Obligatory assignment radiation

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April 29, 2019

Hand in your assignment in a PDF with figures and explanations plus your modified script (it can be handed in as a notebook).

The following files should be downloaded:

atmos.csv	Temperate and pressure profile
r_gass.csv	Mixing ratios for H_2O and O_3
abs_kons.csv	Absorption constants H_2O and O_3
emiss.csv	Emissivity H_2O, O_3 and CO_2

These files can be loaded as pandas dataframes and the variable names should be more or less self explanatory. In addition you should have downloaded 'radiation_model.py' and a notebook with some start help.

1 About the model

The aim of this assignment is to make a simple numerical model which describes how absorption of short wave radiation from the sun and absorption/emission of long wave radiation from the earth surface leads to heating or cooling of the air in a vertical column in the atmosphere. The columns position in terms of latitude and time of year can be changed such that the incoming radiation at the top of the atmosphere changes.
Based on Beers law (equations 4.17, 4.30 and 4.32 in Wallace and Hobbs) for short wave radiation and Schwarzschilds equation (equation 4.41 and 4.42 in W&H) for long wave radiation, we set up a model which describes the flux density of the radiation (F defined in equation 4.5 in W&H) as a function of height, that is we find F(z). The model describes radiation in a vertical column from the surface and up to 50 km (see figure **??**). When $F(z)$ is known, we can compute the heating rates from radiation from equation 4.52, with some simplifications. An important simplification is that we only consider absorption and emission and thus completely neglect scattering and reflection (especially important for short wave radiation).

It is really an impossible task to make a simple radiation model which gives a somewhat correct answer, but this is not too bad! The general picture the model produces should comply with reality.
Even though the radiation equations in them selves are not too hard to understand, it can be tricky to do the calculations. Every equation needs to be solved per wavelength, but in reality we have "infinitely many wavelengths in the incoming radiation at the top of the atmosphere and that the earth itself emits. Different molecules will absorb differently for different wavelengths and this will give different heating- and cooling rates. Since we cannot do "infinitely many calculations, it is common (especially in climate models), to split the radiation into intervals of wavelengths, or so called "bands". Within each band, we can assume e.g. constant emissivity/absorptivity for each gas. We do this in this model.

The most common approach when computing radiation fluxes is a plane parallel one. Every layer is 1 km thick. Within each layer, we assume that the physical properties are **constant in time**: pressure, density, mixing ratios of H_2O, CO_2 and O_3, emissivity for long wave radiation.

The model includes a simple representation of the energy balance at the surface (see e.g. equation 9.18 - 9.21 in W&H.),

$$F* = F_{H_S} + F_{E_S} + F_{G_S} \qquad \text{(eq. 18 W\&H)}$$

where $F*$ is the net radiation density at the surface, F_{H_S} is the sensible heat flux into the air, F_{E_S} is the latent heat flux into the air and F_{G_S} is the heat flux into the ground (positive downwards).

In the model, we omit latent heat and assume $F_{G_S} = 0$ and F_{H_S} is given by:

$$F_{H_S} = \rho c_p C_H \mid V \mid (T_S - T_{air}) \qquad \text{(modified eq. 9.19 W\&H)}$$

Where T_S is the surface temperature, T_{air} is the temperature of the air, C_H is a dimensionless bulk transfer coefficient, ρ is the air density and c_p is the specific heat at constant pressure. (See text under eq 9.19 in W&H for explenation.)

In the assignment you will be modifying a program written in python in the script radiation_model.py. The places you are supposed to modify are marked by exercise number.

