(\theta_3) - 2 a_1 a_3 m_5 \sin(\theta_2) \sin(\theta_3) - 2 \text{lc}_{2,1} \text{lc}_{2,3} m_2 \cos(\theta_2) \sin(\theta_2) - 2 a_1 \text{lc}_{3,1} m_3 \sin(\theta_2) \sin(\theta_3) - \\
& 2 a_2 \text{lc}_{4,2} m_4 \sin(\theta_3) \sin(\theta_4) + 2 a_3 \text{lc}_{5,2} m_5 \sin(\theta_4) \sin(\theta_5) + 4 I_{xx,5} \cos(\theta_2)^2 \cos(\theta_3)^2 \cos(\theta_4)^2 \cos(\theta_5)^2 - \\
& 4 I_{yy,5} \cos(\theta_2)^2 \cos(\theta_3)^2 \cos(\theta_4)^2 \cos(\theta_5)^2 - a_3 a_4 m_4 \cos(\theta_2)^2 \cos(\theta_4)^2 - 2 a_3 a_4 m_5 \cos(\theta_2)^2 \cos(\theta_4)^2 - \\
& 2 a_2 \text{lc}_{3,2} m_3 \cos(\theta_2)^2 \cos(\theta_3)^2 - 2 a_3 \text{lc}_{4,1} m_4 \cos(\theta_2)^2 \cos(\theta_4)^2 - a_2 a_3 m_3 \cos(\theta_2)^2 \sin(\theta_3) - 2 a_2 a_3 m_4 \cos(\theta_2)^2 \sin(\theta_3) - \\
& 2 a_2 a_3 m_5 \cos(\theta_2)^2 \sin(\theta_3) - 2 a_2 \text{lc}_{3,1} m_3 \cos(\theta_2)^2 \sin(\theta_3) + 2 a_3 \text{lc}_{4,2} m_4 \cos(\theta_2)^2 \sin(\theta_4) + 4 I_{xy,4} \cos(\theta_2)^2 \cos(\theta_3) \cos(\theta_4)^2 \sin(\theta_3) + \\
& 4 I_{xy,4} \cos(\theta_2)^2 \cos(\theta_3)^2 \cos(\theta_4) \sin(\theta_4) + 4 I_{xy,5} \cos(\theta_2)^2 \cos(\theta_3) \cos(\theta_4)^2 \sin(\theta_3) + 4 I_{xy,5} \cos(\theta_2)^2 \cos(\theta_3) \cos(\theta_5)^2 \sin(\theta_3) + \\
& 4 I_{xy,5} \cos(\theta_2)^2 \cos(\theta_3)^2 \cos(\theta_4) \sin(\theta_4) + 4 I_{xy,5} \cos(\theta_2)^2 \cos(\theta_4) \cos(\theta_5)^2 \sin(\theta_4) + 4 I_{xy,5} \cos(\theta_2)^2 \cos(\theta_3)^2 \cos(\theta_5) \sin(\theta_5) + \\
& 4 I_{xy,5} \cos(\theta_2)^2 \cos(\theta_4)^2 \cos(\theta_5) \sin(\theta_5) - 8 I_{xy,5} \cos(\theta_2)^2 \cos(\theta_3) \cos(\theta_4)^2 \cos(\theta_5)^2 \sin(\theta_3) - 8 I_{xy,5} \cos(\theta_2)^2 \cos(\theta_3)^2 \cos(\theta_4) \cos(\theta_5)^2 \sin(\theta_4) - \\
& 8 I_{xy,5} \cos(\theta_2)^2 \cos(\theta_3)^2 \cos(\theta_4)^2 \cos(\theta_5) \sin(\theta_5) - 2 I_{xz,5} \cos(\theta_2) \cos(\theta_3) \sin(\theta_2) \sin(\theta_4) \sin(\theta_5) - 2 I_{xz,5} \cos(\theta_2) \cos(\theta_4) \sin(\theta_2) \sin(\theta_3) \sin(\theta_5) - \\
& 2 I_{xz,5} \cos(\theta_2) \cos(\theta_5) \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) + a_2 a_5 m_5 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) + 2 I_{yz,5} \cos(\theta_2) \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) + \\
& 2 a_2 \text{lc}_{5,1} m_5 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) - 2 a_1 \text{lc}_{4,2} m_4 \cos(\theta_3) \cos(\theta_4) \sin(\theta_2) - 2 a_2 \text{lc}_{5,2} m_5 \cos(\theta_3) \cos(\theta_4) \sin(\theta_5) - \\
& 2 a_2 \text{lc}_{5,2} m_5 \cos(\theta_3) \cos(\theta_5) \sin(\theta_4) - 2 a_2 \text{lc}_{5,2} m_5 \cos(\theta_4) \cos(\theta_5) \sin(\theta_3) - a_1 a_4 m_4 \cos(\theta_3) \sin(\theta_2) \sin(\theta_4) - \\
& a_1 a_4 m_4 \cos(\theta_4) \sin(\theta_2) \sin(\theta_3) - 2 a_1 a_4 m_5 \cos(\theta_3) \sin(\theta_2) \sin(\theta_4) - 2 a_1 a_4 m_5 \cos(\theta_4) \sin(\theta_2) \sin(\theta_3) - \\
& a_2 a_5 m_5 \cos(\theta_3) \sin(\theta_4) \sin(\theta_5) - a_2 a_5 m_5 \cos(\theta_4) \sin(\theta_3) \sin(\theta_5) - a_2 a_5 m_5 \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) - \\
& 2 \text{lc}_{3,2} \text{lc}_{3,3} m_3 \cos(\theta_2) \cos(\theta_3) \sin(\theta_2) + 2 a_4 \text{lc}_{4,1} m_4 \cos(\theta_2)^2 \cos(\theta_3)^2 \cos(\theta_4)^2 + 2 a_5 \text{lc}_{5,1} m_5 \cos(\theta_2)^2 \cos(\theta_3)^2 \cos(\theta_4)^2 + \\
& 2 a_5 \text{lc}_{5,1} m_5 \cos(\theta_2)^2 \cos(\theta_3)^2 \cos(\theta_5)^2 + 2 a_5 \text{lc}_{5,1} m_5 \cos(\theta_2)^2 \cos(\theta_4)^2 \cos(\theta_5)^2 - a_3 \text{lc}_{3,3} m_3 \cos(\theta_2) \sin(\theta_2) \sin(\theta_3) - \\
& 2 a_1 \text{lc}_{4,1} m_4 \cos(\theta_3) \sin(\theta_2) \sin(\theta_4) - 2 a_1 \text{lc}_{4,1} m_4 \cos(\theta_4) \sin(\theta_2) \sin(\theta_3) - 2 a_3 \text{lc}_{4,3} m_4 \cos(\theta_2) \sin(\theta_2) \sin(\theta_3) - \\
& 2 a_3 \text{lc}_{5,3} m_5 \cos(\theta_2) \sin(\theta_2) \sin(\theta_3) - 2 a_2 \text{lc}_{5,1} m_5 \cos(\theta_3) \sin(\theta_4) \sin(\theta_5) - 2 a_2 \text{lc}_{5,1} m_5 \cos(\theta_4) \sin(\theta_3) \sin(\theta_5) - \\
& 2 a_2 \text{lc}_{5,1} m_5 \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) - 2 \text{lc}_{3,1} \text{lc}_{3,3} m_3 \cos(\theta_2) \sin(\theta_2) \sin(\theta_3) + 2 a_1 \text{lc}_{4,2} m_4 \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) + \\
& 2 a_2 \text{lc}_{5,2} m_5 \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) + 2 I_{xx,4} \cos(\theta_2)^2 \cos(\theta_3) \cos(\theta_4) \sin(\theta_3) \sin(\theta_4) + 2 I_{xx,5} \cos(\theta_2)^2 \cos(\theta_3) \cos(\theta_4) \sin(\theta_3) \sin(\theta_4) + \\
& 2 I_{xx,5} \cos(\theta_2)^2 \cos(\theta_3) \cos(\theta_5) \sin(\theta_3) \sin(\theta_5) + 2 I_{xx,5} \cos(\theta_2)^2 \cos(\theta_4) \cos(\theta_5) \sin(\theta_4) \sin(\theta_5) - 2 I_{yy,4} \cos(\theta_2)^2 \cos(\theta_3) \cos(\theta_4) \sin(\theta_3) \sin(\theta_4) - \\
& 2 I_{yy,5} \cos(\theta_2)^2 \cos(\theta_3) \cos(\theta_4) \sin(\theta_3) \sin(\theta_4) - 2 I_{yy,5} \cos(\theta_2)^2 \cos(\theta_3) \cos(\theta_5) \sin(\theta_3) \sin(\theta_5) - 2 I_{yy,5} \cos(\theta_2)^2 \cos(\theta_4) \cos(\theta_5) \sin(\theta_4) \sin(\theta_5) - \\
& a_5^2 m_5 \cos(\theta_2)^2 \cos(\theta_3)^2 \cos(\theta_4)^2 \cos(\theta_5)^2 - 4 \text{lc}_{5,1}^2 m_5 \cos(\theta_2)^2 \cos(\theta_3)^2 \cos(\theta_4)^2 \cos(\theta_5)^2 + 4 \text{lc}_{5,2}^2 m_5 \cos(\theta_2)^2 \cos(\theta_3)^2 \cos(\theta_4)^2 \cos(\theta_5)^2 - \\
& 2 a_2 \text{lc}_{4,2} m_4 \cos(\theta_2)^2 \cos(\theta_3) \cos(\theta_4) + 2 a_3 \text{lc}_{5,2} m_5 \cos(\theta_2)^2 \cos(\theta_4) \cos(\theta_5) - a_2 a_4 m_4 \cos(\theta_2)^2 \cos(\theta_3) \sin(\theta_4) - \\
& a_2 a_4 m_4 \cos(\theta_2)^2 \cos(\theta_4) \sin(\theta_3) - 2 a_2 a_4 m_5 \cos(\theta_2)^2 \cos(\theta_3) \sin(\theta_4) - 2 a_2 a_4 m_5 \cos(\theta_2)^2 \cos(\theta_4) \sin(\theta_3) +
\end{aligned}$$

$$\begin{aligned}
M_{12} = & I_{xy,4} \cos(\theta_2) - I_{xy,3} \cos(\theta_2) - I_{xy,2} \cos(\theta_2) + I_{xy,5} \cos(\theta_2) - I_{yz,2} \sin(\theta_2) + I_{xz,3} \cos(\theta_3) \sin(\theta_2) - \\
& I_{yz,3} \sin(\theta_2) \sin(\theta_3) + 2.0 I_{xy,3} \cos(\theta_2) \cos(\theta_3)^2 - 2.0 I_{xy,4} \cos(\theta_2) \cos(\theta_3)^2 - 2.0 I_{xy,4} \cos(\theta_2) \cos(\theta_4)^2 - 2.0 I_{xy,5} \cos(\theta_2) \cos(\theta_3)^2 - \\
& 2.0 I_{xy,5} \cos(\theta_2) \cos(\theta_4)^2 - 2.0 I_{xy,5} \cos(\theta_2) \cos(\theta_5)^2 + I_{xx,3} \cos(\theta_2) \cos(\theta_3) \sin(\theta_3) - I_{xx,4} \cos(\theta_2) \cos(\theta_3) \sin(\theta_3) - \\
& I_{xx,5} \cos(\theta_2) \cos(\theta_3) \sin(\theta_3) - I_{xx,4} \cos(\theta_2) \cos(\theta_4) \sin(\theta_4) - I_{xx,5} \cos(\theta_2) \cos(\theta_4) \sin(\theta_4) + I_{xz,4} \cos(\theta_3) \cos(\theta_4) \sin(\theta_2) - \\
& I_{xx,5} \cos(\theta_2) \cos(\theta_5) \sin(\theta_5) - I_{yy,3} \cos(\theta_2) \cos(\theta_3) \sin(\theta_3) + I_{yy,4} \cos(\theta_2) \cos(\theta_3) \sin(\theta_3) + I_{yy,5} \cos(\theta_2) \cos(\theta_3) \sin(\theta_3) + \\
& I_{yy,4} \cos(\theta_2) \cos(\theta_4) \sin(\theta_4) + I_{yy,5} \cos(\theta_2) \cos(\theta_4) \sin(\theta_4) + I_{yy,5} \cos(\theta_2) \cos(\theta_5) \sin(\theta_5) - I_{yz,4} \cos(\theta_3) \sin(\theta_2) \sin(\theta_4) - \\
& I_{yz,4} \cos(\theta_4) \sin(\theta_2) \sin(\theta_3) - I_{xz,4} \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) + 0.5 a_2 lc_{2,2} m_2 \cos(\theta_2) + 0.5 a_3 lc_{3,2} m_3 \cos(\theta_2) - \\
& 0.5 a_4 lc_{4,2} m_4 \cos(\theta_2) - 0.5 a_5 lc_{5,2} m_5 \cos(\theta_2) + lc_{2,1} lc_{2,2} m_2 \cos(\theta_2) + lc_{3,1} lc_{3,2} m_3 \cos(\theta_2) - lc_{4,1} lc_{4,2} m_4 \cos(\theta_2) - \\
& lc_{5,1} lc_{5,2} m_5 \cos(\theta_2) + lc_{2,2} lc_{2,3} m_2 \sin(\theta_2) + 4.0 I_{xy,4} \cos(\theta_2) \cos(\theta_3)^2 \cos(\theta_4)^2 + 4.0 I_{xy,5} \cos(\theta_2) \cos(\theta_3)^2 \cos(\theta_4)^2 + \\
& 4.0 I_{xy,5} \cos(\theta_2) \cos(\theta_3)^2 \cos(\theta_5)^2 + 4.0 I_{xy,5} \cos(\theta_2) \cos(\theta_4)^2 \cos(\theta_5)^2 - I_{yz,5} \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) \sin(\theta_2) - \\
& 0.25 a_3^2 m_3 \cos(\theta_2) \cos(\theta_3) \sin(\theta_3) - a_3^2 m_4 \cos(\theta_2) \cos(\theta_3) \sin(\theta_3) - a_3^2 m_5 \cos(\theta_2) \cos(\theta_3) \sin(\theta_3) + 0.25 a_4^2 m_4 \cos(\theta_2) \cos(\theta_3) \sin(\theta_3) + \\
& a_4^2 m_5 \cos(\theta_2) \cos(\theta_3) \sin(\theta_3) + 0.25 a_4^2 m_4 \cos(\theta_2) \cos(\theta_4) \sin(\theta_4) + 0.25 a_5^2 m_5 \cos(\theta_2) \cos(\theta_3) \sin(\theta_3) + a_4^2 m_5 \cos(\theta_2) \cos(\theta_4) \sin(\theta_4) + \\
& 0.25 a_5^2 m_5 \cos(\theta_2) \cos(\theta_4) \sin(\theta_4) + 0.25 a_5^2 m_5 \cos(\theta_2) \cos(\theta_5) \sin(\theta_5) - 8.0 I_{xy,5} \cos(\theta_2) \cos(\theta_3)^2 \cos(\theta_4)^2 \cos(\theta_5)^2 - \\
& I_{xz,5} \cos(\theta_3) \cos(\theta_4) \sin(\theta_2) \sin(\theta_5) - I_{xz,5} \cos(\theta_3) \cos(\theta_5) \sin(\theta_2) \sin(\theta_4) - I_{xz,5} \cos(\theta_4) \cos(\theta_5) \sin(\theta_2) \sin(\theta_3) - \\
& lc_{3,1}^2 m_3 \cos(\theta_2) \cos(\theta_3) \sin(\theta_3) + lc_{3,2}^2 m_3 \cos(\theta_2) \cos(\theta_3) \sin(\theta_3) + lc_{4,1}^2 m_4 \cos(\theta_2) \cos(\theta_3) \sin(\theta_3) - lc_{4,2}^2 m_4 \cos(\theta_2) \cos(\theta_3) \sin(\theta_3) + \\
& lc_{4,1}^2 m_4 \cos(\theta_2) \cos(\theta_4) \sin(\theta_4) - lc_{4,2}^2 m_4 \cos(\theta_2) \cos(\theta_4) \sin(\theta_4) + lc_{5,1}^2 m_5 \cos(\theta_2) \cos(\theta_3) \sin(\theta_3) - lc_{5,2}^2 m_5 \cos(\theta_2) \cos(\theta_3) \sin(\theta_3) + \\
& lc_{5,1}^2 m_5 \cos(\theta_2) \cos(\theta_4) \sin(\theta_4) - lc_{5,2}^2 m_5 \cos(\theta_2) \cos(\theta_4) \sin(\theta_4) + lc_{5,1}^2 m_5 \cos(\theta_2) \cos(\theta_5) \sin(\theta_5) - lc_{5,2}^2 m_5 \cos(\theta_2) \cos(\theta_5) \sin(\theta_5) + \\
& I_{yz,5} \cos(\theta_3) \sin(\theta_2) \sin(\theta_4) \sin(\theta_5) + I_{yz,5} \cos(\theta_4) \sin(\theta_2) \sin(\theta_3) \sin(\theta_5) + I_{yz,5} \cos(\theta_5) \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) - \\
& 0.5 a_2 a_3 m_3 \cos(\theta_2) \cos(\theta_3) - a_2 a_3 m_4 \cos(\theta_2) \cos(\theta_3) - a_2 a_3 m_5 \cos(\theta_2) \cos(\theta_3) - 0.5 a_4 a_5 m_5 \cos(\theta_2) \cos(\theta_5) + \\
& I_{xz,5} \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) - a_2 lc_{3,1} m_3 \cos(\theta_2) \cos(\theta_3) + a_3 lc_{4,2} m_4 \cos(\theta_2) \cos(\theta_4) - a_4 lc_{5,1} m_5 \cos(\theta_2) \cos(\theta_5) + \\
& 0.5 a_3 a_4 m_4 \cos(\theta_2) \sin(\theta_4) + a_3 a_4 m_5 \cos(\theta_2) \sin(\theta_4) + a_2 lc_{3,2} m_3 \cos(\theta_2) \sin(\theta_3) - 0.5 a_3 lc_{3,3} m_3 \cos(\theta_3) \sin(\theta_2) + \\
& a_3 lc_{4,1} m_4 \cos(\theta_2) \sin(\theta_4) - a_3 lc_{4,3} m_4 \cos(\theta_3) \sin(\theta_2) - a_3 lc_{5,3} m_5 \cos(\theta_3) \sin(\theta_2) + a_4 lc_{5,2} m_5 \cos(\theta_2) \sin(\theta_5) - \\
& lc_{3,1} lc_{3,3} m_3 \cos(\theta_3) \sin(\theta_2) + lc_{3,2} lc_{3,3} m_3 \sin(\theta_2) \sin(\theta_3) + 2.0 I_{xx,4} \cos(\theta_2) \cos(\theta_3) \cos(\theta_4)^2 \sin(\theta_3) + 2.0 I_{xx,4} \cos(\theta_2) \cos(\theta_3)^2 \cos(\theta_4) \sin(\theta_4) + \\
& 2.0 I_{xx,5} \cos(\theta_2) \cos(\theta_3) \cos(\theta_4)^2 \sin(\theta_3) + 2.0 I_{xx,5} \cos(\theta_2) \cos(\theta_3) \cos(\theta_5)^2 \sin(\theta_3) + 2.0 I_{xx,5} \cos(\theta_2) \cos(\theta_3)^2 \cos(\theta_4) \sin(\theta_4) + \\
& 2.0 I_{xx,5} \cos(\theta_2) \cos(\theta_4) \cos(\theta_5)^2 \sin(\theta_4) + 2.0 I_{xx,5} \cos(\theta_2) \cos(\theta_3)^2 \cos(\theta_5) \sin(\theta_5) + 2.0 I_{xx,5} \cos(\theta_2) \cos(\theta_4)^2 \cos(\theta_5) \sin(\theta_5) - \\
& 2.0 I_{yy,4} \cos(\theta_2) \cos(\theta_3) \cos(\theta_4)^2 \sin(\theta_3) - 2.0 I_{yy,4} \cos(\theta_2) \cos(\theta_3)^2 \cos(\theta_4) \sin(\theta_4) - 2.0 I_{yy,5} \cos(\theta_2) \cos(\theta_3) \cos(\theta_4)^2 \sin(\theta_3) - \\
& 2.0 I_{yy,5} \cos(\theta_2) \cos(\theta_3) \cos(\theta_5)^2 \sin(\theta_3) - 2.0 I_{yy,5} \cos(\theta_2) \cos(\theta_3)^2 \cos(\theta_4) \sin(\theta_4) - 2.0 I_{yy,5} \cos(\theta_2) \cos(\theta_4) \cos(\theta_5)^2 \sin(\theta_4) - \\
& 2.0 I_{yy,5} \cos(\theta_2) \cos(\theta_3)^2 \cos(\theta_5) \sin(\theta_5) - 2.0 I_{yy,5} \cos(\theta_2) \cos(\theta_4)^2 \cos(\theta_5) \sin(\theta_5) - a_3 lc_{3,2} m_3 \cos(\theta_2) \cos(\theta_3)^2 + \\
& a_4 lc_{4,2} m_4 \cos(\theta_2) \cos(\theta_3)^2 + a_4 lc_{4,2} m_4 \cos(\theta_2) \cos(\theta_4)^2 + a_5 lc_{5,2} m_5 \cos(\theta_2) \cos(\theta_3)^2 + a_5 lc_{5,2} m_5 \cos(\theta_2) \cos(\theta_4)^2 + \\
& a_5 lc_{5,2} m_5 \cos(\theta_2) \cos(\theta_5)^2 - 2.0 lc_{3,1} lc_{3,2} m_3 \cos(\theta_2) \cos(\theta_3)^2 + 2.0 lc_{4,1} lc_{4,2} m_4 \cos(\theta_2) \cos(\theta_3)^2 + 2.0 lc_{4,1} lc_{4,2} m_4 \cos(\theta_2) \cos(\theta_4)^2 + \\
& 2.0 lc_{5,1} lc_{5,2} m_5 \cos(\theta_2) \cos(\theta_3)^2 + 2.0 lc_{5,1} lc_{5,2} m_5 \cos(\theta_2) \cos(\theta_4)^2 + 2.0 lc_{5,1} lc_{5,2} m_5 \cos(\theta_2) \cos(\theta_5)^2 - 0.5 a_2 a_4 m_4 \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) - \\
& a_2 a_4 m_5 \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) + 0.5 a_3 a_5 m_5 \cos(\theta_2) \cos(\theta_4) \cos(\theta_5) - a_2 lc_{4,1} m_4 \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) + \\
& a_3 lc_{5,1} m_5 \cos(\theta_2) \cos(\theta_4) \cos(\theta_5) - a_3 lc_{3,1} m_3 \cos(\theta_2) \cos(\theta_3) \sin(\theta_3) + a_2 lc_{4,2} m_4 \cos(\theta_2) \cos(\theta_3) \sin(\theta_4) + \\
& a_2 lc_{4,2} m_4 \cos(\theta_2) \cos(\theta_4) \sin(\theta_3) + a_4 lc_{4,1} m_4 \cos(\theta_2) \cos(\theta_3) \sin(\theta_3) + a_4 lc_{4,1} m_4 \cos(\theta_2) \cos(\theta_4) \sin(\theta_4) - \\
& 0.5 a_4 lc_{4,3} m_4 \cos(\theta_3) \cos(\theta_4) \sin(\theta_2) + a_5 lc_{5,1} m_5 \cos(\theta_2) \cos(\theta_3) \sin(\theta_3) - a_3 lc_{5,2} m_5 \cos(\theta_2) \cos(\theta_4) \sin(\theta_5) - \\
& a_3 lc_{5,2} m_5 \cos(\theta_2) \cos(\theta_5) \sin(\theta_4) - a_4 lc_{5,3} m_5 \cos(\theta_3) \cos(\theta_4) \sin(\theta_2) + a_5 lc_{5,1} m_5 \cos(\theta_2) \cos(\theta_4) \sin(\theta_4) + \\
& a_5 lc_{5,1} m_5 \cos(\theta_2) \cos(\theta_5) \sin(\theta_5) + 0.5 a_2 a_4 m_4 \cos(\theta_2) \sin(\theta_3) \sin(\theta_4) + a_2 a_4 m_5 \cos(\theta_2) \sin(\theta_3) \sin(\theta_4) -
\end{aligned}$$

[illegible]

[illegible]

$$\begin{aligned}
M_{13} = & I_{yz,4} \cos(\theta_2) \sin(\theta_3) \sin(\theta_4) - I_{zz,4} \sin(\theta_2) - I_{zz,5} \sin(\theta_2) - I_{yz,3} \cos(\theta_2) \cos(\theta_3) - I_{xz,3} \cos(\theta_2) \sin(\theta_3) - 0.25 a_3^2 m_3 \sin(\theta_2) - \\
& a_3^2 m_4 \sin(\theta_2) - a_3^2 m_5 \sin(\theta_2) - 0.25 a_4^2 m_4 \sin(\theta_2) - a_4^2 m_5 \sin(\theta_2) - 0.25 a_5^2 m_5 \sin(\theta_2) - lc_{3,1}^2 m_3 \sin(\theta_2) - lc_{3,2}^2 m_3 \sin(\theta_2) - \\
& lc_{4,1}^2 m_4 \sin(\theta_2) - lc_{4,2}^2 m_4 \sin(\theta_2) - lc_{5,1}^2 m_5 \sin(\theta_2) - lc_{5,2}^2 m_5 \sin(\theta_2) - I_{yz,4} \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) - I_{xz,4} \cos(\theta_2) \cos(\theta_3) \sin(\theta_4) - \\
& I_{xz,4} \cos(\theta_2) \cos(\theta_4) \sin(\theta_3) - I_{zz,3} \sin(\theta_2) + a_1 lc_{3,2} m_3 \cos(\theta_3) + 0.5 a_1 a_3 m_3 \sin(\theta_3) + a_1 a_3 m_4 \sin(\theta_3) + a_1 a_3 m_5 \sin(\theta_3) + \\
& a_1 lc_{3,1} m_3 \sin(\theta_3) - a_3 lc_{3,1} m_3 \sin(\theta_2) - a_4 lc_{4,1} m_4 \sin(\theta_2) - a_5 lc_{5,1} m_5 \sin(\theta_2) - I_{xz,5} \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) - \\
& 0.5 a_2 lc_{3,1} m_3 \sin(\theta_3) \sin(\theta_2) + a_3 lc_{4,2} m_4 \sin(\theta_4) \sin(\theta_2) + a_4 lc_{5,1} m_5 \sin(\theta_5) \sin(\theta_2) + I_{yz,5} \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) \sin(\theta_5) + \\
& I_{yz,5} \cos(\theta_2) \cos(\theta_3) \cos(\theta_5) \sin(\theta_4) + I_{yz,5} \cos(\theta_2) \cos(\theta_4) \cos(\theta_5) \sin(\theta_3) + I_{xz,5} \cos(\theta_2) \cos(\theta_3) \sin(\theta_4) \sin(\theta_5) + I_{xz,5} \cos(\theta_2) \cos(\theta_4) \sin(\theta_3) \sin(\theta_5) + \\
& I_{xz,5} \cos(\theta_2) \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) - I_{yz,5} \cos(\theta_2) \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) + 0.25 a_2 a_3 m_3 \cos(\theta_2) \cos(\theta_3) + 0.5 a_2 a_3 m_4 \cos(\theta_2) \cos(\theta_3) + \\
& 0.5 a_2 a_3 m_5 \cos(\theta_2) \cos(\theta_3) + 0.5 a_2 lc_{3,1} m_3 \cos(\theta_2) \cos(\theta_3) + a_1 lc_{4,2} m_4 \cos(\theta_3) \cos(\theta_4) - a_3 lc_{4,2} m_4 \cos(\theta_2) \cos(\theta_4) - a_4 lc_{5,1} m_5 \cos(\theta_2) \cos(\theta_5) + \\
& 0.5 a_1 a_4 m_4 \cos(\theta_3) \sin(\theta_4) + 0.5 a_1 a_4 m_4 \cos(\theta_4) \sin(\theta_3) + a_1 a_4 m_5 \cos(\theta_3) \sin(\theta_4) + a_1 a_4 m_5 \cos(\theta_4) \sin(\theta_3) - 0.5 a_3 a_4 m_4 \cos(\theta_2) \sin(\theta_4) - \\
& 0.5 a_3 a_4 m_4 \cos(\theta_4) \sin(\theta_2) - a_3 a_4 m_5 \cos(\theta_2) \sin(\theta_4) - a_3 a_4 m_5 \cos(\theta_4) \sin(\theta_2) + lc_{3,2} lc_{3,3} m_3 \cos(\theta_2) \cos(\theta_3) - 0.5 a_2 lc_{3,2} m_3 \cos(\theta_2) \sin(\theta_3) - \\
& 0.5 a_2 lc_{3,2} m_3 \cos(\theta_3) \sin(\theta_2) + 0.25 a_3 lc_{3,3} m_3 \cos(\theta_2) \sin(\theta_3) + 0.25 a_3 lc_{3,3} m_3 \cos(\theta_3) \sin(\theta_2) + a_1 lc_{4,1} m_4 \cos(\theta_3) \sin(\theta_4) + \\
& a_1 lc_{4,1} m_4 \cos(\theta_4) \sin(\theta_3) - a_3 lc_{4,1} m_4 \cos(\theta_2) \sin(\theta_4) - a_3 lc_{4,1} m_4 \cos(\theta_4) \sin(\theta_2) + a_3 lc_{4,3} m_4 \cos(\theta_2) \sin(\theta_3) + a_3 lc_{5,3} m_5 \cos(\theta_2) \sin(\theta_3) + \\
& a_4 lc_{5,2} m_5 \cos(\theta_2) \sin(\theta_5) + a_4 lc_{5,2} m_5 \cos(\theta_5) \sin(\theta_2) - 0.25 a_2 a_3 m_3 \sin(\theta_2) \sin(\theta_3) - 0.5 a_2 a_3 m_4 \sin(\theta_2) \sin(\theta_3) - 0.5 a_2 a_3 m_5 \sin(\theta_2) \sin(\theta_3) + \\
& a_4 a_5 m_5 \sin(\theta_2) \sin(\theta_5) + lc_{3,1} lc_{3,3} m_3 \cos(\theta_2) \sin(\theta_3) - 0.5 a_2 lc_{3,1} m_3 \sin(\theta_2) \sin(\theta_3) - a_1 lc_{4,2} m_4 \sin(\theta_3) \sin(\theta_4) + a_3 lc_{4,2} m_4 \sin(\theta_2) \sin(\theta_4) + \\
& a_4 lc_{5,1} m_5 \sin(\theta_2) \sin(\theta_5) - 0.25 a_2 a_3 m_3 \cos(\theta_3) \cos(\theta_2) - 0.5 a_2 a_3 m_4 \cos(\theta_3) \cos(\theta_2) - 0.5 a_2 a_3 m_5 \cos(\theta_3) \cos(\theta_2) - 0.5 a_2 lc_{3,1} m_3 \cos(\theta_3) \cos(\theta_2) + \\
& a_3 lc_{4,2} m_4 \cos(\theta_4) \cos(\theta_2) + a_4 lc_{5,1} m_5 \cos(\theta_5) \cos(\theta_2) - 0.5 a_3 a_4 m_4 \cos(\theta_4) \sin(\theta_2) + 0.5 a_3 a_4 m_4 \sin(\theta_4) \cos(\theta_2) - a_3 a_4 m_5 \cos(\theta_4) \sin(\theta_2) + \\
& a_3 a_4 m_5 \sin(\theta_4) \cos(\theta_2) - 0.5 a_2 lc_{3,2} m_3 \cos(\theta_3) \sin(\theta_2) + 0.5 a_2 lc_{3,2} m_3 \sin(\theta_3) \cos(\theta_2) - 0.25 a_3 lc_{3,3} m_3 \cos(\theta_3) \sin(\theta_2) + \\
& 0.25 a_3 lc_{3,3} m_3 \sin(\theta_3) \cos(\theta_2) - a_3 lc_{4,1} m_4 \cos(\theta_4) \sin(\theta_2) + a_3 lc_{4,1} m_4 \sin(\theta_4) \cos(\theta_2) + a_4 lc_{5,2} m_5 \cos(\theta_5) \sin(\theta_2) - a_4 lc_{5,2} m_5 \sin(\theta_5) \cos(\theta_2) - \\
& 0.25 a_2 a_3 m_3 \sin(\theta_3) \sin(\theta_2) - 0.5 a_2 a_3 m_4 \sin(\theta_3) \sin(\theta_2) - 0.5 a_2 a_3 m_5 \sin(\theta_3) \sin(\theta_2) + 0.5 a_1 a_5 m_5 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) + \\
& a_1 lc_{5,1} m_5 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) + lc_{4,2} lc_{4,3} m_4 \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) - a_2 lc_{4,2} m_4 \cos(\theta_3) \cos(\theta_4) \sin(\theta_2) + 0.5 a_4 lc_{4,3} m_4 \cos(\theta_2) \cos(\theta_3) \sin(\theta_4) + \\
& 0.5 a_4 lc_{4,3} m_4 \cos(\theta_2) \cos(\theta_4) \sin(\theta_3) - a_1 lc_{5,2} m_5 \cos(\theta_3) \cos(\theta_4) \sin(\theta_5) - a_1 lc_{5,2} m_5 \cos(\theta_3) \cos(\theta_5) \sin(\theta_4) - a_1 lc_{5,2} m_5 \cos(\theta_4) \cos(\theta_5) \sin(\theta_3) + \\
& 2.0 a_3 lc_{5,2} m_5 \cos(\theta_4) \cos(\theta_5) \sin(\theta_2) + a_4 lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_3) \sin(\theta_4) + a_4 lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_4) \sin(\theta_3) - 0.5 a_2 a_4 m_4 \cos(\theta_3) \sin(\theta_2) \sin(\theta_4) - \\
& 0.5 a_2 a_4 m_4 \cos(\theta_4) \sin(\theta_2) \sin(\theta_3) - a_2 a_4 m_5 \cos(\theta_3) \sin(\theta_2) \sin(\theta_4) - a_2 a_4 m_5 \cos(\theta_4) \sin(\theta_2) \sin(\theta_3) - 0.5 a_1 a_5 m_5 \cos(\theta_3) \sin(\theta_4) \sin(\theta_5) - \\
& 0.5 a_1 a_5 m_5 \cos(\theta_4) \sin(\theta_3) \sin(\theta_5) - 0.5 a_1 a_5 m_5 \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) + a_3 a_5 m_5 \cos(\theta_4) \sin(\theta_2) \sin(\theta_5) + a_3 a_5 m_5 \cos(\theta_5) \sin(\theta_2) \sin(\theta_4) + \\
& lc_{4,1} lc_{4,3} m_4 \cos(\theta_2) \cos(\theta_3) \sin(\theta_4) + lc_{4,1} lc_{4,3} m_4 \cos(\theta_2) \cos(\theta_4) \sin(\theta_3) - a_2 lc_{4,1} m_4 \cos(\theta_3) \sin(\theta_2) \sin(\theta_4) - a_2 lc_{4,1} m_4 \cos(\theta_4) \sin(\theta_2) \sin(\theta_3) - \\
& a_1 lc_{5,1} m_5 \cos(\theta_3) \sin(\theta_4) \sin(\theta_5) - a_1 lc_{5,1} m_5 \cos(\theta_4) \sin(\theta_3) \sin(\theta_5) - a_1 lc_{5,1} m_5 \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) + 2.0 a_3 lc_{5,1} m_5 \cos(\theta_4) \sin(\theta_2) \sin(\theta_5) + \\
& 2.0 a_3 lc_{5,1} m_5 \cos(\theta_5) \sin(\theta_2) \sin(\theta_4) - lc_{4,2} lc_{4,3} m_4 \cos(\theta_2) \sin(\theta_3) \sin(\theta_4) + a_2 lc_{4,2} m_4 \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) + a_1 lc_{5,2} m_5 \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) - \\
& 2.0 a_3 lc_{5,2} m_5 \sin(\theta_2) \sin(\theta_4) \sin(\theta_5) + lc_{5,1} lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) - a_2 lc_{5,1} m_5 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) \sin(\theta_2) - \\
& lc_{5,2} lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) \sin(\theta_5) - lc_{5,2} lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_3) \cos(\theta_5) \sin(\theta_4) - lc_{5,2} lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_4) \cos(\theta_5) \sin(\theta_3) + \\
& a_2 lc_{5,2} m_5 \cos(\theta_3) \cos(\theta_4) \sin(\theta_2) \sin(\theta_5) + a_2 lc_{5,2} m_5 \cos(\theta_3) \cos(\theta_5) \sin(\theta_2) \sin(\theta_4) + a_2 lc_{5,2} m_5 \cos(\theta_4) \cos(\theta_5) \sin(\theta_2) \sin(\theta_3) - \\
& 0.5 a_5 lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_3) \sin(\theta_4) \sin(\theta_5) - 0.5 a_5 lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_4) \sin(\theta_3) \sin(\theta_5) - 0.5 a_5 lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) + \\
& 0.5 a_2 a_5 m_5 \cos(\theta_3) \sin(\theta_2) \sin(\theta_4) \sin(\theta_5) + 0.5 a_2 a_5 m_5 \cos(\theta_4) \sin(\theta_2) \sin(\theta_3) \sin(\theta_5) + 0.5 a_2 a_5 m_5 \cos(\theta_5) \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) - \\
& lc_{5,1} lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_3) \sin(\theta_4) \sin(\theta_5) - lc_{5,1} lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_4) \sin(\theta_3) \sin(\theta_5) - lc_{5,1} lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) + \\
& a_2 lc_{5,1} m_5 \cos(\theta_3) \sin(\theta_2) \sin(\theta_4) \sin(\theta_5) + a_2 lc_{5,1} m_5 \cos(\theta_4) \sin(\theta_2) \sin(\theta_3) \sin(\theta_5) + a_2 lc_{5,1} m_5 \cos(\theta_5) \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) + \\
& lc_{5,2} lc_{5,3} m_5 \cos(\theta_2) \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) - a_2 lc_{5,2} m_5 \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) + 0.5 a_5 lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) - \\
& 0.5 a_2 a_5 m_5 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) \sin(\theta_2)
\end{aligned}
\tag{A4}$$

