Resources and tasks management in ES

(up to/from here: session 19/20)

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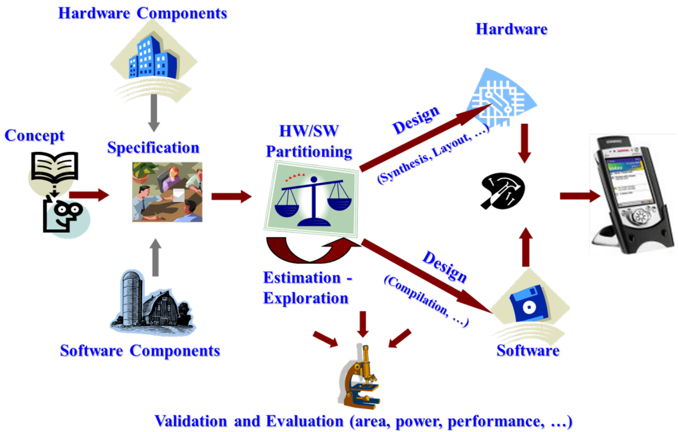
Outline

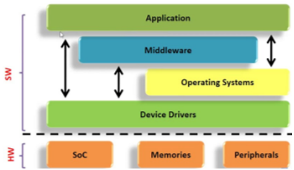
- Preliminaries on scheduling
  - A deeper view on ES software
  - Definitions
  - Classifications of scheduling algorithms
- Real-time scheduling
  - Definitions
  - Different types of real-time scheduling algorithms
  - Uniprocessor real-time scheduling
    - Equal arrival times algorithms
    - Different arrival times
  - Multi-processor real-time scheduling

Embedded system design flow



Reuse of standard software components

- Knowledge from previous designs to be made available in the form of intellectual property (IP, for SW and HW).
    - Operating systems (real-time)
    - Middleware
    - Real-time data bases
    - Standard software (MPEG-x, GSM-kernel, . . .)
- Includes standard approaches for scheduling (requires knowledge about execution times)



Time-triggered systems

- Entirely time-triggered system
  - The temporal control structure of all tasks is established a priori by off-line support-tools
  - Temporal control structure is encoded in a Task-Descriptor List (TDL) that contains the cyclic schedule for all activities of the node
  - This schedule considers the required precedence and mutual exclusion relationships among the tasks such that an explicit coordination of the tasks by the operating system at run time is not necessary

Time-triggered systems

- The dispatcher is activated by the synchronized clock tick
  - It looks at the TDL, and then performs the action that has been planned for this instant [Kopetz]
  - The only practical means of providing predictability
- It can be easily checked if timing constraints are met
  - Response to sporadic events may be poor

Worst case execution time

- **Definition: The worst case execution time (WCET) is an upper bound on the execution times of tasks**
  - Computing such a bound is undecidable
    - Possible for programs without recursion and finite loops
      - **Pipeline hazards, interrupts, caches –> serious overestimates**
- Approaches:
  - For hardware: Typically requires hardware synthesis
  - For software: Requires availability of machine programs; complex analysis

Average execution time

- Estimated cost and performance values
    - Difficult to generate sufficiently precise estimates; Balance between run-time and precision
- Accurate cost and performance values
    - Can be done with normal tools (such as compilers)
        - As precise as the input data is

Which timing metric is more important in real-time ES?

Schedulability

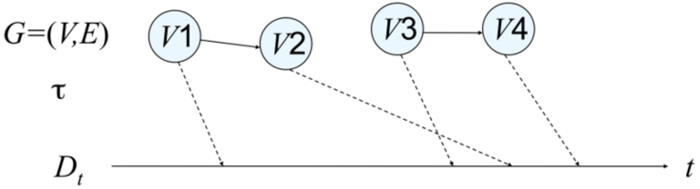
- A set of tasks is said to be schedulable under a given set of constraints, if a schedule exists for that set of tasks and constraints
  - Exact tests are NP-hard in many situations
  - Sufficient tests: Sufficient conditions for guaranteeing a schedule are checked
  - Necessary tests: Checking necessary conditions. Can be used to show that no schedule exists
    - Always not possible to prove even if no schedule exists

Scheduling classifications

- Centralized and distributed scheduling
  - Multiprocessor scheduling either locally on one or distributed on several processors
- Mono- and multi-processor scheduling
  - Simple scheduling algorithms handle single processors, more complex algorithms handle multiple processors
- Online (dynamic) - and offline (static) scheduling:
  - Online scheduling is done at run-time based on the information about the tasks arrived so far
  - Offline scheduling assumes prior knowledge about arrival times, execution times, and deadlines

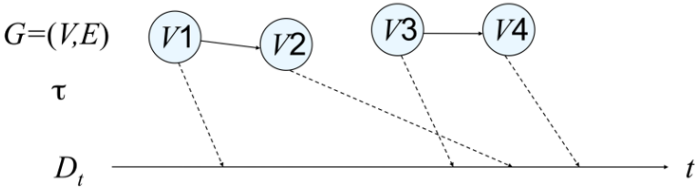
What is real-time scheduling?

- Assume that we have a task graph G=(V,E)
- A schedule of G is a mapping, V –> T, of a set of tasks V to start times from domain T



What is real-time scheduling?

- Schedules have to respect a set of constraints, such as resource, dependency, and deadlines
- Scheduling is the process of finding such a mapping
- During the design of embedded systems, scheduling has to be performed several times
  - Early rough scheduling as well as late precise scheduling



Simple tasks

- Tasks without any inter-process communication are called simple tasks (S-tasks)
  - S-tasks can be either ready or running
- API of an S-task in a TT system: Two OS Calls
  - TERMINATE TASK and ERROR
    - The TERMINATE TASK system call is executed whenever the task has reached its termination point
    - In case of an error that cannot be handled within the application task, the task terminates its operation with the ERROR system call [Kopetz, 1997]

Cost functions

- Cost function: Different algorithms aim at minimizing different functions
  - For example **Maximum lateness** is one of the main metrics that are used in cost functions
- **Definition: Maximum lateness is defined as the difference between the completion time and the deadline, maximized over all tasks**
  - Maximum lateness is negative if all tasks complete before their deadline

