Assignment

Q12. Given three students:

A: 4, 80, 654

B: 6, 85, 726

C: 3, 70, 603

$$\text{Matrix} = \begin{bmatrix} 4 & 80 & 654 \\ 6 & 85 & 726 \\ 3 & 70 & 603 \end{bmatrix}_{3\times3}$$

Q13. Assign weight to the features:

weights = 0.4, 0.3, 0.3

student B = 6, 85, 726

$$\text{Dot product} = (0.4 \times 6) + (0.3 \times 85) + (0.3 \times 726)$$

$$= 2.4 + 25.5 + 217.8$$

$$= 245.7$$

Q.14. Feature Matrix:

$$\begin{bmatrix} 4 & 80 & 654 \\ 6 & 85 & 726 \\ 3 & 70 & 603 \end{bmatrix} \times \begin{bmatrix} 0.4 \\ 0.3 \\ 0.3 \end{bmatrix}$$

$$\Rightarrow a[0] = 4 \times 0.4 + 80 \times 0.3 + 654 \times 0.3$$

$$a[1] = 6 \times 0.4 + 85 \times 0.3 + 726 \times 0.3$$

$$a[2] = 3 \times 0.4 + 70 \times 0.3 + 603 \times 0.3$$

$$= \begin{bmatrix} 1.6 + 24 + 196.2 \\ 2.4 + 25.5 + 217.8 \\ 1.2 + 21 + 180.9 \end{bmatrix}$$

$$= \begin{bmatrix} 221.8 \\ 245.7 \\ 203.1 \end{bmatrix}$$

Q.15. Perform a linear transformation to double the study hours using this matrix:

$$T = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow$ A: 4, 80, 654

$$\Rightarrow \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 80 \\ 654 \end{bmatrix}$$

$$= (4\times2) + (80\times0) + (654\times0) = 8$$

$$(4\times0) + (80\times0) + (654\times0) = 80$$

$$(4\times0) + (80\times0) + (654\times1) = 654$$

$$= \begin{bmatrix} 8 \\ 80 \\ 654 \end{bmatrix}$$

Q.16) Normalise the 'Study hours' in Q2 to Scale them between 0 & 1.

ans) Normalised Value $= \dfrac{x - x_{min}}{x_{max} - x_{min}}$ Min = 3

Max = 6

$$\Rightarrow \frac{4-3}{6-3} = 0.333$$

$$\Rightarrow \frac{6-3}{6-3} = \underline{\underline{1}}$$

$$\Rightarrow \frac{3-3}{6-3} = \underline{\underline{0}}$$

$$\Rightarrow \underline{\underline{[0.333, 1, 0]}}$$

Q.17) Euclidean distance b/w A & B

$A = [4, 80, 654]$

$B = [6, 85, 726]$

$$\text{Euclidean distance} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

$$= \sqrt{(6-4)^2 + (85-80)^2 + (726-654)^2}$$

$$= \sqrt{2^2 + 5^2 + 72^2}$$

$$= \sqrt{4 + 25 + 5184}$$

$$= \sqrt{5213}$$

$$= \underline{\underline{72.201}}$$

Q18. Matrix A is (5,3)

Matrix B is (3,1)

If matrix A weight was (m,n) and matrix B (n,p)

then ⇒ (m,p)

⇒ (5,1)

Q.19). No. of students = 100

No. of passed = 60

$$\text{probability} = \frac{60}{100} = 0.6$$

Q.20) Attended students = 50

passed students = 40

$$\text{probability} = \frac{40}{50} = 0.8$$

Q.21) Probability of passing $= 70\% = 0.7$

Probability of ~~passing~~ fails $= 100\% - 70\% = 30\%$

$= 1 - 0.7$

$= \underline{\underline{0.3}}$

Q.22).