$$\begin{aligned}
M_{21} = & I_{xy,4} \cos(\theta_2) - I_{xy,3} \cos(\theta_2) - I_{xy,2} \cos(\theta_2) + I_{xy,5} \cos(\theta_2) - I_{yz,2} \sin(\theta_2) + I_{xz,3} \cos(\theta_3) \sin(\theta_2) - \\
& I_{yz,3} \sin(\theta_2) \sin(\theta_3) + 2.0 I_{xy,3} \cos(\theta_2) \cos(\theta_3)^2 - 2.0 I_{xy,4} \cos(\theta_2) \cos(\theta_3)^2 - 2.0 I_{xy,4} \cos(\theta_2) \cos(\theta_4)^2 - \\
& 2.0 I_{xy,5} \cos(\theta_2) \cos(\theta_3)^2 - 2.0 I_{xy,5} \cos(\theta_2) \cos(\theta_4)^2 - 2.0 I_{xy,5} \cos(\theta_2) \cos(\theta_5)^2 + I_{xx,3} \cos(\theta_2) \cos(\theta_3) \sin(\theta_3) - \\
& I_{xx,4} \cos(\theta_2) \cos(\theta_3) \sin(\theta_3) - I_{xx,5} \cos(\theta_2) \cos(\theta_3) \sin(\theta_3) - I_{xx,4} \cos(\theta_2) \cos(\theta_4) \sin(\theta_4) - I_{xx,5} \cos(\theta_2) \cos(\theta_4) \sin(\theta_4) + \\
& I_{xz,4} \cos(\theta_3) \cos(\theta_4) \sin(\theta_2) - I_{xx,5} \cos(\theta_2) \cos(\theta_5) \sin(\theta_5) - I_{yy,3} \cos(\theta_2) \cos(\theta_3) \sin(\theta_3) + I_{yy,4} \cos(\theta_2) \cos(\theta_3) \sin(\theta_3) + \\
& I_{yy,5} \cos(\theta_2) \cos(\theta_3) \sin(\theta_3) + I_{yy,4} \cos(\theta_2) \cos(\theta_4) \sin(\theta_4) + I_{yy,5} \cos(\theta_2) \cos(\theta_4) \sin(\theta_4) + I_{yy,5} \cos(\theta_2) \cos(\theta_5) \sin(\theta_5) - \\
& I_{yz,4} \cos(\theta_3) \sin(\theta_2) \sin(\theta_4) - I_{yz,4} \cos(\theta_4) \sin(\theta_2) \sin(\theta_3) - I_{xz,4} \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) + 0.5 a_2 lc_{2,2} m_2 \cos(\theta_2) + \\
& 0.5 a_3 lc_{3,2} m_3 \cos(\theta_2) - 0.5 a_4 lc_{4,2} m_4 \cos(\theta_2) - 0.5 a_5 lc_{5,2} m_5 \cos(\theta_2) + lc_{2,1} lc_{2,2} m_2 \cos(\theta_2) + lc_{3,1} lc_{3,2} m_3 \cos(\theta_2) - \\
& lc_{4,1} lc_{4,2} m_4 \cos(\theta_2) - lc_{5,1} lc_{5,2} m_5 \cos(\theta_2) + lc_{2,2} lc_{2,3} m_2 \sin(\theta_2) + 4.0 I_{xy,4} \cos(\theta_2) \cos(\theta_3)^2 \cos(\theta_4)^2 + 4.0 I_{xy,5} \cos(\theta_2) \cos(\theta_3)^2 \cos(\theta_4)^2 + \\
& 4.0 I_{xy,5} \cos(\theta_2) \cos(\theta_3)^2 \cos(\theta_5)^2 + 4.0 I_{xy,5} \cos(\theta_2) \cos(\theta_4)^2 \cos(\theta_5)^2 - I_{yz,5} \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) \sin(\theta_2) - \\
& 0.25 a_3^2 m_3 \cos(\theta_2) \cos(\theta_3) \sin(\theta_3) - a_3^2 m_4 \cos(\theta_2) \cos(\theta_3) \sin(\theta_3) - a_3^2 m_5 \cos(\theta_2) \cos(\theta_3) \sin(\theta_3) + 0.25 a_4^2 m_4 \cos(\theta_2) \cos(\theta_3) \sin(\theta_3) + \\
& a_4^2 m_5 \cos(\theta_2) \cos(\theta_3) \sin(\theta_3) + 0.25 a_4^2 m_4 \cos(\theta_2) \cos(\theta_4) \sin(\theta_4) + 0.25 a_5^2 m_5 \cos(\theta_2) \cos(\theta_3) \sin(\theta_3) + \\
& a_4^2 m_5 \cos(\theta_2) \cos(\theta_4) \sin(\theta_4) + 0.25 a_5^2 m_5 \cos(\theta_2) \cos(\theta_4) \sin(\theta_4) + 0.25 a_5^2 m_5 \cos(\theta_2) \cos(\theta_5) \sin(\theta_5) - \\
& 8.0 I_{xy,5} \cos(\theta_2) \cos(\theta_3)^2 \cos(\theta_4)^2 \cos(\theta_5)^2 - I_{xz,5} \cos(\theta_3) \cos(\theta_4) \sin(\theta_2) \sin(\theta_5) - I_{xz,5} \cos(\theta_3) \cos(\theta_5) \sin(\theta_2) \sin(\theta_4) - \\
& I_{xz,5} \cos(\theta_4) \cos(\theta_5) \sin(\theta_2) \sin(\theta_3) - lc_{3,1}^2 m_3 \cos(\theta_2) \cos(\theta_3) \sin(\theta_3) + lc_{3,2}^2 m_3 \cos(\theta_2) \cos(\theta_3) \sin(\theta_3) + \\
& lc_{4,1}^2 m_4 \cos(\theta_2) \cos(\theta_3) \sin(\theta_3) - lc_{4,2}^2 m_4 \cos(\theta_2) \cos(\theta_3) \sin(\theta_3) + lc_{4,1}^2 m_4 \cos(\theta_2) \cos(\theta_4) \sin(\theta_4) - lc_{4,2}^2 m_4 \cos(\theta_2) \cos(\theta_4) \sin(\theta_4) + \\
& lc_{5,1}^2 m_5 \cos(\theta_2) \cos(\theta_3) \sin(\theta_3) - lc_{5,2}^2 m_5 \cos(\theta_2) \cos(\theta_3) \sin(\theta_3) + lc_{5,1}^2 m_5 \cos(\theta_2) \cos(\theta_4) \sin(\theta_4) - lc_{5,2}^2 m_5 \cos(\theta_2) \cos(\theta_4) \sin(\theta_4) + \\
& lc_{5,1}^2 m_5 \cos(\theta_2) \cos(\theta_5) \sin(\theta_5) - lc_{5,2}^2 m_5 \cos(\theta_2) \cos(\theta_5) \sin(\theta_5) + I_{yz,5} \cos(\theta_3) \sin(\theta_2) \sin(\theta_4) \sin(\theta_5) + \\
& I_{yz,5} \cos(\theta_4) \sin(\theta_2) \sin(\theta_3) \sin(\theta_5) + I_{yz,5} \cos(\theta_5) \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) - 0.5 a_2 a_3 m_3 \cos(\theta_2) \cos(\theta_3) - \\
& a_2 a_3 m_4 \cos(\theta_2) \cos(\theta_3) - a_2 a_3 m_5 \cos(\theta_2) \cos(\theta_3) - 0.5 a_4 a_5 m_5 \cos(\theta_2) \cos(\theta_5) + I_{xz,5} \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) - \\
& a_2 lc_{3,1} m_3 \cos(\theta_2) \cos(\theta_3) + a_3 lc_{4,2} m_4 \cos(\theta_2) \cos(\theta_4) - a_4 lc_{5,1} m_5 \cos(\theta_2) \cos(\theta_5) + 0.5 a_3 a_4 m_4 \cos(\theta_2) \sin(\theta_4) + \\
& a_3 a_4 m_5 \cos(\theta_2) \sin(\theta_4) + a_2 lc_{3,2} m_3 \cos(\theta_2) \sin(\theta_3) - 0.5 a_3 lc_{3,3} m_3 \cos(\theta_3) \sin(\theta_2) + a_3 lc_{4,1} m_4 \cos(\theta_2) \sin(\theta_4) - \\
& a_3 lc_{4,3} m_4 \cos(\theta_3) \sin(\theta_2) - a_3 lc_{5,3} m_5 \cos(\theta_3) \sin(\theta_2) + a_4 lc_{5,2} m_5 \cos(\theta_2) \sin(\theta_5) - lc_{3,1} lc_{3,3} m_3 \cos(\theta_3) \sin(\theta_2) + \\
& lc_{3,2} lc_{3,3} m_3 \sin(\theta_2) \sin(\theta_3) + 2.0 I_{xx,4} \cos(\theta_2) \cos(\theta_3) \cos(\theta_4)^2 \sin(\theta_3) + 2.0 I_{xx,4} \cos(\theta_2) \cos(\theta_3)^2 \cos(\theta_4) \sin(\theta_4) + \\
& 2.0 I_{xx,5} \cos(\theta_2) \cos(\theta_3) \cos(\theta_4)^2 \sin(\theta_3) + 2.0 I_{xx,5} \cos(\theta_2) \cos(\theta_3) \cos(\theta_5)^2 \sin(\theta_3) + 2.0 I_{xx,5} \cos(\theta_2) \cos(\theta_3)^2 \cos(\theta_4) \sin(\theta_4) + \\
& 2.0 I_{xx,5} \cos(\theta_2) \cos(\theta_4) \cos(\theta_5)^2 \sin(\theta_4) + 2.0 I_{xx,5} \cos(\theta_2) \cos(\theta_3)^2 \cos(\theta_5) \sin(\theta_5) + 2.0 I_{xx,5} \cos(\theta_2) \cos(\theta_4)^2 \cos(\theta_5) \sin(\theta_5) - \\
& 2.0 I_{yy,4} \cos(\theta_2) \cos(\theta_3) \cos(\theta_4)^2 \sin(\theta_3) - 2.0 I_{yy,4} \cos(\theta_2) \cos(\theta_3)^2 \cos(\theta_4) \sin(\theta_4) - 2.0 I_{yy,5} \cos(\theta_2) \cos(\theta_3) \cos(\theta_4)^2 \sin(\theta_3) - \\
& 2.0 I_{yy,5} \cos(\theta_2) \cos(\theta_3) \cos(\theta_5)^2 \sin(\theta_3) - 2.0 I_{yy,5} \cos(\theta_2) \cos(\theta_3)^2 \cos(\theta_4) \sin(\theta_4) - 2.0 I_{yy,5} \cos(\theta_2) \cos(\theta_4) \cos(\theta_5)^2 \sin(\theta_4) - \\
& 2.0 I_{yy,5} \cos(\theta_2) \cos(\theta_3)^2 \cos(\theta_5) \sin(\theta_5) - 2.0 I_{yy,5} \cos(\theta_2) \cos(\theta_4)^2 \cos(\theta_5) \sin(\theta_5) - a_3 lc_{3,2} m_3 \cos(\theta_2) \cos(\theta_3)^2 + \\
& a_4 lc_{4,2} m_4 \cos(\theta_2) \cos(\theta_3)^2 + a_4 lc_{4,2} m_4 \cos(\theta_2) \cos(\theta_4)^2 + a_5 lc_{5,2} m_5 \cos(\theta_2) \cos(\theta_3)^2 + a_5 lc_{5,2} m_5 \cos(\theta_2) \cos(\theta_4)^2 + \\
& a_5 lc_{5,2} m_5 \cos(\theta_2) \cos(\theta_5)^2 - 2.0 lc_{3,1} lc_{3,2} m_3 \cos(\theta_2) \cos(\theta_3)^2 + 2.0 lc_{4,1} lc_{4,2} m_4 \cos(\theta_2) \cos(\theta_3)^2 + 2.0 lc_{4,1} lc_{4,2} m_4 \cos(\theta_2) \cos(\theta_4)^2 + \\
& 2.0 lc_{5,1} lc_{5,2} m_5 \cos(\theta_2) \cos(\theta_3)^2 + 2.0 lc_{5,1} lc_{5,2} m_5 \cos(\theta_2) \cos(\theta_4)^2 + 2.0 lc_{5,1} lc_{5,2} m_5 \cos(\theta_2) \cos(\theta_5)^2 - 0.5 a_2 a_4 m_4 \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) - \\
& a_2 a_4 m_5 \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) + 0.5 a_3 a_5 m_5 \cos(\theta_2) \cos(\theta_4) \cos(\theta_5) - a_2 lc_{4,1} m_4 \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) + \\
& a_3 lc_{5,1} m_5 \cos(\theta_2) \cos(\theta_4) \cos(\theta_5) - a_3 lc_{3,1} m_3 \cos(\theta_2) \cos(\theta_3) \sin(\theta_3) + a_2 lc_{4,2} m_4 \cos(\theta_2) \cos(\theta_3) \sin(\theta_4) + \\
& a_2 lc_{4,2} m_4 \cos(\theta_2) \cos(\theta_4) \sin(\theta_3) + a_4 lc_{4,1} m_4 \cos(\theta_2) \cos(\theta_3) \sin(\theta_3) + a_4 lc_{4,1} m_4 \cos(\theta_2) \cos(\theta_4) \sin(\theta_4) - \\
& 0.5 a_4 lc_{4,3} m_4 \cos(\theta_3) \cos(\theta_4) \sin(\theta_2) + a_5 lc_{5,1} m_5 \cos(\theta_2) \cos(\theta_3) \sin(\theta_3) - a_3 lc_{5,2} m_5 \cos(\theta_2) \cos(\theta_4) \sin(\theta_5) - \\
& a_3 lc_{5,2} m_5 \cos(\theta_2) \cos(\theta_5) \sin(\theta_4) - a_4 lc_{5,3} m_5 \cos(\theta_3) \cos(\theta_4) \sin(\theta_2) + a_5 lc_{5,1} m_5 \cos(\theta_2) \cos(\theta_4) \sin(\theta_4) +
\end{aligned}$$

[illegible]

[illegible]

$$\begin{aligned}
M_{22} = & I_{xx,4} + I_{xx,5} + I_{yy,2} + I_{yy,3} + 0.25 a_2^2 m_2 + a_2^2 m_3 + a_2^2 m_4 + 0.25 a_3^2 m_3 + a_2^2 m_5 + a_3^2 m_4 + a_3^2 m_5 + \\
& lc_{2,1}^2 m_2 + lc_{2,3}^2 m_2 + lc_{3,1}^2 m_3 + lc_{3,3}^2 m_3 + lc_{4,2}^2 m_4 + lc_{4,3}^2 m_4 + lc_{5,2}^2 m_5 + lc_{5,3}^2 m_5 + I_{xx,3} \cos(\theta_3)^2 - \\
& I_{xx,4} \cos(\theta_3)^2 - I_{xx,4} \cos(\theta_4)^2 - I_{xx,5} \cos(\theta_3)^2 - I_{xx,5} \cos(\theta_4)^2 - I_{xx,5} \cos(\theta_5)^2 - I_{yy,3} \cos(\theta_3)^2 + I_{yy,4} \cos(\theta_3)^2 + \\
& I_{yy,4} \cos(\theta_4)^2 + I_{yy,5} \cos(\theta_3)^2 + I_{yy,5} \cos(\theta_4)^2 + I_{yy,5} \cos(\theta_5)^2 + 2.0 I_{xx,4} \cos(\theta_3)^2 \cos(\theta_4)^2 + 2.0 I_{xx,5} \cos(\theta_3)^2 \cos(\theta_4)^2 + \\
& 2.0 I_{xx,5} \cos(\theta_3)^2 \cos(\theta_5)^2 + 2.0 I_{xx,5} \cos(\theta_4)^2 \cos(\theta_5)^2 - 2.0 I_{yy,4} \cos(\theta_3)^2 \cos(\theta_4)^2 - 2.0 I_{yy,5} \cos(\theta_3)^2 \cos(\theta_4)^2 - \\
& 2.0 I_{yy,5} \cos(\theta_3)^2 \cos(\theta_5)^2 - 2.0 I_{yy,5} \cos(\theta_4)^2 \cos(\theta_5)^2 - 2.0 I_{xy,3} \cos(\theta_3) \sin(\theta_3) + 2.0 I_{xy,4} \cos(\theta_3) \sin(\theta_3) + \\
& 2.0 I_{xy,5} \cos(\theta_3) \sin(\theta_3) + 2.0 I_{xy,4} \cos(\theta_4) \sin(\theta_4) + 2.0 I_{xy,5} \cos(\theta_4) \sin(\theta_4) + 2.0 I_{xy,5} \cos(\theta_5) \sin(\theta_5) + a_2 lc_{2,1} m_2 + \\
& a_3 lc_{3,1} m_3 - 0.25 a_3^2 m_3 \cos(\theta_3)^2 - a_3^2 m_4 \cos(\theta_3)^2 - a_3^2 m_5 \cos(\theta_3)^2 + 0.25 a_4^2 m_4 \cos(\theta_3)^2 + 0.25 a_4^2 m_4 \cos(\theta_4)^2 + \\
& a_4^2 m_5 \cos(\theta_3)^2 + a_4^2 m_5 \cos(\theta_4)^2 + 0.25 a_5^2 m_5 \cos(\theta_3)^2 + 0.25 a_5^2 m_5 \cos(\theta_4)^2 + 0.25 a_5^2 m_5 \cos(\theta_5)^2 - lc_{3,1}^2 m_3 \cos(\theta_3)^2 + \\
& lc_{3,2}^2 m_3 \cos(\theta_3)^2 + lc_{4,1}^2 m_4 \cos(\theta_3)^2 + lc_{4,1}^2 m_4 \cos(\theta_4)^2 - lc_{4,2}^2 m_4 \cos(\theta_3)^2 - lc_{4,2}^2 m_4 \cos(\theta_4)^2 + lc_{5,1}^2 m_5 \cos(\theta_3)^2 + \\
& lc_{5,1}^2 m_5 \cos(\theta_4)^2 - lc_{5,2}^2 m_5 \cos(\theta_3)^2 + lc_{5,1}^2 m_5 \cos(\theta_5)^2 - lc_{5,2}^2 m_5 \cos(\theta_4)^2 - lc_{5,2}^2 m_5 \cos(\theta_5)^2 - 2.0 lc_{4,1}^2 m_4 \cos(\theta_3)^2 \cos(\theta_4)^2 + \\
& 2.0 lc_{4,2}^2 m_4 \cos(\theta_3)^2 \cos(\theta_4)^2 - 2.0 lc_{5,1}^2 m_5 \cos(\theta_3)^2 \cos(\theta_4)^2 - 2.0 lc_{5,1}^2 m_5 \cos(\theta_3)^2 \cos(\theta_5)^2 + 2.0 lc_{5,2}^2 m_5 \cos(\theta_3)^2 \cos(\theta_4)^2 - \\
& 2.0 lc_{5,1}^2 m_5 \cos(\theta_4)^2 \cos(\theta_5)^2 + 2.0 lc_{5,2}^2 m_5 \cos(\theta_3)^2 \cos(\theta_5)^2 + 2.0 lc_{5,2}^2 m_5 \cos(\theta_4)^2 \cos(\theta_5)^2 - 4.0 I_{xx,5} \cos(\theta_3)^2 \cos(\theta_4)^2 \cos(\theta_5)^2 + \\
& 4.0 I_{yy,5} \cos(\theta_3)^2 \cos(\theta_4)^2 \cos(\theta_5)^2 + a_3 a_4 m_4 \cos(\theta_4) + 2.0 a_3 a_4 m_5 \cos(\theta_4) + 2.0 a_2 lc_{3,2} m_3 \cos(\theta_3) + 2.0 a_3 lc_{4,1} m_4 \cos(\theta_4) + \\
& a_2 a_3 m_3 \sin(\theta_3) + 2.0 a_2 a_3 m_4 \sin(\theta_3) + 2.0 a_2 a_3 m_5 \sin(\theta_3) + 2.0 a_2 lc_{3,1} m_3 \sin(\theta_3) - 2.0 a_3 lc_{4,2} m_4 \sin(\theta_4) - \\
& 4.0 I_{xy,4} \cos(\theta_3) \cos(\theta_4)^2 \sin(\theta_3) - 4.0 I_{xy,4} \cos(\theta_3)^2 \cos(\theta_4) \sin(\theta_4) - 4.0 I_{xy,5} \cos(\theta_3) \cos(\theta_4)^2 \sin(\theta_3) - 4.0 I_{xy,5} \cos(\theta_3) \cos(\theta_5)^2 \sin(\theta_3) - \\
& 4.0 I_{xy,5} \cos(\theta_3)^2 \cos(\theta_4) \sin(\theta_4) - 4.0 I_{xy,5} \cos(\theta_4) \cos(\theta_5)^2 \sin(\theta_4) - 4.0 I_{xy,5} \cos(\theta_3)^2 \cos(\theta_5) \sin(\theta_5) - 4.0 I_{xy,5} \cos(\theta_4)^2 \cos(\theta_5) \sin(\theta_5) - \\
& a_3 lc_{3,1} m_3 \cos(\theta_3)^2 + a_4 lc_{4,1} m_4 \cos(\theta_3)^2 + a_4 lc_{4,1} m_4 \cos(\theta_4)^2 + a_5 lc_{5,1} m_5 \cos(\theta_3)^2 + a_5 lc_{5,1} m_5 \cos(\theta_4)^2 + \\
& a_5 lc_{5,1} m_5 \cos(\theta_5)^2 - 0.5 a_4^2 m_4 \cos(\theta_3)^2 \cos(\theta_4)^2 - 2.0 a_4^2 m_5 \cos(\theta_3)^2 \cos(\theta_4)^2 - 0.5 a_5^2 m_5 \cos(\theta_3)^2 \cos(\theta_4)^2 - \\
& 0.5 a_5^2 m_5 \cos(\theta_3)^2 \cos(\theta_5)^2 - 0.5 a_5^2 m_5 \cos(\theta_4)^2 \cos(\theta_5)^2 - 2.0 a_4 lc_{4,1} m_4 \cos(\theta_3)^2 \cos(\theta_4)^2 - 2.0 a_5 lc_{5,1} m_5 \cos(\theta_3)^2 \cos(\theta_4)^2 - \\
& 2.0 a_5 lc_{5,1} m_5 \cos(\theta_3)^2 \cos(\theta_5)^2 - 2.0 a_5 lc_{5,1} m_5 \cos(\theta_4)^2 \cos(\theta_5)^2 - 2.0 I_{xx,4} \cos(\theta_3) \cos(\theta_4) \sin(\theta_3) \sin(\theta_4) - \\
& 2.0 I_{xx,5} \cos(\theta_3) \cos(\theta_4) \sin(\theta_3) \sin(\theta_4) - 2.0 I_{xx,5} \cos(\theta_3) \cos(\theta_5) \sin(\theta_3) \sin(\theta_5) - 2.0 I_{xx,5} \cos(\theta_4) \cos(\theta_5) \sin(\theta_4) \sin(\theta_5) + \\
& 2.0 I_{yy,4} \cos(\theta_3) \cos(\theta_4) \sin(\theta_3) \sin(\theta_4) + 2.0 I_{yy,5} \cos(\theta_3) \cos(\theta_4) \sin(\theta_3) \sin(\theta_4) + 2.0 I_{yy,5} \cos(\theta_3) \cos(\theta_5) \sin(\theta_3) \sin(\theta_5) + \\
& 2.0 I_{yy,5} \cos(\theta_4) \cos(\theta_5) \sin(\theta_4) \sin(\theta_5) + a_5^2 m_5 \cos(\theta_3)^2 \cos(\theta_4)^2 \cos(\theta_5)^2 + 4.0 lc_{5,1}^2 m_5 \cos(\theta_3)^2 \cos(\theta_4)^2 \cos(\theta_5)^2 - \\
& 4.0 lc_{5,2}^2 m_5 \cos(\theta_3)^2 \cos(\theta_4)^2 \cos(\theta_5)^2 + 2.0 a_2 lc_{4,2} m_4 \cos(\theta_3) \cos(\theta_4) - 2.0 a_3 lc_{5,2} m_5 \cos(\theta_4) \cos(\theta_5) + a_2 a_4 m_4 \cos(\theta_3) \sin(\theta_4) + \\
& a_2 a_4 m_4 \cos(\theta_4) \sin(\theta_3) + 2.0 a_2 a_4 m_5 \cos(\theta_3) \sin(\theta_4) + 2.0 a_2 a_4 m_5 \cos(\theta_4) \sin(\theta_3) - a_3 a_5 m_5 \cos(\theta_4) \sin(\theta_5) - \\
& a_3 a_5 m_5 \cos(\theta_5) \sin(\theta_4) + a_3 lc_{3,2} m_3 \cos(\theta_3) \sin(\theta_3) + 2.0 a_2 lc_{4,1} m_4 \cos(\theta_3) \sin(\theta_4) + 2.0 a_2 lc_{4,1} m_4 \cos(\theta_4) \sin(\theta_3) - \\
& a_4 lc_{4,2} m_4 \cos(\theta_3) \sin(\theta_3) - a_4 lc_{4,2} m_4 \cos(\theta_4) \sin(\theta_4) - 2.0 a_3 lc_{5,1} m_5 \cos(\theta_4) \sin(\theta_5) - 2.0 a_3 lc_{5,1} m_5 \cos(\theta_5) \sin(\theta_4) - \\
& a_5 lc_{5,2} m_5 \cos(\theta_3) \sin(\theta_3) - a_5 lc_{5,2} m_5 \cos(\theta_4) \sin(\theta_4) - a_5 lc_{5,2} m_5 \cos(\theta_5) \sin(\theta_5) + 2.0 lc_{3,1} lc_{3,2} m_3 \cos(\theta_3) \sin(\theta_3) - \\
& 2.0 lc_{4,1} lc_{4,2} m_4 \cos(\theta_3) \sin(\theta_3) - 2.0 lc_{4,1} lc_{4,2} m_4 \cos(\theta_4) \sin(\theta_4) - 2.0 lc_{5,1} lc_{5,2} m_5 \cos(\theta_3) \sin(\theta_3) - 2.0 lc_{5,1} lc_{5,2} m_5 \cos(\theta_4) \sin(\theta_4) - \\
& 2.0 lc_{5,1} lc_{5,2} m_5 \cos(\theta_5) \sin(\theta_5) - 2.0 a_2 lc_{4,2} m_4 \sin(\theta_3) \sin(\theta_4) + 2.0 a_3 lc_{5,2} m_5 \sin(\theta_4) \sin(\theta_5) - a_3 a_4 m_4 \cos(\theta_3)^2 \cos(\theta_4) - \\
& 2.0 a_3 a_4 m_5 \cos(\theta_3)^2 \cos(\theta_4) - 2.0 a_3 lc_{4,1} m_4 \cos(\theta_3)^2 \cos(\theta_4) - 2.0 a_4 lc_{5,2} m_5 \cos(\theta_3)^2 \cos(\theta_5) - 2.0 a_4 lc_{5,2} m_5 \cos(\theta_4)^2 \cos(\theta_5) - \\
& a_4 a_5 m_5 \cos(\theta_3)^2 \sin(\theta_5) - a_4 a_5 m_5 \cos(\theta_4)^2 \sin(\theta_5) + 2.0 a_3 lc_{4,2} m_4 \cos(\theta_3)^2 \sin(\theta_4) - 2.0 a_4 lc_{5,1} m_5 \cos(\theta_3)^2 \sin(\theta_5) - \\
& 2.0 a_4 lc_{5,1} m_5 \cos(\theta_4)^2 \sin(\theta_5) + 8.0 I_{xy,5} \cos(\theta_3) \cos(\theta_4)^2 \cos(\theta_5)^2 \sin(\theta_3) + 8.0 I_{xy,5} \cos(\theta_3)^2 \cos(\theta_4) \cos(\theta_5)^2 \sin(\theta_4) + \\
& 8.0 I_{xy,5} \cos(\theta_3)^2 \cos(\theta_4)^2 \cos(\theta_5) \sin(\theta_5) + 0.5 a_4^2 m_4 \cos(\theta_3) \cos(\theta_4) \sin(\theta_3) \sin(\theta_4) + 2.0 a_4^2 m_5 \cos(\theta_3) \cos(\theta_4) \sin(\theta_3) \sin(\theta_4) + \\
& 0.5 a_5^2 m_5 \cos(\theta_3) \cos(\theta_4) \sin(\theta_3) \sin(\theta_4) + 0.5 a_5^2 m_5 \cos(\theta_3) \cos(\theta_5) \sin(\theta_3) \sin(\theta_5) + 0.5 a_5^2 m_5 \cos(\theta_4) \cos(\theta_5) \sin(\theta_4) \sin(\theta_5) +
\end{aligned}$$

