# AI1103 ASSIGNMENT 4

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### 1 Question-CSIR UGC NET June 2012,Q.50

Let $X_1, X_2, ....$ be i.i.d N(1,1) random variables.Let $S_n = X_1^2 + X_2^2 + ... + X_n^2$ for $n \geq 1$.Then

$$\lim_{n \to \infty} \frac{Var(S_n)}{n} =$$

(A) 4
(B) 6
(C) 1
(D) 0

### 2 Solution-CSIR UGC NET June 2012,Q.50

**Definition 1** (NON-CENTRAL CHI SQUARE DISTRIBUTION)**.** *Let $X_1, X_2, X_3., X_i, .X_n$ be n independent, normally distributed random variables with means $\mu_i$ and unit variances.Then the random variable*

$$\sum_{i=0}^{n} X_i$$

*is distributed according to the non-central chi square distribution.It has two parameters 'k' which specifies the number of degrees of freedom (i.e. the number of $X_i$), and '$\lambda$' which is called non-centrality parameter given by,*

$$\lambda = \sum_{i=0}^{n} \mu_i^2 \qquad (2.0.1)$$

**Lemma 2.1.** *Moment generating function of a non-central chi square distributed random variable X is given by,*

$$M_X(t) = \frac{e^{\frac{\lambda t}{1-2t}}}{(1-2t)^{\frac{n}{2}}} \qquad (2.0.2)$$

**Lemma 2.2.** *If two normal random variables $X, Y$ are independent then $X^2, Y^2$ are also independent.*

From lemma 2.2,Since $X_1, X_2, X_3, ...$ are mutually independent random variables therefore the random variables $X_1^2, X_2^2, X_3^2, ....$ are also mutually independent.
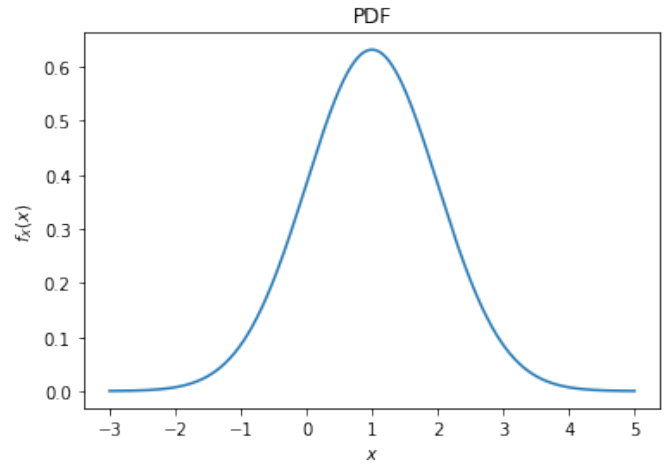


Fig. 4: PDF of $X_1, X_2, ..$

Given,

$$S_n = X_1^2 + X_2^2 + .. + X_n^2 \qquad (2.0.3)$$

From definition 1, Since $X_1, X_2, ...$ are i.i.d N(1,1) normal random variables therefore the distribution of the random variable $S_n = X_1^2 + X_2^2 + ...X_n^2$ is a non-central chi square distribution with 'n' degrees of freedom and non centrality parameter '$\lambda$' given by

$$\lambda = \sum_{i=1}^{n} (\mu_i)^2 \qquad (2.0.4)$$

$$\lambda = \sum_{i=1}^{n} 1 = n \qquad (2.0.5)$$

From Lemma 2.1 Moment generating function of $S_n$ is given by,

$$M_{S_n}(t) = \frac{e^{\frac{nt}{1-2t}}}{(1-2t)^{\frac{n}{2}}} \qquad (2.0.6)$$

and variance of $S_n$,

$$Var(S_n) = E(S_n^2) - (E(S_n))^2 \qquad (2.0.7)$$

**Lemma 2.3.** *The nth moment of a random variable $X$ whose MGF is $M_X(t)\left(\equiv E(e^{tX}) = \sum_{k=0}^{n} \frac{(tX)^k}{k!}\right)$ is given by,*

$$E(X^n) = \frac{d^n}{dt^n}(M_X(t))\,|_{t=0} \qquad (2.0.8)$$

From lemma 2.3 ,

$$E(S_n) = \frac{d}{dt}(M_{S_n}(t))\,|_{t=0} \qquad (2.0.9)$$

$$E(S_n) = \frac{2ne^{\frac{nt}{1-2t}}}{(1-2t)^{\frac{n}{2}+2}}(1-t)|_{t=0} \qquad (2.0.10)$$

$$\implies E(S_n) = 2n \qquad (2.0.11)$$

From lemma 2.3,

$$E(S_n^2) = \frac{d^2}{dt^2}(M_{S_n}(t))\,|_{t=0} \qquad (2.0.12)$$

$$E(S_n^2) = \frac{2ne^{\frac{nt}{1-2t}}}{(1-2t)^{\frac{n}{2}+4}}(n(1-t) - (1-2t)^2$$
$$+ (1-t)(1-2t)(n+4))|_{t=0} \quad (2.0.13)$$

$$E(S_n^2) = \frac{2n}{1^{n/2+2}}\left(n - 1^2 + n + 4\right) \qquad (2.0.14)$$

$$E(S_n^2) = 2n \times (2n + 3) \qquad (2.0.15)$$

$$\implies E(S_n^2) = 4n^2 + 6n \qquad (2.0.16)$$

Substituting equations (2.0.11) and (2.0.16) in (2.0.7),we get

$$Var(S_n) = 4n^2 + 6n - (2n)^2 \qquad (2.0.17)$$

$$\implies Var(S_n) = 6n \qquad (2.0.18)$$

and

$$\lim_{n\to\infty}\frac{Var(S_n)}{n} = \lim_{n\to\infty}\frac{6n}{n} = \lim_{n\to\infty} 6 = 6$$

Hence,option(B) is correct.