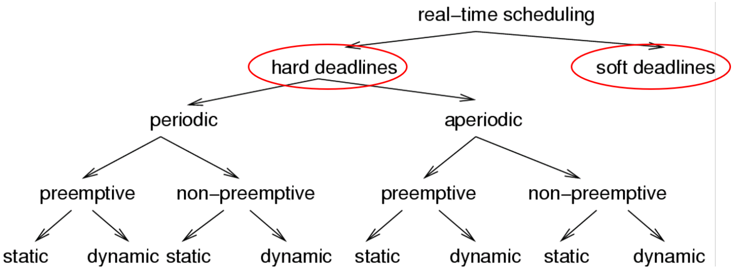
Basic parameters in real-time scheduling (from here: session 21)

- Arrival time a_i
  - Time when a task becomes ready for execution
- Computation time c_i
  - Time necessary for completion of a task
- Absolute deadline d_i
  - Time before which a task should be finished
- Finishing time f_i
  - Time at which a task finishes its execution

Basic parameters in real-time scheduling (up to here: session 20)

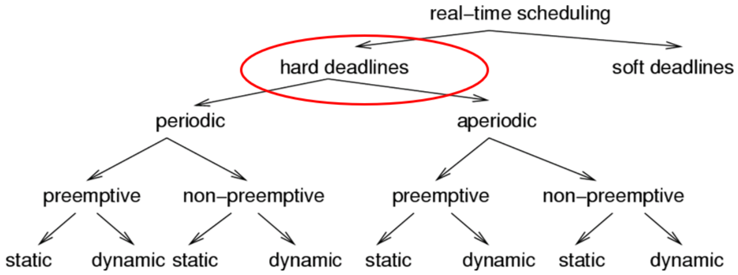
- Derived Parameters
  - Relative deadline $D_i = d_i$ - a_i
  - Response time $R_i = f_i$- a_i
  - Lateness $L_i = f_i$ - d_i delay of a task (can be negative)
  - Slack time (laxity) $s_i = d_i$ - a_i - c_i
    - Maximum time a task can be delayed on its activation to complete within deadline

Classification of real-time scheduling algorithms

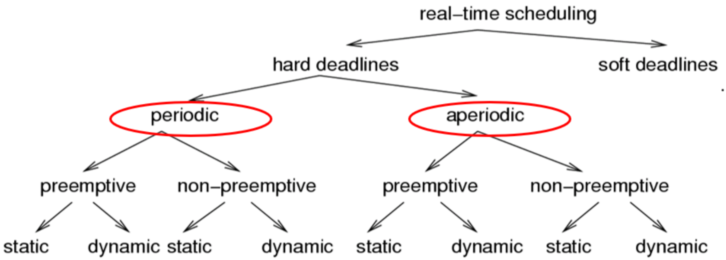


Hard and soft deadlines

- **Definition: A time-constraint (deadline) is called hard if not meeting that constraint could result in a catastrophe [Kopetz, 1997]**
  - All other time constraints are called soft
- We will focus on hard deadlines!

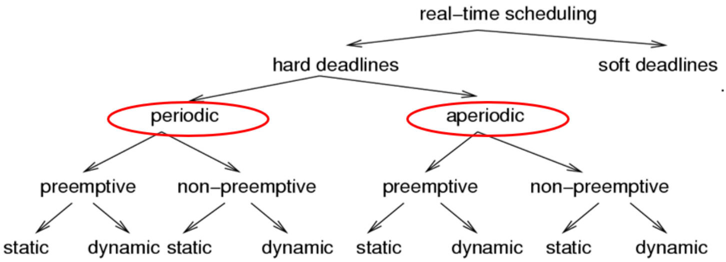


Periodic and aperiodic tasks



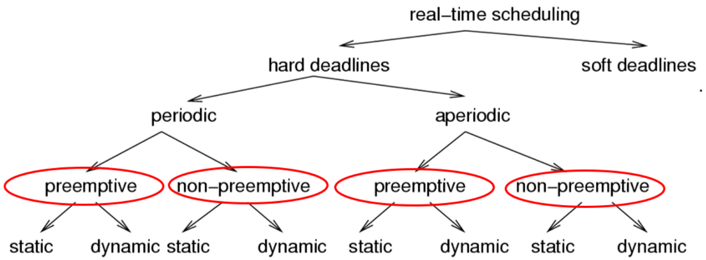
- **Definition: Tasks which must be executed once every p units of time are called periodic tasks**
  - p is called their period
  - Each execution of a periodic task is called a job
  - All other tasks are called aperiodic

Periodic and aperiodic tasks



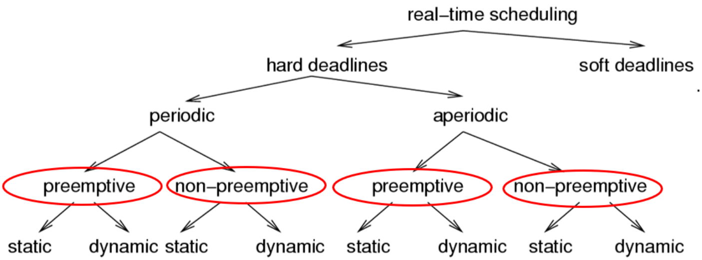
- **Definition: Aperiodic tasks requesting the processor at unpredictable times are called sporadic if there is a minimum separation between the times at which they request the processor**

Preemptive and non-preemptive scheduling



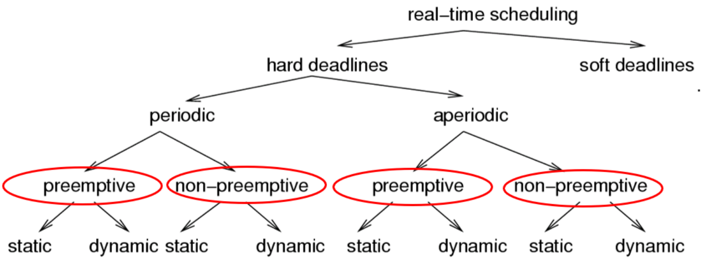
- **Definition: Preemptive and non-preemptive scheduling**
  - Non-preemptive schedulers are based on the assumption that tasks are executed until they are done

Preemptive and non-preemptive scheduling



- For non-preemptive scheduler
  - The response time for external events may be quite long if some tasks have a large execution time