$$\begin{aligned}
M_{31} = & I_{yz,4} \cos(\theta_2) \sin(\theta_3) \sin(\theta_4) - I_{zz,4} \sin(\theta_2) - I_{zz,5} \sin(\theta_2) - I_{yz,3} \cos(\theta_2) \cos(\theta_3) - I_{xz,3} \cos(\theta_2) \sin(\theta_3) - \\
& 0.25 a_3^2 m_3 \sin(\theta_2) - a_3^2 m_4 \sin(\theta_2) - a_3^2 m_5 \sin(\theta_2) - 0.25 a_4^2 m_4 \sin(\theta_2) - a_4^2 m_5 \sin(\theta_2) - 0.25 a_5^2 m_5 \sin(\theta_2) - \\
& lc_{3,1}^2 m_3 \sin(\theta_2) - lc_{3,2}^2 m_3 \sin(\theta_2) - lc_{4,1}^2 m_4 \sin(\theta_2) - lc_{4,2}^2 m_4 \sin(\theta_2) - lc_{5,1}^2 m_5 \sin(\theta_2) - lc_{5,2}^2 m_5 \sin(\theta_2) - \\
& I_{yz,4} \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) - I_{xz,4} \cos(\theta_2) \cos(\theta_3) \sin(\theta_4) - I_{xz,4} \cos(\theta_2) \cos(\theta_4) \sin(\theta_3) - I_{zz,3} \sin(\theta_2) + a_1 lc_{3,2} m_3 \cos(\theta_3) + \\
& 0.5 a_1 a_3 m_3 \sin(\theta_3) + a_1 a_3 m_4 \sin(\theta_3) + a_1 a_3 m_5 \sin(\theta_3) + a_1 lc_{3,1} m_3 \sin(\theta_3) - a_3 lc_{3,1} m_3 \sin(\theta_2) - a_4 lc_{4,1} m_4 \sin(\theta_2) - \\
& a_5 lc_{5,1} m_5 \sin(\theta_2) - I_{xz,5} \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) - 0.5 a_2 lc_{3,1} m_3 \sin(\theta_3) \sin(\theta_2) + a_3 lc_{4,2} m_4 \sin(\theta_4) \sin(\theta_2) + \\
& a_4 lc_{5,1} m_5 \sin(\theta_5) \sin(\theta_2) + I_{yz,5} \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) \sin(\theta_5) + I_{yz,5} \cos(\theta_2) \cos(\theta_3) \cos(\theta_5) \sin(\theta_4) + I_{yz,5} \cos(\theta_2) \cos(\theta_4) \cos(\theta_5) \sin(\theta_3) + \\
& I_{xz,5} \cos(\theta_2) \cos(\theta_3) \sin(\theta_4) \sin(\theta_5) + I_{xz,5} \cos(\theta_2) \cos(\theta_4) \sin(\theta_3) \sin(\theta_5) + I_{xz,5} \cos(\theta_2) \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) - \\
& I_{yz,5} \cos(\theta_2) \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) + 0.25 a_2 a_3 m_3 \cos(\theta_2) \cos(\theta_3) + 0.5 a_2 a_3 m_4 \cos(\theta_2) \cos(\theta_3) + 0.5 a_2 a_3 m_5 \cos(\theta_2) \cos(\theta_3) + \\
& 0.5 a_2 lc_{3,1} m_3 \cos(\theta_2) \cos(\theta_3) + a_1 lc_{4,2} m_4 \cos(\theta_3) \cos(\theta_4) - a_3 lc_{4,2} m_4 \cos(\theta_2) \cos(\theta_4) - a_4 lc_{5,1} m_5 \cos(\theta_2) \cos(\theta_5) + \\
& 0.5 a_1 a_4 m_4 \cos(\theta_3) \sin(\theta_4) + 0.5 a_1 a_4 m_4 \cos(\theta_4) \sin(\theta_3) + a_1 a_4 m_5 \cos(\theta_3) \sin(\theta_4) + a_1 a_4 m_5 \cos(\theta_4) \sin(\theta_3) - \\
& 0.5 a_3 a_4 m_4 \cos(\theta_2) \sin(\theta_4) - 0.5 a_3 a_4 m_4 \cos(\theta_4) \sin(\theta_2) - a_3 a_4 m_5 \cos(\theta_2) \sin(\theta_4) - a_3 a_4 m_5 \cos(\theta_4) \sin(\theta_2) + \\
& lc_{3,2} lc_{3,3} m_3 \cos(\theta_2) \cos(\theta_3) - 0.5 a_2 lc_{3,2} m_3 \cos(\theta_2) \sin(\theta_3) - 0.5 a_2 lc_{3,2} m_3 \cos(\theta_3) \sin(\theta_2) + 0.25 a_3 lc_{3,3} m_3 \cos(\theta_2) \sin(\theta_3) + \\
& 0.25 a_3 lc_{3,3} m_3 \cos(\theta_3) \sin(\theta_2) + a_1 lc_{4,1} m_4 \cos(\theta_3) \sin(\theta_4) + a_1 lc_{4,1} m_4 \cos(\theta_4) \sin(\theta_3) - a_3 lc_{4,1} m_4 \cos(\theta_2) \sin(\theta_4) - \\
& a_3 lc_{4,1} m_4 \cos(\theta_4) \sin(\theta_2) + a_3 lc_{4,3} m_4 \cos(\theta_2) \sin(\theta_3) + a_3 lc_{5,3} m_5 \cos(\theta_2) \sin(\theta_3) + a_4 lc_{5,2} m_5 \cos(\theta_2) \sin(\theta_5) + \\
& a_4 lc_{5,2} m_5 \cos(\theta_5) \sin(\theta_2) - 0.25 a_2 a_3 m_3 \sin(\theta_2) \sin(\theta_3) - 0.5 a_2 a_3 m_4 \sin(\theta_2) \sin(\theta_3) - 0.5 a_2 a_3 m_5 \sin(\theta_2) \sin(\theta_3) + \\
& a_4 a_5 m_5 \sin(\theta_2) \sin(\theta_5) + lc_{3,1} lc_{3,3} m_3 \cos(\theta_2) \sin(\theta_3) - 0.5 a_2 lc_{3,1} m_3 \sin(\theta_2) \sin(\theta_3) - a_1 lc_{4,2} m_4 \sin(\theta_3) \sin(\theta_4) + \\
& a_3 lc_{4,2} m_4 \sin(\theta_2) \sin(\theta_4) + a_4 lc_{5,1} m_5 \sin(\theta_2) \sin(\theta_5) - 0.25 a_2 a_3 m_3 \cos(\theta_3) \cos(\theta_2) - 0.5 a_2 a_3 m_4 \cos(\theta_3) \cos(\theta_2) - \\
& 0.5 a_2 a_3 m_5 \cos(\theta_3) \cos(\theta_2) - 0.5 a_2 lc_{3,1} m_3 \cos(\theta_3) \cos(\theta_2) + a_3 lc_{4,2} m_4 \cos(\theta_4) \cos(\theta_2) + a_4 lc_{5,1} m_5 \cos(\theta_5) \cos(\theta_2) - \\
& 0.5 a_3 a_4 m_4 \cos(\theta_4) \sin(\theta_2) + 0.5 a_3 a_4 m_4 \sin(\theta_4) \cos(\theta_2) - a_3 a_4 m_5 \cos(\theta_4) \sin(\theta_2) + a_3 a_4 m_5 \sin(\theta_4) \cos(\theta_2) - \\
& 0.5 a_2 lc_{3,2} m_3 \cos(\theta_3) \sin(\theta_2) + 0.5 a_2 lc_{3,2} m_3 \sin(\theta_3) \cos(\theta_2) - 0.25 a_3 lc_{3,3} m_3 \cos(\theta_3) \sin(\theta_2) + 0.25 a_3 lc_{3,3} m_3 \sin(\theta_3) \cos(\theta_2) - \\
& a_3 lc_{4,1} m_4 \cos(\theta_4) \sin(\theta_2) + a_3 lc_{4,1} m_4 \sin(\theta_4) \cos(\theta_2) + a_4 lc_{5,2} m_5 \cos(\theta_5) \sin(\theta_2) - a_4 lc_{5,2} m_5 \sin(\theta_5) \cos(\theta_2) - \\
& 0.25 a_2 a_3 m_3 \sin(\theta_3) \sin(\theta_2) - 0.5 a_2 a_3 m_4 \sin(\theta_3) \sin(\theta_2) - 0.5 a_2 a_3 m_5 \sin(\theta_3) \sin(\theta_2) + 0.5 a_1 a_5 m_5 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) + \\
& a_1 lc_{5,1} m_5 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) + lc_{4,2} lc_{4,3} m_4 \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) - a_2 lc_{4,2} m_4 \cos(\theta_3) \cos(\theta_4) \sin(\theta_2) + \\
& 0.5 a_4 lc_{4,3} m_4 \cos(\theta_2) \cos(\theta_3) \sin(\theta_4) + 0.5 a_4 lc_{4,3} m_4 \cos(\theta_2) \cos(\theta_4) \sin(\theta_3) - a_1 lc_{5,2} m_5 \cos(\theta_3) \cos(\theta_4) \sin(\theta_5) - \\
& a_1 lc_{5,2} m_5 \cos(\theta_3) \cos(\theta_5) \sin(\theta_4) - a_1 lc_{5,2} m_5 \cos(\theta_4) \cos(\theta_5) \sin(\theta_3) + 2.0 a_3 lc_{5,2} m_5 \cos(\theta_4) \cos(\theta_5) \sin(\theta_2) + \\
& a_4 lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_3) \sin(\theta_4) + a_4 lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_4) \sin(\theta_3) - 0.5 a_2 a_4 m_4 \cos(\theta_3) \sin(\theta_2) \sin(\theta_4) - \\
& 0.5 a_2 a_4 m_4 \cos(\theta_4) \sin(\theta_2) \sin(\theta_3) - a_2 a_4 m_5 \cos(\theta_3) \sin(\theta_2) \sin(\theta_4) - a_2 a_4 m_5 \cos(\theta_4) \sin(\theta_2) \sin(\theta_3) - 0.5 a_1 a_5 m_5 \cos(\theta_3) \sin(\theta_4) \sin(\theta_5) - \\
& 0.5 a_1 a_5 m_5 \cos(\theta_4) \sin(\theta_3) \sin(\theta_5) - 0.5 a_1 a_5 m_5 \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) + a_3 a_5 m_5 \cos(\theta_4) \sin(\theta_2) \sin(\theta_5) + a_3 a_5 m_5 \cos(\theta_5) \sin(\theta_2) \sin(\theta_4) + \\
& lc_{4,1} lc_{4,3} m_4 \cos(\theta_2) \cos(\theta_3) \sin(\theta_4) + lc_{4,1} lc_{4,3} m_4 \cos(\theta_2) \cos(\theta_4) \sin(\theta_3) - a_2 lc_{4,1} m_4 \cos(\theta_3) \sin(\theta_2) \sin(\theta_4) - \\
& a_2 lc_{4,1} m_4 \cos(\theta_4) \sin(\theta_2) \sin(\theta_3) - a_1 lc_{5,1} m_5 \cos(\theta_3) \sin(\theta_4) \sin(\theta_5) - a_1 lc_{5,1} m_5 \cos(\theta_4) \sin(\theta_3) \sin(\theta_5) - a_1 lc_{5,1} m_5 \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) + \\
& 2.0 a_3 lc_{5,1} m_5 \cos(\theta_4) \sin(\theta_2) \sin(\theta_5) + 2.0 a_3 lc_{5,1} m_5 \cos(\theta_5) \sin(\theta_2) \sin(\theta_4) - lc_{4,2} lc_{4,3} m_4 \cos(\theta_2) \sin(\theta_3) \sin(\theta_4) + \\
& a_2 lc_{4,2} m_4 \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) + a_1 lc_{5,2} m_5 \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) - 2.0 a_3 lc_{5,2} m_5 \sin(\theta_2) \sin(\theta_4) \sin(\theta_5) + \\
& lc_{5,1} lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) - a_2 lc_{5,1} m_5 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) \sin(\theta_2) - lc_{5,2} lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) \sin(\theta_5) - \\
& lc_{5,2} lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_3) \cos(\theta_5) \sin(\theta_4) - lc_{5,2} lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_4) \cos(\theta_5) \sin(\theta_3) + a_2 lc_{5,2} m_5 \cos(\theta_3) \cos(\theta_4) \sin(\theta_2) \sin(\theta_5) + \\
& a_2 lc_{5,2} m_5 \cos(\theta_3) \cos(\theta_5) \sin(\theta_2) \sin(\theta_4) + a_2 lc_{5,2} m_5 \cos(\theta_4) \cos(\theta_5) \sin(\theta_2) \sin(\theta_3) - 0.5 a_5 lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_3) \sin(\theta_4) \sin(\theta_5) - \\
& 0.5 a_5 lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_4) \sin(\theta_3) \sin(\theta_5) - 0.5 a_5 lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) + 0.5 a_2 a_5 m_5 \cos(\theta_3) \sin(\theta_2) \sin(\theta_4) \sin(\theta_5) + \\
& 0.5 a_2 a_5 m_5 \cos(\theta_4) \sin(\theta_2) \sin(\theta_3) \sin(\theta_5) + 0.5 a_2 a_5 m_5 \cos(\theta_5) \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) - lc_{5,1} lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_3) \sin(\theta_4) \sin(\theta_5) - \\
& lc_{5,1} lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_4) \sin(\theta_3) \sin(\theta_5) - lc_{5,1} lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) + a_2 lc_{5,1} m_5 \cos(\theta_3) \sin(\theta_2) \sin(\theta_4) \sin(\theta_5) + \\
& a_2 lc_{5,1} m_5 \cos(\theta_4) \sin(\theta_2) \sin(\theta_3) \sin(\theta_5) + a_2 lc_{5,1} m_5 \cos(\theta_5) \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) + lc_{5,2} lc_{5,3} m_5 \cos(\theta_2) \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) - \\
& a_2 lc_{5,2} m_5 \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) + 0.5 a_5 lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) - 0.5 a_2 a_5 m_5 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) \sin(\theta_2)
\end{aligned} \tag{A12}$$

$$\begin{aligned}
M_{32} = & I_{yz,3} \sin(\theta_3) - I_{xz,3} \cos(\theta_3) - I_{xz,4} \cos(\theta_3) \cos(\theta_4) + I_{yz,4} \cos(\theta_3) \sin(\theta_4) + I_{yz,4} \cos(\theta_4) \sin(\theta_3) + \\
& I_{xz,4} \sin(\theta_3) \sin(\theta_4) + I_{yz,5} \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) + I_{xz,5} \cos(\theta_3) \cos(\theta_4) \sin(\theta_5) + I_{xz,5} \cos(\theta_3) \cos(\theta_5) \sin(\theta_4) + \\
& I_{xz,5} \cos(\theta_4) \cos(\theta_5) \sin(\theta_3) - I_{yz,5} \cos(\theta_3) \sin(\theta_4) \sin(\theta_5) - I_{yz,5} \cos(\theta_4) \sin(\theta_3) \sin(\theta_5) - I_{yz,5} \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) - \\
& I_{xz,5} \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) + 0.5 a_3 lc_{3,3} m_3 \cos(\theta_3) + a_3 lc_{4,3} m_4 \cos(\theta_3) + a_3 lc_{5,3} m_5 \cos(\theta_3) + lc_{3,1} lc_{3,3} m_3 \cos(\theta_3) - \\
& lc_{3,2} lc_{3,3} m_3 \sin(\theta_3) + 0.5 a_4 lc_{4,3} m_4 \cos(\theta_3) \cos(\theta_4) + a_4 lc_{5,3} m_5 \cos(\theta_3) \cos(\theta_4) + lc_{4,1} lc_{4,3} m_4 \cos(\theta_3) \cos(\theta_4) - \\
& lc_{4,2} lc_{4,3} m_4 \cos(\theta_3) \sin(\theta_4) - lc_{4,2} lc_{4,3} m_4 \cos(\theta_4) \sin(\theta_3) - 0.5 a_4 lc_{4,3} m_4 \sin(\theta_3) \sin(\theta_4) - a_4 lc_{5,3} m_5 \sin(\theta_3) \sin(\theta_4) - \\
& lc_{4,1} lc_{4,3} m_4 \sin(\theta_3) \sin(\theta_4) - lc_{5,2} lc_{5,3} m_5 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) - 0.5 a_5 lc_{5,3} m_5 \cos(\theta_3) \cos(\theta_4) \sin(\theta_5) - \\
& 0.5 a_5 lc_{5,3} m_5 \cos(\theta_3) \cos(\theta_5) \sin(\theta_4) - 0.5 a_5 lc_{5,3} m_5 \cos(\theta_4) \cos(\theta_5) \sin(\theta_3) - lc_{5,1} lc_{5,3} m_5 \cos(\theta_3) \cos(\theta_4) \sin(\theta_5) - \\
& lc_{5,1} lc_{5,3} m_5 \cos(\theta_3) \cos(\theta_5) \sin(\theta_4) - lc_{5,1} lc_{5,3} m_5 \cos(\theta_4) \cos(\theta_5) \sin(\theta_3) + lc_{5,2} lc_{5,3} m_5 \cos(\theta_3) \sin(\theta_4) \sin(\theta_5) + \\
& lc_{5,2} lc_{5,3} m_5 \cos(\theta_4) \sin(\theta_3) \sin(\theta_5) + lc_{5,2} lc_{5,3} m_5 \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) + 0.5 a_5 lc_{5,3} m_5 \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) + \\
& lc_{5,1} lc_{5,3} m_5 \sin(\theta_3) \sin(\theta_4) \sin(\theta_5)
\end{aligned} \tag{A13}$$

$$\begin{aligned}
M_{33} = & I_{zz,3} + I_{zz,4} + I_{zz,5} + 0.25 a_3^2 m_3 + a_3^2 m_4 + a_3^2 m_5 + 0.25 a_4^2 m_4 + a_4^2 m_5 + 0.25 a_5^2 m_5 + \\
& lc_{3,1}^2 m_3 + lc_{3,2}^2 m_3 + lc_{4,1}^2 m_4 + lc_{4,2}^2 m_4 + lc_{5,1}^2 m_5 + lc_{5,2}^2 m_5 + a_3 lc_{3,1} m_3 + a_4 lc_{4,1} m_4 + a_5 lc_{5,1} m_5 + \\
& a_3 a_4 m_4 \cos(\theta_4) + 2.0 a_3 a_4 m_5 \cos(\theta_4) + 2.0 a_3 lc_{4,1} m_4 \cos(\theta_4) - 2.0 a_4 lc_{5,2} m_5 \cos(\theta_5) - a_4 a_5 m_5 \sin(\theta_5) - \\
& 2.0 a_3 lc_{4,2} m_4 \sin(\theta_4) - 2.0 a_4 lc_{5,1} m_5 \sin(\theta_5) - 2.0 a_3 lc_{5,2} m_5 \cos(\theta_4) \cos(\theta_5) - a_3 a_5 m_5 \cos(\theta_4) \sin(\theta_5) - \\
& a_3 a_5 m_5 \cos(\theta_5) \sin(\theta_4) - 2.0 a_3 lc_{5,1} m_5 \cos(\theta_4) \sin(\theta_5) - 2.0 a_3 lc_{5,1} m_5 \cos(\theta_5) \sin(\theta_4) + 2.0 a_3 lc_{5,2} m_5 \sin(\theta_4) \sin(\theta_5)
\end{aligned} \tag{A14}$$

$$\begin{aligned}
M_{34} = & I_{zz,4} + I_{zz,5} + 0.25 a_4^2 m_4 + a_4^2 m_5 + 0.25 a_5^2 m_5 + lc_{4,1}^2 m_4 + lc_{4,2}^2 m_4 + lc_{5,1}^2 m_5 + lc_{5,2}^2 m_5 + \\
& a_4 lc_{4,1} m_4 + a_5 lc_{5,1} m_5 + 0.5 a_3 a_4 m_4 \cos(\theta_4) + a_3 a_4 m_5 \cos(\theta_4) + a_3 lc_{4,1} m_4 \cos(\theta_4) - 2.0 a_4 lc_{5,2} m_5 \cos(\theta_5) - \\
& a_4 a_5 m_5 \sin(\theta_5) - a_3 lc_{4,2} m_4 \sin(\theta_4) - 2.0 a_4 lc_{5,1} m_5 \sin(\theta_5) - a_3 lc_{5,2} m_5 \cos(\theta_4) \cos(\theta_5) - 0.5 a_3 a_5 m_5 \cos(\theta_4) \sin(\theta_5) - \\
& 0.5 a_3 a_5 m_5 \cos(\theta_5) \sin(\theta_4) - a_3 lc_{5,1} m_5 \cos(\theta_4) \sin(\theta_5) - a_3 lc_{5,1} m_5 \cos(\theta_5) \sin(\theta_4) + a_3 lc_{5,2} m_5 \sin(\theta_4) \sin(\theta_5)
\end{aligned} \tag{A15}$$

$$\begin{aligned}
M_{35} = & I_{zz,5} + 0.25 a_5^2 m_5 + lc_{5,1}^2 m_5 + lc_{5,2}^2 m_5 + a_5 lc_{5,1} m_5 - a_4 lc_{5,2} m_5 \cos(\theta_5) - 0.5 a_4 a_5 m_5 \sin(\theta_5) - \\
& a_4 lc_{5,1} m_5 \sin(\theta_5) - a_3 lc_{5,2} m_5 \cos(\theta_4) \cos(\theta_5) - 0.5 a_3 a_5 m_5 \cos(\theta_4) \sin(\theta_5) - 0.5 a_3 a_5 m_5 \cos(\theta_5) \sin(\theta_4) - \\
& a_3 lc_{5,1} m_5 \cos(\theta_4) \sin(\theta_5) - a_3 lc_{5,1} m_5 \cos(\theta_5) \sin(\theta_4) + a_3 lc_{5,2} m_5 \sin(\theta_4) \sin(\theta_5)
\end{aligned} \tag{A16}$$

$$\begin{aligned}
M_{41} = & I_{yz,4} \cos(\theta_2) \sin(\theta_3) \sin(\theta_4) - I_{zz,5} \sin(\theta_2) - 0.25 a_4^2 m_4 \sin(\theta_2) - a_4^2 m_5 \sin(\theta_2) - 0.25 a_5^2 m_5 \sin(\theta_2) - \\
& lc_{4,1}^2 m_4 \sin(\theta_2) - lc_{4,2}^2 m_4 \sin(\theta_2) - lc_{5,1}^2 m_5 \sin(\theta_2) - lc_{5,2}^2 m_5 \sin(\theta_2) - I_{yz,4} \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) - \\
& I_{xz,4} \cos(\theta_2) \cos(\theta_3) \sin(\theta_4) - I_{xz,4} \cos(\theta_2) \cos(\theta_4) \sin(\theta_3) - I_{zz,4} \sin(\theta_2) - a_4 lc_{4,1} m_4 \sin(\theta_2) - a_5 lc_{5,1} m_5 \sin(\theta_2) - \\
& I_{xz,5} \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) + I_{yz,5} \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) \sin(\theta_5) + I_{yz,5} \cos(\theta_2) \cos(\theta_3) \cos(\theta_5) \sin(\theta_4) + \\
& I_{yz,5} \cos(\theta_2) \cos(\theta_4) \cos(\theta_5) \sin(\theta_3) + I_{xz,5} \cos(\theta_2) \cos(\theta_3) \sin(\theta_4) \sin(\theta_5) + I_{xz,5} \cos(\theta_2) \cos(\theta_4) \sin(\theta_3) \sin(\theta_5) + \\
& I_{xz,5} \cos(\theta_2) \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) - I_{yz,5} \cos(\theta_2) \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) + a_1 lc_{4,2} m_4 \cos(\theta_3) \cos(\theta_4) + \\
& 0.5 a_1 a_4 m_4 \cos(\theta_3) \sin(\theta_4) + 0.5 a_1 a_4 m_4 \cos(\theta_4) \sin(\theta_3) + a_1 a_4 m_5 \cos(\theta_3) \sin(\theta_4) + a_1 a_4 m_5 \cos(\theta_4) \sin(\theta_3) - \\
& 0.5 a_3 a_4 m_4 \cos(\theta_4) \sin(\theta_2) - a_3 a_4 m_5 \cos(\theta_4) \sin(\theta_2) + a_1 lc_{4,1} m_4 \cos(\theta_3) \sin(\theta_4) + a_1 lc_{4,1} m_4 \cos(\theta_4) \sin(\theta_3) - \\
& a_3 lc_{4,1} m_4 \cos(\theta_4) \sin(\theta_2) + 2.0 a_4 lc_{5,2} m_5 \cos(\theta_5) \sin(\theta_2) + a_4 a_5 m_5 \sin(\theta_2) \sin(\theta_5) - a_1 lc_{4,2} m_4 \sin(\theta_3) \sin(\theta_4) + \\
& a_3 lc_{4,2} m_4 \sin(\theta_2) \sin(\theta_4) + 2.0 a_4 lc_{5,1} m_5 \sin(\theta_2) \sin(\theta_5) + 0.5 a_1 a_5 m_5 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) + a_1 lc_{5,1} m_5 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) + \\
& lc_{4,2} lc_{4,3} m_4 \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) - a_2 lc_{4,2} m_4 \cos(\theta_3) \cos(\theta_4) \sin(\theta_2) + 0.5 a_4 lc_{4,3} m_4 \cos(\theta_2) \cos(\theta_3) \sin(\theta_4) + \\
& 0.5 a_4 lc_{4,3} m_4 \cos(\theta_2) \cos(\theta_4) \sin(\theta_3) - a_1 lc_{5,2} m_5 \cos(\theta_3) \cos(\theta_4) \sin(\theta_5) - a_1 lc_{5,2} m_5 \cos(\theta_3) \cos(\theta_5) \sin(\theta_4) - \\
& a_1 lc_{5,2} m_5 \cos(\theta_4) \cos(\theta_5) \sin(\theta_3) + a_3 lc_{5,2} m_5 \cos(\theta_4) \cos(\theta_5) \sin(\theta_2) + a_4 lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_3) \sin(\theta_4) + \\
& a_4 lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_4) \sin(\theta_3) - 0.5 a_2 a_4 m_4 \cos(\theta_3) \sin(\theta_2) \sin(\theta_4) - 0.5 a_2 a_4 m_4 \cos(\theta_4) \sin(\theta_2) \sin(\theta_3) - \\
& a_2 a_4 m_5 \cos(\theta_3) \sin(\theta_2) \sin(\theta_4) - a_2 a_4 m_5 \cos(\theta_4) \sin(\theta_2) \sin(\theta_3) - 0.5 a_1 a_5 m_5 \cos(\theta_3) \sin(\theta_4) \sin(\theta_5) - \\
& 0.5 a_1 a_5 m_5 \cos(\theta_4) \sin(\theta_3) \sin(\theta_5) - 0.5 a_1 a_5 m_5 \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) + 0.5 a_3 a_5 m_5 \cos(\theta_4) \sin(\theta_2) \sin(\theta_5) + \\
& 0.5 a_3 a_5 m_5 \cos(\theta_5) \sin(\theta_2) \sin(\theta_4) + lc_{4,1} lc_{4,3} m_4 \cos(\theta_2) \cos(\theta_3) \sin(\theta_4) + lc_{4,1} lc_{4,3} m_4 \cos(\theta_2) \cos(\theta_4) \sin(\theta_3) - \\
& a_2 lc_{4,1} m_4 \cos(\theta_3) \sin(\theta_2) \sin(\theta_4) - a_2 lc_{4,1} m_4 \cos(\theta_4) \sin(\theta_2) \sin(\theta_3) - a_1 lc_{5,1} m_5 \cos(\theta_3) \sin(\theta_4) \sin(\theta_5) - \\
& a_1 lc_{5,1} m_5 \cos(\theta_4) \sin(\theta_3) \sin(\theta_5) - a_1 lc_{5,1} m_5 \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) + a_3 lc_{5,1} m_5 \cos(\theta_4) \sin(\theta_2) \sin(\theta_5) + \\
& a_3 lc_{5,1} m_5 \cos(\theta_5) \sin(\theta_2) \sin(\theta_4) - lc_{4,2} lc_{4,3} m_4 \cos(\theta_2) \sin(\theta_3) \sin(\theta_4) + a_2 lc_{4,2} m_4 \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) + \\
& a_1 lc_{5,2} m_5 \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) - a_3 lc_{5,2} m_5 \sin(\theta_2) \sin(\theta_4) \sin(\theta_5) + lc_{5,1} lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) - \\
& a_2 lc_{5,1} m_5 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) \sin(\theta_2) - lc_{5,2} lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) \sin(\theta_5) - lc_{5,2} lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_3) \cos(\theta_5) \sin(\theta_4) - \\
& lc_{5,2} lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_4) \cos(\theta_5) \sin(\theta_3) + a_2 lc_{5,2} m_5 \cos(\theta_3) \cos(\theta_4) \sin(\theta_2) \sin(\theta_5) + a_2 lc_{5,2} m_5 \cos(\theta_3) \cos(\theta_5) \sin(\theta_2) \sin(\theta_4) + \\
& a_2 lc_{5,2} m_5 \cos(\theta_4) \cos(\theta_5) \sin(\theta_2) \sin(\theta_3) - 0.5 a_5 lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_3) \sin(\theta_4) \sin(\theta_5) - 0.5 a_5 lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_4) \sin(\theta_3) \sin(\theta_5) - \\
& 0.5 a_5 lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) + 0.5 a_2 a_5 m_5 \cos(\theta_3) \sin(\theta_2) \sin(\theta_4) \sin(\theta_5) + 0.5 a_2 a_5 m_5 \cos(\theta_4) \sin(\theta_2) \sin(\theta_3) \sin(\theta_5) + \\
& 0.5 a_2 a_5 m_5 \cos(\theta_5) \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) - lc_{5,1} lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_3) \sin(\theta_4) \sin(\theta_5) - lc_{5,1} lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_4) \sin(\theta_3) \sin(\theta_5) - \\
& lc_{5,1} lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) + a_2 lc_{5,1} m_5 \cos(\theta_3) \sin(\theta_2) \sin(\theta_4) \sin(\theta_5) + a_2 lc_{5,1} m_5 \cos(\theta_4) \sin(\theta_2) \sin(\theta_3) \sin(\theta_5) + \\
& a_2 lc_{5,1} m_5 \cos(\theta_5) \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) + lc_{5,2} lc_{5,3} m_5 \cos(\theta_2) \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) - a_2 lc_{5,2} m_5 \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) + \\
& 0.5 a_5 lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) - 0.5 a_2 a_5 m_5 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) \sin(\theta_2)
\end{aligned}$$

(A17)

$$\begin{aligned}
M_{42} = & I_{yz,4} \cos(\theta_3) \sin(\theta_4) - I_{xz,4} \cos(\theta_3) \cos(\theta_4) + I_{yz,4} \cos(\theta_4) \sin(\theta_3) + I_{xz,4} \sin(\theta_3) \sin(\theta_4) + I_{yz,5} \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) + \\
& I_{xz,5} \cos(\theta_3) \cos(\theta_4) \sin(\theta_5) + I_{xz,5} \cos(\theta_3) \cos(\theta_5) \sin(\theta_4) + I_{xz,5} \cos(\theta_4) \cos(\theta_5) \sin(\theta_3) - I_{yz,5} \cos(\theta_3) \sin(\theta_4) \sin(\theta_5) - \\
& I_{yz,5} \cos(\theta_4) \sin(\theta_3) \sin(\theta_5) - I_{yz,5} \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) - I_{xz,5} \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) + 0.5 a_4 lc_{4,3} m_4 \cos(\theta_3) \cos(\theta_4) + \\
& a_4 lc_{5,3} m_5 \cos(\theta_3) \cos(\theta_4) + lc_{4,1} lc_{4,3} m_4 \cos(\theta_3) \cos(\theta_4) - lc_{4,2} lc_{4,3} m_4 \cos(\theta_3) \sin(\theta_4) - lc_{4,2} lc_{4,3} m_4 \cos(\theta_4) \sin(\theta_3) - \\
& 0.5 a_4 lc_{4,3} m_4 \sin(\theta_3) \sin(\theta_4) - a_4 lc_{5,3} m_5 \sin(\theta_3) \sin(\theta_4) - lc_{4,1} lc_{4,3} m_4 \sin(\theta_3) \sin(\theta_4) - lc_{5,2} lc_{5,3} m_5 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) - \\
& 0.5 a_5 lc_{5,3} m_5 \cos(\theta_3) \cos(\theta_4) \sin(\theta_5) - 0.5 a_5 lc_{5,3} m_5 \cos(\theta_3) \cos(\theta_5) \sin(\theta_4) - 0.5 a_5 lc_{5,3} m_5 \cos(\theta_4) \cos(\theta_5) \sin(\theta_3) - \\
& lc_{5,1} lc_{5,3} m_5 \cos(\theta_3) \cos(\theta_4) \sin(\theta_5) - lc_{5,1} lc_{5,3} m_5 \cos(\theta_3) \cos(\theta_5) \sin(\theta_4) - lc_{5,1} lc_{5,3} m_5 \cos(\theta_4) \cos(\theta_5) \sin(\theta_3) + \\
& lc_{5,2} lc_{5,3} m_5 \cos(\theta_3) \sin(\theta_4) \sin(\theta_5) + lc_{5,2} lc_{5,3} m_5 \cos(\theta_4) \sin(\theta_3) \sin(\theta_5) + lc_{5,2} lc_{5,3} m_5 \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) + \\
& 0.5 a_5 lc_{5,3} m_5 \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) + lc_{5,1} lc_{5,3} m_5 \sin(\theta_3) \sin(\theta_4) \sin(\theta_5)
\end{aligned} \tag{A18}$$

$$\begin{aligned}
M_{43} = & I_{zz,4} + I_{zz,5} + 0.25 a_4^2 m_4 + a_4^2 m_5 + 0.25 a_5^2 m_5 + lc_{4,1}^2 m_4 + lc_{4,2}^2 m_4 + lc_{5,1}^2 m_5 + lc_{5,2}^2 m_5 + \\
& a_4 lc_{4,1} m_4 + a_5 lc_{5,1} m_5 + 0.5 a_3 a_4 m_4 \cos(\theta_4) + a_3 a_4 m_5 \cos(\theta_4) + a_3 lc_{4,1} m_4 \cos(\theta_4) - 2.0 a_4 lc_{5,2} m_5 \cos(\theta_5) - \\
& a_4 a_5 m_5 \sin(\theta_5) - a_3 lc_{4,2} m_4 \sin(\theta_4) - 2.0 a_4 lc_{5,1} m_5 \sin(\theta_5) - a_3 lc_{5,2} m_5 \cos(\theta_4) \cos(\theta_5) - 0.5 a_3 a_5 m_5 \cos(\theta_4) \sin(\theta_5) - \\
& 0.5 a_3 a_5 m_5 \cos(\theta_5) \sin(\theta_4) - a_3 lc_{5,1} m_5 \cos(\theta_4) \sin(\theta_5) - a_3 lc_{5,1} m_5 \cos(\theta_5) \sin(\theta_4) + a_3 lc_{5,2} m_5 \sin(\theta_4) \sin(\theta_5)
\end{aligned} \tag{A19}$$

$$\begin{aligned}
M_{44} = & I_{zz,4} + I_{zz,5} + 0.25 a_4^2 m_4 + a_4^2 m_5 + 0.25 a_5^2 m_5 + lc_{4,1}^2 m_4 + lc_{4,2}^2 m_4 + lc_{5,1}^2 m_5 + lc_{5,2}^2 m_5 + \\
& a_4 lc_{4,1} m_4 + a_5 lc_{5,1} m_5 - 2.0 a_4 lc_{5,2} m_5 \cos(\theta_5) - a_4 a_5 m_5 \sin(\theta_5) - 2.0 a_4 lc_{5,1} m_5 \sin(\theta_5)
\end{aligned} \tag{A20}$$

$$\begin{aligned}
M_{45} = & 0.25 m_5 a_5^2 + m_5 a_5 lc_{5,1} - 0.5 a_4 m_5 \sin(\theta_5) a_5 + m_5 lc_{5,1}^2 - a_4 m_5 \sin(\theta_5) lc_{5,1} + m_5 lc_{5,2}^2 - \\
& a_4 m_5 \cos(\theta_5) lc_{5,2} + I_{zz,5}
\end{aligned} \tag{A21}$$

$$\begin{aligned}
M_{51} = & I_{yz,5} \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) \sin(\theta_5) - 0.25 a_5^2 m_5 \sin(\theta_2) - lc_{5,1}^2 m_5 \sin(\theta_2) - lc_{5,2}^2 m_5 \sin(\theta_2) - \\
& a_5 lc_{5,1} m_5 \sin(\theta_2) - I_{xz,5} \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) - I_{zz,5} \sin(\theta_2) + I_{yz,5} \cos(\theta_2) \cos(\theta_3) \cos(\theta_5) \sin(\theta_4) + \\
& I_{yz,5} \cos(\theta_2) \cos(\theta_4) \cos(\theta_5) \sin(\theta_3) + I_{xz,5} \cos(\theta_2) \cos(\theta_3) \sin(\theta_4) \sin(\theta_5) + I_{xz,5} \cos(\theta_2) \cos(\theta_4) \sin(\theta_3) \sin(\theta_5) + \\
& I_{xz,5} \cos(\theta_2) \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) - I_{yz,5} \cos(\theta_2) \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) + a_4 lc_{5,2} m_5 \cos(\theta_5) \sin(\theta_2) + 0.5 a_4 a_5 m_5 \sin(\theta_2) \sin(\theta_5) + \\
& a_4 lc_{5,1} m_5 \sin(\theta_2) \sin(\theta_5) + 0.5 a_1 a_5 m_5 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) + a_1 lc_{5,1} m_5 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) - a_1 lc_{5,2} m_5 \cos(\theta_3) \cos(\theta_4) \sin(\theta_5) - \\
& a_1 lc_{5,2} m_5 \cos(\theta_3) \cos(\theta_5) \sin(\theta_4) - a_1 lc_{5,2} m_5 \cos(\theta_4) \cos(\theta_5) \sin(\theta_3) + a_3 lc_{5,2} m_5 \cos(\theta_4) \cos(\theta_5) \sin(\theta_2) - \\
& 0.5 a_1 a_5 m_5 \cos(\theta_3) \sin(\theta_4) \sin(\theta_5) - 0.5 a_1 a_5 m_5 \cos(\theta_4) \sin(\theta_3) \sin(\theta_5) - 0.5 a_1 a_5 m_5 \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) + \\
& 0.5 a_3 a_5 m_5 \cos(\theta_4) \sin(\theta_2) \sin(\theta_5) + 0.5 a_3 a_5 m_5 \cos(\theta_5) \sin(\theta_2) \sin(\theta_4) - a_1 lc_{5,1} m_5 \cos(\theta_3) \sin(\theta_4) \sin(\theta_5) - \\
& a_1 lc_{5,1} m_5 \cos(\theta_4) \sin(\theta_3) \sin(\theta_5) - a_1 lc_{5,1} m_5 \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) + a_3 lc_{5,1} m_5 \cos(\theta_4) \sin(\theta_2) \sin(\theta_5) + \\
& a_3 lc_{5,1} m_5 \cos(\theta_5) \sin(\theta_2) \sin(\theta_4) + a_1 lc_{5,2} m_5 \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) - a_3 lc_{5,2} m_5 \sin(\theta_2) \sin(\theta_4) \sin(\theta_5) + lc_{5,1} lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) - \\
& a_2 lc_{5,1} m_5 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) \sin(\theta_2) - lc_{5,2} lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) \sin(\theta_5) - lc_{5,2} lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_3) \cos(\theta_5) \sin(\theta_4) - \\
& lc_{5,2} lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_4) \cos(\theta_5) \sin(\theta_3) + a_2 lc_{5,2} m_5 \cos(\theta_3) \cos(\theta_4) \sin(\theta_2) \sin(\theta_5) + a_2 lc_{5,2} m_5 \cos(\theta_3) \cos(\theta_5) \sin(\theta_2) \sin(\theta_4) + \\
& a_2 lc_{5,2} m_5 \cos(\theta_4) \cos(\theta_5) \sin(\theta_2) \sin(\theta_3) - 0.5 a_5 lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_3) \sin(\theta_4) \sin(\theta_5) - 0.5 a_5 lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_4) \sin(\theta_3) \sin(\theta_5) - \\
& 0.5 a_5 lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) + 0.5 a_2 a_5 m_5 \cos(\theta_3) \sin(\theta_2) \sin(\theta_4) \sin(\theta_5) + 0.5 a_2 a_5 m_5 \cos(\theta_4) \sin(\theta_2) \sin(\theta_3) \sin(\theta_5) + \\
& 0.5 a_2 a_5 m_5 \cos(\theta_5) \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) - lc_{5,1} lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_3) \sin(\theta_4) \sin(\theta_5) - lc_{5,1} lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_4) \sin(\theta_3) \sin(\theta_5) - \\
& lc_{5,1} lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) + a_2 lc_{5,1} m_5 \cos(\theta_3) \sin(\theta_2) \sin(\theta_4) \sin(\theta_5) + a_2 lc_{5,1} m_5 \cos(\theta_4) \sin(\theta_2) \sin(\theta_3) \sin(\theta_5) + \\
& a_2 lc_{5,1} m_5 \cos(\theta_5) \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) + lc_{5,2} lc_{5,3} m_5 \cos(\theta_2) \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) - a_2 lc_{5,2} m_5 \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) + \\
& 0.5 a_5 lc_{5,3} m_5 \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) - 0.5 a_2 a_5 m_5 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) \sin(\theta_2)
\end{aligned} \tag{A22}$$

$$\begin{aligned}
M_{52} = & I_{yz,5} \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) + I_{xz,5} \cos(\theta_3) \cos(\theta_4) \sin(\theta_5) + I_{xz,5} \cos(\theta_3) \cos(\theta_5) \sin(\theta_4) + I_{xz,5} \cos(\theta_4) \cos(\theta_5) \sin(\theta_3) - \\
& I_{yz,5} \cos(\theta_3) \sin(\theta_4) \sin(\theta_5) - I_{yz,5} \cos(\theta_4) \sin(\theta_3) \sin(\theta_5) - I_{yz,5} \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) - I_{xz,5} \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) - \\
& lc_{5,2} lc_{5,3} m_5 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) - 0.5 a_5 lc_{5,3} m_5 \cos(\theta_3) \cos(\theta_4) \sin(\theta_5) - 0.5 a_5 lc_{5,3} m_5 \cos(\theta_3) \cos(\theta_5) \sin(\theta_4) - \\
& 0.5 a_5 lc_{5,3} m_5 \cos(\theta_4) \cos(\theta_5) \sin(\theta_3) - lc_{5,1} lc_{5,3} m_5 \cos(\theta_3) \cos(\theta_4) \sin(\theta_5) - lc_{5,1} lc_{5,3} m_5 \cos(\theta_3) \cos(\theta_5) \sin(\theta_4) - \\
& lc_{5,1} lc_{5,3} m_5 \cos(\theta_4) \cos(\theta_5) \sin(\theta_3) + lc_{5,2} lc_{5,3} m_5 \cos(\theta_3) \sin(\theta_4) \sin(\theta_5) + lc_{5,2} lc_{5,3} m_5 \cos(\theta_4) \sin(\theta_3) \sin(\theta_5) + \\
& lc_{5,2} lc_{5,3} m_5 \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) + 0.5 a_5 lc_{5,3} m_5 \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) + lc_{5,1} lc_{5,3} m_5 \sin(\theta_3) \sin(\theta_4) \sin(\theta_5)
\end{aligned} \tag{A23}$$

$$\begin{aligned}
M_{53} = & I_{zz,5} + 0.25 a_5^2 m_5 + lc_{5,1}^2 m_5 + lc_{5,2}^2 m_5 + a_5 lc_{5,1} m_5 - a_4 lc_{5,2} m_5 \cos(\theta_5) - 0.5 a_4 a_5 m_5 \sin(\theta_5) - \\
& a_4 lc_{5,1} m_5 \sin(\theta_5) - a_3 lc_{5,2} m_5 \cos(\theta_4) \cos(\theta_5) - 0.5 a_3 a_5 m_5 \cos(\theta_4) \sin(\theta_5) - 0.5 a_3 a_5 m_5 \cos(\theta_5) \sin(\theta_4) - \\
& a_3 lc_{5,1} m_5 \cos(\theta_4) \sin(\theta_5) - a_3 lc_{5,1} m_5 \cos(\theta_5) \sin(\theta_4) + a_3 lc_{5,2} m_5 \sin(\theta_4) \sin(\theta_5)
\end{aligned} \tag{A24}$$

$$\begin{aligned}
M_{54} = & 0.25 m_5 a_5^2 + m_5 a_5 lc_{5,1} - 0.5 a_4 m_5 \sin(\theta_5) a_5 + m_5 lc_{5,1}^2 - a_4 m_5 \sin(\theta_5) lc_{5,1} + m_5 lc_{5,2}^2 - a_4 m_5 \cos(\theta_5) lc_{5,2} + \\
& I_{zz,5}
\end{aligned} \tag{A25}$$

$$M_{55} = 0.25 m_5 a_5^2 + m_5 a_5 lc_{5,1} + m_5 lc_{5,1}^2 + m_5 lc_{5,2}^2 + I_{zz,5} \tag{A26}$$

In the context of the coupled human-exoskeleton system, m_i represents the mass, a_i denotes the length, and $I_{xx,i}, I_{yy,i}, I_{zz,i}$ are the moments of inertia for the i^{th} link. Additionally, $I_{xy,i}, I_{xz,i}, I_{yz,i}$ refer to the products of inertia. The coordinates $l_{ci,x}, l_{ci,y}, l_{ci,z}$ specify the center of mass along the x, y, and z axes, respectively, while θ_i signifies the angular position.