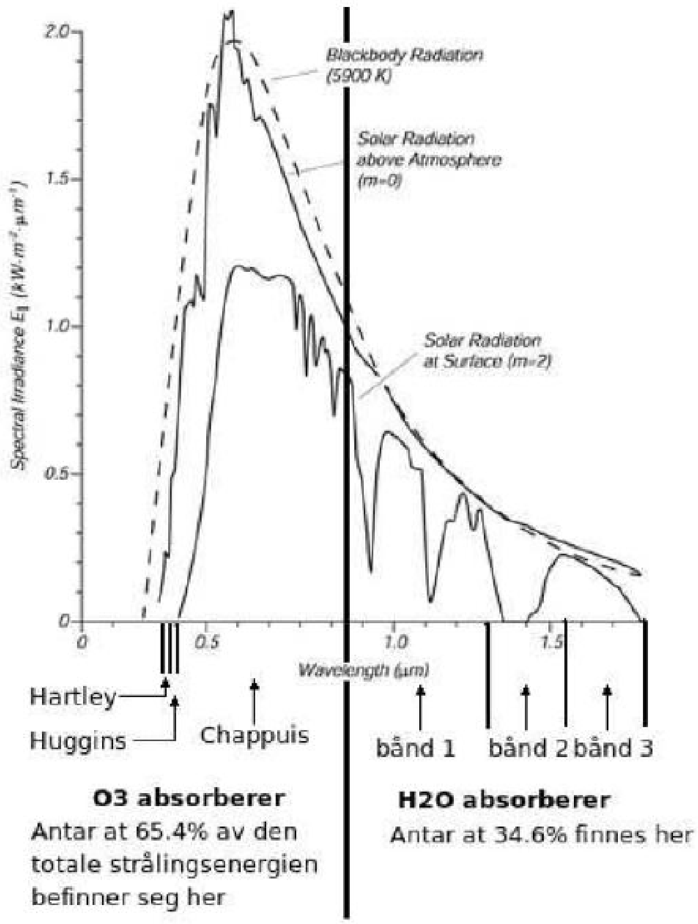

Figure 1: Incoming radiation at the top of the atmosphere and at the surface.

1.1 Short wave radiation

The solar constant is the amount of solar insolation per area measured at the top of the Earth's atmosphere in a plane normal to the incoming radiation (the "sun beam"). The solar constant contains all wavelengths, not only the visible light. The solar constant varies little and is set to be equal to 1367 W/m^2 in the model. Because gases absorb radiation at given wavelengths (ref section 4.4.3. in W&H) the model contains 6 different bands for short wave radiation, 3 for water vapor and 3 for ozone. We further assume that the water vapor can only absorb in 34% of the incoming solar radiation. This does not mean that water vapor absorbs 34% of the incoming radiation, but that the 34% of the total radiation is contained within the

Name of band	Wavelength interval	Available energy for band
H_2O 1	0.85 - 1.3 μm	73%
H_2O 2	1.3 - 1.55 μm	20%
H_2O 3	1.55 - 1.8 μm	7%

Table 1: Wavelenth bands H_2O

Name of band	Wavelength interval	Available energy for band
Hartley band 1	0.2 - 0.31 μm	0.2%
Huggins band	0.31- 0.4 μm	1%
Chappuis band	0.4 - 0.85 μm	98.8 %

Table 2: Wavelength bands ozone

wavelengths where water vapor absorbs. We further assume that every band absorbs only a fraction of this radiation. The same type of assumptions are done for O_3.

1.1.1 H_2O

Available energy for H_2O is 34.6% of the solar constant. This energy is distributed between the three bands.

1.1.2 Ozone

Available energy for O_3 is 65.4 % of the solar constant.

These are used in exercise 2 in the partitioning of the solar constant when you calculate the short wave flux.

1.2 Long wave radiation

We say that two gases overlap when they absorb radiation at the same wavelength. In the calculation of flux density for long wave radiation, we assume there is no overlap between the gases. This is not completely true, but we use this assumption the make the calculations easier.
You will also be calculating heating/cooling rates for each gas. This gives three calculations, one for each gas, but the equations are identical. They should have different emissivity, as this varies from gas to gas.

In the program the fluxes (SW_down, LW_up and LW_down) are divided into fluxes for each band. The flux for e.g. SW band for the ozone Hartley band is found in variable SW_down['O3_Hartley'] (the variable SW_down is a dictionary).

The variable declination angle in the program can vary between -23 and 23. Value -23 is winter solstice and +23 is summer solstice. Latitude is also given and is defined as positive on the northern hemisphere.