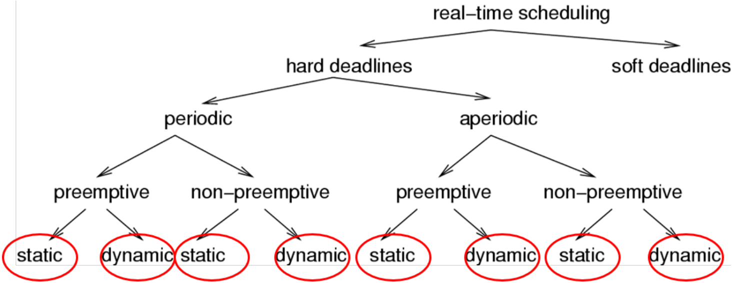
Preemptive and non-preemptive scheduling



- Preemptive schedulers have to be used if some tasks have long execution times or if the response time for external events is required to be short

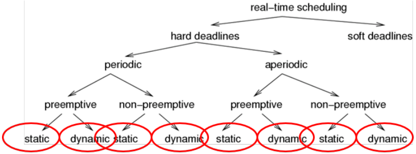
Static and dynamic scheduling

- **Definition: Dynamic scheduling**
  - **Processor allocation decisions (scheduling) done at run-time**

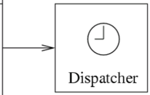


Static and dynamic scheduling

- **Definition: Static scheduling**
  - **Processor allocation decisions (scheduling) done at design-time**
    - Dispatcher allocates processor when interrupted by a timer
    - The timer is controlled by a table generated at design time

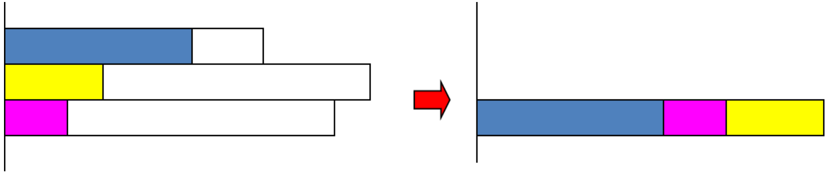


Time	Action	WCET
10	start T1	12
17	send M5	
22	stop T1	
38	start T2	20
47	send M3	



Earliest Due Date (EDD)

- Given a set of n independent tasks, any algorithm that executes the tasks in order of non-decreasing deadlines is optimal with respect to minimizing the maximum lateness
  - Proof: See Dertouzos, M. L.: "Control Robotics: the Procedural Control of Physical Processes", Information Processing 74, North-Holland Publishing Company, 1974
- EDD requires all tasks to be sorted by their deadlines



In-class assignment: Question

- Consider the following job's information table
  - Draw all possible scheduling Gannet chart
  - Which one is better in terms of minimizing maximum lateness?

	1	2	← job number
t_j	3	2	← time required
d_j	1	3	← deadline

In-class assignment: Answer

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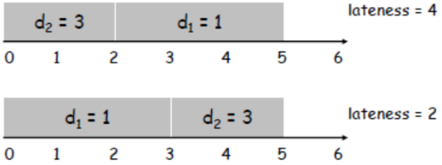


Earliest Deadline First (EDF)

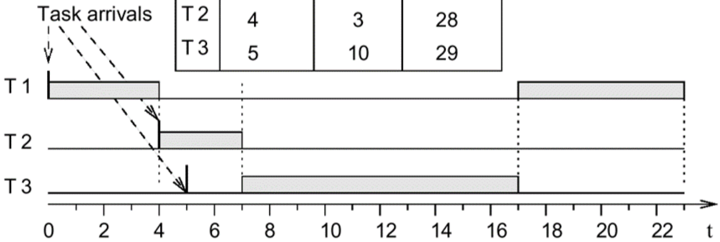
- Theorem **[Horn74]: Given a set of n independent tasks with arbitrary arrival times, any algorithm that at any instant executes the task with the** earliest absolute deadline **among all the ready tasks is optimal with respect to minimizing the maximum lateness**

Earliest Deadline First (EDF)

- Each time a new ready task arrives, it is inserted into a queue of ready tasks, sorted by their deadlines
  - If a newly arrived task is inserted at the head of the queue, the currently executing task is preempted
    - Preemption potentially reduces lateness

Earliest Deadline First (EDF)

	arrival	duration	deadline
T 1	0	10	33
T 2	4	3	28
T 3	5	10	29



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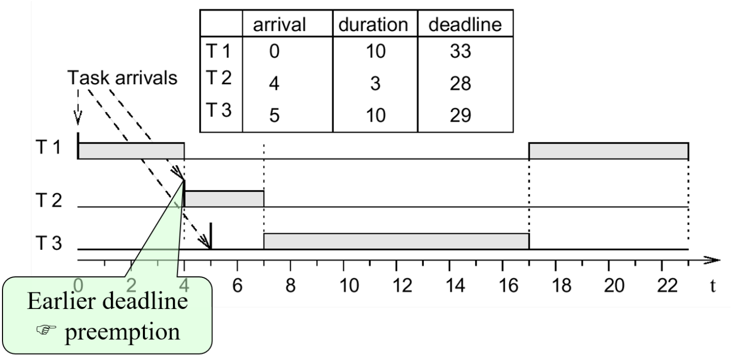
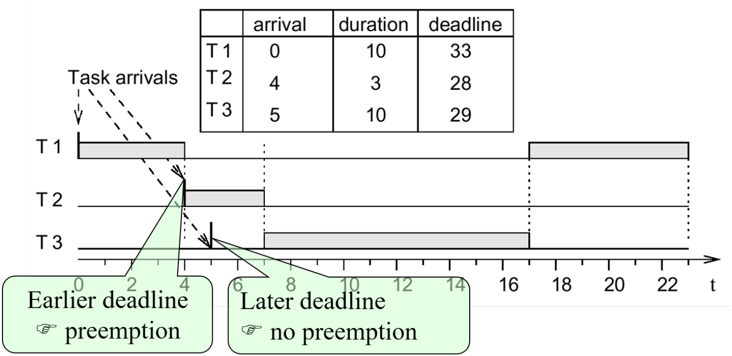