B. CORIOLIS AND CENTRIPETAL MATRIX ($C(q, \dot{q})$)

Regarding the Coriolis and centripetal matrix, the following formulations are derived:

$$C(q, \dot{q}) = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} \end{bmatrix} \quad (A27)$$

where

$$\begin{aligned} C_{11} = & 0.25 I_{xy,3} \dot{\theta}_2 + 0.25 I_{xy,3} \dot{\theta}_3 - 0.25 I_{xy,4} \dot{\theta}_2 + 0.25 I_{xy,4} \dot{\theta}_3 - 0.25 I_{xy,5} \dot{\theta}_2 + 0.25 I_{xy,4} \dot{\theta}_4 + 0.25 I_{xy,5} \dot{\theta}_3 + \\ & 0.25 I_{xy,5} \dot{\theta}_4 + 0.25 I_{xy,5} \dot{\theta}_5 - 0.5 I_{xy,3} \dot{\theta}_2 \cos(\theta_2)^2 - 0.5 I_{xy,3} \dot{\theta}_2 \cos(\theta_3)^2 - 0.5 I_{xy,3} \dot{\theta}_3 \cos(\theta_2)^2 + 0.5 I_{xy,4} \dot{\theta}_2 \cos(\theta_2)^2 - \\ & 0.5 I_{xy,3} \dot{\theta}_3 \cos(\theta_3)^2 + 0.5 I_{xy,4} \dot{\theta}_2 \cos(\theta_3)^2 + 0.5 I_{xy,4} \dot{\theta}_3 \cos(\theta_2)^2 + 0.5 I_{xy,5} \dot{\theta}_2 \cos(\theta_2)^2 + 0.5 I_{xy,4} \dot{\theta}_2 \cos(\theta_4)^2 - \\ & 0.5 I_{xy,4} \dot{\theta}_3 \cos(\theta_3)^2 + 0.5 I_{xy,4} \dot{\theta}_4 \cos(\theta_2)^2 + 0.5 I_{xy,5} \dot{\theta}_2 \cos(\theta_3)^2 + 0.5 I_{xy,5} \dot{\theta}_3 \cos(\theta_2)^2 - 0.5 I_{xy,4} \dot{\theta}_3 \cos(\theta_4)^2 - \\ & 0.5 I_{xy,4} \dot{\theta}_4 \cos(\theta_3)^2 + 0.5 I_{xy,5} \dot{\theta}_2 \cos(\theta_4)^2 - 0.5 I_{xy,5} \dot{\theta}_3 \cos(\theta_3)^2 + 0.5 I_{xy,5} \dot{\theta}_4 \cos(\theta_2)^2 - 0.5 I_{xy,4} \dot{\theta}_4 \cos(\theta_4)^2 + \\ & 0.5 I_{xy,5} \dot{\theta}_2 \cos(\theta_5)^2 - 0.5 I_{xy,5} \dot{\theta}_3 \cos(\theta_4)^2 - 0.5 I_{xy,5} \dot{\theta}_4 \cos(\theta_3)^2 + 0.5 I_{xy,5} \dot{\theta}_5 \cos(\theta_2)^2 - 0.5 I_{xy,5} \dot{\theta}_3 \cos(\theta_5)^2 - \\ & 0.5 I_{xy,5} \dot{\theta}_4 \cos(\theta_4)^2 - 0.5 I_{xy,5} \dot{\theta}_5 \cos(\theta_3)^2 - 0.5 I_{xy,5} \dot{\theta}_4 \cos(\theta_5)^2 - 0.5 I_{xy,5} \dot{\theta}_5 \cos(\theta_4)^2 - 0.5 I_{xy,5} \dot{\theta}_5 \cos(\theta_5)^2 + \\ & I_{xz,2} \dot{\theta}_2 \cos(2.0 \theta_2) + 0.25 I_{xy,3} \dot{\theta}_2 \cos(2.0 \theta_3) + 0.25 I_{xy,3} \dot{\theta}_3 \cos(2.0 \theta_3) - 0.5 I_{xx,2} \dot{\theta}_2 \sin(2.0 \theta_2) - 0.25 I_{xx,3} \dot{\theta}_2 \sin(2.0 \theta_2) + \\ & 0.125 I_{xx,3} \dot{\theta}_2 \sin(2.0 \theta_3) - 0.25 I_{xx,4} \dot{\theta}_2 \sin(2.0 \theta_2) + 0.125 I_{xx,3} \dot{\theta}_3 \sin(2.0 \theta_3) - 0.25 I_{xx,5} \dot{\theta}_2 \sin(2.0 \theta_2) - 0.25 I_{yy,3} \dot{\theta}_2 \sin(2.0 \theta_2) - \\ & 0.125 I_{yy,3} \dot{\theta}_2 \sin(2.0 \theta_3) - 0.25 I_{yy,4} \dot{\theta}_2 \sin(2.0 \theta_2) - 0.125 I_{yy,3} \dot{\theta}_3 \sin(2.0 \theta_3) - 0.25 I_{yy,5} \dot{\theta}_2 \sin(2.0 \theta_2) + 0.5 I_{zz,2} \dot{\theta}_2 \sin(2.0 \theta_2) + \\ & 0.5 I_{zz,3} \dot{\theta}_2 \sin(2.0 \theta_2) + 0.5 I_{zz,4} \dot{\theta}_2 \sin(2.0 \theta_2) + 0.5 I_{zz,5} \dot{\theta}_2 \sin(2.0 \theta_2) + 0.25 I_{xy,4} \dot{\theta}_2 \cos(2.0 \theta_3) \cos(2.0 \theta_4) - 0.25 I_{xy,4} \dot{\theta}_3 \cos(2.0 \theta_3) \cos(2.0 \theta_4) - \\ & 0.25 I_{xy,4} \dot{\theta}_4 \cos(2.0 \theta_3) \cos(2.0 \theta_4) + 0.5 I_{yz,3} \dot{\theta}_2 \cos(2.0 \theta_2) \cos(\theta_3) - 0.25 I_{yz,3} \dot{\theta}_3 \cos(2.0 \theta_2) \cos(\theta_3) - 0.125 a_3 lc_{3,2} m_3 \dot{\theta}_2 - \\ & 0.125 a_3 lc_{3,2} m_3 \dot{\theta}_3 + 0.125 a_4 lc_{4,2} m_4 \dot{\theta}_2 - 0.125 a_4 lc_{4,2} m_4 \dot{\theta}_3 - 0.125 a_4 lc_{4,2} m_4 \dot{\theta}_4 + 0.125 a_5 lc_{5,2} m_5 \dot{\theta}_2 - 0.125 a_5 lc_{5,2} m_5 \dot{\theta}_3 - \\ & 0.125 a_5 lc_{5,2} m_5 \dot{\theta}_4 - 0.125 a_5 lc_{5,2} m_5 \dot{\theta}_5 - 0.25 lc_{3,1} lc_{3,2} m_3 \dot{\theta}_2 - 0.25 lc_{3,1} lc_{3,2} m_3 \dot{\theta}_3 + 0.25 lc_{4,1} lc_{4,2} m_4 \dot{\theta}_2 - 0.25 lc_{4,1} lc_{4,2} m_4 \dot{\theta}_3 - \\ & 0.25 lc_{4,1} lc_{4,2} m_4 \dot{\theta}_4 + 0.25 lc_{5,1} lc_{5,2} m_5 \dot{\theta}_2 - 0.25 lc_{5,1} lc_{5,2} m_5 \dot{\theta}_3 - 0.25 lc_{5,1} lc_{5,2} m_5 \dot{\theta}_4 - 0.25 lc_{5,1} lc_{5,2} m_5 \dot{\theta}_5 + \\ & 0.5 I_{xz,3} \dot{\theta}_2 \cos(2.0 \theta_2) \sin(\theta_3) - 0.5 I_{xz,3} \dot{\theta}_2 \cos(\theta_3) \sin(2.0 \theta_2) - 0.25 I_{xz,3} \dot{\theta}_3 \cos(2.0 \theta_2) \sin(\theta_3) + 0.25 I_{xz,3} \dot{\theta}_3 \cos(\theta_3) \sin(2.0 \theta_2) + \\ & 0.125 I_{xx,4} \dot{\theta}_2 \cos(2.0 \theta_3) \sin(2.0 \theta_4) + 0.125 I_{xx,4} \dot{\theta}_2 \cos(2.0 \theta_4) \sin(2.0 \theta_3) - 0.125 I_{xx,4} \dot{\theta}_3 \cos(2.0 \theta_3) \sin(2.0 \theta_4) - \\ & 0.125 I_{xx,4} \dot{\theta}_3 \cos(2.0 \theta_4) \sin(2.0 \theta_3) - 0.125 I_{xx,4} \dot{\theta}_4 \cos(2.0 \theta_3) \sin(2.0 \theta_4) - 0.125 I_{xx,4} \dot{\theta}_4 \cos(2.0 \theta_4) \sin(2.0 \theta_3) - \\ & 0.125 I_{yy,4} \dot{\theta}_2 \cos(2.0 \theta_3) \sin(2.0 \theta_4) - 0.125 I_{yy,4} \dot{\theta}_2 \cos(2.0 \theta_4) \sin(2.0 \theta_3) + 0.125 I_{yy,4} \dot{\theta}_3 \cos(2.0 \theta_3) \sin(2.0 \theta_4) + \\ & 0.125 I_{yy,4} \dot{\theta}_3 \cos(2.0 \theta_4) \sin(2.0 \theta_3) + 0.125 I_{yy,4} \dot{\theta}_4 \cos(2.0 \theta_3) \sin(2.0 \theta_4) + 0.125 I_{yy,4} \dot{\theta}_4 \cos(2.0 \theta_4) \sin(2.0 \theta_3) - \\ & 0.25 I_{xy,4} \dot{\theta}_2 \sin(2.0 \theta_3) \sin(2.0 \theta_4) + 0.25 I_{xy,4} \dot{\theta}_3 \sin(2.0 \theta_3) \sin(2.0 \theta_4) + 0.25 I_{xy,4} \dot{\theta}_4 \sin(2.0 \theta_3) \sin(2.0 \theta_4) + 0.5 I_{yz,3} \dot{\theta}_2 \sin(2.0 \theta_2) \sin(\theta_3) - \end{aligned}$$

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$$\begin{aligned}
C_{12} = & I_{xy,2} \dot{\theta}_2 \sin(\theta_2) + I_{xz,2} \dot{\theta}_1 \cos(2.0\theta_2) - 0.5 I_{xx,2} \dot{\theta}_1 \sin(2.0\theta_2) - 0.25 I_{xx,3} \dot{\theta}_1 \sin(2.0\theta_2) - 0.25 I_{xx,4} \dot{\theta}_1 \sin(2.0\theta_2) - \\
& 0.25 I_{xx,5} \dot{\theta}_1 \sin(2.0\theta_2) - 0.25 I_{yy,3} \dot{\theta}_1 \sin(2.0\theta_2) - 0.25 I_{yy,4} \dot{\theta}_1 \sin(2.0\theta_2) - 0.25 I_{yy,5} \dot{\theta}_1 \sin(2.0\theta_2) + 0.5 I_{zz,2} \dot{\theta}_1 \sin(2.0\theta_2) + \\
& 0.5 I_{zz,3} \dot{\theta}_1 \sin(2.0\theta_2) + 0.5 I_{zz,4} \dot{\theta}_1 \sin(2.0\theta_2) + 0.5 I_{zz,5} \dot{\theta}_1 \sin(2.0\theta_2) - I_{yz,2} \dot{\theta}_2 \cos(\theta_2) - 0.5 I_{zz,3} \dot{\theta}_3 \cos(\theta_2) - \\
& 0.5 I_{zz,4} \dot{\theta}_3 \cos(\theta_2) - 0.5 I_{zz,5} \dot{\theta}_3 \cos(\theta_2) - 0.5 I_{zz,5} \dot{\theta}_4 \cos(\theta_2) - 0.5 I_{zz,5} \dot{\theta}_5 \cos(\theta_2) + 0.25 I_{xx,3} \dot{\theta}_1 \cos(2.0\theta_3) \sin(2.0\theta_2) - \\
& 0.25 I_{yy,3} \dot{\theta}_1 \cos(2.0\theta_3) \sin(2.0\theta_2) - 0.5 I_{xy,3} \dot{\theta}_1 \sin(2.0\theta_2) \sin(2.0\theta_3) + 0.5 I_{xx,3} \dot{\theta}_3 \cos(2.0\theta_3) \cos(\theta_2) + I_{yz,3} \dot{\theta}_1 \cos(2.0\theta_2) \cos(\theta_3) - \\
& 0.5 I_{yy,3} \dot{\theta}_3 \cos(2.0\theta_3) \cos(\theta_2) - I_{xy,3} \dot{\theta}_2 \cos(2.0\theta_3) \sin(\theta_2) + I_{xz,3} \dot{\theta}_1 \cos(2.0\theta_2) \sin(\theta_3) - I_{xy,3} \dot{\theta}_3 \sin(2.0\theta_3) \cos(\theta_2) + \\
& 0.125 a_2^2 m_2 \dot{\theta}_1 \sin(2.0\theta_2) + 0.5 a_2^2 m_3 \dot{\theta}_1 \sin(2.0\theta_2) + 0.5 a_2^2 m_4 \dot{\theta}_1 \sin(2.0\theta_2) + 0.0625 a_3^2 m_3 \dot{\theta}_1 \sin(2.0\theta_2) + \\
& 0.5 a_2^2 m_5 \dot{\theta}_1 \sin(2.0\theta_2) + 0.25 a_3^2 m_4 \dot{\theta}_1 \sin(2.0\theta_2) + 0.25 a_3^2 m_5 \dot{\theta}_1 \sin(2.0\theta_2) + 0.0625 a_4^2 m_4 \dot{\theta}_1 \sin(2.0\theta_2) + \\
& 0.25 a_4^2 m_5 \dot{\theta}_1 \sin(2.0\theta_2) + 0.0625 a_5^2 m_5 \dot{\theta}_1 \sin(2.0\theta_2) - 0.5 I_{xx,3} \dot{\theta}_2 \sin(2.0\theta_3) \sin(\theta_2) + 0.5 I_{yy,3} \dot{\theta}_2 \sin(2.0\theta_3) \sin(\theta_2) + \\
& 0.5 lc_{2,1}^2 m_2 \dot{\theta}_1 \sin(2.0\theta_2) - 0.5 lc_{2,3}^2 m_2 \dot{\theta}_1 \sin(2.0\theta_2) + 0.25 lc_{3,1}^2 m_3 \dot{\theta}_1 \sin(2.0\theta_2) + 0.25 lc_{3,2}^2 m_3 \dot{\theta}_1 \sin(2.0\theta_2) - \\
& 0.5 lc_{3,3}^2 m_3 \dot{\theta}_1 \sin(2.0\theta_2) + 0.25 lc_{4,1}^2 m_4 \dot{\theta}_1 \sin(2.0\theta_2) + 0.25 lc_{4,2}^2 m_4 \dot{\theta}_1 \sin(2.0\theta_2) - 0.5 lc_{4,3}^2 m_4 \dot{\theta}_1 \sin(2.0\theta_2) + \\
& 0.25 lc_{5,1}^2 m_5 \dot{\theta}_1 \sin(2.0\theta_2) + 0.25 lc_{5,2}^2 m_5 \dot{\theta}_1 \sin(2.0\theta_2) - 0.5 lc_{5,3}^2 m_5 \dot{\theta}_1 \sin(2.0\theta_2) + I_{xz,3} \dot{\theta}_2 \cos(\theta_2) \cos(\theta_3) - \\
& 0.125 a_3^2 m_3 \dot{\theta}_3 \cos(\theta_2) - 0.5 a_3^2 m_4 \dot{\theta}_3 \cos(\theta_2) - 0.5 a_3^2 m_5 \dot{\theta}_3 \cos(\theta_2) - 0.125 a_4^2 m_4 \dot{\theta}_3 \cos(\theta_2) - 0.125 a_4^2 m_4 \dot{\theta}_4 \cos(\theta_2) - \\
& 0.5 a_4^2 m_5 \dot{\theta}_3 \cos(\theta_2) - 0.5 a_4^2 m_5 \dot{\theta}_4 \cos(\theta_2) - 0.125 a_5^2 m_5 \dot{\theta}_3 \cos(\theta_2) - 0.125 a_5^2 m_5 \dot{\theta}_4 \cos(\theta_2) - 0.125 a_5^2 m_5 \dot{\theta}_5 \cos(\theta_2) - \\
& I_{yz,3} \dot{\theta}_2 \cos(\theta_2) \sin(\theta_3) - 0.5 lc_{3,1}^2 m_3 \dot{\theta}_3 \cos(\theta_2) - 0.5 lc_{3,2}^2 m_3 \dot{\theta}_3 \cos(\theta_2) - 0.5 lc_{4,1}^2 m_4 \dot{\theta}_3 \cos(\theta_2) - 0.5 lc_{4,1}^2 m_4 \dot{\theta}_4 \cos(\theta_2) - \\
& 0.5 lc_{4,2}^2 m_4 \dot{\theta}_3 \cos(\theta_2) - 0.5 lc_{4,2}^2 m_4 \dot{\theta}_4 \cos(\theta_2) - 0.5 lc_{5,1}^2 m_5 \dot{\theta}_3 \cos(\theta_2) - 0.5 lc_{5,1}^2 m_5 \dot{\theta}_4 \cos(\theta_2) - 0.5 lc_{5,2}^2 m_5 \dot{\theta}_3 \cos(\theta_2) - \\
& 0.5 lc_{5,1}^2 m_5 \dot{\theta}_5 \cos(\theta_2) - 0.5 lc_{5,2}^2 m_5 \dot{\theta}_4 \cos(\theta_2) - 0.5 lc_{5,2}^2 m_5 \dot{\theta}_5 \cos(\theta_2) - lc_{2,1} lc_{2,2} m_2 \dot{\theta}_2 \sin(\theta_2) - 0.5 I_{xx,4} \dot{\theta}_2 \cos(2.0\theta_3) \sin(2.0\theta_4) \sin(\theta_2) - \\
& 0.5 I_{xx,4} \dot{\theta}_2 \cos(2.0\theta_4) \sin(2.0\theta_3) \sin(\theta_2) - 0.5 I_{xx,4} \dot{\theta}_3 \sin(2.0\theta_3) \sin(2.0\theta_4) \cos(\theta_2) - 0.5 I_{xx,4} \dot{\theta}_4 \sin(2.0\theta_3) \sin(2.0\theta_4) \cos(\theta_2) + \\
& 0.5 I_{yy,4} \dot{\theta}_2 \cos(2.0\theta_3) \sin(2.0\theta_4) \sin(\theta_2) + 0.5 I_{yy,4} \dot{\theta}_2 \cos(2.0\theta_4) \sin(2.0\theta_3) \sin(\theta_2) + 0.5 I_{yy,4} \dot{\theta}_3 \sin(2.0\theta_3) \sin(2.0\theta_4) \cos(\theta_2) + \\
& 0.5 I_{yy,4} \dot{\theta}_4 \sin(2.0\theta_3) \sin(2.0\theta_4) \cos(\theta_2) - 0.25 lc_{3,1}^2 m_3 \dot{\theta}_1 \cos(2.0\theta_3) \sin(2.0\theta_2) + 0.25 lc_{3,2}^2 m_3 \dot{\theta}_1 \cos(2.0\theta_3) \sin(2.0\theta_2) + \\
& I_{xy,4} \dot{\theta}_2 \sin(2.0\theta_3) \sin(2.0\theta_4) \sin(\theta_2) + I_{yz,4} \dot{\theta}_1 \cos(2.0\theta_2) \cos(\theta_3) \cos(\theta_4) - 0.125 a_3^2 m_3 \dot{\theta}_3 \cos(2.0\theta_3) \cos(\theta_2) - \\
& 0.5 a_3^2 m_4 \dot{\theta}_3 \cos(2.0\theta_3) \cos(\theta_2) - 0.5 a_3^2 m_5 \dot{\theta}_3 \cos(2.0\theta_3) \cos(\theta_2) + I_{xz,4} \dot{\theta}_1 \cos(2.0\theta_2) \cos(\theta_3) \sin(\theta_4) + I_{xz,4} \dot{\theta}_1 \cos(2.0\theta_2) \cos(\theta_4) \sin(\theta_3) - \\
& 0.5 lc_{3,1}^2 m_3 \dot{\theta}_3 \cos(2.0\theta_3) \cos(\theta_2) + 0.5 lc_{3,2}^2 m_3 \dot{\theta}_3 \cos(2.0\theta_3) \cos(\theta_2) - I_{yz,4} \dot{\theta}_1 \cos(2.0\theta_2) \sin(\theta_3) \sin(\theta_4) + \\
& 0.125 a_3^2 m_3 \dot{\theta}_2 \sin(2.0\theta_3) \sin(\theta_2) + 0.5 a_3^2 m_4 \dot{\theta}_2 \sin(2.0\theta_3) \sin(\theta_2) + 0.5 a_3^2 m_5 \dot{\theta}_2 \sin(2.0\theta_3) \sin(\theta_2) + 0.5 lc_{3,1}^2 m_3 \dot{\theta}_2 \sin(2.0\theta_3) \sin(\theta_2) - \\
& 0.5 lc_{3,2}^2 m_3 \dot{\theta}_2 \sin(2.0\theta_3) \sin(\theta_2) - 0.5 a_2 lc_{2,3} m_2 \dot{\theta}_1 \cos(2.0\theta_2) - a_2 lc_{3,3} m_3 \dot{\theta}_1 \cos(2.0\theta_2) - a_2 lc_{4,3} m_4 \dot{\theta}_1 \cos(2.0\theta_2) - \\
& a_2 lc_{5,3} m_5 \dot{\theta}_1 \cos(2.0\theta_2) - lc_{2,1} lc_{2,3} m_2 \dot{\theta}_1 \cos(2.0\theta_2) + 0.25 I_{xx,4} \dot{\theta}_1 \cos(2.0\theta_3) \cos(2.0\theta_4) \sin(2.0\theta_2) - 0.25 I_{yy,4} \dot{\theta}_1 \cos(2.0\theta_3) \cos(2.0\theta_4) \sin(2.0\theta_2) + \\
& 0.5 a_2 lc_{2,1} m_2 \dot{\theta}_1 \sin(2.0\theta_2) + 0.25 a_3 lc_{3,1} m_3 \dot{\theta}_1 \sin(2.0\theta_2) + 0.25 a_4 lc_{4,1} m_4 \dot{\theta}_1 \sin(2.0\theta_2) + 0.25 a_5 lc_{5,1} m_5 \dot{\theta}_1 \sin(2.0\theta_2) - \\
& 0.5 I_{xy,4} \dot{\theta}_1 \cos(2.0\theta_3) \sin(2.0\theta_2) \sin(2.0\theta_4) - 0.5 I_{xy,4} \dot{\theta}_1 \cos(2.0\theta_4) \sin(2.0\theta_2) \sin(2.0\theta_3) + I_{xz,4} \dot{\theta}_2 \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) - \\
& 0.25 I_{xx,4} \dot{\theta}_1 \sin(2.0\theta_2) \sin(2.0\theta_3) \sin(2.0\theta_4) + 0.25 I_{yy,4} \dot{\theta}_1 \sin(2.0\theta_2) \sin(2.0\theta_3) \sin(2.0\theta_4) - I_{yz,4} \dot{\theta}_2 \cos(\theta_2) \cos(\theta_3) \sin(\theta_4) - \\
& I_{yz,4} \dot{\theta}_2 \cos(\theta_2) \cos(\theta_4) \sin(\theta_3) - I_{xz,4} \dot{\theta}_2 \cos(\theta_2) \sin(\theta_3) \sin(\theta_4) - 0.5 a_1 a_2 m_2 \dot{\theta}_1 \cos(\theta_2) - a_1 a_2 m_3 \dot{\theta}_1 \cos(\theta_2) - \\
& a_1 a_2 m_4 \dot{\theta}_1 \cos(\theta_2) - a_1 a_2 m_5 \dot{\theta}_1 \cos(\theta_2) + 0.5 I_{xx,4} \dot{\theta}_3 \cos(2.0\theta_3) \cos(2.0\theta_4) \cos(\theta_2) + 0.5 I_{xx,4} \dot{\theta}_4 \cos(2.0\theta_3) \cos(2.0\theta_4) \cos(\theta_2) - \\
& 0.5 I_{yy,4} \dot{\theta}_3 \cos(2.0\theta_3) \cos(2.0\theta_4) \cos(\theta_2) - 0.5 I_{yy,4} \dot{\theta}_4 \cos(2.0\theta_3) \cos(2.0\theta_4) \cos(\theta_2) - a_1 lc_{2,1} m_2 \dot{\theta}_1 \cos(\theta_2) - \\
& 0.5 a_3 lc_{3,1} m_3 \dot{\theta}_3 \cos(\theta_2) - 0.5 a_4 lc_{4,1} m_4 \dot{\theta}_3 \cos(\theta_2) - 0.5 a_4 lc_{4,1} m_4 \dot{\theta}_4 \cos(\theta_2) - 0.5 a_5 lc_{5,1} m_5 \dot{\theta}_3 \cos(\theta_2) - 0.5 a_5 lc_{5,1} m_5 \dot{\theta}_4 \cos(\theta_2) - \\
& 0.5 a_5 lc_{5,1} m_5 \dot{\theta}_5 \cos(\theta_2) + lc_{2,2} lc_{2,3} m_2 \dot{\theta}_2 \cos(\theta_2) - I_{xy,4} \dot{\theta}_2 \cos(2.0\theta_3) \cos(2.0\theta_4) \sin(\theta_2) - I_{xy,4} \dot{\theta}_3 \cos(2.0\theta_3) \sin(2.0\theta_4) \cos(\theta_2) - \\
& I_{xy,4} \dot{\theta}_3 \cos(2.0\theta_4) \sin(2.0\theta_3) \cos(\theta_2) - I_{xy,4} \dot{\theta}_4 \cos(2.0\theta_3) \sin(2.0\theta_4) \cos(\theta_2) - I_{xy,4} \dot{\theta}_4 \cos(2.0\theta_4) \sin(2.0\theta_3) \cos(\theta_2) - \\
& a_1 lc_{2,3} m_2 \dot{\theta}_1 \sin(\theta_2) - 0.5 a_2 lc_{2,2} m_2 \dot{\theta}_2 \sin(\theta_2) - a_1 lc_{3,3} m_3 \dot{\theta}_1 \sin(\theta_2) - a_1 lc_{4,3} m_4 \dot{\theta}_1 \sin(\theta_2) - a_1 lc_{5,3} m_5 \dot{\theta}_1 \sin(\theta_2) -
\end{aligned}$$