$A = \{2, 4, 6\}$

$B = \{4, 5, 6\}$

$$P(A) = \frac{\text{fav. outcome}}{\text{total}} = \frac{3}{6} = \frac{1}{2} = \underline{\underline{0.5}}$$

$$P(B) = \frac{3}{6} = \frac{1}{2} = \underline{\underline{0.5}}$$

$P(A \cap B) \Rightarrow A \cap B = \{4, 6\}$

$$\Rightarrow P(A \cap B) = \frac{2}{6} = \underline{\underline{\frac{1}{3}}}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/3}{1/2} = \underline{\underline{\frac{2}{3}}}$$

Q. 23) A) $\begin{bmatrix} 2x+y & 3 & 10 \\ y+1 & -2 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 10 \\ 3 & -2 & 0 \end{bmatrix}$

$\Rightarrow 2x+y = 2$ $\quad y+1=3$

$2x+2 = \cancel{0}$ $\quad 2x=0$ $\quad x=0$ $\quad y=3-1$

$x+y = \frac{2}{2} = \underline{1}$ $\quad y=2$

$y+1 = 3 \Rightarrow y = 3-1 = \underline{\underline{2}}$

$x+y=1, \; y=2$

$\Rightarrow x+2=1 \Rightarrow x = 2-2 = \underline{0}$

$x = 0, \; y=2$

B) $\begin{bmatrix} 6 & -4 & -6 & x-y \end{bmatrix} = -2\begin{bmatrix} -3 & 2 & 2x+2y & 13 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 6 & -4 & -6 & x-y \end{bmatrix} = \begin{bmatrix} 6 & -4 & -4x-4y & -26 \end{bmatrix}$

$x-y = -26$

$-4x-4y = -6$

$\Rightarrow x = -26+y \quad \times 4$

$-4x = -6+4y$

$\Rightarrow 4x = -104 + \cancel{4y}$

$-4x = -6 + \cancel{4y}$

$8x = -98$

$$= x = \frac{-78}{8}$$

$$x = -12.25 \quad \text{—— (1)}$$

$$x - y = -26$$

$$\Rightarrow \cancel{x + 12.25 = -26}$$

$$\cancel{x = -26 - 12.25}$$

$$= \cancel{-38.5}$$

$$\cancel{x} - 12.25 - y = -26$$

$$= -26 + 12.25 = y$$

$$= y = 13.75$$

$$x = (-12.25)\ \cancel{13.}\ (y = 13.75)$$

*. $A = \begin{bmatrix} -1 & 1 & -2 \\ 0 & -2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 2 & 0 \\ 0 & -3 & 4 \\ -1 & -2 & 3 \end{bmatrix}$

1). AB

$$(-1)(-1) + (1)(0) + (-2)(-1)$$

$$= 1 + 0 + 2 = \underline{3}$$

First column Second row

$(-1)(0) + (1)(4) + (-2)(3)$

$= 0 + 4 - 6$

$= \underline{\underline{-2}}$

$(0)(-1) + (2)(0) + (1)(-1)$

$= 0 + 0 - 1 = \underline{\underline{-1}}$

$(0)(2) + (-2)(3) + (1)(-2)$

$0 + 6 - 2 = \underline{\underline{4}}$

$(0)(0) + (-2)(4) + (1)(3)$

$= 0 - 8 + 3$

$= \underline{\underline{-5}}$

2) Not Possible

3) Not Possible

4) Not possible

5). FE

$(3)(-1) + (0)(2) + (11)(1)$

$= -3 + 0 + 11$

$= \underline{\underline{8}}$

$(0)(3) + (-3)(0) + (4)(1)$

$= 0 + 0 + 44$

$= \underline{44}$

$(2)(3) + (4)(0) + (-3)(1)$

$= 6 + 0 - 33$

$= \underline{\underline{-27}}$

14) Given matrix :

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

Find the eigen values and eigen vectors.

Eigen values = $\det(A - \lambda I) = 0$

$$\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = 0$$

$$\begin{bmatrix} 2-\lambda & 0 \\ 0 & 3-\lambda \end{bmatrix} = 0$$

$\underline{\underline{\lambda = 2}}$ or $\underline{\underline{\lambda = 3}}$

Eigen Vectors

For $\lambda = 2$

$(A - \lambda I)x = 0$

$$\begin{bmatrix} 2 & -2 & 0 \\ 0 & & 3-2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$x_2 = 0$

$x_1 = 1$

$x = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

for $\lambda = 3$

$$\begin{bmatrix} 2-3 & 0 \\ 0 & 3-3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$x_1 = 0$

$x_2 = 1$

$x = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

15) Given Matrix:

$$B = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$$

Find the eigen values;