Earliest Deadline First (EDF)



	arrival	duration	deadline
T 1	0	10	33
T 2	4	3	28
T 3	5	10	29

Scheduling with no precedence constraints

- Let T_i be a set of tasks and
  - c_i be the execution time of T_i
  - d_i be the **deadline interval**, that is, the time between T_i becoming available and the time until which T_i has to finish execution
  - l_i be the **laxity** or **slack**, defined as $l_i = d_i$ - c_i

Least Laxity First (LLF) or Least Slack Time First (LSF)

- Priorities = Decreasing function of the laxity
  - The less laxity, the higher the priority
  - Dynamically changing priority
  - Preemptive

	arrival	duration	deadline
T 1	0	10	33
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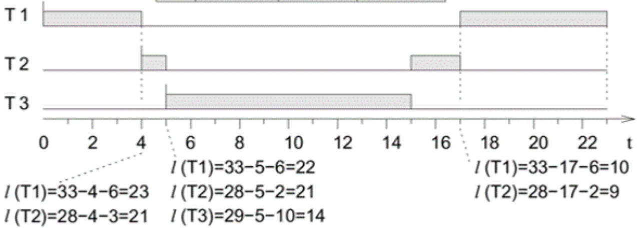
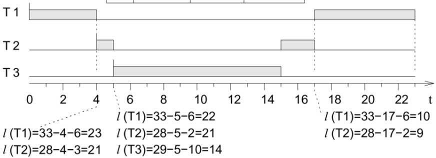


Least Laxity First (LLF) or Least Slack Time First (LSF)

- Priorities = Decreasing function of the laxity
  - The less laxity, the higher the priority
  - Dynamically changing priority
  - Preemptive

✓ Requires calling the scheduler periodically, and to recompute the laxity. Overhead for many calls of the scheduler and many context switches.
✓ Detects missed deadlines early

	arrival	duration	deadline
T 1	0	10	33
T 2	4	3	28
T 3	5	10	29

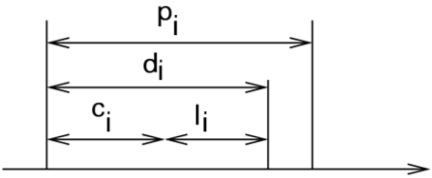


LLF/LSF properties

- LLF/LSF is also an optimal scheduling for uni-processor systems
  - BUT... uses dynamic priorities
    - Therefore cannot be used with a fixed priority OS
      - Fixed-priority preemptive scheduling is a scheduling system commonly used in real-time systems
- LLF/LSF scheduling requires the knowledge of the execution time
  - May not know this in advance!

Periodic scheduling (from here: session 22)

- Let
  - p_i be the period of task T_i,
  - c_i be the execution time of T_i,
  - d_i be the deadline interval, that is, the time between a job of T_i becoming available and the time after which the same job T_i has to finish execution
  - l_i be the **laxity** or **slack**, defined as $l_i = d_i$ - c_i



Periodic scheduling (up to here: session 21)

- Accumulated utilization
  - Accumulated execution time divided by period

$$\mu = \sum_{i=1}^{n} \frac{c_i}{p_i}$$

Necessary condition for schedulability (with m=number of processors):

$$\mu \leq m$$

Rate Monotonic (RM)

- Well-known technique for scheduling independent periodic tasks [Liu, 1973]
- Assumptions:
  - All tasks that have hard deadlines are periodic
  - All tasks are independent
  - $d_i = p_i$, for all tasks
  - c_i is constant and is known for all tasks
  - The time required for context switching is negligible

Rate Monotonic (RM)

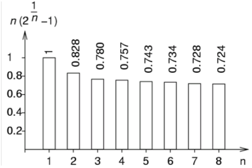
- RM schedulability condition
  - For a single processor with n tasks, the following equation must hold for the accumulated utilization μ

$$\mu = \sum_{i=1}^{n} \frac{c_i}{p_i} \leq n(2^{1/n} - 1)$$

Rate Monotonic (RM)

- The priority of a task is a monotonically decreasing function of its period
  - Low period: High priority
- At any time, a highest priority task among all those that are ready for execution is allocated
- If all assumptions are met, schedulability is guaranteed

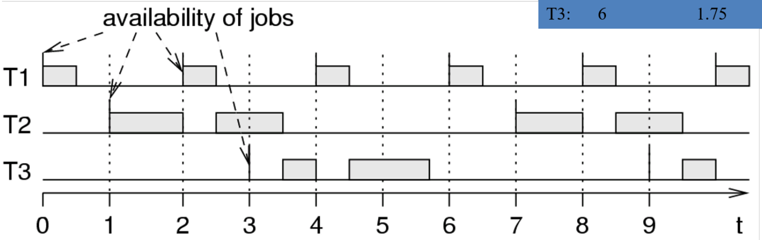
Maximum utilization as a function of the number of tasks⇨



Example: RM-generated schedule

- T1 preempts T2 and T3
- T2 and T3 do not preempt each other

	Period	Execution Time
T1:	2	0.5
T2:	6	2
T3:	6	1.75

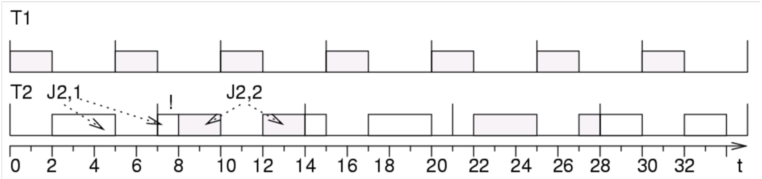


Case of failing RM scheduling

Task 1: period 5, execution time 2
Task 2: period 7, execution time 4
μ=2/5+4/7=34/35 ≈ 0.97
$2(2^{1/2}-1) \approx 0.828$

$$\mu = \sum_{i=1}^{n} \frac{c_i}{p_i} \leq n(2^{1/n} - 1)$$

Not enough idle time



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$\mu = 2/5 + 4/7 = 34/35 \approx 0.97$
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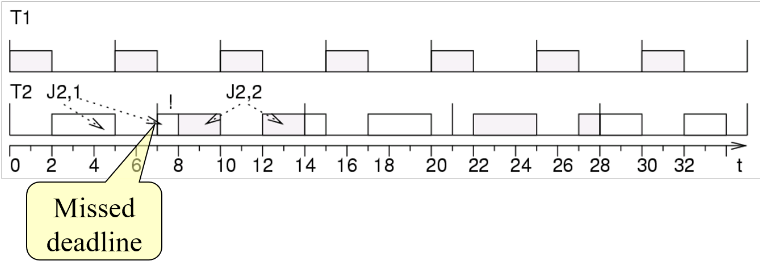
Not enough idle time