$$\begin{aligned}
&0.25 a_4^2 m_5 \dot{\theta}_1 \sin(2.0 \theta_2) \sin(2.0 \theta_3) \sin(2.0 \theta_4) - 0.5 I_{xx,5} \dot{\theta}_2 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \sin(2.0 \theta_5) \sin(\theta_2) + 0.5 I_{yy,5} \dot{\theta}_2 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \sin(2.0 \theta_5) \sin(\theta_2) + \\
&I_{yz,5} \dot{\theta}_2 \cos(\theta_2) \cos(\theta_3) \sin(\theta_4) \sin(\theta_5) + I_{yz,5} \dot{\theta}_2 \cos(\theta_2) \cos(\theta_4) \sin(\theta_3) \sin(\theta_5) + I_{yz,5} \dot{\theta}_2 \cos(\theta_2) \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) + \\
&0.25 lc_{4,1}^2 m_4 \dot{\theta}_1 \sin(2.0 \theta_2) \sin(2.0 \theta_3) \sin(2.0 \theta_4) - 0.25 lc_{4,2}^2 m_4 \dot{\theta}_1 \sin(2.0 \theta_2) \sin(2.0 \theta_3) \sin(2.0 \theta_4) + I_{xz,5} \dot{\theta}_2 \cos(\theta_2) \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) - \\
&0.5 a_3 a_4 m_4 \dot{\theta}_3 \cos(\theta_2) \cos(\theta_4) - 0.25 a_3 a_4 m_4 \dot{\theta}_4 \cos(\theta_2) \cos(\theta_4) - a_3 a_4 m_5 \dot{\theta}_3 \cos(\theta_2) \cos(\theta_4) - 0.5 a_3 a_4 m_5 \dot{\theta}_4 \cos(\theta_2) \cos(\theta_4) - \\
&a_1 lc_{3,2} m_3 \dot{\theta}_1 \cos(\theta_2) \cos(\theta_3) - 0.5 a_3 lc_{3,3} m_3 \dot{\theta}_2 \cos(\theta_2) \cos(\theta_3) - a_3 lc_{4,1} m_4 \dot{\theta}_3 \cos(\theta_2) \cos(\theta_4) - a_3 lc_{4,3} m_4 \dot{\theta}_2 \cos(\theta_2) \cos(\theta_3) - \\
&0.5 a_3 lc_{4,1} m_4 \dot{\theta}_4 \cos(\theta_2) \cos(\theta_4) - a_3 lc_{5,3} m_5 \dot{\theta}_2 \cos(\theta_2) \cos(\theta_3) + a_4 lc_{5,2} m_5 \dot{\theta}_3 \cos(\theta_2) \cos(\theta_5) + a_4 lc_{5,2} m_5 \dot{\theta}_4 \cos(\theta_2) \cos(\theta_5) + \\
&0.5 a_4 lc_{5,2} m_5 \dot{\theta}_5 \cos(\theta_2) \cos(\theta_5) - 0.5 a_1 a_3 m_3 \dot{\theta}_1 \cos(\theta_2) \sin(\theta_3) - a_1 a_3 m_4 \dot{\theta}_1 \cos(\theta_2) \sin(\theta_3) - a_1 a_3 m_5 \dot{\theta}_1 \cos(\theta_2) \sin(\theta_3) + \\
&0.5 a_2 a_3 m_3 \dot{\theta}_2 \cos(\theta_3) \sin(\theta_2) + a_2 a_3 m_4 \dot{\theta}_2 \cos(\theta_3) \sin(\theta_2) + a_2 a_3 m_5 \dot{\theta}_2 \cos(\theta_3) \sin(\theta_2) + 0.5 a_4 a_5 m_5 \dot{\theta}_3 \cos(\theta_2) \sin(\theta_5) + \\
&0.5 a_4 a_5 m_5 \dot{\theta}_4 \cos(\theta_2) \sin(\theta_5) + 0.25 a_4 a_5 m_5 \dot{\theta}_5 \cos(\theta_2) \sin(\theta_5) - 0.125 a_4^2 m_4 \dot{\theta}_3 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(\theta_2) - \\
&0.125 a_4^2 m_4 \dot{\theta}_4 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(\theta_2) - 0.5 a_4^2 m_5 \dot{\theta}_3 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(\theta_2) - 0.5 a_4^2 m_5 \dot{\theta}_4 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(\theta_2) - \\
&lc_{3,1} lc_{3,3} m_3 \dot{\theta}_2 \cos(\theta_2) \cos(\theta_3) - a_1 lc_{3,1} m_3 \dot{\theta}_1 \cos(\theta_2) \sin(\theta_3) + a_2 lc_{3,1} m_3 \dot{\theta}_2 \cos(\theta_3) \sin(\theta_2) + a_3 lc_{4,2} m_4 \dot{\theta}_3 \cos(\theta_2) \sin(\theta_4) + \\
&0.5 a_3 lc_{4,2} m_4 \dot{\theta}_4 \cos(\theta_2) \sin(\theta_4) + a_4 lc_{5,1} m_5 \dot{\theta}_3 \cos(\theta_2) \sin(\theta_5) + a_4 lc_{5,1} m_5 \dot{\theta}_4 \cos(\theta_2) \sin(\theta_5) + 0.5 a_4 lc_{5,1} m_5 \dot{\theta}_5 \cos(\theta_2) \sin(\theta_5) - \\
&0.5 lc_{4,1}^2 m_4 \dot{\theta}_3 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(\theta_2) - 0.5 lc_{4,1}^2 m_4 \dot{\theta}_4 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(\theta_2) + 0.5 lc_{4,2}^2 m_4 \dot{\theta}_3 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(\theta_2) + \\
&0.5 lc_{4,2}^2 m_4 \dot{\theta}_4 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(\theta_2) + lc_{3,2} lc_{3,3} m_3 \dot{\theta}_2 \cos(\theta_2) \sin(\theta_3) - a_2 lc_{3,2} m_3 \dot{\theta}_2 \sin(\theta_2) \sin(\theta_3) + \\
&0.125 a_4^2 m_4 \dot{\theta}_2 \cos(2.0 \theta_3) \sin(2.0 \theta_4) \sin(\theta_2) + 0.125 a_4^2 m_4 \dot{\theta}_2 \cos(2.0 \theta_4) \sin(2.0 \theta_3) \sin(\theta_2) + 0.125 a_4^2 m_4 \dot{\theta}_3 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) + \\
&0.5 a_4^2 m_5 \dot{\theta}_2 \cos(2.0 \theta_3) \sin(2.0 \theta_4) \sin(\theta_2) + 0.5 a_4^2 m_5 \dot{\theta}_2 \cos(2.0 \theta_4) \sin(2.0 \theta_3) \sin(\theta_2) + 0.125 a_4^2 m_4 \dot{\theta}_4 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) + \\
&0.5 a_4^2 m_5 \dot{\theta}_3 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) + 0.5 a_4^2 m_5 \dot{\theta}_4 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) + 0.5 lc_{4,1}^2 m_4 \dot{\theta}_2 \cos(2.0 \theta_3) \sin(2.0 \theta_4) \sin(\theta_2) + \\
&0.5 lc_{4,1}^2 m_4 \dot{\theta}_2 \cos(2.0 \theta_4) \sin(2.0 \theta_3) \sin(\theta_2) + 0.5 lc_{4,1}^2 m_4 \dot{\theta}_3 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) - 0.5 lc_{4,2}^2 m_4 \dot{\theta}_2 \cos(2.0 \theta_3) \sin(2.0 \theta_4) \sin(\theta_2) - \\
&0.5 lc_{4,2}^2 m_4 \dot{\theta}_2 \cos(2.0 \theta_4) \sin(2.0 \theta_3) \sin(\theta_2) + 0.5 lc_{4,1}^2 m_4 \dot{\theta}_4 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) - 0.5 lc_{4,2}^2 m_4 \dot{\theta}_3 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) - \\
&0.5 lc_{4,2}^2 m_4 \dot{\theta}_4 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) + 0.5 a_3 lc_{4,1} m_4 \dot{\theta}_1 \sin(2.0 \theta_2) \sin(2.0 \theta_3) \sin(\theta_4) - 0.5 a_4 lc_{4,2} m_4 \dot{\theta}_2 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \sin(\theta_2) - \\
&0.0625 a_5^2 m_5 \dot{\theta}_1 \cos(2.0 \theta_3) \sin(2.0 \theta_2) \sin(2.0 \theta_4) \sin(2.0 \theta_5) - 0.0625 a_5^2 m_5 \dot{\theta}_1 \cos(2.0 \theta_4) \sin(2.0 \theta_2) \sin(2.0 \theta_3) \sin(2.0 \theta_5) - \\
&0.0625 a_5^2 m_5 \dot{\theta}_1 \cos(2.0 \theta_5) \sin(2.0 \theta_2) \sin(2.0 \theta_3) \sin(2.0 \theta_4) - lc_{4,1} lc_{4,2} m_4 \dot{\theta}_2 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \sin(\theta_2) - \\
&0.25 lc_{5,1}^2 m_5 \dot{\theta}_1 \cos(2.0 \theta_3) \sin(2.0 \theta_2) \sin(2.0 \theta_4) \sin(2.0 \theta_5) - 0.25 lc_{5,1}^2 m_5 \dot{\theta}_1 \cos(2.0 \theta_4) \sin(2.0 \theta_2) \sin(2.0 \theta_3) \sin(2.0 \theta_5) - \\
&0.25 lc_{5,1}^2 m_5 \dot{\theta}_1 \cos(2.0 \theta_5) \sin(2.0 \theta_2) \sin(2.0 \theta_3) \sin(2.0 \theta_4) + 0.25 lc_{5,2}^2 m_5 \dot{\theta}_1 \cos(2.0 \theta_3) \sin(2.0 \theta_2) \sin(2.0 \theta_4) \sin(2.0 \theta_5) + \\
&0.25 lc_{5,2}^2 m_5 \dot{\theta}_1 \cos(2.0 \theta_4) \sin(2.0 \theta_2) \sin(2.0 \theta_3) \sin(2.0 \theta_5) + 0.25 lc_{5,2}^2 m_5 \dot{\theta}_1 \cos(2.0 \theta_5) \sin(2.0 \theta_2) \sin(2.0 \theta_3) \sin(2.0 \theta_4) - \\
&0.5 a_3 a_4 m_4 \dot{\theta}_3 \cos(2.0 \theta_3) \cos(\theta_2) \cos(\theta_4) - 0.25 a_3 a_4 m_4 \dot{\theta}_4 \cos(2.0 \theta_3) \cos(\theta_2) \cos(\theta_4) - a_3 a_4 m_5 \dot{\theta}_3 \cos(2.0 \theta_3) \cos(\theta_2) \cos(\theta_4) - \\
&0.5 a_3 a_4 m_5 \dot{\theta}_4 \cos(2.0 \theta_3) \cos(\theta_2) \cos(\theta_4) - a_3 lc_{4,1} m_4 \dot{\theta}_3 \cos(2.0 \theta_3) \cos(\theta_2) \cos(\theta_4) - 0.5 a_3 lc_{4,1} m_4 \dot{\theta}_4 \cos(2.0 \theta_3) \cos(\theta_2) \cos(\theta_4) + \\
&0.125 a_5^2 m_5 \dot{\theta}_3 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(2.0 \theta_5) \cos(\theta_2) + 0.125 a_5^2 m_5 \dot{\theta}_4 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(2.0 \theta_5) \cos(\theta_2) + \\
&0.125 a_5^2 m_5 \dot{\theta}_5 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(2.0 \theta_5) \cos(\theta_2) - lc_{4,2} lc_{4,3} m_4 \dot{\theta}_1 \cos(2.0 \theta_2) \cos(\theta_3) \cos(\theta_4) + a_2 lc_{4,2} m_4 \dot{\theta}_1 \sin(2$$

[illegible]

[illegible]

[illegible]

$$\begin{aligned}
& 0.5 a_3 \text{lc}_{5,2} m_5 \dot{\theta}_1 \cos(2.0 \theta_3) \sin(2.0 \theta_2) \sin(\theta_4) \sin(\theta_5) - 0.5 a_3 \text{lc}_{5,2} m_5 \dot{\theta}_1 \sin(2.0 \theta_2) \sin(2.0 \theta_3) \cos(\theta_4) \sin(\theta_5) - \\
& 0.5 a_3 \text{lc}_{5,2} m_5 \dot{\theta}_1 \sin(2.0 \theta_2) \sin(2.0 \theta_3) \cos(\theta_5) \sin(\theta_4) - a_4 \text{lc}_{5,1} m_5 \dot{\theta}_2 \cos(2.0 \theta_3) \sin(2.0 \theta_4) \sin(\theta_2) \sin(\theta_5) - \\
& a_4 \text{lc}_{5,1} m_5 \dot{\theta}_2 \cos(2.0 \theta_4) \sin(2.0 \theta_3) \sin(\theta_2) \sin(\theta_5) - a_4 \text{lc}_{5,1} m_5 \dot{\theta}_2 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_5) \sin(\theta_2) - a_4 \text{lc}_{5,1} m_5 \dot{\theta}_3 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) \sin(\theta_5) - \\
& a_4 \text{lc}_{5,1} m_5 \dot{\theta}_4 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) \sin(\theta_5) - 0.5 a_4 \text{lc}_{5,1} m_5 \dot{\theta}_5 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) \sin(\theta_5) - \\
& 0.25 a_3 a_5 m_5 \dot{\theta}_1 \sin(2.0 \theta_2) \sin(2.0 \theta_3) \sin(\theta_4) \sin(\theta_5) - 0.5 a_3 \text{lc}_{5,1} m_5 \dot{\theta}_1 \sin(2.0 \theta_2) \sin(2.0 \theta_3) \sin(\theta_4) \sin(\theta_5) + \\
& a_4 \text{lc}_{5,2} m_5 \dot{\theta}_2 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \sin(\theta_2) \sin(\theta_5) + 0.25 a_5 \text{lc}_{5,1} m_5 \dot{\theta}_1 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(2.0 \theta_5) \sin(2.0 \theta_2)
\end{aligned} \tag{A29}$$

[illegible]

$$\begin{aligned}
& 0.25 a_3 a_5 m_5 \dot{\theta}_1 \cos(2.0 \theta_2) \cos(2.0 \theta_3) \sin(\theta_4) \sin(\theta_5) + 0.25 a_3 a_5 m_5 \dot{\theta}_1 \cos(2.0 \theta_2) \sin(2.0 \theta_3) \cos(\theta_4) \sin(\theta_5) + \\
& 0.25 a_3 a_5 m_5 \dot{\theta}_1 \cos(2.0 \theta_2) \sin(2.0 \theta_3) \cos(\theta_5) \sin(\theta_4) + 0.5 a_3 \text{lc}_{5,1} m_5 \dot{\theta}_1 \cos(2.0 \theta_2) \cos(2.0 \theta_3) \sin(\theta_4) \sin(\theta_5) + \\
& 0.5 a_3 \text{lc}_{5,1} m_5 \dot{\theta}_1 \cos(2.0 \theta_2) \sin(2.0 \theta_3) \cos(\theta_4) \sin(\theta_5) + 0.5 a_3 \text{lc}_{5,1} m_5 \dot{\theta}_1 \cos(2.0 \theta_2) \sin(2.0 \theta_3) \cos(\theta_5) \sin(\theta_4) - \\
& a_4 \text{lc}_{5,2} m_5 \dot{\theta}_2 \cos(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) \sin(\theta_5) - a_4 \text{lc}_{5,2} m_5 \dot{\theta}_2 \cos(2.0 \theta_4) \sin(2.0 \theta_3) \cos(\theta_2) \sin(\theta_5) - a_4 \text{lc}_{5,2} m_5 \dot{\theta}_2 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) \cos(\theta_5) - \\
& 0.5 a_4 a_5 m_5 \dot{\theta}_2 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) \sin(\theta_5) - 0.5 a_3 \text{lc}_{5,2} m_5 \dot{\theta}_1 \cos(2.0 \theta_2) \sin(2.0 \theta_3) \sin(\theta_4) \sin(\theta_5) - \\
& a_4 \text{lc}_{5,1} m_5 \dot{\theta}_2 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) \sin(\theta_5) + 0.25 a_5 \text{lc}_{5,2} m_5 \dot{\theta}_1 \cos(2.0 \theta_2) \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(2.0 \theta_5) + \\
& 0.5 \text{lc}_{5,1} \text{lc}_{5,2} m_5 \dot{\theta}_1 \cos(2.0 \theta_2) \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(2.0 \theta_5) + 0.25 a_5 \text{lc}_{5,1} m_5 \dot{\theta}_1 \cos(2.0 \theta_2) \cos(2.0 \theta_3) \cos(2.0 \theta_4) \sin(2.0 \theta_5) + \\
& 0.25 a_5 \text{lc}_{5,1} m_5 \dot{\theta}_1 \cos(2.0 \theta_2) \cos(2.0 \theta_3) \cos(2.0 \theta_5) \sin(2.0 \theta_4) + 0.25 a_5 \text{lc}_{5,1} m_5 \dot{\theta}_1 \cos(2.0 \theta_2) \cos(2.0 \theta_4) \cos(2.0 \theta_5) \sin(2.0 \theta_3)
\end{aligned} \tag{A30}$$

[illegible]

[illegible]

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[illegible]

$$\begin{aligned}
C_{21} = & -I_{xz,2} \dot{\theta}_1 \cos(2.0 \theta_2) + 0.5 I_{xx,2} \dot{\theta}_1 \sin(2.0 \theta_2) + 0.25 I_{xx,3} \dot{\theta}_1 \sin(2.0 \theta_2) + 0.25 I_{xx,4} \dot{\theta}_1 \sin(2.0 \theta_2) + \\
& 0.25 I_{xx,5} \dot{\theta}_1 \sin(2.0 \theta_2) + 0.25 I_{yy,3} \dot{\theta}_1 \sin(2.0 \theta_2) + 0.25 I_{yy,4} \dot{\theta}_1 \sin(2.0 \theta_2) + 0.25 I_{yy,5} \dot{\theta}_1 \sin(2.0 \theta_2) - 0.5 I_{zz,2} \dot{\theta}_1 \sin(2.0 \theta_2) - \\
& 0.5 I_{zz,3} \dot{\theta}_1 \sin(2.0 \theta_2) - 0.5 I_{zz,4} \dot{\theta}_1 \sin(2.0 \theta_2) - 0.5 I_{zz,5} \dot{\theta}_1 \sin(2.0 \theta_2) + 0.5 I_{zz,3} \dot{\theta}_3 \cos(\theta_2) + 0.5 I_{zz,4} \dot{\theta}_3 \cos(\theta_2) + \\
& 0.5 I_{zz,4} \dot{\theta}_4 \cos(\theta_2) + 0.5 I_{zz,5} \dot{\theta}_3 \cos(\theta_2) + 0.5 I_{zz,5} \dot{\theta}_4 \cos(\theta_2) + 0.5 I_{zz,5} \dot{\theta}_5 \cos(\theta_2) - 0.25 I_{xx,3} \dot{\theta}_1 \cos(2.0 \theta_3) \sin(2.0 \theta_2) + \\
& 0.25 I_{yy,3} \dot{\theta}_1 \cos(2.0 \theta_3) \sin(2.0 \theta_2) + 0.5 I_{xy,3} \dot{\theta}_1 \sin(2.0 \theta_2) \sin(2.0 \theta_3) + 0.5 I_{xx,3} \dot{\theta}_3 \cos(2.0 \theta_3) \cos(\theta_2) - I_{yz,3} \dot{\theta}_1 \cos(2.0 \theta_2) \cos(\theta_3) - \\
& 0.5 I_{yy,3} \dot{\theta}_3 \cos(2.0 \theta_3) \cos(\theta_2) - I_{xz,3} \dot{\theta}_1 \cos(2.0 \theta_2) \sin(\theta_3) - I_{xy,3} \dot{\theta}_3 \sin(2.0 \theta_3) \cos(\theta_2) - 0.125 a_2^2 m_2 \dot{\theta}_1 \sin(2.0 \theta_2) - \\
& 0.5 a_2^2 m_3 \dot{\theta}_1 \sin(2.0 \theta_2) - 0.5 a_2^2 m_4 \dot{\theta}_1 \sin(2.0 \theta_2) - 0.0625 a_3^2 m_3 \dot{\theta}_1 \sin(2.0 \theta_2) - 0.5 a_2^2 m_5 \dot{\theta}_1 \sin(2.0 \theta_2) - \\
& 0.25 a_3^2 m_4 \dot{\theta}_1 \sin(2.0 \theta_2) - 0.25 a_3^2 m_5 \dot{\theta}_1 \sin(2.0 \theta_2) - 0.0625 a_4^2 m_4 \dot{\theta}_1 \sin(2.0 \theta_2) - 0.25 a_4^2 m_5 \dot{\theta}_1 \sin(2.0 \theta_2) - \\
& 0.0625 a_5^2 m_5 \dot{\theta}_1 \sin(2.0 \theta_2) - 0.5 lc_{2,1}^2 m_2 \dot{\theta}_1 \sin(2.0 \theta_2) + 0.5 lc_{2,3}^2 m_2 \dot{\theta}_1 \sin(2.0 \theta_2) - 0.25 lc_{3,1}^2 m_3 \dot{\theta}_1 \sin(2.0 \theta_2) - \\
& 0.25 lc_{3,2}^2 m_3 \dot{\theta}_1 \sin(2.0 \theta_2) + 0.5 lc_{3,3}^2 m_3 \dot{\theta}_1 \sin(2.0 \theta_2) - 0.25 lc_{4,1}^2 m_4 \dot{\theta}_1 \sin(2.0 \theta_2) - 0.25 lc_{4,2}^2 m_4 \dot{\theta}_1 \sin(2.0 \theta_2) + \\
& 0.5 lc_{4,3}^2 m_4 \dot{\theta}_1 \sin(2.0 \theta_2) - 0.25 lc_{5,1}^2 m_5 \dot{\theta}_1 \sin(2.0 \theta_2) - 0.25 lc_{5,2}^2 m_5 \dot{\theta}_1 \sin(2.0 \theta_2) + 0.5 lc_{5,3}^2 m_5 \dot{\theta}_1 \sin(2.0 \theta_2) + \\
& 0.125 a_3^2 m_3 \dot{\theta}_3 \cos(\theta_2) + 0.5 a_3^2 m_4 \dot{\theta}_3 \cos(\theta_2) + 0.5 a_3^2 m_5 \dot{\theta}_3 \cos(\theta_2) + 0.125 a_4^2 m_4 \dot{\theta}_3 \cos(\theta_2) + 0.125 a_4^2 m_4 \dot{\theta}_4 \cos(\theta_2) + \\
& 0.5 a_4^2 m_5 \dot{\theta}_3 \cos(\theta_2) + 0.5 a_4^2 m_5 \dot{\theta}_4 \cos(\theta_2) + 0.125 a_5^2 m_5 \dot{\theta}_3 \cos(\theta_2) + 0.125 a_5^2 m_5 \dot{\theta}_4 \cos(\theta_2) + 0.125 a_5^2 m_5 \dot{\theta}_5 \cos(\theta_2) - \\
& I_{yz,3} \dot{\theta}_3 \cos(\theta_3) \sin(\theta_2) + 0.5 lc_{3,1}^2 m_3 \dot{\theta}_3 \cos(\theta_2) + 0.5 lc_{3,2}^2 m_3 \dot{\theta}_3 \cos(\theta_2) + 0.5 lc_{4,1}^2 m_4 \dot{\theta}_3 \cos(\theta_2) + 0.5 lc_{4,1}^2 m_4 \dot{\theta}_4 \cos(\theta_2) + \\
& 0.5 lc_{4,2}^2 m_4 \dot{\theta}_3 \cos(\theta_2) + 0.5 lc_{4,2}^2 m_4 \dot{\theta}_4 \cos(\theta_2) + 0.5 lc_{5,1}^2 m_5 \dot{\theta}_3 \cos(\theta_2) + 0.5 lc_{5,1}^2 m_5 \dot{\theta}_4 \cos(\theta_2) + 0.5 lc_{5,2}^2 m_5 \dot{\theta}_3 \cos(\theta_2) + \\
& 0.5 lc_{5,1}^2 m_5 \dot{\theta}_5 \cos(\theta_2) + 0.5 lc_{5,2}^2 m_5 \dot{\theta}_4 \cos(\theta_2) + 0.5 lc_{5,2}^2 m_5 \dot{\theta}_5 \cos(\theta_2) - I_{xz,3} \dot{\theta}_3 \sin(\theta_2) \sin(\theta_3) - 0.5 I_{xx,4} \dot{\theta}_3 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) - \\
& 0.5 I_{xx,4} \dot{\theta}_4 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) + 0.5 I_{yy,4} \dot{\theta}_3 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) + 0.5 I_{yy,4} \dot{\theta}_4 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) + \\
& 0.25 lc_{3,1}^2 m_3 \dot{\theta}_1 \cos(2.0 \theta_3) \sin(2.0 \theta_2) - 0.25 lc_{3,2}^2 m_3 \dot{\theta}_1 \cos(2.0 \theta_3) \sin(2.0 \theta_2) - I_{yz,4} \dot{\theta}_1 \cos(2.0 \theta_2) \cos(\theta_3) \cos(\theta_4) - \\
& 0.125 a_3^2 m_3 \dot{\theta}_3 \cos(2.0 \theta_3) \cos(\theta_2) - 0.5 a_3^2 m_4 \dot{\theta}_3 \cos(2.0 \theta_3) \cos(\theta_2) - 0.5 a_3^2 m_5 \dot{\theta}_3 \cos(2.0 \theta_3) \cos(\theta_2) - \\
& I_{xz,4} \dot{\theta}_1 \cos(2.0 \theta_2) \cos(\theta_3) \sin(\theta_4) - I_{xz,4} \dot{\theta}_1 \cos(2.0 \theta_2) \cos(\theta_4) \sin(\theta_3) - 0.5 lc_{3,1}^2 m_3 \dot{\theta}_3 \cos(2.0 \theta_3) \cos(\theta_2) + \\
& 0.5 lc_{3,2}^2 m_3 \dot{\theta}_3 \cos(2.0 \theta_3) \cos(\theta_2) + I_{yz,4} \dot{\theta}_1 \cos(2.0 \theta_2) \sin(\theta_3) \sin(\theta_4) + 0.5 a_2 lc_{2,3} m_2 \dot{\theta}_1 \cos(2.0 \theta_2) + a_2 lc_{3,3} m_3 \dot{\theta}_1 \cos(2.0 \theta_2) + \\
& a_2 lc_{4,3} m_4 \dot{\theta}_1 \cos(2.0 \theta_2) + a_2 lc_{5,3} m_5 \dot{\theta}_1 \cos(2.0 \theta_2) + lc_{2,1} lc_{2,3} m_2 \dot{\theta}_1 \cos(2.0 \theta_2) - 0.25 I_{xx,4} \dot{\theta}_1 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \sin(2.0 \theta_2) + \\
& 0.25 I_{yy,4} \dot{\theta}_1 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \sin(2.0 \theta_2) - 0.5 a_2 lc_{2,1} m_2 \dot{\theta}_1 \sin(2.0 \theta_2) - 0.25 a_3 lc_{3,1} m_3 \dot{\theta}_1 \sin(2.0 \theta_2) - \\
& 0.25 a_4 lc_{4,1} m_4 \dot{\theta}_1 \sin(2.0 \theta_2) - 0.25 a_5 lc_{5,1} m_5 \dot{\theta}_1 \sin(2.0 \theta_2) + 0.5 I_{xy,4} \dot{\theta}_1 \cos(2.0 \theta_3) \sin(2.0 \theta_2) \sin(2.0 \theta_4) + \\
& 0.5 I_{xy,4} \dot{\theta}_1 \cos(2.0 \theta_4) \sin(2.0 \theta_2) \sin(2.0 \theta_3) + 0.25 I_{xx,4} \dot{\theta}_1 \sin(2.0 \theta_2) \sin(2.0 \theta_3) \sin(2.0 \theta_4) - 0.25 I_{yy,4} \dot{\theta}_1 \sin(2.0 \theta_2) \sin(2.0 \theta_3) \sin(2.0 \theta_4) - \\
& I_{yz,4} \dot{\theta}_3 \cos(\theta_3) \cos(\theta_4) \sin(\theta_2) - I_{yz,4} \dot{\theta}_4 \cos(\theta_3) \cos(\theta_4) \sin(\theta_2) - I_{xz,4} \dot{\theta}_3 \cos(\theta_3) \sin(\theta_2) \sin(\theta_4) - I_{xz,4} \dot{\theta}_3 \cos(\theta_4) \sin(\theta_2) \sin(\theta_3) - \\
& I_{xz,4} \dot{\theta}_4 \cos(\theta_3) \sin(\theta_2) \sin(\theta_4) - I_{xz,4} \dot{\theta}_4 \cos(\theta_4) \sin(\theta_2) \sin(\theta_3) + I_{yz,4} \dot{\theta}_3 \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) + I_{yz,4} \dot{\theta}_4 \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) + \\
& 0.5 a_1 a_2 m_2 \dot{\theta}_1 \cos(\theta_2) + a_1 a_2 m_3 \dot{\theta}_1 \cos(\theta_2) + a_1 a_2 m_4 \dot{\theta}_1 \cos(\theta_2) + a_1 a_2 m_5 \dot{\theta}_1 \cos(\theta_2) + 0.5 I_{xx,4} \dot{\theta}_3 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(\theta_2) + \\
& 0.5 I_{xx,4} \dot{\theta}_4 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(\theta_2) - 0.5 I_{yy,4} \dot{\theta}_3 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(\theta_2) - 0.5 I_{yy,4} \dot{\theta}_4 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(\theta_2) + \\
& a_1 lc_{2,1} m_2 \dot{\theta}_1 \cos(\theta_2) + 0.5 a_3 lc_{3,1} m_3 \dot{\theta}_3 \cos(\theta_2) + 0.5 a_4 lc_{4,1} m_4 \dot{\theta}_3 \cos(\theta_2) + 0.5 a_4 lc_{4,1} m_4 \dot{\theta}_4 \cos(\theta_2) + 0.5 a_5 lc_{5,1} m_5 \dot{\theta}_3 \cos(\theta_2) + \\
& 0.5 a_5 lc_{5,1} m_5 \dot{\theta}_4 \cos(\theta_2) + 0.5 a_5 lc_{5,1} m_5 \dot{\theta}_5 \cos(\theta_2) - I_{xy,4} \dot{\theta}_3 \cos(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) - I_{xy,4} \dot{\theta}_3 \cos(2.0 \theta_4) \sin(2.0 \theta_3) \cos(\theta_2) - \\
& I_{xy,4} \dot{\theta}_4 \cos(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) - I_{xy,4} \dot{\theta}_4 \cos(2.0 \theta_4) \sin(2.0 \theta_3) \cos(\theta_2) + a_1 lc_{2,3} m_2 \dot{\theta}_1 \sin(\theta_2) + a_1 lc_{3,3} m_3 \dot{\theta}_1 \sin(\theta_2) + \\
& a_1 lc_{4,3} m_4 \dot{\theta}_1 \sin(\theta_2) + a_1 lc_{5,3} m_5 \dot{\theta}_1 \sin(\theta_2) + 0.0625 a_3^2 m_3 \dot{\theta}_1 \cos(2.0 \theta_3) \sin(2.0 \theta_2) + 0.25 a_3^2 m_4 \dot{\theta}_1 \cos(2.0 \theta_3) \sin(2.0 \theta_2) + \\
& 0.25 a_3^2 m_5 \dot{\theta}_1 \cos(2.0 \theta_3) \sin(2.0 \theta_2) + 0.25 I_{xx,5} \dot{\theta}_1 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(2.0 \theta_5) \sin(2.0 \theta_2) - 0.25 I_{yy,5} \dot{\theta}_1 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(2.0 \theta_5) \sin(2.0 \theta_2) + \\
& 0.25 a_3 lc_{3,1} m_3 \dot{\theta}_1 \cos(2.0 \theta_3) \sin(2.0 \theta_2) - 0.5 I_{xy,5} \dot{\theta}_1 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \sin(2.0 \theta_2) \sin(2.0 \theta_5) - 0.5 I_{xy,5} \dot{\theta}_1 \cos(2.0 \theta_3) \cos(2.0 \theta_5) \sin(2.0 \theta_2) \sin(2.0 \theta_4) - \\
& 0.5 I_{xy,5} \dot{\theta}_1 \cos(2.0 \theta_4) \cos(2.0 \theta_5) \sin(2.0 \theta_2) \sin(2.0 \theta_3) - I_{xz,5} \dot{\theta}_1 \cos(2.0 \theta_2) \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) - 0.25 a_3 lc_{3,2} m_3 \dot{\theta}_1 \sin(2.0 \theta_2) \sin(2.0 \theta_3) -
\end{aligned}$$

[illegible]

[illegible]

$$\begin{aligned}
& 0.5 a_4 \text{lc}_{5,2} m_5 \dot{\theta}_5 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) \cos(\theta_5) - 0.5 a_4 a_5 m_5 \dot{\theta}_3 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) \sin(\theta_5) - \\
& 0.5 a_4 a_5 m_5 \dot{\theta}_4 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) \sin(\theta_5) - 0.25 a_4 a_5 m_5 \dot{\theta}_5 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) \sin(\theta_5) + \\
& 0.5 a_3 \text{lc}_{5,2} m_5 \dot{\theta}_1 \cos(2.0 \theta_3) \sin(2.0 \theta_2) \sin(\theta_4) \sin(\theta_5) + 0.5 a_3 \text{lc}_{5,2} m_5 \dot{\theta}_1 \sin(2.0 \theta_2) \sin(2.0 \theta_3) \cos(\theta_4) \sin(\theta_5) + \\
& 0.5 a_3 \text{lc}_{5,2} m_5 \dot{\theta}_1 \sin(2.0 \theta_2) \sin(2.0 \theta_3) \cos(\theta_5) \sin(\theta_4) - a_4 \text{lc}_{5,1} m_5 \dot{\theta}_3 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) \sin(\theta_5) - \\
& a_4 \text{lc}_{5,1} m_5 \dot{\theta}_4 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) \sin(\theta_5) - 0.5 a_4 \text{lc}_{5,1} m_5 \dot{\theta}_5 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) \sin(\theta_5) + \\
& 0.25 a_3 a_5 m_5 \dot{\theta}_1 \sin(2.0 \theta_2) \sin(2.0 \theta_3) \sin(\theta_4) \sin(\theta_5) + 0.5 a_3 \text{lc}_{5,1} m_5 \dot{\theta}_1 \sin(2.0 \theta_2) \sin(2.0 \theta_3) \sin(\theta_4) \sin(\theta_5) - \\
& 0.25 a_5 \text{lc}_{5,1} m_5 \dot{\theta}_1 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(2.0 \theta_5) \sin(2.0 \theta_2)
\end{aligned} \tag{A33}$$

[illegible]

[illegible]

[illegible]

[illegible]