$(B - \lambda I) = 0$

$$\begin{bmatrix} (4-\lambda) & 1 \\ 2 & (3-\lambda) \end{bmatrix} = 0$$

$= (4-\lambda)(3-\lambda) - (1)(2)$

$= 12 - 4\lambda - 3\lambda + \lambda^2 - 2$

$= \lambda^2 - 7\lambda + 10$

$\therefore \lambda = \underline{\underline{5 \cdot \text{or } 2}}$

16). Find Eigen values & Eigen vectors:

$$A = \begin{bmatrix} -2 & 1 \\ 12 & 3 \end{bmatrix}$$

$\det(A - \lambda I) = 0$

$$\begin{bmatrix} -2-\lambda & 1 \\ 12 & 3-\lambda \end{bmatrix} = 0$$

$$(-2-\lambda)(-3-\lambda) - (1)(12) = 0$$

$$6 + 2\lambda + 3\lambda + \lambda^2 - 12 = 0$$

$$\lambda^2 + 5\lambda - 6 = 0$$

$$\lambda_1 = 1 \text{ or } \lambda_2 = -6 \,//$$

For $\lambda_1 = 1$

$$(A - \lambda I)x = 0$$

$$\begin{bmatrix} -2-1 & 1 \\ 12 & -3-1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -3 & 1 \\ 12 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-3x_1 + x_2 = 0$$

$$x_2 = 3x_1$$

let $x_1 = 1$

$$x_2 = 3$$

$$x = \begin{bmatrix} 1 \\ 3 \end{bmatrix} //$$

For $\lambda_2 = -6$

$$\begin{bmatrix} -2-(-6) & 1 \\ 12 & -3-(-6) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 1 \\ 12 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$4x_1 + x_2 = 0 \qquad x_2 = -4x_1$

let $x_1 = 0 \Rightarrow x_2 = -4$

$\therefore x = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$ //

Q. 17) 1st & 2nd

i) $(4\times3 + 2\times1 + 1\times0)$

$= 12 + 2 = \underline{\underline{14}}$

ii) 1st & 3rd

$(4\times1 + 2\times3 + 1\times2)$

$= 4 + 6 + 2$

$= \underline{\underline{12}}$

Better Match = product A = 14 //

18). Distance (km), traffic level (0-1)

weight vector : weights (w) = $\begin{bmatrix} 0.5 \\ 1.2 \\ -0.8 \end{bmatrix}$

Delivery location Matrix :

$$x = \begin{bmatrix} 10 & 0.5 & 0.6 \\ 15 & 0.8 & 0.3 \\ 8 & 0.3 & 0.9 \end{bmatrix}$$

Wx, Predicted delivery times ;

$$wx = 10 \times 0.5 + (0.5)(1.2) + (0.6)(-0.8)$$

$$= 5 + 0.6 - 0.48$$

$$= \underline{\underline{5.12}}$$

$$(15 \times 0.5) + (0.8 \times 1.2) + (0.3)\cancel{\times}(-0.8)$$

$$= 7.5 + 0.96 - 0.24$$

$$= \underline{\underline{8.22}}$$

$$= (8)(0.5) + (0.3)(1.2) + (0.9)(-0.8)$$

$= 4 + 0.36 - 0.72$

$= \underline{\underline{3.69}}$

predicted delivery times $= \begin{bmatrix} 5.12 \\ 8.22 \\ 3.64 \end{bmatrix}$ //

19). $P(\text{cloudy} \mid \text{Rainy}) = \left(\frac{25}{30}\right)$

$P(\text{cloudy}) = \frac{40}{100}$

Using Bayer's Theorum ;

$$P(\text{Rain} \mid \text{cloudy}) = \frac{P(\text{cloudy} \mid \text{Rain}) \cdot P(\text{Rain})}{P(\text{cloudy})}$$

$$= \frac{(25/30)(30/100)}{(40/100)} = \frac{25}{40}$$

$= \underline{\underline{0.625}}$

20). Customer = [3, 1, 12 5]

weights for prediction model : [1.5, 2.0, -0.3]

Compute the charn score using dot product.

charn Score = $1.5 \times 3 + 2.0 \times 1 + (-0.3) \times 12$

$= 4.5 + 2 - 3.6$

$= \underline{\underline{2.9}}$

21) a) P (study > 5 hours)

$\frac{50}{100} = 0.5$ //

b) P (Pass)

$= \frac{70}{100} = \underline{\underline{0.7}}$

c) P (pass | study > 5)

$\rightarrow \frac{B(40/100)}{50/100}$

$= \underline{\underline{0.8}}$