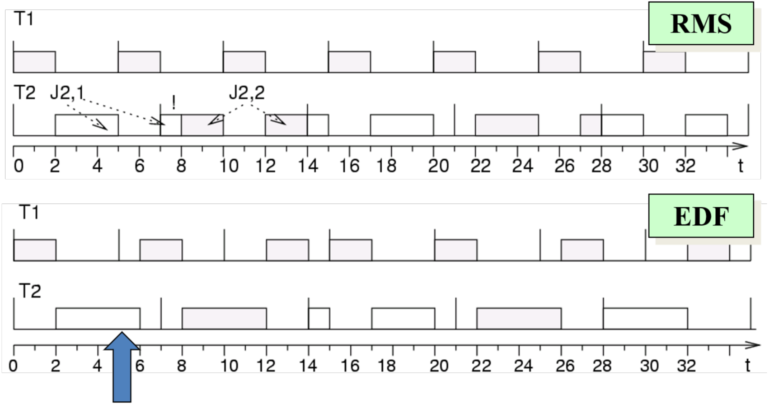
Properties of RM scheduling

- RM scheduling is based on static priorities
  - This allows RM scheduling to be used in standard OS
    - Such as Windows NT
  - A huge number of variations of RM scheduling exists
- In the context of RM scheduling, many formal proofs exist.
- The idle capacity is not required if the period of all tasks is a multiple of the period of the highest priority task
  - Necessary condition for schedulability: $\mu <= 1$

EDF in periodic scheduling

- EDF can also be applied to periodic scheduling
- EDF optimal for every period
  - Optimal for periodic scheduling
    - Trivially!
  - EDF must be able to schedule the example in which RMS failed
- EDF requires dynamic priorities
  - EDF cannot be used with a standard operating system just providing static priorities

Comparison of EDF and RMS



T2 not preempted, due to its earlier deadline

Scheduling without preemption

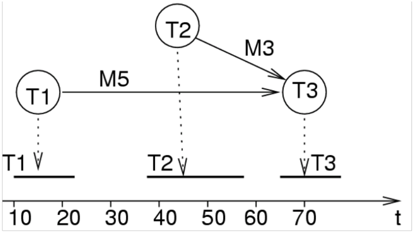
- Optimal schedules may leave processor idle to finish tasks with early deadlines arriving late
  - Knowledge about the future is needed for optimal scheduling algorithms
    - No online algorithm can decide whether or not to keep idle

Scheduling without preemption

- EDF is optimal among all scheduling algorithms not keeping the processor idle at certain times
- If arrival times are known a priori, the scheduling problem becomes NP-hard in general
  - Branch and bound techniques are typically used

Scheduling with precedence constraints

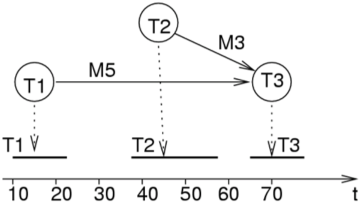
- Task graph and possible schedule



Schedule can be stored in a table
(can be used by dispatcher/OS)

Synchronous arrival times

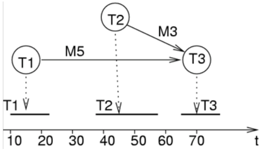
- Optimal algorithm for minimum latency
  - Latest Deadline First (LDF)
- LDF [Lawler, 1973]: Generation of total order compatible with the partial order described by the task graph (LDF performs a topological sort)



Synchronous arrival times

- LDF reads the task graph from tail to head and inserts tasks with no successors into a queue. It then repeats this process, putting tasks whose successor have all been selected into the queue
  - At run-time, the tasks are executed in from head to tail
  - LDF is non-preemptive and is optimal for uni-processors

Latest Deadline First (LDF)



Asynchronous arrival times

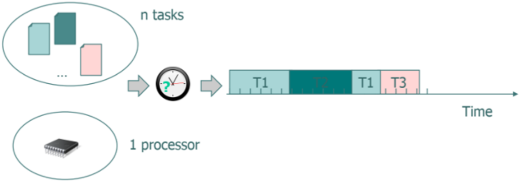
- This case can be handled with a modified EDF algorithm
    - The key idea is to transform the problem from a given set of dependent tasks into a set of independent tasks with different timing parameters [Chetto90]
- This algorithm is optimal for uni-processor systems
- If preemption is not allowed, the heuristic algorithm developed by [Stankovic and Ramamritham 1991] can be used

Sporadic tasks

- If sporadic tasks were connected to interrupts, the execution time of other tasks would become very unpredictable
  - Introduction of a sporadic task server,periodically checking for ready sporadic tasks
    - Sporadic tasks are essentially turned into periodic tasks

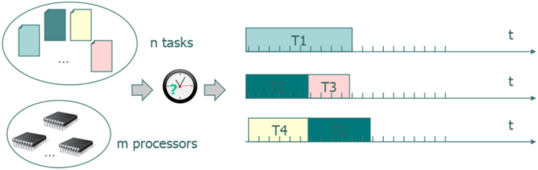
Introduction (from here: session 23)

- Mono-processor scheduling: One-dimension problem
  - Temporal organization



Introduction

- Multi-processor (multi-core) scheduling: Two dimension problem
  - Temporal organization +
  - Spatial organization
    - On which processor execute every task?