$$\begin{aligned}
C_{23} = & I_{xz,3} \dot{\theta}_3 \sin(\theta_3) - I_{xy,3} \dot{\theta}_2 \cos(2.0\theta_3) - 0.5 I_{xx,3} \dot{\theta}_2 \sin(2.0\theta_3) + 0.5 I_{yy,3} \dot{\theta}_2 \sin(2.0\theta_3) + I_{yz,3} \dot{\theta}_3 \cos(\theta_3) + \\
& 0.5 I_{zz,3} \dot{\theta}_1 \cos(\theta_2) + 0.5 I_{zz,4} \dot{\theta}_1 \cos(\theta_2) + 0.5 I_{zz,5} \dot{\theta}_1 \cos(\theta_2) - I_{xy,4} \dot{\theta}_2 \cos(2.0\theta_3) \cos(2.0\theta_4) - 0.5 I_{xx,4} \dot{\theta}_2 \cos(2.0\theta_3) \sin(2.0\theta_4) - \\
& 0.5 I_{xx,4} \dot{\theta}_2 \cos(2.0\theta_4) \sin(2.0\theta_3) + 0.5 I_{yy,4} \dot{\theta}_2 \cos(2.0\theta_3) \sin(2.0\theta_4) + 0.5 I_{yy,4} \dot{\theta}_2 \cos(2.0\theta_4) \sin(2.0\theta_3) + \\
& I_{xy,4} \dot{\theta}_2 \sin(2.0\theta_3) \sin(2.0\theta_4) + 0.5 I_{xx,3} \dot{\theta}_1 \cos(2.0\theta_3) \cos(\theta_2) - 0.5 I_{yy,3} \dot{\theta}_1 \cos(2.0\theta_3) \cos(\theta_2) - I_{xy,3} \dot{\theta}_1 \sin(2.0\theta_3) \cos(\theta_2) + \\
& 0.125 a_3^2 m_3 \dot{\theta}_2 \sin(2.0\theta_3) + 0.5 a_3^2 m_4 \dot{\theta}_2 \sin(2.0\theta_3) + 0.5 a_3^2 m_5 \dot{\theta}_2 \sin(2.0\theta_3) + 0.5 lc_{3,1}^2 m_3 \dot{\theta}_2 \sin(2.0\theta_3) - \\
& 0.5 lc_{3,2}^2 m_3 \dot{\theta}_2 \sin(2.0\theta_3) + I_{yz,4} \dot{\theta}_3 \cos(\theta_3) \cos(\theta_4) + I_{yz,4} \dot{\theta}_4 \cos(\theta_3) \cos(\theta_4) + 0.125 a_3^2 m_3 \dot{\theta}_1 \cos(\theta_2) + 0.5 a_3^2 m_4 \dot{\theta}_1 \cos(\theta_2) + \\
& 0.5 a_3^2 m_5 \dot{\theta}_1 \cos(\theta_2) + 0.125 a_4^2 m_4 \dot{\theta}_1 \cos(\theta_2) + 0.5 a_4^2 m_5 \dot{\theta}_1 \cos(\theta_2) + 0.125 a_5^2 m_5 \dot{\theta}_1 \cos(\theta_2) + I_{xz,4} \dot{\theta}_3 \cos(\theta_3) \sin(\theta_4) + \\
& I_{xz,4} \dot{\theta}_3 \cos(\theta_4) \sin(\theta_3) + I_{xz,4} \dot{\theta}_4 \cos(\theta_3) \sin(\theta_4) + I_{xz,4} \dot{\theta}_4 \cos(\theta_4) \sin(\theta_3) - I_{yz,3} \dot{\theta}_1 \cos(\theta_3) \sin(\theta_2) + 0.5 lc_{3,1}^2 m_3 \dot{\theta}_1 \cos(\theta_2) + \\
& 0.5 lc_{3,2}^2 m_3 \dot{\theta}_1 \cos(\theta_2) + 0.5 lc_{4,1}^2 m_4 \dot{\theta}_1 \cos(\theta_2) + 0.5 lc_{4,2}^2 m_4 \dot{\theta}_1 \cos(\theta_2) + 0.5 lc_{5,1}^2 m_5 \dot{\theta}_1 \cos(\theta_2) + 0.5 lc_{5,2}^2 m_5 \dot{\theta}_1 \cos(\theta_2) - \\
& I_{xz,3} \dot{\theta}_1 \sin(\theta_2) \sin(\theta_3) - I_{yz,4} \dot{\theta}_3 \sin(\theta_3) \sin(\theta_4) - I_{yz,4} \dot{\theta}_4 \sin(\theta_3) \sin(\theta_4) - lc_{3,1} lc_{3,3} m_3 \dot{\theta}_3 \sin(\theta_3) - 0.5 I_{xx,4} \dot{\theta}_1 \sin(2.0\theta_3) \sin(2.0\theta_4) \cos(\theta_2) + \\
& 0.5 I_{yy,4} \dot{\theta}_1 \sin(2.0\theta_3) \sin(2.0\theta_4) \cos(\theta_2) + 0.5 lc_{4,1}^2 m_4 \dot{\theta}_2 \cos(2.0\theta_3) \sin(2.0\theta_4) + 0.5 lc_{4,1}^2 m_4 \dot{\theta}_2 \cos(2.0\theta_4) \sin(2.0\theta_3) - \\
& 0.5 lc_{4,2}^2 m_4 \dot{\theta}_2 \cos(2.0\theta_3) \sin(2.0\theta_4) - 0.5 lc_{4,2}^2 m_4 \dot{\theta}_2 \cos(2.0\theta_4) \sin(2.0\theta_3) - 0.125 a_3^2 m_3 \dot{\theta}_1 \cos(2.0\theta_3) \cos(\theta_2) - \\
& 0.5 a_3^2 m_4 \dot{\theta}_1 \cos(2.0\theta_3) \cos(\theta_2) - 0.5 a_3^2 m_5 \dot{\theta}_1 \cos(2.0\theta_3) \cos(\theta_2) - 0.5 lc_{3,1}^2 m_3 \dot{\theta}_1 \cos(2.0\theta_3) \cos(\theta_2) + 0.5 lc_{3,2}^2 m_3 \dot{\theta}_1 \cos(2.0\theta_3) \cos(\theta_2) + \\
& I_{xy,5} \dot{\theta}_2 \cos(2.0\theta_3) \cos(2.0\theta_4) \cos(2.0\theta_5) + 0.5 a_3 lc_{3,2} m_3 \dot{\theta}_2 \cos(2.0\theta_3) + lc_{3,1} lc_{3,2} m_3 \dot{\theta}_2 \cos(2.0\theta_3) + 0.5 I_{xx,5} \dot{\theta}_2 \cos(2.0\theta_3) \cos(2.0\theta_4) \sin(2.0\theta_5) + \\
& 0.5 I_{xx,5} \dot{\theta}_2 \cos(2.0\theta_3) \cos(2.0\theta_5) \sin(2.0\theta_4) + 0.5 I_{xx,5} \dot{\theta}_2 \cos(2.0\theta_4) \cos(2.0\theta_5) \sin(2.0\theta_3) - 0.5 I_{yy,5} \dot{\theta}_2 \cos(2.0\theta_3) \cos(2.0\theta_4) \sin(2.0\theta_5) - \\
& 0.5 I_{yy,5} \dot{\theta}_2 \cos(2.0\theta_3) \cos(2.0\theta_5) \sin(2.0\theta_4) - 0.5 I_{yy,5} \dot{\theta}_2 \cos(2.0\theta_4) \cos(2.0\theta_5) \sin(2.0\theta_3) + 0.5 a_3 lc_{3,1} m_3 \dot{\theta}_2 \sin(2.0\theta_3) - \\
& I_{xy,5} \dot{\theta}_2 \cos(2.0\theta_3) \sin(2.0\theta_4) \sin(2.0\theta_5) - I_{xy,5} \dot{\theta}_2 \cos(2.0\theta_4) \sin(2.0\theta_3) \sin(2.0\theta_5) - I_{xy,5} \dot{\theta}_2 \cos(2.0\theta_5) \sin(2.0\theta_3) \sin(2.0\theta_4) + \\
& I_{xz,5} \dot{\theta}_3 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) + I_{xz,5} \dot{\theta}_4 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) + I_{xz,5} \dot{\theta}_5 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) - 0.5 I_{xx,5} \dot{\theta}_2 \sin(2.0\theta_3) \sin(2.0\theta_4) \sin(2.0\theta_5) + \\
& 0.5 I_{yy,5} \dot{\theta}_2 \sin(2.0\theta_3) \sin(2.0\theta_4) \sin(2.0\theta_5) - I_{yz,4} \dot{\theta}_1 \cos(\theta_3) \cos(\theta_4) \sin(\theta_2) - I_{yz,5} \dot{\theta}_3 \cos(\theta_3) \cos(\theta_4) \sin(\theta_5) - \\
& I_{yz,5} \dot{\theta}_3 \cos(\theta_3) \cos(\theta_5) \sin(\theta_4) - I_{yz,5} \dot{\theta}_3 \cos(\theta_4) \cos(\theta_5) \sin(\theta_3) - I_{yz,5} \dot{\theta}_4 \cos(\theta_3) \cos(\theta_4) \sin(\theta_5) - I_{yz,5} \dot{\theta}_4 \cos(\theta_3) \cos(\theta_5) \sin(\theta_4) - \\
& I_{yz,5} \dot{\theta}_4 \cos(\theta_4) \cos(\theta_5) \sin(\theta_3) - I_{yz,5} \dot{\theta}_5 \cos(\theta_3) \cos(\theta_4) \sin(\theta_5) - I_{yz,5} \dot{\theta}_5 \cos(\theta_3) \cos(\theta_5) \sin(\theta_4) - I_{yz,5} \dot{\theta}_5 \cos(\theta_4) \cos(\theta_5) \sin(\theta_3) - \\
& I_{xz,4} \dot{\theta}_1 \cos(\theta_3) \sin(\theta_2) \sin(\theta_4) - I_{xz,4} \dot{\theta}_1 \cos(\theta_4) \sin(\theta_2) \sin(\theta_3) - I_{xz,5} \dot{\theta}_3 \cos(\theta_3) \sin(\theta_4) \sin(\theta_5) - I_{xz,5} \dot{\theta}_3 \cos(\theta_4) \sin(\theta_3) \sin(\theta_5) - \\
& I_{xz,5} \dot{\theta}_3 \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) - I_{xz,5} \dot{\theta}_4 \cos(\theta_3) \sin(\theta_4) \sin(\theta_5) - I_{xz,5} \dot{\theta}_4 \cos(\theta_4) \sin(\theta_3) \sin(\theta_5) - I_{xz,5} \dot{\theta}_4 \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) - \\
& I_{xz,5} \dot{\theta}_5 \cos(\theta_3) \sin(\theta_4) \sin(\theta_5) - I_{xz,5} \dot{\theta}_5 \cos(\theta_4) \sin(\theta_3) \sin(\theta_5) - I_{xz,5} \dot{\theta}_5 \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) + I_{yz,4} \dot{\theta}_1 \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) + \\
& I_{yz,5} \dot{\theta}_3 \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) + I_{yz,5} \dot{\theta}_4 \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) + I_{yz,5} \dot{\theta}_5 \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) + 0.5 a_2 a_3 m_3 \dot{\theta}_2 \cos(\theta_3) + \\
& a_2 a_3 m_4 \dot{\theta}_2 \cos(\theta_3) + a_2 a_3 m_5 \dot{\theta}_2 \cos(\theta_3) + 0.5 I_{xx,4} \dot{\theta}_1 \cos(2.0\theta_3) \cos(2.0\theta_4) \cos(\theta_2) - 0.5 I_{yy,4} \dot{\theta}_1 \cos(2.0\theta_3) \cos(2.0\theta_4) \cos(\theta_2) + \\
& 0.5 a_3 lc_{3,1} m_3 \dot{\theta}_1 \cos(\theta_2) + a_2 lc_{3,1} m_3 \dot{\theta}_2 \cos(\theta_3) + 0.5 a_4 lc_{4,1} m_4 \dot{\theta}_1 \cos(\theta_2) + 0.5 a_5 lc_{5,1} m_5 \dot{\theta}_1 \cos(\theta_2) - lc_{3,2} lc_{3,3} m_3 \dot{\theta}_3 \cos(\theta_3) - \\
& I_{xy,4} \dot{\theta}_1 \cos(2.0\theta_3) \sin(2.0\theta_4) \cos(\theta_2) - I_{xy,4} \dot{\theta}_1 \cos(2.0\theta_4) \sin(2.0\theta_3) \cos(\theta_2) - a_2 lc_{3,2} m_3 \dot{\theta}_2 \sin(\theta_3) - 0.5 a_3 lc_{3,3} m_3 \dot{\theta}_3 \sin(\theta_3) - \\
& a_3 lc_{4,3} m_4 \dot{\theta}_3 \sin(\theta_3) - a_3 lc_{5,3} m_5 \dot{\theta}_3 \sin(\theta_3) + 0.125 a_4^2 m_4 \dot{\theta}_2 \cos(2.0\theta_3) \sin(2.0\theta_4) + 0.125 a_4^2 m_4 \dot{\theta}_2 \cos(2.0\theta_4) \sin(2.0\theta_3) + \\
& 0.5 a_4^2 m_5 \dot{\theta}_2 \cos(2.0\theta_3) \sin(2.0\theta_4) + 0.5 a_4^2 m_5 \dot{\theta}_2 \cos(2.0\theta_4) \sin(2.0\theta_3) + lc_{4,1} lc_{4,2} m_4 \dot{\theta}_2 \cos(2.0\theta_3) \cos(2.0\theta_4) + \\
& 0.5 a_4 lc_{4,1} m_4 \dot{\theta}_2 \cos(2.0\theta_3) \sin(2.0\theta_4) + 0.5 a_4 lc_{4,1} m_4 \dot{\theta}_2 \cos(2.0\theta_4) \sin(2.0\theta_3) - 0.5 a_4 lc_{4,2} m_4 \dot{\theta}_2 \sin(2.0\theta_3) \sin(2.0\theta_4) - \\
& lc_{4,1} lc_{4,2} m_4 \dot{\theta}_2 \sin(2.0\theta_3) \sin(2.0\theta_4) - 0.5 I_{xx,5} \dot{\theta}_1 \cos(2.0\theta_3) \cos(2.0\theta_4) \cos(2.0\theta_5) \cos(\theta_2) + 0.5 I_{yy,5} \dot{\theta}_1 \cos(2.0\theta_3) \cos(2.0\theta_4) \cos(2.0\theta_5) \cos(\theta_2) - \\
& 0.5 a_3 lc_{3,1} m_3 \dot{\theta}_1 \cos(2.0\theta_3) \cos(\theta_2) + a_3 lc_{4,2} m_4 \dot{\theta}_2 \cos(2.0\theta_3) \cos(\theta_4) + 0.5 a_3 a_4 m_4 \dot{\theta}_2 \cos(2.0\theta_3) \sin(\theta_4) + \\
& 0.5 a_3 a_4 m_4 \dot{\theta}_2 \sin(2.0\theta_3) \cos(\theta_4) + a_3 a_4 m_5 \dot{\theta}_2 \cos(2.0\theta_3) \sin(\theta_4) + a_3 a_4 m_5 \dot{\theta}_2 \sin(2.0\theta_3) \cos(\theta_4) + I_{xy,5} \dot{\theta}_1 \cos(2.0\theta_3) \cos(2.0\theta_4) \sin(2.0\theta_5) \cos(\theta_2) + \\
& I_{xy,5} \dot{\theta}_1 \cos(2.0\theta_3) \cos(2.0\theta_5) \sin(2.0\theta_4) \cos(\theta_2) + I_{xy,5} \dot{\theta}_1 \cos(2.0\theta_4) \cos(2.0\theta_5) \sin(2.0\theta_3) \cos(\theta_2) + 0.5 a_3 lc_{3,2} m_3 \dot{\theta}_1 \sin(2.0\theta_3) \cos(\theta_2) +
\end{aligned}$$

$$\begin{aligned}
& a_3 lc_{5,1} m_5 \dot{\theta}_1 \cos(2.0 \theta_3) \cos(\theta_2) \sin(\theta_4) \sin(\theta_5) - a_3 lc_{5,2} m_5 \dot{\theta}_1 \sin(2.0 \theta_3) \cos(\theta_2) \cos(\theta_5) \sin(\theta_4) - \\
& 0.5 a_3 a_5 m_5 \dot{\theta}_1 \sin(2.0 \theta_3) \cos(\theta_2) \sin(\theta_4) \sin(\theta_5) + 0.5 a_5 lc_{5,1} m_5 \dot{\theta}_1 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(2.0 \theta_5) \cos(\theta_2) - \\
& 0.5 a_5 lc_{5,2} m_5 \dot{\theta}_1 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \sin(2.0 \theta_5) \cos(\theta_2) - 0.5 a_5 lc_{5,2} m_5 \dot{\theta}_1 \cos(2.0 \theta_3) \cos(2.0 \theta_5) \sin(2.0 \theta_4) \cos(\theta_2) - \\
& 0.5 a_5 lc_{5,2} m_5 \dot{\theta}_1 \cos(2.0 \theta_4) \cos(2.0 \theta_5) \sin(2.0 \theta_3) \cos(\theta_2) + 0.5 a_2 a_5 m_5 \dot{\theta}_1 \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) - \\
& lc_{5,1} lc_{5,2} m_5 \dot{\theta}_1 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \sin(2.0 \theta_5) \cos(\theta_2) - lc_{5,1} lc_{5,2} m_5 \dot{\theta}_1 \cos(2.0 \theta_3) \cos(2.0 \theta_5) \sin(2.0 \theta_4) \cos(\theta_2) - \\
& lc_{5,1} lc_{5,2} m_5 \dot{\theta}_1 \cos(2.0 \theta_4) \cos(2.0 \theta_5) \sin(2.0 \theta_3) \cos(\theta_2) - 0.5 a_5 lc_{5,1} m_5 \dot{\theta}_1 \cos(2.0 \theta_3) \sin(2.0 \theta_4) \sin(2.0 \theta_5) \cos(\theta_2) - \\
& 0.5 a_5 lc_{5,1} m_5 \dot{\theta}_1 \cos(2.0 \theta_4) \sin(2.0 \theta_3) \sin(2.0 \theta_5) \cos(\theta_2) - 0.5 a_5 lc_{5,1} m_5 \dot{\theta}_1 \cos(2.0 \theta_5) \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) + \\
& a_2 lc_{5,1} m_5 \dot{\theta}_1 \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) + 0.5 a_5 lc_{5,2} m_5 \dot{\theta}_1 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \sin(2.0 \theta_5) \cos(\theta_2) - \\
& a_2 lc_{5,2} m_5 \dot{\theta}_1 \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) \sin(\theta_5) - a_2 lc_{5,2} m_5 \dot{\theta}_1 \cos(\theta_2) \cos(\theta_3) \cos(\theta_5) \sin(\theta_4) - a_2 lc_{5,2} m_5 \dot{\theta}_1 \cos(\theta_2) \cos(\theta_4) \cos(\theta_5) \sin(\theta_3) + \\
& 0.5 a_5 lc_{5,3} m_5 \dot{\theta}_1 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) \sin(\theta_2) - 0.5 a_2 a_5 m_5 \dot{\theta}_1 \cos(\theta_2) \cos(\theta_3) \sin(\theta_4) \sin(\theta_5) - 0.5 a_2 a_5 m_5 \dot{\theta}_1 \cos(\theta_2) \cos(\theta_4) \sin(\theta_3) \sin(\theta_5) - \\
& 0.5 a_2 a_5 m_5 \dot{\theta}_1 \cos(\theta_2) \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) + lc_{5,1} lc_{5,2} m_5 \dot{\theta}_1 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \sin(2.0 \theta_5) \cos(\theta_2) + \\
& lc_{5,1} lc_{5,3} m_5 \dot{\theta}_1 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) \sin(\theta_2) - a_2 lc_{5,1} m_5 \dot{\theta}_1 \cos(\theta_2) \cos(\theta_3) \sin(\theta_4) \sin(\theta_5) - a_2 lc_{5,1} m_5 \dot{\theta}_1 \cos(\theta_2) \cos(\theta_4) \sin(\theta_3) \sin(\theta_5) - \\
& a_2 lc_{5,1} m_5 \dot{\theta}_1 \cos(\theta_2) \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) - lc_{5,2} lc_{5,3} m_5 \dot{\theta}_1 \cos(\theta_3) \cos(\theta_4) \sin(\theta_2) \sin(\theta_5) - lc_{5,2} lc_{5,3} m_5 \dot{\theta}_1 \cos(\theta_3) \cos(\theta_5) \sin(\theta_2) \sin(\theta_4) - \\
& lc_{5,2} lc_{5,3} m_5 \dot{\theta}_1 \cos(\theta_4) \cos(\theta_5) \sin(\theta_2) \sin(\theta_3) + a_2 lc_{5,2} m_5 \dot{\theta}_1 \cos(\theta_2) \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) - 0.5 a_5 lc_{5,3} m_5 \dot{\theta}_1 \cos(\theta_3) \sin(\theta_2) \sin(\theta_4) \sin(\theta_5) - \\
& 0.5 a_5 lc_{5,3} m_5 \dot{\theta}_1 \cos(\theta_4) \sin(\theta_2) \sin(\theta_3) \sin(\theta_5) - 0.5 a_5 lc_{5,3} m_5 \dot{\theta}_1 \cos(\theta_5) \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) - lc_{5,1} lc_{5,3} m_5 \dot{\theta}_1 \cos(\theta_3) \sin(\theta_2) \sin(\theta_4) \sin(\theta_5) - \\
& lc_{5,1} lc_{5,3} m_5 \dot{\theta}_1 \cos(\theta_4) \sin(\theta_2) \sin(\theta_3) \sin(\theta_5) - lc_{5,1} lc_{5,3} m_5 \dot{\theta}_1 \cos(\theta_5) \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) + a_4 lc_{5,2} m_5 \dot{\theta}_1 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(\theta_2) \cos(\theta_5) + \\
& 0.5 a_4 a_5 m_5 \dot{\theta}_1 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(\theta_2) \sin(\theta_5) + 0.5 a_4 a_5 m_5 \dot{\theta}_1 \cos(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) \cos(\theta_5) + \\
& 0.5 a_4 a_5 m_5 \dot{\theta}_1 \cos(2.0 \theta_4) \sin(2.0 \theta_3) \cos(\theta_2) \cos(\theta_5) + lc_{5,2} lc_{5,3} m_5 \dot{\theta}_1 \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) + a_4 lc_{5,1} m_5 \dot{\theta}_1 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(\theta_2) \sin(\theta_5) + \\
& a_4 lc_{5,1} m_5 \dot{\theta}_1 \cos(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) \cos(\theta_5) + a_4 lc_{5,1} m_5 \dot{\theta}_1 \cos(2.0 \theta_4) \sin(2.0 \theta_3) \cos(\theta_2) \cos(\theta_5) - \\
& a_4 lc_{5,2} m_5 \dot{\theta}_1 \cos(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) \sin(\theta_5) - a_4 lc_{5,2} m_5 \dot{\theta}_1 \cos(2.0 \theta_4) \sin(2.0 \theta_3) \cos(\theta_2) \sin(\theta_5) - \\
& a_4 lc_{5,2} m_5 \dot{\theta}_1 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) \cos(\theta_5) - 0.5 a_4 a_5 m_5 \dot{\theta}_1 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) \sin(\theta_5) - \\
& a_4 lc_{5,1} m_5 \dot{\theta}_1 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) \sin(\theta_5)
\end{aligned} \tag{A35}$$

$$\begin{aligned}
C_{24} = & 2.0 I_{xy,4} \dot{\theta}_2 \cos(\theta_3)^2 - I_{xy,4} \dot{\theta}_2 - I_{xy,4} \dot{\theta}_2 + 2.0 I_{xy,4} \dot{\theta}_2 \cos(\theta_4)^2 + 2 I_{xy,5} \dot{\theta}_2 \cos(\theta_3)^2 + 2 I_{xy,5} \dot{\theta}_2 \cos(\theta_4)^2 + \\
& 2 I_{xy,5} \dot{\theta}_2 \cos(\theta_5)^2 + 0.5 I_{zz,4} \dot{\theta}_1 \cos(\theta_2) + 0.5 I_{zz,5} \dot{\theta}_1 \cos(\theta_2) + 0.5 a_4 l_{c4,2} m_4 \dot{\theta}_2 + 0.5 a_5 l_{c5,2} m_5 \dot{\theta}_2 + l_{c4,1} l_{c4,2} m_4 \dot{\theta}_2 + \\
& l_{c5,1} l_{c5,2} m_5 \dot{\theta}_2 - 4.0 I_{xy,4} \dot{\theta}_2 \cos(\theta_3)^2 \cos(\theta_4)^2 - 4 I_{xy,5} \dot{\theta}_2 \cos(\theta_3)^2 \cos(\theta_4)^2 - 4 I_{xy,5} \dot{\theta}_2 \cos(\theta_3)^2 \cos(\theta_5)^2 - 4 I_{xy,5} \dot{\theta}_2 \cos(\theta_4)^2 \cos(\theta_5)^2 + \\
& I_{yz,4} \dot{\theta}_3 \cos(\theta_3) \cos(\theta_4) + I_{yz,4} \dot{\theta}_4 \cos(\theta_3) \cos(\theta_4) + 0.125 a_4^2 m_4 \dot{\theta}_1 \cos(\theta_2) + 0.5 a_4^2 m_5 \dot{\theta}_1 \cos(\theta_2) + 0.125 a_5^2 m_5 \dot{\theta}_1 \cos(\theta_2) + \\
& I_{xx,4} \dot{\theta}_2 \cos(\theta_3) \sin(\theta_3) + I_{xx,5} \dot{\theta}_2 \cos(\theta_3) \sin(\theta_3) + I_{xx,4} \dot{\theta}_2 \cos(\theta_4) \sin(\theta_4) + I_{xx,5} \dot{\theta}_2 \cos(\theta_4) \sin(\theta_4) + I_{xz,4} \dot{\theta}_3 \cos(\theta_3) \sin(\theta_4) + \\
& I_{xz,4} \dot{\theta}_3 \cos(\theta_4) \sin(\theta_3) + I_{xx,5} \dot{\theta}_2 \cos(\theta_5) \sin(\theta_5) + I_{xz,4} \dot{\theta}_4 \cos(\theta_3) \sin(\theta_4) + I_{xz,4} \dot{\theta}_4 \cos(\theta_4) \sin(\theta_3) - I_{yy,4} \dot{\theta}_2 \cos(\theta_3) \sin(\theta_3) - \\
& I_{yy,5} \dot{\theta}_2 \cos(\theta_3) \sin(\theta_3) - I_{yy,4} \dot{\theta}_2 \cos(\theta_4) \sin(\theta_4) - I_{yy,5} \dot{\theta}_2 \cos(\theta_4) \sin(\theta_4) - I_{yy,5} \dot{\theta}_2 \cos(\theta_5) \sin(\theta_5) + 0.5 l_{c4,1}^2 m_4 \dot{\theta}_1 \cos(\theta_2) + \\
& 0.5 l_{c4,2}^2 m_4 \dot{\theta}_1 \cos(\theta_2) + 0.5 l_{c5,1}^2 m_5 \dot{\theta}_1 \cos(\theta_2) + 0.5 l_{c5,2}^2 m_5 \dot{\theta}_1 \cos(\theta_2) - I_{yz,4} \dot{\theta}_3 \sin(\theta_3) \sin(\theta_4) - I_{yz,4} \dot{\theta}_4 \sin(\theta_3) \sin(\theta_4) - \\
& 0.5 I_{xx,4} \dot{\theta}_1 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) + 0.5 I_{yy,4} \dot{\theta}_1 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) - 2.0 I_{xx,4} \dot{\theta}_2 \cos(\theta_3) \cos(\theta_4)^2 \sin(\theta_3) - \\
& 2.0 I_{xx,4} \dot{\theta}_2 \cos(\theta_3)^2 \cos(\theta_4) \sin(\theta_4) - 2.0 I_{xx,5} \dot{\theta}_2 \cos(\theta_3) \cos(\theta_4)^2 \sin(\theta_3) - 2.0 I_{xx,5} \dot{\theta}_2 \cos(\theta_3) \cos(\theta_5)^2 \sin(\theta_3) - \\
& 2.0 I_{xx,5} \dot{\theta}_2 \cos(\theta_3)^2 \cos(\theta_4) \sin(\theta_4) - 2.0 I_{xx,5} \dot{\theta}_2 \cos(\theta_4) \cos(\theta_5)^2 \sin(\theta_4) - 2.0 I_{xx,5} \dot{\theta}_2 \cos(\theta_3)^2 \cos(\theta_5) \sin(\theta_5) - \\
& 2.0 I_{xx,5} \dot{\theta}_2 \cos(\theta_4)^2 \cos(\theta_5) \sin(\theta_5) + 2.0 I_{yy,4} \dot{\theta}_2 \cos(\theta_3) \cos(\theta_4)^2 \sin(\theta_3) + 2.0 I_{yy,4} \dot{\theta}_2 \cos(\theta_3)^2 \cos(\theta_4) \sin(\theta_4) + \\
& 2.0 I_{yy,5} \dot{\theta}_2 \cos(\theta_3) \cos(\theta_4)^2 \sin(\theta_3) + 2.0 I_{yy,5} \dot{\theta}_2 \cos(\theta_3) \cos(\theta_5)^2 \sin(\theta_3) + 2.0 I_{yy,5} \dot{\theta}_2 \cos(\theta_3)^2 \cos(\theta_4) \sin(\theta_4) + \\
& 2.0 I_{yy,5} \dot{\theta}_2 \cos(\theta_4) \cos(\theta_5)^2 \sin(\theta_4) + 2.0 I_{yy,5} \dot{\theta}_2 \cos(\theta_3)^2 \cos(\theta_5) \sin(\theta_5) + 2.0 I_{yy,5} \dot{\theta}_2 \cos(\theta_4)^2 \cos(\theta_5) \sin(\theta_5) - \\
& a_4 l_{c4,2} m_4 \dot{\theta}_2 \cos(\theta_3)^2 - a_4 l_{c4,2} m_4 \dot{\theta}_2 \cos(\theta_4)^2 - a_5 l_{c5,2} m_5 \dot{\theta}_2 \cos(\theta_3)^2 - a_5 l_{c5,2} m_5 \dot{\theta}_2 \cos(\theta_4)^2 - a_5 l_{c5,2} m_5 \dot{\theta}_2 \cos(\theta_5)^2 - \\
& 2 l_{c4,1} l_{c4,2} m_4 \dot{\theta}_2 \cos(\theta_3)^2 - 2 l_{c4,1} l_{c4,2} m_4 \dot{\theta}_2 \cos(\theta_4)^2 - 2.0 l_{c5,1} l_{c5,2} m_5 \dot{\theta}_2 \cos(\theta_3)^2 - 2.0 l_{c5,1} l_{c5,2} m_5 \dot{\theta}_2 \cos(\theta_4)^2 - \\
& 2.0 l_{c5,1} l_{c5,2} m_5 \dot{\theta}_2 \cos(\theta_5)^2 + I_{xz,5} \dot{\theta}_3 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) + I_{xz,5} \dot{\theta}_4 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) + I_{xz,5} \dot{\theta}_5 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) - \\
& I_{yz,4} \dot{\theta}_1 \cos(\theta_3) \cos(\theta_4) \sin(\theta_2) - I_{yz,5} \dot{\theta}_3 \cos(\theta_3) \cos(\theta_4) \sin(\theta_5) - I_{yz,5} \dot{\theta}_3 \cos(\theta_3) \cos(\theta_5) \sin(\theta_4) - I_{yz,5} \dot{\theta}_3 \cos(\theta_4) \cos(\theta_5) \sin(\theta_3) - \\
& I_{yz,5} \dot{\theta}_4 \cos(\theta_3) \cos(\theta_4) \sin(\theta_5) - I_{yz,5} \dot{\theta}_4 \cos(\theta_3) \cos(\theta_5) \sin(\theta_4) - I_{yz,5} \dot{\theta}_4 \cos(\theta_4) \cos(\theta_5) \sin(\theta_3) - I_{yz,5} \dot{\theta}_5 \cos(\theta_3) \cos(\theta_4) \sin(\theta_5) - \\
& I_{yz,5} \dot{\theta}_5 \cos(\theta_3) \cos(\theta_5) \sin(\theta_4) - I_{yz,5} \dot{\theta}_5 \cos(\theta_4) \cos(\theta_5) \sin(\theta_3) - 0.25 a_4^2 m_4 \dot{\theta}_2 \cos(\theta_3) \sin(\theta_3) - a_4^2 m_5 \dot{\theta}_2 \cos(\theta_3) \sin(\theta_3) - \\
& 0.25 a_4^2 m_4 \dot{\theta}_2 \cos(\theta_4) \sin(\theta_4) - 0.25 a_5^2 m_5 \dot{\theta}_2 \cos(\theta_3) \sin(\theta_3) - a_4^2 m_5 \dot{\theta}_2 \cos(\theta_4) \sin(\theta_4) - 0.25 a_5^2 m_5 \dot{\theta}_2 \cos(\theta_4) \sin(\theta_4) - \\
& 0.25 a_5^2 m_5 \dot{\theta}_2 \cos(\theta_5) \sin(\theta_5) + 8 I_{xy,5} \dot{\theta}_2 \cos(\theta_3)^2 \cos(\theta_4)^2 \cos(\theta_5)^2 - I_{xz,4} \dot{\theta}_1 \cos(\theta_3) \sin(\theta_2) \sin(\theta_4) - I_{xz,4} \dot{\theta}_1 \cos(\theta_4) \sin(\theta_2) \sin(\theta_3) - \\
& I_{xz,5} \dot{\theta}_3 \cos(\theta_3) \sin(\theta_4) \sin(\theta_5) - I_{xz,5} \dot{\theta}_3 \cos(\theta_4) \sin(\theta_3) \sin(\theta_5) - I_{xz,5} \dot{\theta}_3 \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) - I_{xz,5} \dot{\theta}_4 \cos(\theta_3) \sin(\theta_4) \sin(\theta_5) - \\
& I_{xz,5} \dot{\theta}_4 \cos(\theta_4) \sin(\theta_3) \sin(\theta_5) - I_{xz,5} \dot{\theta}_4 \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) - I_{xz,5} \dot{\theta}_5 \cos(\theta_3) \sin(\theta_4) \sin(\theta_5) - I_{xz,5} \dot{\theta}_5 \cos(\theta_4) \sin(\theta_3) \sin(\theta_5) - \\
& I_{xz,5} \dot{\theta}_5 \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) - l_{c4,1}^2 m_4 \dot{\theta}_2 \cos(\theta_3) \sin(\theta_3) + l_{c4,2}^2 m_4 \dot{\theta}_2 \cos(\theta_3) \sin(\theta_3) - l_{c4,1}^2 m_4 \dot{\theta}_2 \cos(\theta_4) \sin(\theta_4) + \\
& l_{c4,2}^2 m_4 \dot{\theta}_2 \cos(\theta_4) \sin(\theta_4) - l_{c5,1}^2 m_5 \dot{\theta}_2 \cos(\theta_3) \sin(\theta_3) + l_{c5,2}^2 m_5 \dot{\theta}_2 \cos(\theta_3) \sin(\theta_3) - l_{c5,1}^2 m_5 \dot{\theta}_2 \cos(\theta_4) \sin(\theta_4) + \\
& l_{c5,2}^2 m_5 \dot{\theta}_2 \cos(\theta_4) \sin(\theta_4) - l_{c5,1}^2 m_5 \dot{\theta}_2 \cos(\theta_5) \sin(\theta_5) + l_{c5,2}^2 m_5 \dot{\theta}_2 \cos(\theta_5) \sin(\theta_5) + I_{yz,4} \dot{\theta}_1 \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) + \\
& I_{yz,5} \dot{\theta}_3 \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) + I_{yz,5} \dot{\theta}_4 \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) + I_{yz,5} \dot{\theta}_5 \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) + 0.5 I_{xx,4} \dot{\theta}_1 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(\theta_2) - \\
& 0.5 I_{yy,4} \dot{\theta}_1 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(\theta_2) + 0.5 a_4 l_{c4,1} m_4 \dot{\theta}_1 \cos(\theta_2) - 0.5 a_3 l_{c4,2} m_4 \dot{\theta}_2 \cos(\theta_4) + 0.5 a_5 l_{c5,1} m_5 \dot{\theta}_1 \cos(\theta_2) - \\
& 0.25 a_3 a_4 m_4 \dot{\theta}_2 \sin(\theta_4) - 0.5 a_3 a_4 m_5 \dot{\theta}_2 \sin(\theta_4) - I_{xy,4} \dot{\theta}_1 \cos(2.0 \theta_3) \sin(2.0 \theta_4)$$