1.2.1 Surface temperature

In the model, we account for the flux of sensible heat from the surface to the atmosphere (SH0), but we neglect latent heat (assuming we have a dry column). The flux of sensible heat is calculated following a bulk formulae (eq. 9.19a W&H). In the atmosphere the added heat is mixed vertically through turbulence in the boundary layer. Turbulence is not accounted for in our simplified model, so we assume the flux of sensible heat is reduced vertically by a scalar height of H_SH which is given a constant value (4000 m in our model). When we have solved problem 1-5 below, the model has all the radiative fluxes at the surface. If we neglect the term for horizontal heat flux into the ground, we now have all the terms in eq. 9.18 in W&H and we can calculate how the surface temperature changes over time:

$$\frac{dT_S}{dt} = \frac{1}{C_S}(F * -SH0) \tag{1}$$

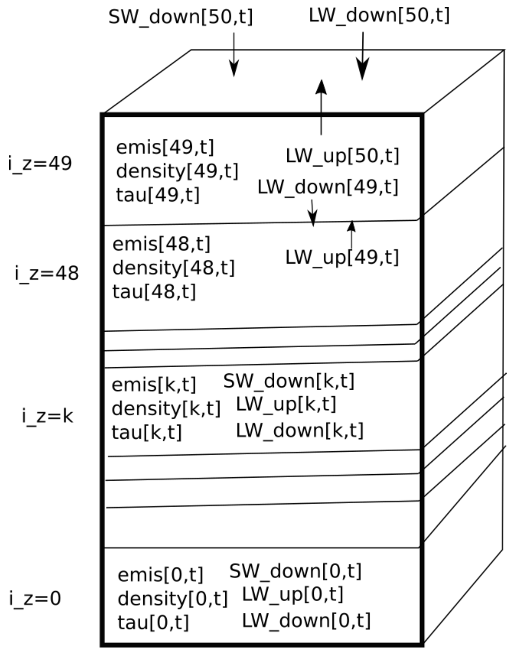

Figure 2: The structure of the model. We have 50 layers where each layer is 1000m thick. Emissivity, density, absorption constants and mixing ratio of the gases are constant within each layer. Temperature in the atmosphere T and the radiative fluxes change with time, t (the timestep is i_t in the program).

Problem 1: Optical thickness for short wave radiation

Since we assume a plane parallel atmosphere, we can use the expression for normal (vertical) optical thickness (τ):

$$\tau = \int_{z}^{z+\Delta z} k \cdot \rho \cdot r \cdot dz \tag{2}$$

The variables k and r are read in for each gas from 'r_gass.csv' (mixing ratios) and 'abs_kons.csv' (absorption constants).

a) Calculate the normal (i.e. in the vertical) optical thickness for each layer for each of the 6 band for water vapor and ozone by using a modified version of equation 4.32 in the book. The variable for each is called tau[band] where band is in ['H2O_1', 'H2O_2', 'H2O_3', 'O3_Hartley', 'O3_Huggins', 'O3_Chappuis'].

b) Plot the vertical distribution of tau for the first water vapor band and the Hartley band. Explain the vertical difference.

Problem 2: Flux of short wave radiation

a) Based on the optical thickness you calculated in Problem 1, calculate the flux density downwards of short wave radiation. We neglect reflection and scattering, so short wave radiation upwards is not calculated. The short wave flux density down for each band is kept in the variable 'SW_down[band]' where band is in ['H2O_1', 'H2O_2', 'H2O_3', 'O3_Hartley', 'O3_Huggins', 'O3_Chappuis']. 'SW_down[band][i_z, i_t]' is the value for layer index i_z and time index i_t.
Hint: Customize equation 4.31 to calculate SW_down from the top downwards. Cosine to the zenith angle is calculated in mu[i_t].

b) Make a figure of the flux density for two of the bands: band 1 for H_2O and Huggins for ozone.