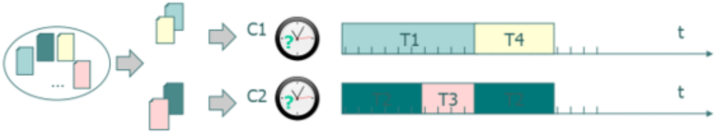


Classification (up to here: session 22)

- Partitioned scheduling
  - Each of the two dimensions is dealt with separately
- Global scheduling
  - Temporal and spatial dimensions are deal with jointly
- Semi-partitioned scheduling
  - Hybrid

Classification: partitioned scheduling

- Each of the two dimensions is dealt with separately
  - Spatial organization: the n tasks are partitioned onto the m cores
    - No task migration at run-time
  - Temporal organization: Mono-processor scheduling is used on each core



Classification: partitioned scheduling

- Two points of view
  - Number of processors to be determined: Optimization problem (bin-packing problem)
    - Bin = task, size = utilization (or other expression obtained from the task temporal parameters)
    - Boxes = processors, size = ability to host tasks
  - Fixed number of processors: search problem (knapsack problem)
- Both problems are NP-hard

Classification: partitioned scheduling

- Optimal mono-processor scheduling strategies: XX
  - RM, DM (Deadline Monotonic: Like rate monotonic but consider task's deadlines instead of task's periods)
  - EDF, LLF (see uni-processor scheduling section)
- Bin-packing heuristics: YY
  - FF: First-Fit
  - BF: Best-Fit
  - WF: Worst-Fit
  - NF: Next-Fit
  - FFD, BFD, WFD: First/Best/Worst-Fit Decreasing

Partitioning algorithms
XX-YY

Classification: partitioned scheduling

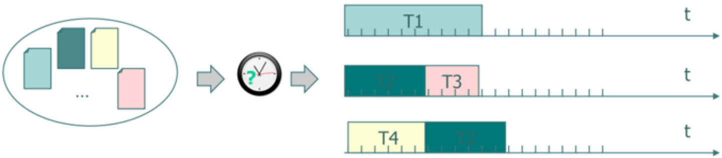
- Benefits
  - Implementation: local schedulers are independent
  - No migration costs
  - Direct reuse of mono-processor schedulability tests
  - Isolation between processors in case of overload
- Limits

Classification: partitioned scheduling

- Benefits
- Limits
  - Rigid: suited to static configurations
  - NP-hard task partitioning
  - Largest utilization bound for any partitioning algorithm [Andersson, 2001]
    - m+1 tasks of execution time "1 + ϵ" and period 2: $\frac{m+1}{2}$

Classification: global scheduling

- Temporal and spatial dimensions are dealt with jointly
  - Global unique scheduler and run queue
  - At each scheduling point, the scheduler decides when and where schedule at most m tasks
  - Task migration allowed

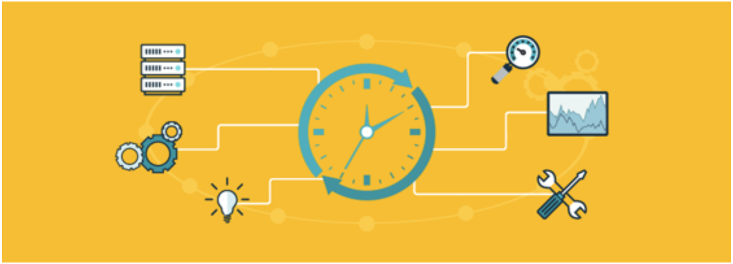


Classification: global scheduling

- Benefits
  - Suited to dynamic configurations
  - Dominates all other scheduling policies
    - If we consider unconstrained migrations + dynamic priorities
  - Optimal schedulers exist
  - Overloads/underloads spread on all processors
- Drawbacks
  - System overheads: migrations, mutual exclusion for sharing the run queue

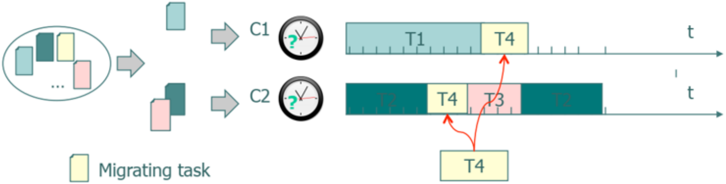
Classification: global scheduling

- Global RM/DM/EDF (preemptive): definition
  - Task priorities assigned according to RM/DM/EDF
  - Scheduling algorithm: The m higher priority tasks are executed on the m processors



Classification: semi-partitioned scheduling

- Partitioned scheduling as far as possible
- Some statically determined tasks may migrate
  - Constraint: Migrating tasks (T4 on the example) must execute on a single processor at a time



Terminology

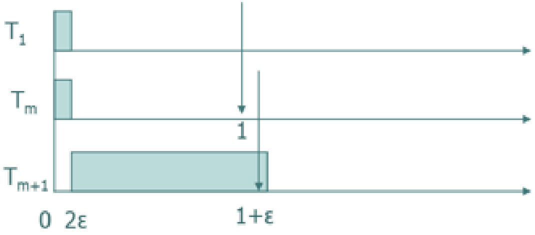
- Priorities
  - Fixed per task (FPT)
  - Fixed per job (FPJ)
  - Dynamic per job (DPJ)

Overview of global scheduling policies (from here: session 24)

- Assumptions
  - Tasks
    - Periodic tasks (p_i)
    - Implicit deadlines ($d_i = p_i$)
    - Synchronous tasks
    - Independent tasks
    - A single job of a task can be active at a time
  - Architecture
    - Identical processors
    - Costs are neglected (preemption, migration, scheduling policy)

Scheduling anomalies

- Dhall's effect [Dhall Liu, 1978]
  - Periodic task sets with utilization close to 1 are unschedulable using global RM/EDF
  - n=m+1 , p_i=1 , c_i=2ϵ ,u_i=2ϵ for all 1$\leq$ i $\leq$m
  - P_{m+1}=1+ϵ , C_{m+1}=1 , $u_{m+1}=\frac{1}{1+\epsilon}$
  - Task m+1 misses its deadline although U very close to 1
  - We assumed m processor cores

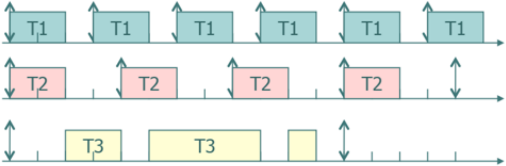


Scheduling anomalies (up to here: session 23)