$$\begin{aligned}
& 2.0 \text{lc}_{5,2}^2 m_5 \dot{\theta}_2 \cos(\theta_3)^2 \cos(\theta_5) \sin(\theta_5) - 2.0 \text{lc}_{5,2}^2 m_5 \dot{\theta}_2 \cos(\theta_4)^2 \cos(\theta_5) \sin(\theta_5) + 4.0 I_{xx,5} \dot{\theta}_2 \cos(\theta_3) \cos(\theta_4)^2 \cos(\theta_5)^2 \sin(\theta_3) + \\
& 4.0 I_{xx,5} \dot{\theta}_2 \cos(\theta_3)^2 \cos(\theta_4) \cos(\theta_5)^2 \sin(\theta_4) + 4.0 I_{xx,5} \dot{\theta}_2 \cos(\theta_3)^2 \cos(\theta_4)^2 \cos(\theta_5) \sin(\theta_5) - 4.0 I_{yy,5} \dot{\theta}_2 \cos(\theta_3) \cos(\theta_4)^2 \cos(\theta_5)^2 \sin(\theta_3) - \\
& 4.0 I_{yy,5} \dot{\theta}_2 \cos(\theta_3)^2 \cos(\theta_4) \cos(\theta_5)^2 \sin(\theta_4) - 4.0 I_{yy,5} \dot{\theta}_2 \cos(\theta_3)^2 \cos(\theta_4)^2 \cos(\theta_5) \sin(\theta_5) + 0.125 a_3 a_4 m_4 \dot{\theta}_1 \cos(\theta_4) \cos(\theta_2) + \\
& 0.25 a_3 a_4 m_5 \dot{\theta}_1 \cos(\theta_4) \cos(\theta_2) - 0.5 I_{xx,5} \dot{\theta}_1 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(2.0 \theta_5) \cos(\theta_2) + 0.5 I_{yy,5} \dot{\theta}_1 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(2.0 \theta_5) \cos(\theta_2) + \\
& 0.25 a_3 \text{lc}_{4,1} m_4 \dot{\theta}_1 \cos(\theta_4) \cos(\theta_2) + 0.5 a_3 \text{lc}_{4,2} m_4 \dot{\theta}_2 \cos(2.0 \theta_3) \cos(\theta_4) - 0.5 a_4 \text{lc}_{5,2} m_5 \dot{\theta}_1 \cos(\theta_5) \cos(\theta_2) + \\
& 0.25 a_3 a_4 m_4 \dot{\theta}_2 \cos(2.0 \theta_3) \sin(\theta_4) + 0.25 a_3 a_4 m_4 \dot{\theta}_2 \sin(2.0 \theta_3) \cos(\theta_4) + 0.5 a_3 a_4 m_5 \dot{\theta}_2 \cos(2.0 \theta_3) \sin(\theta_4) + \\
& 0.5 a_3 a_4 m_5 \dot{\theta}_2 \sin(2.0 \theta_3) \cos(\theta_4) + 0.25 a_4 a_5 m_5 \dot{\theta}_1 \cos(\theta_5) \sin(\theta_2) - 0.25 a_4 a_5 m_5 \dot{\theta}_1 \sin(\theta_5) \cos(\theta_2) + I_{xy,5} \dot{\theta}_1 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \sin(2.0 \theta_5) \cos(\theta_2) + \\
& I_{xy,5} \dot{\theta}_1 \cos(2.0 \theta_3) \cos(2.0 \theta_5) \sin(2.0 \theta_4) \cos(\theta_2) + I_{xy,5} \dot{\theta}_1 \cos(2.0 \theta_4) \cos(2.0 \theta_5) \sin(2.0 \theta_3) \cos(\theta_2) + 0.25 a_3 \text{lc}_{4,2} m_4 \dot{\theta}_1 \cos(\theta_4) \sin(\theta_2) - \\
& 0.25 a_3 \text{lc}_{4,2} m_4 \dot{\theta}_1 \sin(\theta_4) \cos(\theta_2) + 0.5 a_3 \text{lc}_{4,1} m_4 \dot{\theta}_2 \cos(2.0 \theta_3) \sin(\theta_4) + 0.5 a_3 \text{lc}_{4,1} m_4 \dot{\theta}_2 \sin(2.0 \theta_3) \cos(\theta_4) + \\
& 0.5 a_4 \text{lc}_{5,1} m_5 \dot{\theta}_1 \cos(\theta_5) \sin(\theta_2) - 0.5 a_4 \text{lc}_{5,1} m_5 \dot{\theta}_1 \sin(\theta_5) \cos(\theta_2) + 0.125 a_3 a_4 m_4 \dot{\theta}_1 \sin(\theta_4) \sin(\theta_2) + 0.25 a_3 a_4 m_5 \dot{\theta}_1 \sin(\theta_4) \sin(\theta_2) + \\
& 0.5 I_{xx,5} \dot{\theta}_1 \cos(2.0 \theta_3) \sin(2.0 \theta_4) \sin(2.0 \theta_5) \cos(\theta_2) + 0.5 I_{xx,5} \dot{\theta}_1 \cos(2.0 \theta_4) \sin(2.0 \theta_3) \sin(2.0 \theta_5) \cos(\theta_2) + \\
& 0.5 I_{xx,5} \dot{\theta}_1 \cos(2.0 \theta_5) \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) - 0.5 I_{yy,5} \dot{\theta}_1 \cos(2.0 \theta_3) \sin(2.0 \theta_4) \sin(2.0 \theta_5) \cos(\theta_2) - \\
& 0.5 I_{yy,5} \dot{\theta}_1 \cos(2.0 \theta_4) \sin(2.0 \theta_3) \sin(2.0 \theta_5) \cos(\theta_2) - 0.5 I_{yy,5} \dot{\theta}_1 \cos(2.0 \theta_5) \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) + \\
& 2.0 a_4 \text{lc}_{4,2} m_4 \dot{\theta}_2 \cos(\theta_3)^2 \cos(\theta_4)^2 + 2.0 a_5 \text{lc}_{5,2} m_5 \dot{\theta}_2 \cos(\theta_3)^2 \cos(\theta_4)^2 + 2.0 a_5 \text{lc}_{5,2} m_5 \dot{\theta}_2 \cos(\theta_3)^2 \cos(\theta_5)^2 + \\
& 2.0 a_5 \text{lc}_{5,2} m_5 \dot{\theta}_2 \cos(\theta_4)^2 \cos(\theta_5)^2 + 0.25 a_3 \text{lc}_{4,1} m_4 \dot{\theta}_1 \sin(\theta_4) \sin(\theta_2) - 0.5 a_3 \text{lc}_{4,2} m_4 \dot{\theta}_2 \sin(2.0 \theta_3) \sin(\theta_4) - \\
& 0.5 a_4 \text{lc}_{5,2} m_5 \dot{\theta}_1 \sin(\theta_5) \sin(\theta_2) + 4 \text{lc}_{4,1} \text{lc}_{4,2} m_4 \dot{\theta}_2 \cos(\theta_3)^2 \cos(\theta_4)^2 + 4.0 \text{lc}_{5,1} \text{lc}_{5,2} m_5 \dot{\theta}_2 \cos(\theta_3)^2 \cos(\theta_4)^2 + \\
& 4.0 \text{lc}_{5,1} \text{lc}_{5,2} m_5 \dot{\theta}_2 \cos(\theta_3)^2 \cos(\theta_5)^2 + 4.0 \text{lc}_{5,1} \text{lc}_{5,2} m_5 \dot{\theta}_2 \cos(\theta_4)^2 \cos(\theta_5)^2 - I_{xy,5} \dot{\theta}_1 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \sin(2.0 \theta_5) \cos(\theta_2) - \\
& I_{xz,5} \dot{\theta}_1 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) \sin(\theta_2) + 4.0 I_{xy,4} \dot{\theta}_2 \cos(\theta_3) \cos(\theta_4) \sin(\theta_3) \sin(\theta_4) + 4 I_{xy,5} \dot{\theta}_2 \cos(\theta_3) \cos(\theta_4) \sin(\theta_3) \sin(\theta_4) + \\
& 4 I_{xy,5} \dot{\theta}_2 \cos(\theta_3) \cos(\theta_5) \sin(\theta_3) \sin(\theta_5) + 4 I_{xy,5} \dot{\theta}_2 \cos(\theta_4) \cos(\theta_5) \sin(\theta_4) \sin(\theta_5) + I_{yz,5} \dot{\theta}_1 \cos(\theta_3) \cos(\theta_4) \sin(\theta_2) \sin(\theta_5) + \\
& I_{yz,5} \dot{\theta}_1 \cos(\theta_3) \cos(\theta_5) \sin(\theta_2) \sin(\theta_4) + I_{yz,5} \dot{\theta}_1 \cos(\theta_4) \cos(\theta_5) \sin(\theta_2) \sin(\theta_3) + I_{xz,5} \dot{\theta}_1 \cos(\theta_3) \sin(\theta_2) \sin(\theta_4) \sin(\theta_5) + \\
& I_{xz,5} \dot{\theta}_1 \cos(\theta_4) \sin(\theta_2) \sin(\theta_3) \sin(\theta_5) + I_{xz,5} \dot{\theta}_1 \cos(\theta_5) \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) + 0.125 a_3 a_4 m_4 \dot{\theta}_1 \cos(\theta_2) \cos(\theta_4) + \\
& 0.5 a_2 a_4 m_4 \dot{\theta}_2 \cos(\theta_3) \cos(\theta_4) + 0.25 a_3 a_4 m_5 \dot{\theta}_1 \cos(\theta_2) \cos(\theta_4) + a_2 a_4 m_5 \dot{\theta}_2 \cos(\theta_3) \cos(\theta_4) - 0.25 a_3 a_5 m_5 \dot{\theta}_2 \cos(\theta_4) \cos(\theta_5) - \\
& I_{yz,5} \dot{\theta}_1 \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) + 0.25 a_3 \text{lc}_{4,1} m_4 \dot{\theta}_1 \cos(\theta_2) \cos(\theta_4) + a_2 \text{lc}_{4,1} m_4 \dot{\theta}_2 \cos(\theta_3) \cos(\theta_4) - \\
& 0.5 a_4 \text{lc}_{5,2} m_5 \dot{\theta}_1 \cos(\theta_2) \cos(\theta_5) - 0.5 a_3 \text{lc}_{5,1} m_5 \dot{\theta}_2 \cos(\theta_4) \cos(\theta_5) - 0.25 a_4 a_5 m_5 \dot{\theta}_1 \cos(\theta_2) \sin(\theta_5) - 0.25 a_4 a_5 m_5 \dot{\theta}_1 \cos(\theta_5) \sin(\theta_2) - \\
& 0.125 a_4^2 m_4 \dot{\theta}_1 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(\theta_2) - 0.5 a_4^2 m_5 \dot{\theta}_1 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(\theta_2) - \text{lc}_{4,2} \text{lc}_{4,3} m_4 \dot{\theta}_3 \cos(\theta_3) \cos(\theta_4) - \\
& \text{lc}_{4,2} \text{lc}_{4,3} m_4 \dot{\theta}_4 \cos(\theta_3) \cos(\theta_4) - 0.25 a_3 \text{lc}_{4,2} m_4 \dot{\theta}_1 \cos(\theta_2) \sin(\theta_4) - 0.25 a_3 \text{lc}_{4,2} m_4 \dot{\theta}_1 \cos(\theta_4) \sin(\theta_2) - a_2 \text{lc}_{4,2} m_4 \dot{\theta}_2 \cos(\theta_3) \sin(\theta_4) - \\
& a_2 \text{lc}_{4,2} m_4 \dot{\theta}_2 \cos(\theta_4) \sin(\theta_3) - a_4 \text{lc}_{4,1} m_4 \dot{\theta}_2 \cos(\theta_3) \sin(\theta_3) - a_4 \text{lc}_{4,1} m_4 \dot{\theta}_2 \cos(\theta_4) \sin(\theta_4) - 0.5 a_4 \text{lc}_{4,3} m_4 \dot{\theta}_3 \cos(\theta_3) \sin(\theta_4) - \\
& 0.5 a_4 \text{lc}_{4,3} m_4 \dot{\theta}_3 \cos(\theta_4) \sin(\theta_3) - 0.5 a_4 \text{lc}_{4,3} m_4 \dot{\theta}_4 \cos(\theta_3) \sin(\theta_4) - 0.5 a_4 \text{lc}_{4,3} m_4 \dot{\theta}_4 \cos(\theta_4) \sin(\theta_3) - 0.5 a_4 \text{lc}_{5,1} m_5 \dot{\theta}_1 \cos(\theta_2) \sin(\theta_5) - \\
& 0.5 a_4 \text{lc}_{5,1} m_5 \dot{\theta}_1 \cos(\theta_5) \sin(\theta_2) - a_5 \text{lc}_{5,1} m_5 \dot{\theta}_2 \cos(\theta_3) \sin(\theta_3) + 0.5 a_3 \text{lc}_{5,2} m_5 \dot{\theta}_2 \cos(\theta_4) \sin(\theta_5) + 0.5 a_3 \text{lc}_{5,2} m_5 \dot{\theta}_2 \cos(\theta_5) \sin(\theta_4) - \\
& a_5 \text{lc}_{5,1} m_5 \dot{\theta}_2 \cos(\theta_4) \sin(\theta_4$$

$$\begin{aligned}
& a_2 \text{lc}_{5,1} m_5 \dot{\theta}_1 \cos(\theta_2) \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) - 2.0 a_5 \text{lc}_{5,2} m_5 \dot{\theta}_2 \cos(\theta_3) \cos(\theta_4) \sin(\theta_3) \sin(\theta_4) - 2.0 a_5 \text{lc}_{5,2} m_5 \dot{\theta}_2 \cos(\theta_3) \cos(\theta_5) \sin(\theta_3) \sin(\theta_5) - \\
& 2.0 a_5 \text{lc}_{5,2} m_5 \dot{\theta}_2 \cos(\theta_4) \cos(\theta_5) \sin(\theta_4) \sin(\theta_5) - 4 \text{lc}_{4,1} \text{lc}_{4,2} m_4 \dot{\theta}_2 \cos(\theta_3) \cos(\theta_4) \sin(\theta_3) \sin(\theta_4) - 4.0 \text{lc}_{5,1} \text{lc}_{5,2} m_5 \dot{\theta}_2 \cos(\theta_3) \cos(\theta_4) \sin(\theta_3) \sin(\theta_4) - \\
& \text{lc}_{5,2} \text{lc}_{5,3} m_5 \dot{\theta}_1 \cos(\theta_3) \cos(\theta_4) \sin(\theta_2) \sin(\theta_5) - \text{lc}_{5,2} \text{lc}_{5,3} m_5 \dot{\theta}_1 \cos(\theta_3) \cos(\theta_5) \sin(\theta_2) \sin(\theta_4) - \text{lc}_{5,2} \text{lc}_{5,3} m_5 \dot{\theta}_1 \cos(\theta_4) \cos(\theta_5) \sin(\theta_2) \sin(\theta_3) - \\
& 4.0 \text{lc}_{5,1} \text{lc}_{5,2} m_5 \dot{\theta}_2 \cos(\theta_3) \cos(\theta_5) \sin(\theta_3) \sin(\theta_5) - 4.0 \text{lc}_{5,1} \text{lc}_{5,2} m_5 \dot{\theta}_2 \cos(\theta_4) \cos(\theta_5) \sin(\theta_4) \sin(\theta_5) + a_2 \text{lc}_{5,2} m_5 \dot{\theta}_1 \cos(\theta_2) \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) - \\
& 0.5 a_5 \text{lc}_{5,3} m_5 \dot{\theta}_1 \cos(\theta_3) \sin(\theta_2) \sin(\theta_4) \sin(\theta_5) - 0.5 a_5 \text{lc}_{5,3} m_5 \dot{\theta}_1 \cos(\theta_4) \sin(\theta_2) \sin(\theta_3) \sin(\theta_5) - 0.5 a_5 \text{lc}_{5,3} m_5 \dot{\theta}_1 \cos(\theta_5) \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) - \\
& \text{lc}_{5,1} \text{lc}_{5,3} m_5 \dot{\theta}_1 \cos(\theta_3) \sin(\theta_2) \sin(\theta_4) \sin(\theta_5) - \text{lc}_{5,1} \text{lc}_{5,3} m_5 \dot{\theta}_1 \cos(\theta_4) \sin(\theta_2) \sin(\theta_3) \sin(\theta_5) - \text{lc}_{5,1} \text{lc}_{5,3} m_5 \dot{\theta}_1 \cos(\theta_5) \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) + \\
& a_4 \text{lc}_{5,2} m_5 \dot{\theta}_1 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(\theta_2) \cos(\theta_5) + 0.5 a_4 a_5 m_5 \dot{\theta}_1 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(\theta_2) \sin(\theta_5) + \\
& 0.5 a_4 a_5 m_5 \dot{\theta}_1 \cos(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) \cos(\theta_5) + 0.5 a_4 a_5 m_5 \dot{\theta}_1 \cos(2.0 \theta_4) \sin(2.0 \theta_3) \cos(\theta_2) \cos(\theta_5) + \\
& \text{lc}_{5,2} \text{lc}_{5,3} m_5 \dot{\theta}_1 \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) + a_4 \text{lc}_{5,1} m_5 \dot{\theta}_1 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(\theta_2) \sin(\theta_5) + a_4 \text{lc}_{5,1} m_5 \dot{\theta}_1 \cos(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) \cos(\theta_5) + \\
& a_4 \text{lc}_{5,1} m_5 \dot{\theta}_1 \cos(2.0 \theta_4) \sin(2.0 \theta_3) \cos(\theta_2) \cos(\theta_5) - a_4 \text{lc}_{5,2} m_5 \dot{\theta}_1 \cos(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) \sin(\theta_5) - \\
& a_4 \text{lc}_{5,2} m_5 \dot{\theta}_1 \cos(2.0 \theta_4) \sin(2.0 \theta_3) \cos(\theta_2) \sin(\theta_5) - a_4 \text{lc}_{5,2} m_5 \dot{\theta}_1 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) \cos(\theta_5) - \\
& 0.5 a_4 a_5 m_5 \dot{\theta}_1 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) \sin(\theta_5) - a_4 \text{lc}_{5,1} m_5 \dot{\theta}_1 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) \sin(\theta_5) + \\
& a_5^2 m_5 \dot{\theta}_2 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) + 4.0 \text{lc}_{5,1}^2 m_5 \dot{\theta}_2 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) - \\
& 4.0 \text{lc}_{5,2}^2 m_5 \dot{\theta}_2 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) + 4.0 a_5 \text{lc}_{5,2} m_5 \dot{\theta}_2 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5)^2 \sin(\theta_3) \sin(\theta_4) + \\
& 4.0 a_5 \text{lc}_{5,2} m_5 \dot{\theta}_2 \cos(\theta_3) \cos(\theta_4)^2 \cos(\theta_5) \sin(\theta_3) \sin(\theta_5) + 4.0 a_5 \text{lc}_{5,2} m_5 \dot{\theta}_2 \cos(\theta_3)^2 \cos(\theta_4) \cos(\theta_5) \sin(\theta_4) \sin(\theta_5) + \\
& 8.0 \text{lc}_{5,1} \text{lc}_{5,2} m_5 \dot{\theta}_2 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5)^2 \sin(\theta_3) \sin(\theta_4) + 8.0 \text{lc}_{5,1} \text{lc}_{5,2} m_5 \dot{\theta}_2 \cos(\theta_3) \cos(\theta_4)^2 \cos(\theta_5) \sin(\theta_3) \sin(\theta_5) + \\
& 8.0 \text{lc}_{5,1} \text{lc}_{5,2} m_5 \dot{\theta}_2 \cos(\theta_3)^2 \cos(\theta_4) \cos(\theta_5) \sin(\theta_4) \sin(\theta_5) + 4.0 a_5 \text{lc}_{5,1} m_5 \dot{\theta}_2 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) \sin(\theta_5)
\end{aligned} \tag{A36}$$

[illegible]

[illegible]

$$\begin{aligned}
& 0.5 a_4 \text{lc}_{5,1} m_5 \dot{\theta}_1 \cos(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) \cos(\theta_5) + 0.5 a_4 \text{lc}_{5,1} m_5 \dot{\theta}_1 \cos(2.0 \theta_4) \sin(2.0 \theta_3) \cos(\theta_2) \cos(\theta_5) - \\
& 0.5 a_4 \text{lc}_{5,2} m_5 \dot{\theta}_1 \cos(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) \sin(\theta_5) - 0.5 a_4 \text{lc}_{5,2} m_5 \dot{\theta}_1 \cos(2.0 \theta_4) \sin(2.0 \theta_3) \cos(\theta_2) \sin(\theta_5) - \\
& 0.5 a_4 \text{lc}_{5,2} m_5 \dot{\theta}_1 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) \cos(\theta_5) - 0.25 a_4 a_5 m_5 \dot{\theta}_1 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) \sin(\theta_5) - \\
& 0.5 a_4 \text{lc}_{5,1} m_5 \dot{\theta}_1 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) \sin(\theta_5) + a_5^2 m_5 \dot{\theta}_2 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) + \\
& 4.0 \text{lc}_{5,1}^2 m_5 \dot{\theta}_2 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) - 4.0 \text{lc}_{5,2}^2 m_5 \dot{\theta}_2 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) + \\
& 4.0 a_5 \text{lc}_{5,2} m_5 \dot{\theta}_2 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5)^2 \sin(\theta_3) \sin(\theta_4) + 4.0 a_5 \text{lc}_{5,2} m_5 \dot{\theta}_2 \cos(\theta_3) \cos(\theta_4)^2 \cos(\theta_5) \sin(\theta_3) \sin(\theta_5) + \\
& 4.0 a_5 \text{lc}_{5,2} m_5 \dot{\theta}_2 \cos(\theta_3)^2 \cos(\theta_4) \cos(\theta_5) \sin(\theta_4) \sin(\theta_5) + 8.0 \text{lc}_{5,1} \text{lc}_{5,2} m_5 \dot{\theta}_2 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5)^2 \sin(\theta_3) \sin(\theta_4) + \\
& 8.0 \text{lc}_{5,1} \text{lc}_{5,2} m_5 \dot{\theta}_2 \cos(\theta_3) \cos(\theta_4)^2 \cos(\theta_5) \sin(\theta_3) \sin(\theta_5) + 8.0 \text{lc}_{5,1} \text{lc}_{5,2} m_5 \dot{\theta}_2 \cos(\theta_3)^2 \cos(\theta_4) \cos(\theta_5) \sin(\theta_4) \sin(\theta_5) + \\
& 4.0 a_5 \text{lc}_{5,1} m_5 \dot{\theta}_2 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) \sin(\theta_5)
\end{aligned} \tag{A37}$$

$$\begin{aligned}
C_{31} = & -0.5 I_{xy,3} \dot{\theta}_1 \cos(2.0 \theta_3) - 0.25 I_{xx,3} \dot{\theta}_1 \sin(2.0 \theta_3) + 0.25 I_{yy,3} \dot{\theta}_1 \sin(2.0 \theta_3) - 0.5 I_{zz,3} \dot{\theta}_2 \cos(\theta_2) - \\
& 0.5 I_{zz,4} \dot{\theta}_2 \cos(\theta_2) - 0.5 I_{zz,5} \dot{\theta}_2 \cos(\theta_2) - 0.5 I_{xy,3} \dot{\theta}_1 \cos(2.0 \theta_2) \cos(2.0 \theta_3) - 0.5 I_{xy,4} \dot{\theta}_1 \cos(2.0 \theta_3) \cos(2.0 \theta_4) - \\
& 0.25 I_{xx,3} \dot{\theta}_1 \cos(2.0 \theta_2) \sin(2.0 \theta_3) - 0.25 I_{xx,4} \dot{\theta}_1 \cos(2.0 \theta_3) \sin(2.0 \theta_4) - 0.25 I_{xx,4} \dot{\theta}_1 \cos(2.0 \theta_4) \sin(2.0 \theta_3) + \\
& 0.25 I_{yy,3} \dot{\theta}_1 \cos(2.0 \theta_2) \sin(2.0 \theta_3) + 0.25 I_{yy,4} \dot{\theta}_1 \cos(2.0 \theta_3) \sin(2.0 \theta_4) + 0.25 I_{yy,4} \dot{\theta}_1 \cos(2.0 \theta_4) \sin(2.0 \theta_3) + \\
& 0.5 I_{xy,4} \dot{\theta}_1 \sin(2.0 \theta_3) \sin(2.0 \theta_4) - 0.5 I_{xx,3} \dot{\theta}_2 \cos(2.0 \theta_3) \cos(\theta_2) + 0.5 I_{yy,3} \dot{\theta}_2 \cos(2.0 \theta_3) \cos(\theta_2) + 1.0 I_{xy,3} \dot{\theta}_2 \sin(2.0 \theta_3) \cos(\theta_2) - \\
& 0.5 I_{xz,3} \dot{\theta}_1 \sin(2.0 \theta_2) \cos(\theta_3) + 0.0625 a_3^2 m_3 \dot{\theta}_1 \sin(2.0 \theta_3) + 0.25 a_3^2 m_4 \dot{\theta}_1 \sin(2.0 \theta_3) + 0.25 a_3^2 m_5 \dot{\theta}_1 \sin(2.0 \theta_3) + \\
& 0.5 I_{yz,3} \dot{\theta}_1 \sin(2.0 \theta_2) \sin(\theta_3) + 0.25 l_{c3,1}^2 m_3 \dot{\theta}_1 \sin(2.0 \theta_3) - 0.25 l_{c3,2}^2 m_3 \dot{\theta}_1 \sin(2.0 \theta_3) - 0.125 a_3^2 m_3 \dot{\theta}_2 \cos(\theta_2) - \\
& 0.5 a_3^2 m_4 \dot{\theta}_2 \cos(\theta_2) - 0.5 a_3^2 m_5 \dot{\theta}_2 \cos(\theta_2) - 0.125 a_4^2 m_4 \dot{\theta}_2 \cos(\theta_2) - 0.5 a_4^2 m_5 \dot{\theta}_2 \cos(\theta_2) - 0.125 a_5^2 m_5 \dot{\theta}_2 \cos(\theta_2) + \\
& 1.0 I_{yz,3} \dot{\theta}_2 \cos(\theta_3) \sin(\theta_2) - 0.5 l_{c3,1}^2 m_3 \dot{\theta}_2 \cos(\theta_2) - 0.5 l_{c3,2}^2 m_3 \dot{\theta}_2 \cos(\theta_2) - 0.5 l_{c4,1}^2 m_4 \dot{\theta}_2 \cos(\theta_2) - 0.5 l_{c4,2}^2 m_4 \dot{\theta}_2 \cos(\theta_2) - \\
& 0.5 l_{c5,1}^2 m_5 \dot{\theta}_2 \cos(\theta_2) - 0.5 l_{c5,2}^2 m_5 \dot{\theta}_2 \cos(\theta_2) + 1.0 I_{xz,3} \dot{\theta}_2 \sin(\theta_2) \sin(\theta_3) + 0.5 I_{xx,4} \dot{\theta}_2 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) - \\
& 0.5 I_{yy,4} \dot{\theta}_2 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) + 0.25 l_{c3,1}^2 m_3 \dot{\theta}_1 \cos(2.0 \theta_2) \sin(2.0 \theta_3) - 0.25 l_{c3,2}^2 m_3 \dot{\theta}_1 \cos(2.0 \theta_2) \sin(2.0 \theta_3) + \\
& 0.25 l_{c4,1}^2 m_4 \dot{\theta}_1 \cos(2.0 \theta_3) \sin(2.0 \theta_4) + 0.25 l_{c4,1}^2 m_4 \dot{\theta}_1 \cos(2.0 \theta_4) \sin(2.0 \theta_3) - 0.25 l_{c4,2}^2 m_4 \dot{\theta}_1 \cos(2.0 \theta_3) \sin(2.0 \theta_4) - \\
& 0.25 l_{c4,2}^2 m_4 \dot{\theta}_1 \cos(2.0 \theta_4) \sin(2.0 \theta_3) + 0.125 a_3^2 m_3 \dot{\theta}_2 \cos(2.0 \theta_3) \cos(\theta_2) + 0.5 a_3^2 m_4 \dot{\theta}_2 \cos(2.0 \theta_3) \cos(\theta_2) + \\
& 0.5 a_3^2 m_5 \dot{\theta}_2 \cos(2.0 \theta_3) \cos(\theta_2) - 0.5 I_{xz,4} \dot{\theta}_1 \sin(2.0 \theta_2) \cos(\theta_3) \cos(\theta_4) + 0.5 l_{c3,1}^2 m_3 \dot{\theta}_2 \cos(2.0 \theta_3) \cos(\theta_2) - \\
& 0.5 l_{c3,2}^2 m_3 \dot{\theta}_2 \cos(2.0 \theta_3) \cos(\theta_2) + 0.5 I_{yz,4} \dot{\theta}_1 \sin(2.0 \theta_2) \cos(\theta_3) \sin(\theta_4) + 0.5 I_{yz,4} \dot{\theta}_1 \sin(2.0 \theta_2) \cos(\theta_4) \sin(\theta_3) + \\
& 0.5 I_{xz,4} \dot{\theta}_1 \sin(2.0 \theta_2) \sin(\theta_3) \sin(\theta_4) - 0.5 I_{xy,4} \dot{\theta}_1 \cos(2.0 \theta_2) \cos(2.0 \theta_3) \cos(2.0 \theta_4) + 0.5 I_{xy,5} \dot{\theta}_1 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(2.0 \theta_5) + \\
& 0.25 a_3 l_{c3,2} m_3 \dot{\theta}_1 \cos(2.0 \theta_3) + 0.5 l_{c3,1} l_{c3,2} m_3 \dot{\theta}_1 \cos(2.0 \theta_2) \cos(2.0 \theta_3) \sin(2.0 \theta_4) - \\
& 0.25 I_{xx,4} \dot{\theta}_1 \cos(2.0 \theta_2) \cos(2.0 \theta_4) \sin(2.0 \theta_3) + 0.25 I_{xx,5} \dot{\theta}_1 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \sin(2.0 \theta_5) + 0.25 I_{xx,5} \dot{\theta}_1 \cos(2.0 \theta_3) \cos(2.0 \theta_5) \sin(2.0 \theta_4) + \\
& 0.25 I_{xx,5} \dot{\theta}_1 \cos(2.0 \theta_4) \cos(2.0 \theta_5) \sin(2.0 \theta_3) + 0.25 I_{yy,4} \dot{\theta}_1 \cos(2.0 \theta_2) \cos(2.0 \theta_3) \sin(2.0 \theta_4) + 0.25 I_{yy,4} \dot{\theta}_1 \cos(2.0 \theta_2) \cos(2.0 \theta_4) \sin(2.0 \theta_3) - \\
& 0.25 I_{yy,5} \dot{\theta}_1 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \sin(2.0 \theta_5) - 0.25 I_{yy,5} \dot{\theta}_1 \cos(2.0 \theta_3) \cos(2.0 \theta_5) \sin(2.0 \theta_4) - 0.25 I_{yy,5} \dot{\theta}_1 \cos(2.0 \theta_4) \cos(2.0 \theta_5) \sin(2.0 \theta_3) + \\
& 0.25 a_3 l_{c3,1} m_3 \dot{\theta}_1 \sin(2.0 \theta_3) + 0.5 I_{xy,4} \dot{\theta}_1 \cos(2.0 \theta_2) \sin(2.0 \theta_3) \sin(2.0 \theta_4) - 0.5 I_{xy,5} \dot{\theta}_1 \cos(2.0 \theta_3) \sin(2.0 \theta_4) \sin(2.0 \theta_5) - \\
& 0.5 I_{xy,5} \dot{\theta}_1 \cos(2.0 \theta_4) \sin(2.0 \theta_3) \sin(2.0 \theta_5) - 0.5 I_{xy,5} \dot{\theta}_1 \cos(2.0 \theta_5) \sin(2.0 \theta_3) \sin(2.0 \theta_4) - 0.25 I_{xx,5} \dot{\theta}_1 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \sin(2.0 \theta_5) + \\
& 0.25 I_{yy,5} \dot{\theta}_1 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \sin(2.0 \theta_5) + 1.0 I_{yz,4} \dot{\theta}_2 \cos(\theta_3) \cos(\theta_4) \sin(\theta_2) + 1.0 I_{xz,4} \dot{\theta}_2 \cos(\theta_3) \sin(\theta_2) \sin(\theta_4) + \\
& 1.0 I_{xz,4} \dot{\theta}_2 \cos(\theta_4) \sin(\theta_2) \sin(\theta_3) - 1.0 I_{yz,4} \dot{\theta}_2 \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) - 0.25 a_2 a_3 m_3 \dot{\theta}_1 \cos(\theta_3) - 0.5 a_2 a_3 m_4 \dot{\theta}_1 \cos(\theta_3) - \\
& 0.5 a_2 a_3 m_5 \dot{\theta}_1 \cos(\theta_3) - 0.5 I_{xx,4} \dot{\theta}_2 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(\theta_2) + 0.5 I_{yy,4} \dot{\theta}_2 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(\theta_2) - \\
& 0.5 a_2 l_{c3,1} m_3 \dot{\theta}_1 \cos(\theta_3) - 0.5 a_3 l_{c3,1} m_3 \dot{\theta}_2 \cos(\theta_2) - 0.5 a_4 l_{c4,1} m_4 \dot{\theta}_2 \cos(\theta_2) - 0.5 a_5 l_{c5,1} m_5 \dot{\theta}_2 \cos(\theta_2) + \\
& 1.0 I_{xy,4} \dot{\theta}_2 \cos(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) + 1.0 I_{xy,4} \dot{\theta}_2 \cos(2.0 \theta_4) \sin(2.0 \theta_3) \cos(\theta_2) + 0.5 a_2 l_{c3,2} m_3 \dot{\theta}_1 \sin(\theta_3) + \\
& 0.0625 a_3^2 m_3 \dot{\theta}_1 \cos(2.0 \theta_2) \sin(2.0 \theta_3) + 0.25 a_3^2 m_4 \dot{\theta}_1 \cos(2.0 \theta_2) \sin(2.0 \theta_3) + 0.25 a_3^2 m_5 \dot{\theta}_1 \cos(2.0 \theta_2) \sin(2.0 \theta_3) + \\
& 0.0625 a_4^2 m_4 \dot{\theta}_1 \cos(2.0 \theta_3) \sin(2.0 \theta_4) + 0.0625 a_4^2 m_4 \dot{\theta}_1 \cos(2.0 \theta_4) \sin(2.0 \theta_3) + 0.25 a_4^2 m_5 \dot{\theta}_1 \cos(2.0 \theta_3) \sin(2.0 \theta_4) + \\
& 0.25 a_4^2 m_5 \dot{\theta}_1 \cos(2.0 \theta_4) \sin(2.0 \theta_3) + 0.5 l_{c3,1} l_{c3,2} m_3 \dot{\theta}_1 \cos(2.0 \theta_2) \cos(2.0 \theta_3) + 0.5 l_{c4,1} l_{c4,2} m_4 \dot{\theta}_1 \cos(2.0 \theta_3) \cos(2.0 \theta_4) + \\$$

$$\begin{aligned}
& 0.5 a_3 \text{lc}_{5,2} m_5 \dot{\theta}_1 \cos(2.0 \theta_2) \cos(2.0 \theta_3) \cos(\theta_4) \sin(\theta_5) - 0.5 a_3 \text{lc}_{5,2} m_5 \dot{\theta}_1 \cos(2.0 \theta_2) \cos(2.0 \theta_3) \cos(\theta_5) \sin(\theta_4) - \\
& 0.5 a_3 \text{lc}_{5,2} m_5 \dot{\theta}_1 \cos(2.0 \theta_2) \sin(2.0 \theta_3) \cos(\theta_4) \cos(\theta_5) - 1.0 a_4 \text{lc}_{5,1} m_5 \dot{\theta}_2 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(\theta_2) \sin(\theta_5) - \\
& 1.0 a_4 \text{lc}_{5,1} m_5 \dot{\theta}_2 \cos(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) \cos(\theta_5) - 1.0 a_4 \text{lc}_{5,1} m_5 \dot{\theta}_2 \cos(2.0 \theta_4) \sin(2.0 \theta_3) \cos(\theta_2) \cos(\theta_5) - \\
& 0.25 a_3 a_5 m_5 \dot{\theta}_1 \cos(2.0 \theta_2) \cos(2.0 \theta_3) \sin(\theta_4) \sin(\theta_5) - 0.25 a_3 a_5 m_5 \dot{\theta}_1 \cos(2.0 \theta_2) \sin(2.0 \theta_3) \cos(\theta_4) \sin(\theta_5) - \\
& 0.25 a_3 a_5 m_5 \dot{\theta}_1 \cos(2.0 \theta_2) \sin(2.0 \theta_3) \cos(\theta_5) \sin(\theta_4) - 0.5 a_3 \text{lc}_{5,1} m_5 \dot{\theta}_1 \cos(2.0 \theta_2) \cos(2.0 \theta_3) \sin(\theta_4) \sin(\theta_5) - \\
& 0.5 a_3 \text{lc}_{5,1} m_5 \dot{\theta}_1 \cos(2.0 \theta_2) \sin(2.0 \theta_3) \cos(\theta_4) \sin(\theta_5) - 0.5 a_3 \text{lc}_{5,1} m_5 \dot{\theta}_1 \cos(2.0 \theta_2) \sin(2.0 \theta_3) \cos(\theta_5) \sin(\theta_4) + \\
& 1.0 a_4 \text{lc}_{5,2} m_5 \dot{\theta}_2 \cos(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) \sin(\theta_5) + 1.0 a_4 \text{lc}_{5,2} m_5 \dot{\theta}_2 \cos(2.0 \theta_4) \sin(2.0 \theta_3) \cos(\theta_2) \sin(\theta_5) + \\
& 1.0 a_4 \text{lc}_{5,2} m_5 \dot{\theta}_2 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) \cos(\theta_5) + 0.5 a_4 a_5 m_5 \dot{\theta}_2 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) \sin(\theta_5) + \\
& 0.5 a_3 \text{lc}_{5,2} m_5 \dot{\theta}_1 \cos(2.0 \theta_2) \sin(2.0 \theta_3) \sin(\theta_4) \sin(\theta_5) + 1.0 a_4 \text{lc}_{5,1} m_5 \dot{\theta}_2 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) \sin(\theta_5) - \\
& 0.25 a_5 \text{lc}_{5,2} m_5 \dot{\theta}_1 \cos(2.0 \theta_2) \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(2.0 \theta_5) - 0.5 \text{lc}_{5,1} \text{lc}_{5,2} m_5 \dot{\theta}_1 \cos(2.0 \theta_2) \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(2.0 \theta_5) - \\
& 0.25 a_5 \text{lc}_{5,1} m_5 \dot{\theta}_1 \cos(2.0 \theta_2) \cos(2.0 \theta_3) \cos(2.0 \theta_4) \sin(2.0 \theta_5) - 0.25 a_5 \text{lc}_{5,1} m_5 \dot{\theta}_1 \cos(2.0 \theta_2) \cos(2.0 \theta_3) \cos(2.0 \theta_5) \sin(2.0 \theta_4) - \\
& 0.25 a_5 \text{lc}_{5,1} m_5 \dot{\theta}_1 \cos(2.0 \theta_2) \cos(2.0 \theta_4) \cos(2.0 \theta_5) \sin(2.0 \theta_3)
\end{aligned} \tag{A38}$$

