Hamming Code

min value of 'k' parity bits required are

$$2^k \geq n + k + 1$$

n is number of bits in message

k is number of parity bits.

∴ Number of bits in Hamming Code are $n+k$.

Let us find Hamming code for message

d_4	d_3	d_2	d_1
1	0	0	0

Find number of Parity bits.

$$2^k \geq n + k + 1$$

$$\therefore 2^k \geq 4 + k + 1$$

∴ If $k=1$ $\quad 2 \geq 6$ ×

$k=2$ $\quad 4 \geq 7$

$k=3$ $\quad 8 \geq 7$ ✓

So correct value of $\underline{k=3}$

∴ parity bits required are P_1, P_2, P_3

∴ 7 bit hamming code is

7	6	5	4	3	2	1
d_4	d_3	d_2	P_3	d_1	P_2	P_1

7	6	5	4	3	2	1
1	0	0	P_3	0	P_2	P_1

$P_1 = d_3 \oplus d_5 \oplus d_7 = 0 \oplus 0 \oplus 1 = 1$

$P_2 = d_3 \oplus d_6 \oplus d_7 = 0 \oplus 0 \oplus 1 = 1$

$P_4 = d_5 \oplus d_6 \oplus d_7 = 0 \oplus 0 \oplus 1 = 1$

Hamming Code = 1 0 0 1 0 1 1

Error Correction

If 1 0 0 1 0 1 1 is transmitted

and 1 0 0 1 1 1 1 is received

7 6 5 4 3 2 1

1 bit error

Check bit $c_1 = D_1 \oplus D_3 \oplus D_5 \oplus D_7 = 1 \oplus 1 \oplus 1 \oplus 0 \oplus 1 = 1$

Check bit $c_2 = D_2 \oplus D_3 \oplus D_6 \oplus D_7 = 1 \oplus 1 \oplus 0 \oplus 1 = 1$

Check bit $c_4 = D_4 \oplus D_5 \oplus D_6 \oplus D_7 = 1 \oplus 0 \oplus 0 \oplus 1 = 0$

c_4	c_2	c_1
0	1	1

=

3rd bit is faulty.

Hamming Code is Error Detection & Correction code.

Let n be the number of bits in message

k, be the number of parity bits to be added.

Number of bits to be added are given by formulae

$$2^k \geq n + k + 1$$

∴ total number of bits in hamming code or n+k.

Let us say you want to transmit 4 bits of data.

∴ no of parity bits required are 3.

as $2^k \geq n + k + 1$

for k = 3 equation gets satisfied.

We have to find minimum value of k.

So representation for Hamming Code Data will

D7	D6	D5	P4	D3	P2	P1

where D7, D6, D5, D4 are data bits. $b_4\ b_3\ b_2\ b_1$

P4, P2, P1 are Parity Bits.

$P_4 = D_5 \oplus D_6 \oplus D_7 =$

$P_2 = D_3 \oplus D_6 \oplus D_7 =$

$P_1 = D_3 \oplus D_5 \oplus D_7 =$

Suppose Data to be transmitted, is

1 1 0 0

Put in format

D_7	D_6	D_5	P_4	D_3	P_2	P_1
1	1	0	P_4	0	P_2	P_1

$P_1 = D_3 \oplus P_5 \oplus D_7 = 0 \oplus 0 \oplus 1 = 1$

$P_2 = D_3 \oplus D_6 \oplus D_7 = 0 \oplus 1 \oplus 1 = 0$

$P_4 = D_5 \oplus D_6 \oplus D_7 = 0 \oplus 1 \oplus 1 = 0$

∴ Final Data is

D_7	D_6	D_5	P_4	D_3	P_2	P_1
1	1	0	0	0	0	1

IF Data received is Faulty.

Say

D_7	D_6	D_5	P_4	D_3	P_2	P_1
1	1	1	0	0	0	1

Calculate check bits.

$C_4 = P_4 \oplus D_5 \oplus D_6 \oplus D_7$
$= 0 \oplus 1 \oplus 1 \oplus 1 = 1$

$C_2 = P_2 \oplus D_3 \oplus D_5 \oplus D_7$
$= 0 \oplus 0 \oplus 1 \oplus 1 = 0$

$C_1 = P_1 \oplus D_3 \oplus D_5 \oplus D_7$
$= 1 \oplus 0 \oplus 1 \oplus 1 = 1$

∴ $C_4 C_2 C_1 = (1\,0\,1)_2$ ∴ error in 5th bit