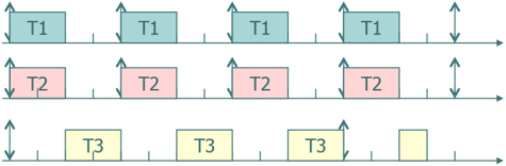
- For G-RM, there may be situations in which schedules exist for a certain task system, but deadlines are violated if periods are extended [Anderson, 2003]
  - n = 3 , m = 2 , ($P_1 = 3$, C_1 =2),($P_2 = 4$, $C_2 = 2$),($P_3 = 12$, $C_3 = 7$)
  - Schedulable under global RM
  - If P_1 is increased to 4 and priorities stay the same, T3 misses its deadline

Scheduling anomalies

- $(P_1 = 3\ ,\ C_1 = 2\ ),(P_2 = 4\ ,\ C_2 = 2),(P_3 = 12,\ C_3 = 7)$

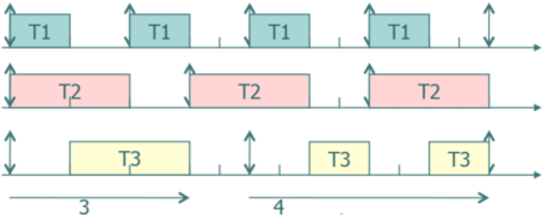


- $(P_1 = 4\ ,\ C_1 = 2\ ),(P_2 = 4\ ,\ C_2 = 2),(P_3 = 12,\ C_3 = 7)$



Scheduling anomalies

- Critical instant not necessarily the simultaneous release of higher priority tasks
  - n=3, m=2
  - $(P_1 = 2\ ,\ C_1 = 1\ ),(P_2 = 3\ ,\ C_2 = 2),(P_3 = 4,\ C_3 = 2)$
  - Under RM scheduling
    - Response time of T3 higher at time 4 than at time 0

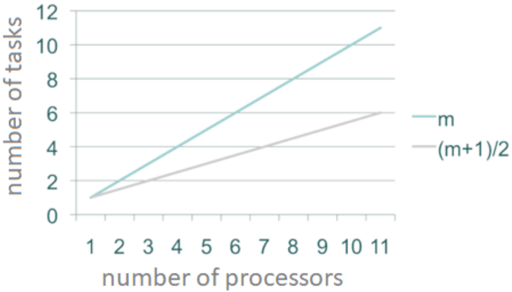


General properties of multiprocessor scheduling

- Exact schedulability condition
  - U $\leq$ m and $u_{max} \leq 1$
    - U = Total utilization
    - u_{max} = Maximum utilization
  - Does not tell for which scheduling algorithm!
- Schedule is cyclic on the hyperperiod H (PPCM(P_i)) for:
  - Deterministic tasks
  - Without memory scheduling algorithms

General properties of multiprocessor scheduling

- Theorem [Srinavasan Baruah, 2002]
  - Non existence of FPJ (FPJ+FPT) scheduling with utilization bound strictly larger than $\frac{m+1}{2}$ for implicit deadline periodic task sets!



Global multiprocessor scheduling: detailed outline

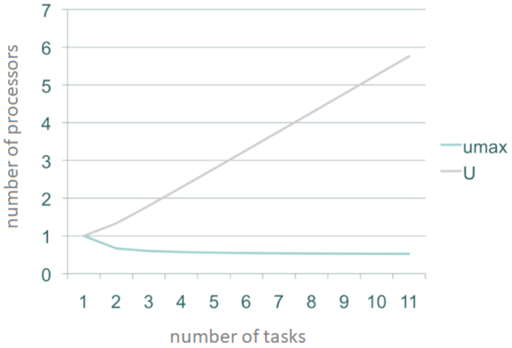
- Transposition of uni-processor algorithms
- Extensions of uni-processor algorithms
    - US (Utilization Threshold)
    - EDF(k)
    - ZL (Zero Laxity)
- Pfair approaches (Proportional Fair)

Transposition of uni-processor algorithms

- Main algorithms
  - RM (Rate Monotonic) –> G-RM, Global RM
  - EDF (Earliest Deadline First) –> G-EDF, Global EDF
  - Not optimal anymore
  - Sufficient schedulability tests (depend on u_{max})

G-RM	G-EDF
$u_{max} \leq m/(3m-2)$ and $U \leq m^2/(3m-2)$	$u_{max} \leq m/(2m+1)$ and $U \leq m^2/(2m+2)$
$u_{max} \leq 1/3$ and $U \leq m/3$	$u_{max} \leq 1/2$ and $U \leq (m+1)/2$
$U \leq m/2 * (1-u_{max}) + u_{max}$	$U \leq m - (m-1)\, u_{max}$

Transposition of uni-processor algorithms



Exten. of global RM/EDF: US (Utilization Threshold)

- Priority assignment depend on an utilization threshold ξ
  - If $u_{max} > \xi$ then T_i is assigned maximal priority
    - Else, T_i's priority assigned as in original algorithm (RM/EDF)
- Remarks
  - Still non optimal
  - Outperforms the base policy that is used
  - Defies Dhall's effect

Exten. of global RM/EDF: US (Utilization Threshold)

- Example: RM-US[$\xi = \frac{1}{2}$]

	Ci	Pi	Ui	Prio
T1	4	10	2/5	2
T2	3	10	3/10	2
T3	8	12	2/3	∞
T4	5	12	5/12	1
T5	7	12	7/12	∞

Exten. of global RM/EDF: US (Utilization Threshold)

- Utilization bounds

RM-US		EDF-US	
$\xi=m/(3m-2)$	$U \le m^2/(3m-2)$	$\xi=m/(2m-1)$	$U \le m^2/(2m-1)$
$\xi=1/3$	$U \le (m+1)/3$	$\xi=1/2$	$U \le (m+1)/2$

- Remarks
  - Utilization bounds do not depend on u_{max} more
  - EDF-US[$\xi=\frac{1}{2}$] attains the best utilization bound possible for FPJ ($\frac{m+1}{2}$)

Exten. of global RM/EDF: EDF(k)

- Task indices by decreasing utilization
    - $u_i \geq u_{i+1}$ for all i in [1,n]
- Priority assignment depends on a threshold on task index
    - i<k,then maximum priority
    - Else, priority assignment according to original algorithm

Exten. of global RM/EDF: EDF(k)

- Example, EDF(4)