Problem 3: Heating rate short wave

a) In Problem 2 we found the flux of short wave radiation downwards in each height. Use this to calculate the heating rate (in K/hour) for each layer and each hour of the day with equation 4.52 in W&H. The variable for the change in SW flux density is DeltaF_SW_down[band] and the SW heating rate is SW_heating_rates where band in ['H2O_1', 'H2O_2', 'H2O_3', 'O3_Hartley', 'O3_Huggins', 'O3_Chappuis'].
The density of the air is in self.density[i_z]. The heat capacity is variable C_p for air and is defined in the beginning of the script.

b) Let the model run in 24 hours.
Make a figure where you plot the heating after 24 hours. The figures should show the heating rate in Kelvin per hour. Explain why the heating rate varies with height as it does.
Hint: equations 4.52 in W&H gives heating rates per second. We want per hour. In the notebook you download, there is a script for plotting heating rates in K/day. These should be similar to figure 4.29 in W&H.

c) Compare the plots with the optical thickness from Problem 1. Is maximal heating in the same height as maximal optical thickness? Why/why not? Explain!

Problem 4 Flux upwards of long wave (LW) radiation

For each long wave radiation we must also take into account the emission of LW radiation from the atmosphere itself as a source term in each layer (the atmosphere is not warm enough to emit SW+ radiation!). (Ref Schwarzschilds equation 4.42 in W&H).
For each layer we can write the change in upward flux density vertically as:

$$\frac{dF}{dz} = E - A \tag{3}$$

where E is the emission of LW radiation and A is absorption.
The emission term can be written as

$$E = \sigma T^4 \cdot \varepsilon \tag{4}$$

where E has units W/m^2, and ε is the flux emissivity per vertical length unit.
The absorption term can be written as

$$A = F \cdot \alpha = F \cdot \varepsilon \tag{5}$$

because of Kirchoff's law which tells us $\alpha = \varepsilon$.
We then have that the equation for long wave radiation in each layer can be written as:

$$\frac{dF}{dz} = \sigma T^4 \cdot \varepsilon - F(z) \cdot \varepsilon. \tag{6}$$

This is equation 4.41 in W&H.
Since we are considering each layer separately and assume that the temperature and flux emissivity is constant within each layer, this differential equation can be solved as long as we have the boundary conditions at the bottom of the layer. For the surface we have that the upward flux of LW radiation is given by Stefan Boltzmann's law (we assume the surface is a 100% black body).

a) Solve the differential equation over each layer upwards (use the flux density upwards at the top of the layer under as a boundary condition for the next layer) and calculate the upwards flux of LW radiation for each layer for each gas. The variable is LW_up[band] for band in ['H2O','O3','CO2'].
Hint 1: Maybe the easiest way to solve it is by separating the variables:

$$\frac{dF}{d(T^4 - F(z))} = \varepsilon \cdot dz \tag{7}$$

over the interval $[z, z + \Delta z]$.
Hint 2: Substitute $\sigma T^4 - F(z)$ with a new variable and integrate wrt. z.

b) Let the model run for 24 hours and plot the vertical distribution of the heating rate for each gas. Explain what you see.

Problem 5: Flux downwards for long wave radiation

a) As in Problem 4, calculate the flux density *downwards* of long wave radiation (LW_down). We now need the boundary condition at the top of the model. This is set to be 10 W/m^2 (variable In set in the program). That is LW_down[band][50,i_t]=In for each band in ['H2O','O3','CO2'].

b) Let the model run for 24 hours and make a plot of the vertical distribution of LW_down[band] for each band. Explain what you see.

Problem 6: Heating rate long wave radiation

a) In the same way as in Problem 3, calculate the heating rate from the long wave flux. You calculate for each gas separately (LW_heating_rates[band] for each band in ['H2O','O3','CO2']).
Hint: For long wave radiation, we also have a source term (at the surface and in the atmosphere) which is temperature dependent (Stefan-Boltzmann's law). In the program the temperature is initialized, but changes over time to reach a new equilibrium. You thus need to make a judgment on how long you need to run the model.

b) Make a plot of the long wave heating rates for each gas in K/h. How is the diurnal variation in the long wave heating rate compared to what you found in Problem 3? Explain (You need to find a way to plot the diurnal variation here).

The notebook includes a plot of heating rates per day. These should be similar to figure 4.29 in W&H.