

Q1

1

$$\text{Weight matrix} = \begin{bmatrix} 0 & 4 & 0 & 0 \\ 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 4 & 0 \end{bmatrix}$$

$$[1, 1, 1, 1] \rightarrow -(4 + 4 + 4 + 4) = -16$$

$$[-1, -1, -1, -1] \rightarrow -(4 + 4 + 4 + 4) = -16$$

$$[1, 1, -1, -1] \rightarrow -(4 + 4 + 4 + 4) = -16$$

$$[-1, -1, 1, 1] \rightarrow -(4 + 4 + 4 + 4) = -16$$

$$[1, -1, 1, 1] \rightarrow -(-4 - 4 + 4 + 4) = 0$$

$$[1, -1, -1, 1] \rightarrow (4 + 4 + 4 + 4) = 16$$

The local minima are highlighted, because they have the lowest energy level (there can't be an energy level less than -16 with the weight matrix above). These patterns are stable and can be saved in the network.

2

The weight matrix is first calculated by making a vertical vector from a horizontal one and using inner multiplication on its transposition. After training on the patterns, we give the first vector as input and using the sign function, calculate the pattern it yields. We see that it is the same input patterns and with calculating the energy, we see that it has the least energy of all it's neighbors (with edit distance = 1). The second vector however doesn't result in the same behavior. The output is another vector which concludes that the input is not a stable pattern for the network, and it does not have the least level of energy either.

3

First, I resized the images to have the same dimensions and created binary images. This way, all the letters have the same contribution to the weight matrix. The rest is pretty much the same as the last question with the addition of the noise function. The noise function creates two lists of random samples, the samples being the indices of the original data. These random pixels of the images were then inverted to add noise. After feeding the noisy samples to the trained network, the accuracy was calculated by dividing the number of correct pixels to the number of all of them.

	10%	30%	60%
16	89.93	76.38	64.93
32	90.16	77.44	65.93
64	90.6	77.58	64.7

The accuracy reported in the table above is the median of the accuracy of all 10 letters. As expected, the noisier the images got, the less accurate they were. Also, when the font size increased, the images became more detailed so that contributed in increasing the accuracy as well.