	Ci	Pi	Ui	Prio
T1	4	10	2/5	EDF
T2	3	10	3/10	EDF
T3	8	12	2/3	∞
T4	5	12	5/12	∞
T5	7	12	7/12	∞

Exten. of global RM/EDF: EDF(k)

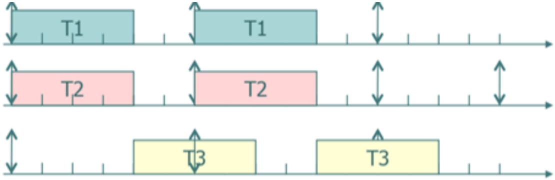
- Sufficient schedulability test
  - m$\geq$(k-1)-$\lceil \frac{\sum_{i=k+1}^{n} u_i}{1-u_k} \rceil$
  - k_{min} = value minimizing right side of the equation
  - With k = k_{min} , utilization bound of $\frac{m+1}{2}$ (the best possible for FPJ)
  - Comparison with EDF[ξ=$\frac{1}{2}$]
    - Same utilization bound
    - EDF(k_{min}) dominates EDF[ξ=$\frac{1}{2}$]

Exten. of global RM/EDF: ZL (Zero Laxity) policies

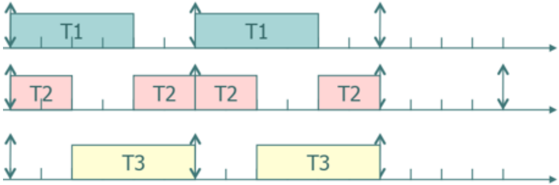
- XX-ZL: Apply policy XX until Zero Laxity
  - Maximal priority when laxity reaches zero (regardless of the currently running job), original priority assignment for the others
  - In category DPJ (dynamic job scheduling)
  - Policies: EDZL [Lee, 1994], RMZL [Kato al, 2009], FPZL [Davis et al, 2010]
  - Utilization bound: $\frac{m+1}{2}$
  - Dominates G-EDF

Exten. of global RM/EDF: ZL (Zero Laxity) policies

- Example: n=3,m=2; all P_i to 6, all C_i to 4
  - G-EDF: T3 misses its deadline



  - EDZL: OK



Pfair algorithms: principle (up to/from here: session 24/25)

- Pfair: "Proportionate Fair" [Baruah et al, 1996]
  - Allocate time slots to tasks as close as possible to a "fluid" system, proportional to their utilization factor
- Example
  - $C_1 = C_2 = 3, P_1 = P_2 = 6$ $(u_1 = u_2 = \frac{1}{2})$
  - Each task will be "approximately" allocated 1 slot out of 2 (whatever the processor)

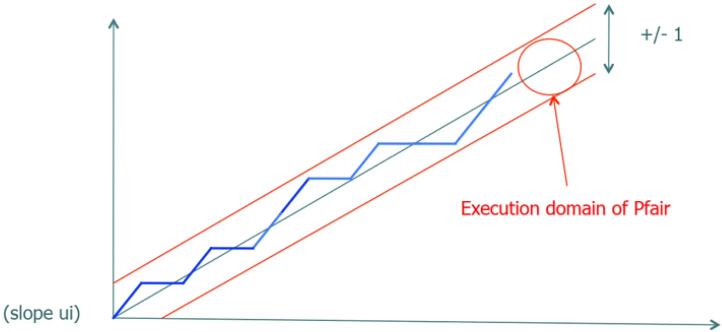
Pfair algorithms: principle

- Lag function: Difference between real and fluid execution
  - Discrete time, successive time slots [t,t+1]
  - Weight of a task: $\omega_i = u_i$
- Lag
  - $lag(T_i, t) = \omega_i t - \sum_{u=0}^{t-1} S(T_i, u)$
  - First term: Fluid execution
  - Second term: real execution, with S(T_i,u)=1 if Ti executed in slot u, else 0

Pfair schedule: for all time t, lag in interval [-1,1]

Pfair algorithms: principle

- Example



Pfair algorithms: principle

- Property
  - If a Pfair schedule exists, deadlines are met
- Exact test of existence of a Pfair schedule
  - $\sum_{i=1}^{n} u_i < m$

Full processor utilization!

Pfair algorithms: construction of a Pfair schedule

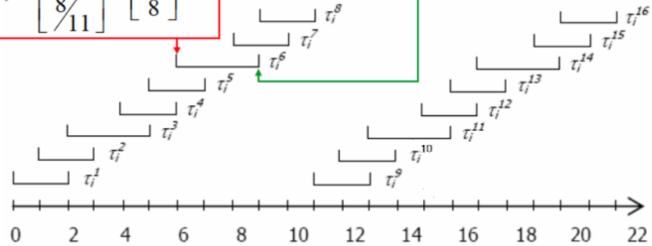
- Divide tasks in unity-length sub-tasks
  - Pfair condition: each subtask j executes in a time window between a pseudo-arrival and a pseudo deadline
  - Pseudo-arrival
    - r(T_i^j)=$\lfloor \frac{j-1}{\omega_i} \rfloor$
  - Pseudo-deadline
    - d(T_i^j)=$\lceil \frac{j}{\omega_i} \rceil$

Pfair algorithms: construction of a Pfair schedule

- Example (to be fixed)
  - $(C_i = 8, T_i = 11) \rightarrow \omega_i = u_i = 8/11$

$$r(\tau_i^6) = \left\lfloor \frac{6-1}{8/11} \right\rfloor = \left\lfloor \frac{55}{8} \right\rfloor = 6$$

$$d(\tau_i^6) = \left\lceil \frac{6}{8/11} \right\rceil = \left\lceil \frac{33}{4} \right\rceil = 9$$



Pfair algorithms: scheduling algorithms

- EPDF (Earliest Pseudo-Deadline First)
  - Apply EDF to pseudo-deadlines
  - Optimal only for m=2 (2 processors)
- Ongoing works
  - Reduce numbers of context switches and migrations while maintaining optimality

Conclusion

- Multi-processor scheduling is an active research area
- Ongoing works
  - Global multi-core scheduling
  - Semi-partitioned scheduling
  - Determining upper bounds of practical factors (preemption, migration, ...)
  - Implementation in real-time operating systems

The End