$$\begin{aligned}
& 1.0 I_{yz,5} \dot{\theta}_1 \cos(\theta_3) \cos(\theta_5) \sin(\theta_2) \sin(\theta_4) - 1.0 I_{yz,5} \dot{\theta}_1 \cos(\theta_4) \cos(\theta_5) \sin(\theta_2) \sin(\theta_3) - 1.0 I_{xz,5} \dot{\theta}_1 \cos(\theta_4) \sin(\theta_2) \sin(\theta_3) \sin(\theta_5) + \\
& 0.5 l c_{5,2}^2 m_5 \dot{\theta}_2 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \sin(2.0 \theta_5) - 1.0 I_{xz,5} \dot{\theta}_1 \cos(\theta_3) \sin(\theta_2) \sin(\theta_4) \sin(\theta_5) - 1.0 I_{xz,5} \dot{\theta}_1 \cos(\theta_4) \sin(\theta_2) \sin(\theta_3) \sin(\theta_5) - \\
& 1.0 I_{xz,5} \dot{\theta}_1 \cos(\theta_5) \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) - 0.5 a_3 a_4 m_4 \dot{\theta}_1 \cos(\theta_2) \cos(\theta_4) - 0.5 a_2 a_4 m_4 \dot{\theta}_2 \cos(\theta_3) \cos(\theta_4) - \\
& 1.0 a_3 a_4 m_5 \dot{\theta}_1 \cos(\theta_2) \cos(\theta_4) - 1.0 a_2 a_4 m_5 \dot{\theta}_2 \cos(\theta_3) \cos(\theta_4) + 1.0 I_{yz,5} \dot{\theta}_1 \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) - \\
& 1.0 a_2 l c_{3,2} m_3 \dot{\theta}_1 \cos(\theta_2) \cos(\theta_3) - 1.0 a_3 l c_{4,1} m_4 \dot{\theta}_1 \cos(\theta_2) \cos(\theta_4) - 1.0 a_2 l c_{4,1} m_4 \dot{\theta}_2 \cos(\theta_3) \cos(\theta_4) + 1.0 a_4 l c_{5,2} m_5 \dot{\theta}_1 \cos(\theta_2) \cos(\theta_5) - \\
& 0.5 a_2 a_3 m_3 \dot{\theta}_1 \cos(\theta_2) \sin(\theta_3) - 1.0 a_2 a_3 m_4 \dot{\theta}_1 \cos(\theta_2) \sin(\theta_3) - 1.0 a_2 a_3 m_5 \dot{\theta}_1 \cos(\theta_2) \sin(\theta_3) + 0.5 a_4 a_5 m_5 \dot{\theta}_1 \cos(\theta_2) \sin(\theta_5) + \\
& 0.125 a_4^2 m_4 \dot{\theta}_1 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(\theta_2) + 0.5 a_4^2 m_5 \dot{\theta}_1 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(\theta_2) - 1.0 a_2 l c_{3,1} m_3 \dot{\theta}_1 \cos(\theta_2) \sin(\theta_3) + \\
& 1.0 a_3 l c_{4,2} m_4 \dot{\theta}_1 \cos(\theta_2) \sin(\theta_4) + 1.0 a_2 l c_{4,2} m_4 \dot{\theta}_2 \cos(\theta_3) \sin(\theta_4) + 1.0 a_2 l c_{4,2} m_4 \dot{\theta}_2 \cos(\theta_4) \sin(\theta_3) + 1.0 a_4 l c_{5,1} m_5 \dot{\theta}_1 \cos(\theta_2) \sin(\theta_5) + \\
& 0.5 a_2 a_4 m_4 \dot{\theta}_2 \sin(\theta_3) \sin(\theta_4) + 1.0 a_2 a_4 m_5 \dot{\theta}_2 \sin(\theta_3) \sin(\theta_4) + 0.5 l c_{4,1}^2 m_4 \dot{\theta}_1 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(\theta_2) - \\
& 0.5 l c_{4,2}^2 m_4 \dot{\theta}_1 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(\theta_2) - 1.0 l c_{3,2} l c_{3,3} m_3 \dot{\theta}_1 \cos(\theta_3) \sin(\theta_2) - 0.5 a_3 l c_{3,3} m_3 \dot{\theta}_1 \sin(\theta_2) \sin(\theta_3) + \\
& 1.0 a_2 l c_{4,1} m_4 \dot{\theta}_2 \sin(\theta_3) \sin(\theta_4) - 1.0 a_3 l c_{4,3} m_4 \dot{\theta}_1 \sin(\theta_2) \sin(\theta_3) - 1.0 a_3 l c_{5,3} m_5 \dot{\theta}_1 \sin(\theta_2) \sin(\theta_3) - 0.125 a_4^2 m_4 \dot{\theta}_1 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) - \\
& 0.5 a_4^2 m_5 \dot{\theta}_1 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) - 1.0 l c_{3,1} l c_{3,3} m_3 \dot{\theta}_1 \sin(\theta_2) \sin(\theta_3) - 0.5 l c_{4,1}^2 m_4 \dot{\theta}_1 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) + \\
& 0.5 l c_{4,2}^2 m_4 \dot{\theta}_1 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) - 0.5 a_4 l c_{4,2} m_4 \dot{\theta}_2 \cos(2.0 \theta_3) \cos(2.0 \theta_4) - 1.0 a_4 l c_{5,2} m_5 \dot{\theta}_2 \sin(2.0 \theta_3) \sin(2.0 \theta_4) \sin(\theta_5) + \\
& 0.5 a_3 a_4 m_4 \dot{\theta}_1 \cos(2.0 \theta_3) \cos(\theta_2) \cos(\theta_4) + 1.0 a_3 a_4 m_5 \dot{\theta}_1 \cos(2.0 \theta_3) \cos(\theta_2) \cos(\theta_4) - 0.5 a_3 a_5 m_5 \dot{\theta}_2 \cos(2.0 \theta_3) \cos(\theta_4) \cos(\theta_5) + \\
& 1.0 a_3 l c_{4,1} m_4 \dot{\theta}_1 \cos(2.0 \theta_3) \cos(\theta_2) \cos(\theta_4) - 1.0 a_3 l c_{5,1} m_5 \dot{\theta}_2 \cos(2.0 \theta_3) \cos(\theta_4) \cos(\theta_5) - 0.125 a_5^2 m_5 \dot{\theta}_1 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(2.0 \theta_5) \cos(\theta_2) - \\
& 1.0 a_3 l c_{4,2} m_4 \dot{\theta}_1 \cos(2.0 \theta_3) \cos(\theta_2) \sin(\theta_4) - 1.0 a_3 l c_{4,2} m_4 \dot{\theta}_1 \sin(2.0 \theta_3) \cos(\theta_2) \cos(\theta_4) + 1.0 a_3 l c_{5,2} m_5 \dot{\theta}_2 \cos(2.0 \theta_3) \cos(\theta_4) \sin(\theta_5) + \\
& 1.0 a_3 l c_{5,2} m_5 \dot{\theta}_2 \cos(2.0 \theta_3) \cos(\theta_5) \sin(\theta_4) + 1.0 a_3 l c_{5,2} m_5 \dot{\theta}_2 \sin(2.0 \theta_3) \cos(\theta_4) \cos(\theta_5) - 0.5 a_3 a_4 m_4 \dot{\theta}_1 \sin(2.0 \theta_3) \cos(\theta_2) \sin(\theta_4) - \\
& 1.0 a_3 a_4 m_5 \dot{\theta}_1 \sin(2.0 \theta_3) \cos(\theta_2) \sin(\theta_4) + 0.5 a_3 a_5 m_5 \dot{\theta}_2 \cos(2.0 \theta_3) \sin(\theta_4) \sin(\theta_5) + 0.5 a_3 a_5 m_5 \dot{\theta}_2 \sin(2.0 \theta_3) \cos(\theta_4) \sin(\theta_5) + \\
& 0.5 a_3 a_5 m_5 \dot{\theta}_2 \sin(2.0 \theta_3) \cos(\theta_5) \sin(\theta_4) - 0.5 l c_{5,1}^2 m_5 \dot{\theta}_1 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(2.0 \theta_5) \cos(\theta_2) + 0.5 l c_{5,2}^2 m_5 \dot{\theta}_1 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(2.0 \theta_5) \cos(\theta_2) - \\
& 1.0 a_3 l c_{4,1} m_4 \dot{\theta}_1 \sin(2.0 \theta_3) \cos(\theta_2) \sin(\theta_4) + 1.0 a_3 l c_{5,1} m_5 \dot{\theta}_2 \cos(2.0 \theta_3) \sin(\theta_4) \sin(\theta_5) + 1.0 a_3 l c_{5,1} m_5 \dot{\theta}_2 \sin(2.0 \theta_3) \cos(\theta_4) \sin(\theta_5) + \\
& 1.0 a_3 l c_{5,1} m_5 \dot{\theta}_2 \sin(2.0 \theta_3) \cos(\theta_5) \sin(\theta_4) + 0.125 a_5^2 m_5 \dot{\theta}_1 \cos(2.0 \theta_3) \sin(2.0 \theta_4) \sin(2.0 \theta_5) \cos(\theta_2) + 0.125 a_5^2 m_5 \dot{\theta}_1 \cos(2.0 \theta_4) \sin(2.0 \theta_3) \sin(2.0 \theta_5) \cos(\theta_2) + \\
& 0.125 a_5^2 m_5 \dot{\theta}_1 \cos(2.0 \theta_5) \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) - 1.0 a_3 l c_{5,2} m_5 \dot{\theta}_2 \sin(2.0 \theta_3) \sin(\theta_4) \sin(\theta_5) + 0.5 l c_{5,1}^2 m_5 \dot{\theta}_1 \cos(2.0 \theta_3) \sin(2.0 \theta_4) \sin(2.0 \theta_5) \cos(\theta_2) + \\
& 0.5 l c_{5,1}^2 m_5 \dot{\theta}_1 \cos(2.0 \theta_4) \sin(2.0 \theta_3) \sin(2.0 \theta_5) \cos(\theta_2) + 0.5 l c_{5,1}^2 m_5 \dot{\theta}_1 \cos(2.0 \theta_5) \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) - \\
& 0.5 l c_{5,2}^2 m_5 \dot{\theta}_1 \cos(2.0 \theta_3) \sin(2.0 \theta_4) \sin(2.0 \theta_5) \cos(\theta_2) - 0.5 l c_{5,2}^2 m_5 \dot{\theta}_1 \cos(2.0 \theta_4) \sin(2.0 \theta_3) \sin(2.0 \theta_5) \cos(\theta_2) - \\
& 0.5 l c_{5,2}^2 m_5 \dot{\theta}_1 \cos(2.0 \theta_5) \sin(2.0 \theta_3) \sin(2.0 \theta_4) \cos(\theta_2) + 0.5 a_5 l c_{5,2} m_5 \dot{\theta}_2 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(2.0 \theta_5) + \\
& 1.0 l c_{5,1} l c_{5,2} m_5 \dot{\theta}_2 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \cos(2.0 \theta_5) + 0.5 a_5 l c_{5,1} m_5 \dot{\theta}_2 \cos(2.0 \theta_3) \cos(2.0 \theta_4) \sin(2.0 \theta_5) + \\
& 0.5 a_5 l c_{5,1} m_5 \dot{\theta}_2 \cos(2.0 \theta_3) \cos(2.0 \theta_5) \sin(2.0 \theta_4) + 0.5 a_5 l c_{5,1} m_5 \dot{\theta}_2 \cos(2.0 \theta_4) \cos(2.0 \theta_5) \sin(2.0 \theta_3) - 0.5 a_5 l c_{5,2} m_5 \dot{\theta}_2 \cos(2.0 \theta_3) \sin(2.0 \theta_4) \sin(2.0 \theta_5) - \\
& 0.5 a_5 l c_{5,2} m_5 \dot{\theta}_2 \cos(2.0 \theta_4) \sin(2.0 \theta_3) \sin(2.0 \theta_5) - 0.5 a_5 l c_{5,2} m_5 \dot{\theta}_2 \cos(2.0 \theta_5) \sin(2.0 \theta_3) \sin(2.0 \theta_4) - 1.0 a_2 l c_{4,2} m_4 \dot{\theta}_1 \cos$$

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[illegible]

[illegible]

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$$\begin{aligned}
C_{43} = & a_3 \text{lc}_{4,2} m_4 \dot{\theta}_3 \cos(\theta_4) - \frac{a_4 a_5 m_5 \dot{\theta}_5 \cos(\theta_5)}{2} - 1.0 a_4 \text{lc}_{5,1} m_5 \dot{\theta}_5 \cos(\theta_5) + 0.5 a_3 a_4 m_4 \dot{\theta}_3 \sin(\theta_4) + a_3 a_4 m_5 \dot{\theta}_3 \sin(\theta_4) + \\
& a_3 \text{lc}_{4,1} m_4 \dot{\theta}_3 \sin(\theta_4) + a_4 \text{lc}_{5,2} m_5 \dot{\theta}_5 \sin(\theta_5) + 0.5 a_3 a_5 m_5 \dot{\theta}_3 \cos(\theta_4) \cos(\theta_5) + a_3 \text{lc}_{5,1} m_5 \dot{\theta}_3 \cos(\theta_4) \cos(\theta_5) - \\
& 1.0 a_3 \text{lc}_{4,2} m_4 \dot{\theta}_1 \cos(\theta_4) \sin(\theta_2) - 1.0 a_3 \text{lc}_{5,2} m_5 \dot{\theta}_3 \cos(\theta_4) \sin(\theta_5) - 1.0 a_3 \text{lc}_{5,2} m_5 \dot{\theta}_3 \cos(\theta_5) \sin(\theta_4) - \frac{a_3 a_4 m_4 \dot{\theta}_1 \sin(\theta_2) \sin(\theta_4)}{2} - \\
& 1.0 a_3 a_4 m_5 \dot{\theta}_1 \sin(\theta_2) \sin(\theta_4) - \frac{a_3 a_5 m_5 \dot{\theta}_3 \sin(\theta_4) \sin(\theta_5)}{2} - 1.0 a_3 \text{lc}_{4,1} m_4 \dot{\theta}_1 \sin(\theta_2) \sin(\theta_4) - 1.0 a_3 \text{lc}_{5,1} m_5 \dot{\theta}_3 \sin(\theta_4) \sin(\theta_5) - \\
& \frac{a_3 a_5 m_5 \dot{\theta}_1 \cos(\theta_4) \cos(\theta_5) \sin(\theta_2)}{2} - 1.0 a_3 \text{lc}_{5,1} m_5 \dot{\theta}_1 \cos(\theta_4) \cos(\theta_5) \sin(\theta_2) + a_3 \text{lc}_{5,2} m_5 \dot{\theta}_1 \cos(\theta_4) \sin(\theta_2) \sin(\theta_5) + \\
& a_3 \text{lc}_{5,2} m_5 \dot{\theta}_1 \cos(\theta_5) \sin(\theta_2) \sin(\theta_4) + 0.5 a_3 a_5 m_5 \dot{\theta}_1 \sin(\theta_2) \sin(\theta_4) \sin(\theta_5) + a_3 \text{lc}_{5,1} m_5 \dot{\theta}_1 \sin(\theta_2) \sin(\theta_4) \sin(\theta_5)
\end{aligned} \tag{A45}$$

$$C_{44} = 1.0 a_4 \text{lc}_{5,2} m_5 \dot{\theta}_5 \sin(\theta_5) - 1.0 a_4 \text{lc}_{5,1} m_5 \dot{\theta}_5 \cos(\theta_5) - \frac{a_4 a_5 m_5 \dot{\theta}_5 \cos(\theta_5)}{2} \tag{A46}$$

$$\begin{aligned}
C_{45} = & 1.0 a_4 \text{lc}_{5,2} m_5 \dot{\theta}_3 \sin(\theta_5) - 0.5 a_4 a_5 m_5 \dot{\theta}_4 \cos(\theta_5) - 0.5 a_4 a_5 m_5 \dot{\theta}_5 \cos(\theta_5) - 1.0 a_4 \text{lc}_{5,1} m_5 \dot{\theta}_3 \cos(\theta_5) - \\
& 1.0 a_4 \text{lc}_{5,1} m_5 \dot{\theta}_4 \cos(\theta_5) - 1.0 a_4 \text{lc}_{5,1} m_5 \dot{\theta}_5 \cos(\theta_5) - 0.5 a_4 a_5 m_5 \dot{\theta}_3 \cos(\theta_5) + 1.0 a_4 \text{lc}_{5,2} m_5 \dot{\theta}_4 \sin(\theta_5) + 1.0 a_4 \text{lc}_{5,2} m_5 \dot{\theta}_5 \sin(\theta_5) + \\
& 0.5 a_4 a_5 m_5 \dot{\theta}_1 \cos(\theta_5) \sin(\theta_2) + 1.0 a_4 \text{lc}_{5,1} m_5 \dot{\theta}_1 \cos(\theta_5) \sin(\theta_2) - 1.0 a_4 \text{lc}_{5,2} m_5 \dot{\theta}_1 \sin(\theta_2) \sin(\theta_5)
\end{aligned} \tag{A47}$$

[illegible]

(A48)

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$$\begin{aligned}
C_{53} = & 0.5 a_4 a_5 m_5 \dot{\theta}_3 \cos(\theta_5) + 0.5 a_4 a_5 m_5 \dot{\theta}_4 \cos(\theta_5) + 1.0 a_4 \text{lc}_{5,1} m_5 \dot{\theta}_3 \cos(\theta_5) + 1.0 a_4 \text{lc}_{5,1} m_5 \dot{\theta}_4 \cos(\theta_5) - \\
& 1.0 a_4 \text{lc}_{5,2} m_5 \dot{\theta}_3 \sin(\theta_5) - 1.0 a_4 \text{lc}_{5,2} m_5 \dot{\theta}_4 \sin(\theta_5) + 0.5 a_3 a_5 m_5 \dot{\theta}_3 \cos(\theta_4) \cos(\theta_5) + 1.0 a_3 \text{lc}_{5,1} m_5 \dot{\theta}_3 \cos(\theta_4) \cos(\theta_5) - \\
& 0.5 a_4 a_5 m_5 \dot{\theta}_1 \cos(\theta_5) \sin(\theta_2) - 1.0 a_4 \text{lc}_{5,1} m_5 \dot{\theta}_1 \cos(\theta_5) \sin(\theta_2) - 1.0 a_3 \text{lc}_{5,2} m_5 \dot{\theta}_3 \cos(\theta_4) \sin(\theta_5) - 1.0 a_3 \text{lc}_{5,2} m_5 \dot{\theta}_3 \cos(\theta_5) \sin(\theta_4) - \\
& 0.5 a_3 a_5 m_5 \dot{\theta}_3 \sin(\theta_4) \sin(\theta_5) + 1.0 a_4 \text{lc}_{5,2} m_5 \dot{\theta}_1 \sin(\theta_2) \sin(\theta_5) - 1.0 a_3 \text{lc}_{5,1} m_5 \dot{\theta}_3 \sin(\theta_4) \sin(\theta_5) - 0.5 a_3 a_5 m_5 \dot{\theta}_1 \cos(\theta_4) \cos(\theta_5) \sin(\theta_2) - \\
& 1.0 a_3 \text{lc}_{5,1} m_5 \dot{\theta}_1 \cos(\theta_4) \cos(\theta_5) \sin(\theta_2) + 1.0 a_3 \text{lc}_{5,2} m_5 \dot{\theta}_1 \cos(\theta_4) \sin(\theta_2) \sin(\theta_5) + 1.0 a_3 \text{lc}_{5,2} m_5 \dot{\theta}_1 \cos(\theta_5) \sin(\theta_2) \sin(\theta_4) + \\
& 0.5 a_3 a_5 m_5 \dot{\theta}_1 \sin(\theta_2) \sin(\theta_4) \sin(\theta_5) + 1.0 a_3 \text{lc}_{5,1} m_5 \dot{\theta}_1 \sin(\theta_2) \sin(\theta_4) \sin(\theta_5)
\end{aligned} \tag{A50}$$

$$\begin{aligned}
C_{54} = & 0.5 a_4 a_5 m_5 \dot{\theta}_3 \cos(\theta_5) + 0.5 a_4 a_5 m_5 \dot{\theta}_4 \cos(\theta_5) + 1.0 a_4 \text{lc}_{5,1} m_5 \dot{\theta}_3 \cos(\theta_5) + 1.0 a_4 \text{lc}_{5,1} m_5 \dot{\theta}_4 \cos(\theta_5) - \\
& 1.0 a_4 \text{lc}_{5,2} m_5 \dot{\theta}_3 \sin(\theta_5) - 1.0 a_4 \text{lc}_{5,2} m_5 \dot{\theta}_4 \sin(\theta_5) - 0.5 a_4 a_5 m_5 \dot{\theta}_1 \cos(\theta_5) \sin(\theta_2) - 1.0 a_4 \text{lc}_{5,1} m_5 \dot{\theta}_1 \cos(\theta_5) \sin(\theta_2) + \\
& 1.0 a_4 \text{lc}_{5,2} m_5 \dot{\theta}_1 \sin(\theta_2) \sin(\theta_5)
\end{aligned} \tag{A51}$$

$$C_{55} = 0 \tag{A52}$$

C. Gravity Vector ($G(q)$)

We can derive the gravity vector as follows:

$$G(q) = \begin{bmatrix} G_1 \\ G_2 \\ G_3 \\ G_4 \\ G_5 \end{bmatrix} \quad (\text{A53})$$

where

$$\begin{aligned} G_1 = & 0.5 a_1 g m_1 \cos(\theta_1) + 1.0 a_1 g m_2 \cos(\theta_1) + 1.0 a_1 g m_3 \cos(\theta_1) + 1.0 a_1 g m_4 \cos(\theta_1) + 1.0 a_1 g m_5 \cos(\theta_1) + \\ & 1.0 g l c_{1,1} m_1 \cos(\theta_1) - 1.0 g l c_{2,2} m_2 \sin(\theta_1) - 1.0 g l c_{3,3} m_3 \sin(\theta_1) + 1.0 g l c_{2,3} m_2 \cos(\theta_1) \cos(\theta_2) + 1.0 g l c_{3,3} m_3 \cos(\theta_1) \cos(\theta_2) + \\ & 1.0 g l c_{4,3} m_4 \cos(\theta_1) \cos(\theta_2) + 1.0 g l c_{5,3} m_5 \cos(\theta_1) \cos(\theta_2) - 0.5 a_2 g m_2 \cos(\theta_1) \sin(\theta_2) - 1.0 a_2 g m_3 \cos(\theta_1) \sin(\theta_2) - \\ & 1.0 a_2 g m_4 \cos(\theta_1) \sin(\theta_2) - 1.0 a_2 g m_5 \cos(\theta_1) \sin(\theta_2) + 0.5 a_3 g m_3 \cos(\theta_3) \sin(\theta_1) + 1.0 a_3 g m_4 \cos(\theta_3) \sin(\theta_1) + \\ & 1.0 a_3 g m_5 \cos(\theta_3) \sin(\theta_1) - 1.0 g l c_{2,1} m_2 \cos(\theta_1) \sin(\theta_2) + 1.0 g l c_{3,1} m_3 \cos(\theta_3) \sin(\theta_1) - 1.0 g l c_{3,2} m_3 \sin(\theta_1) \sin(\theta_3) + \\ & 0.5 a_4 g m_4 \cos(\theta_3) \cos(\theta_4) \sin(\theta_1) + 1.0 a_4 g m_5 \cos(\theta_3) \cos(\theta_4) \sin(\theta_1) - 1.0 g l c_{3,2} m_3 \cos(\theta_1) \cos(\theta_3) \sin(\theta_2) + \\ & 1.0 g l c_{4,1} m_4 \cos(\theta_3) \cos(\theta_4) \sin(\theta_1) - 0.5 a_3 g m_3 \cos(\theta_1) \sin(\theta_2) \sin(\theta_3) - 1.0 a_3 g m_4 \cos(\theta_1) \sin(\theta_2) \sin(\theta_3) - \\ & 1.0 a_3 g m_5 \cos(\theta_1) \sin(\theta_2) \sin(\theta_3) - 1.0 g l c_{3,1} m_3 \cos(\theta_1) \sin(\theta_2) \sin(\theta_3) - 1.0 g l c_{4,2} m_4 \cos(\theta_3) \sin(\theta_1) \sin(\theta_4) - \\ & 1.0 g l c_{4,2} m_4 \cos(\theta_4) \sin(\theta_1) \sin(\theta_3) - 0.5 a_4 g m_4 \sin(\theta_1) \sin(\theta_3) \sin(\theta_4) - 1.0 a_4 g m_5 \sin(\theta_1) \sin(\theta_3) \sin(\theta_4) - \\ & 1.0 g l c_{4,1} m_4 \sin(\theta_1) \sin(\theta_3) \sin(\theta_4) - 1.0 g l c_{4,2} m_4 \cos(\theta_1) \cos(\theta_3) \cos(\theta_4) \sin(\theta_2) - 1.0 g l c_{5,2} m_5 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) \sin(\theta_1) - \\ & 0.5 a_4 g m_4 \cos(\theta_1) \cos(\theta_3) \sin(\theta_2) \sin(\theta_4) - 0.5 a_4 g m_4 \cos(\theta_1) \cos(\theta_4) \sin(\theta_2) \sin(\theta_3) - 1.0 a_4 g m_5 \cos(\theta_1) \cos(\theta_3) \sin(\theta_2) \sin(\theta_4) - \\ & 1.0 a_4 g m_5 \cos(\theta_1) \cos(\theta_4) \sin(\theta_2) \sin(\theta_3) - 0.5 a_5 g m_5 \cos(\theta_3) \cos(\theta_4) \sin(\theta_1) \sin(\theta_5) - 0.5 a_5 g m_5 \cos(\theta_3) \cos(\theta_5) \sin(\theta_1) \sin(\theta_4) - \\ & 0.5 a_5 g m_5 \cos(\theta_4) \cos(\theta_5) \sin(\theta_1) \sin(\theta_3) - 1.0 g l c_{4,1} m_4 \cos(\theta_1) \cos(\theta_3) \sin(\theta_2) \sin(\theta_4) - 1.0 g l c_{4,1} m_4 \cos(\theta_1) \cos(\theta_4) \sin(\theta_2) \sin(\theta_3) - \\ & 1.0 g l c_{5,1} m_5 \cos(\theta_3) \cos(\theta_4) \sin(\theta_1) \sin(\theta_5) - 1.0 g l c_{5,1} m_5 \cos(\theta_3) \cos(\theta_5) \sin(\theta_1) \sin(\theta_4) - 1.0 g l c_{5,1} m_5 \cos(\theta_4) \cos(\theta_5) \sin(\theta_1) \sin(\theta_3) + \\ & 1.0 g l c_{4,2} m_4 \cos(\theta_1) \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) + 1.0 g l c_{5,2} m_5 \cos(\theta_3) \sin(\theta_1) \sin(\theta_4) \sin(\theta_5) + 1.0 g l c_{5,2} m_5 \cos(\theta_4) \sin(\theta_1) \sin(\theta_3) \sin(\theta_5) + \\ & 1.0 g l c_{5,2} m_5 \cos(\theta_5) \sin(\theta_1) \sin(\theta_3) \sin(\theta_4) + 0.5 a_5 g m_5 \sin(\theta_1) \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) + 1.0 g l c_{5,1} m_5 \sin(\theta_1) \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) + \\ & 1.0 g l c_{5,1} m_5 \cos(\theta_1) \cos(\theta_3) \sin(\theta_2) \sin(\theta_4) \sin(\theta_5) + 1.0 g l c_{5,1} m_5 \cos(\theta_1) \cos(\theta_4) \sin(\theta_2) \sin(\theta_3) \sin(\theta_5) + \\ & 1.0 g l c_{5,1} m_5 \cos(\theta_1) \cos(\theta_5) \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) - 1.0 g l c_{5,2} m_5 \cos(\theta_1) \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) - \\ & 0.5 a_5 g m_5 \cos(\theta_1) \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) \sin(\theta_2) - 1.0 g l c_{5,1} m_5 \cos(\theta_1) \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) \sin(\theta_2) + \\ & 1.0 g l c_{5,2} m_5 \cos(\theta_1) \cos(\theta_3) \cos(\theta_4) \sin(\theta_2) \sin(\theta_5) + 1.0 g l c_{5,2} m_5 \cos(\theta_1) \cos(\theta_3) \cos(\theta_5) \sin(\theta_2) \sin(\theta_4) + \\ & 1.0 g l c_{5,2} m_5 \cos(\theta_1) \cos(\theta_4) \cos(\theta_5) \sin(\theta_2) \sin(\theta_3) + 0.5 a_5 g m_5 \cos(\theta_1) \cos(\theta_3) \sin(\theta_2) \sin(\theta_4) \sin(\theta_5) + \\ & 0.5 a_5 g m_5 \cos(\theta_1) \cos(\theta_4) \sin(\theta_2) \sin(\theta_3) \sin(\theta_5) + 0.5 a_5 g m_5 \cos(\theta_1) \cos(\theta_5) \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) \end{aligned} \quad (\text{A54})$$

$$\begin{aligned}
G_2 = & 1.0 g \text{lc}_{4,2} m_4 \cos(\theta_2) \sin(\theta_1) \sin(\theta_3) \sin(\theta_4) - 1.0 a_2 g m_3 \cos(\theta_2) \sin(\theta_1) - 1.0 a_2 g m_4 \cos(\theta_2) \sin(\theta_1) - \\
& 1.0 a_2 g m_5 \cos(\theta_2) \sin(\theta_1) - 1.0 g \text{lc}_{2,1} m_2 \cos(\theta_2) \sin(\theta_1) - 1.0 g \text{lc}_{2,3} m_2 \sin(\theta_1) \sin(\theta_2) - 1.0 g \text{lc}_{3,3} m_3 \sin(\theta_1) \sin(\theta_2) - \\
& 1.0 g \text{lc}_{4,3} m_4 \sin(\theta_1) \sin(\theta_2) - 1.0 g \text{lc}_{5,3} m_5 \sin(\theta_1) \sin(\theta_2) - 1.0 g \text{lc}_{3,2} m_3 \cos(\theta_2) \cos(\theta_3) \sin(\theta_1) - \frac{a_3 g m_3 \cos(\theta_2) \sin(\theta_1) \sin(\theta_3)}{2} - \\
& 1.0 a_3 g m_4 \cos(\theta_2) \sin(\theta_1) \sin(\theta_3) - 1.0 a_3 g m_5 \cos(\theta_2) \sin(\theta_1) \sin(\theta_3) - 1.0 g \text{lc}_{3,1} m_3 \cos(\theta_2) \sin(\theta_1) \sin(\theta_3) - \\
& 1.0 g \text{lc}_{4,2} m_4 \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) \sin(\theta_1) - \frac{a_4 g m_4 \cos(\theta_2) \cos(\theta_3) \sin(\theta_1) \sin(\theta_4)}{2} - \frac{a_4 g m_4 \cos(\theta_2) \cos(\theta_4) \sin(\theta_1) \sin(\theta_3)}{2} - \\
& 1.0 a_4 g m_5 \cos(\theta_2) \cos(\theta_3) \sin(\theta_1) \sin(\theta_4) - 1.0 a_4 g m_5 \cos(\theta_2) \cos(\theta_4) \sin(\theta_1) \sin(\theta_3) - 1.0 g \text{lc}_{4,1} m_4 \cos(\theta_2) \cos(\theta_3) \sin(\theta_1) \sin(\theta_4) - \\
& 1.0 g \text{lc}_{4,1} m_4 \cos(\theta_2) \cos(\theta_4) \sin(\theta_1) \sin(\theta_3) - \frac{a_2 g m_2 \cos(\theta_2) \sin(\theta_1)}{2} + 1.0 g \text{lc}_{5,1} m_5 \cos(\theta_2) \cos(\theta_3) \sin(\theta_1) \sin(\theta_4) \sin(\theta_5) + \\
& 1.0 g \text{lc}_{5,1} m_5 \cos(\theta_2) \cos(\theta_4) \sin(\theta_1) \sin(\theta_3) \sin(\theta_5) + 1.0 g \text{lc}_{5,1} m_5 \cos(\theta_2) \cos(\theta_5) \sin(\theta_1) \sin(\theta_3) \sin(\theta_4) - \\
& 1.0 g \text{lc}_{5,2} m_5 \cos(\theta_2) \sin(\theta_1) \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) - \frac{a_5 g m_5 \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) \sin(\theta_1)}{2} - 1.0 g \text{lc}_{5,1} m_5 \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) \sin(\theta_1) + \\
& 1.0 g \text{lc}_{5,2} m_5 \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) \sin(\theta_1) \sin(\theta_5) + 1.0 g \text{lc}_{5,2} m_5 \cos(\theta_2) \cos(\theta_3) \cos(\theta_5) \sin(\theta_1) \sin(\theta_4) + \\
& 1.0 g \text{lc}_{5,2} m_5 \cos(\theta_2) \cos(\theta_4) \cos(\theta_5) \sin(\theta_1) \sin(\theta_3) + \frac{a_5 g m_5 \cos(\theta_2) \cos(\theta_3) \sin(\theta_1) \sin(\theta_4) \sin(\theta_5)}{2} + \frac{a_5 g m_5 \cos(\theta_2) \cos(\theta_4) \sin(\theta_1) \sin(\theta_3) \sin(\theta_5)}{2} + \\
& \frac{a_5 g m_5 \cos(\theta_2) \cos(\theta_5) \sin(\theta_1) \sin(\theta_3) \sin(\theta_4)}{2}
\end{aligned} \tag{A55}$$

$$\begin{aligned}
G_3 = & g \text{lc}_{3,2} m_3 \cos(\theta_1) \cos(\theta_3) + 0.5 a_3 g m_3 \cos(\theta_1) \sin(\theta_3) + a_3 g m_4 \cos(\theta_1) \sin(\theta_3) + a_3 g m_5 \cos(\theta_1) \sin(\theta_3) + \\
& g \text{lc}_{3,1} m_3 \cos(\theta_1) \sin(\theta_3) + g \text{lc}_{4,2} m_4 \cos(\theta_1) \cos(\theta_3) \cos(\theta_4) + 0.5 a_4 g m_4 \cos(\theta_1) \cos(\theta_3) \sin(\theta_4) + 0.5 a_4 g m_4 \cos(\theta_1) \cos(\theta_4) \sin(\theta_3) + \\
& a_4 g m_5 \cos(\theta_1) \cos(\theta_3) \sin(\theta_4) + a_4 g m_5 \cos(\theta_1) \cos(\theta_4) \sin(\theta_3) + g \text{lc}_{4,1} m_4 \cos(\theta_1) \cos(\theta_3) \sin(\theta_4) + g \text{lc}_{4,1} m_4 \cos(\theta_1) \cos(\theta_4) \sin(\theta_3) - \\
& 0.5 a_3 g m_3 \cos(\theta_3) \sin(\theta_1) \sin(\theta_2) - 1.0 a_3 g m_4 \cos(\theta_3) \sin(\theta_1) \sin(\theta_2) - 1.0 a_3 g m_5 \cos(\theta_3) \sin(\theta_1) \sin(\theta_2) - \\
& 1.0 g \text{lc}_{3,1} m_3 \cos(\theta_3) \sin(\theta_1) \sin(\theta_2) - g \text{lc}_{4,2} m_4 \cos(\theta_1) \sin(\theta_3) \sin(\theta_4) + g \text{lc}_{3,2} m_3 \sin(\theta_1) \sin(\theta_2) \sin(\theta_3) - \\
& 1.0 g \text{lc}_{5,2} m_5 \cos(\theta_1) \cos(\theta_3) \cos(\theta_4) \sin(\theta_5) - 1.0 g \text{lc}_{5,2} m_5 \cos(\theta_1) \cos(\theta_3) \cos(\theta_5) \sin(\theta_4) - 1.0 g \text{lc}_{5,2} m_5 \cos(\theta_1) \cos(\theta_4) \cos(\theta_5) \sin(\theta_3) - \\
& 0.5 a_4 g m_4 \cos(\theta_3) \cos(\theta_4) \sin(\theta_1) \sin(\theta_2) - 1.0 a_4 g m_5 \cos(\theta_3) \cos(\theta_4) \sin(\theta_1) \sin(\theta_2) - 0.5 a_5 g m_5 \cos(\theta_1) \cos(\theta_3) \sin(\theta_4) \sin(\theta_5) - \\
& 0.5 a_5 g m_5 \cos(\theta_1) \cos(\theta_4) \sin(\theta_3) \sin(\theta_5) - 0.5 a_5 g m_5 \cos(\theta_1) \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) - 1.0 g \text{lc}_{4,1} m_4 \cos(\theta_3) \cos(\theta_4) \sin(\theta_1) \sin(\theta_2) - \\
& g \text{lc}_{5,1} m_5 \cos(\theta_1) \cos(\theta_3) \sin(\theta_4) \sin(\theta_5) - g \text{lc}_{5,1} m_5 \cos(\theta_1) \cos(\theta_4) \sin(\theta_3) \sin(\theta_5) - g \text{lc}_{5,1} m_5 \cos(\theta_1) \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) + \\
& g \text{lc}_{4,2} m_4 \cos(\theta_3) \sin(\theta_1) \sin(\theta_2) \sin(\theta_4) + g \text{lc}_{4,2} m_4 \cos(\theta_4) \sin(\theta_1) \sin(\theta_2) \sin(\theta_3) + 1.0 g \text{lc}_{5,2} m_5 \cos(\theta_1) \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) + \\
& 0.5 a_4 g m_4 \sin(\theta_1) \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) + 1.0 a_4 g m_5 \sin(\theta_1) \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) + 1.0 g \text{lc}_{4,1} m_4 \sin(\theta_1) \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) + \\
& 0.5 a_5 g m_5 \cos(\theta_1) \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) + g \text{lc}_{5,1} m_5 \cos(\theta_1) \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) + g \text{lc}_{5,1} m_5 \cos(\theta_3) \cos(\theta_4) \sin(\theta_1) \sin(\theta_2) \sin(\theta_5) + \\
& g \text{lc}_{5,1} m_5 \cos(\theta_3) \cos(\theta_5) \sin(\theta_1) \sin(\theta_2) \sin(\theta_4) + g \text{lc}_{5,1} m_5 \cos(\theta_4) \cos(\theta_5) \sin(\theta_1) \sin(\theta_2) \sin(\theta_3) - g \text{lc}_{5,2} m_5 \cos(\theta_3) \sin(\theta_1) \sin(\theta_2) \sin(\theta_4) \sin(\theta_5) - \\
& g \text{lc}_{5,2} m_5 \cos(\theta_4) \sin(\theta_1) \sin(\theta_2) \sin(\theta_3) \sin(\theta_5) - g \text{lc}_{5,2} m_5 \cos(\theta_5) \sin(\theta_1) \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) - 0.5 a_5 g m_5 \sin(\theta_1) \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) - \\
& g \text{lc}_{5,1} m_5 \sin(\theta_1) \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) + g \text{lc}_{5,2} m_5 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) \sin(\theta_1) \sin(\theta_2) + 0.5 a_5 g m_5 \cos(\theta_3) \cos(\theta_4) \sin(\theta_1) \sin(\theta_2) \sin(\theta_5) + \\
& 0.5 a_5 g m_5 \cos(\theta_3) \cos(\theta_5) \sin(\theta_1) \sin(\theta_2) \sin(\theta_4) + 0.5 a_5 g m_5 \cos(\theta_4) \cos(\theta_5) \sin(\theta_1) \sin(\theta_2) \sin(\theta_3)
\end{aligned} \tag{A56}$$

