AI1103 ASSIGNMENT 4

Name:MANNAM SARANDEEP,Rollno:CS20BTECH11030

Download the python code from

```
https://github.com/sarandeepmannam/
    ASSIGNMENT4/blob/main/Assignment4.py
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and latex-tikz code from

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https://github.com/sarandeepmannam/
    ASSIGNMENT4/blob/main/Assignment4.tex
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1 Question-CSIR UGC NET June 2012,Q.50

Let $X_1, X_2, ...$ be i.i.d N(1,1) random variables.Let $S_n = X_1^2 + X_2^2 + ... + X_n^2$ for $n \geq 1$.Then

$$\lim_{n\to\infty} \frac{Var(S_n)}{n} =$$

(A) 4
(B) 6
(C) 1
(D) 0

2 Solution-CSIR UGC NET June 2012,Q.50

Definition 1 (NON-CENTRAL CHI SQUARE DISTRIBUTION)**.** *Let $X_1, X_2, X_3., X_i, .X_n$ be n independent, normally distributed random variables with means μ_i and unit variances.Then the random variable*

$$\sum_{i=0}^{n} X_i$$

is distributed according to the non-central chi square distribution.It has two parameters 'k' which specifies the number of degrees of freedom (i.e. the number of X_i), and 'λ' which is called non-centrality parameter given by,

$$\lambda = \sum_{i=0}^{n} \mu_i^2 \tag{2.0.1}$$

Lemma 2.1. *Moment generating function of a non-central chi square distributed random variable X is given by,*

$$M_X(t) = \frac{e^{\frac{\lambda t}{1-2t}}}{(1-2t)^{\frac{n}{2}}} \tag{2.0.2}$$

We know that if two normal random variables X, Y are independent then X^2, Y^2 are also independent. Since $X_1, X_2, X_3, ...$ are mutually independent random variables therefore the random variables $X_1^2, X_2^2, X_3^2, ...$ are also mutually independent.

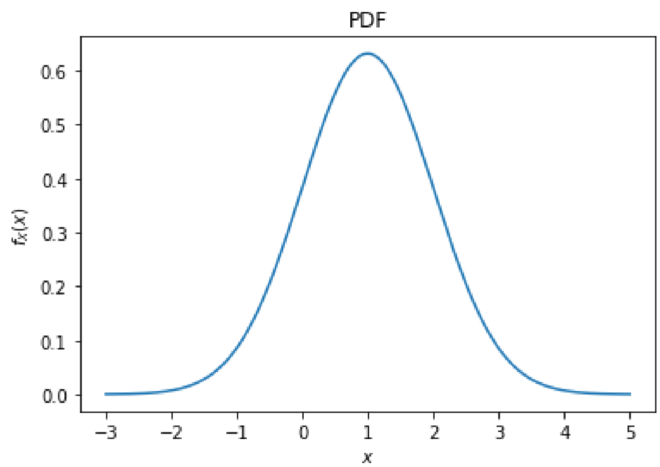

Fig. 4: PDF of $X_1, X_2, ..$

Given,

$$S_n = X_1^2 + X_2^2 + .. + X_n^2 \tag{2.0.3}$$

From definition 1, Since $X_1, X_2, ...$ are i.i.d N(1,1) normal random variables therefore the distribution of the random variable $S_n = X_1^2 + X_2^2 + ...X_n^2$ is a non-central chi square distribution with 'n' degrees of freedom and non centrality parameter 'λ' given by

$$\lambda = \sum_{i=1}^{n} (\mu_i)^2 \tag{2.0.4}$$

$$\lambda = \sum_{i=1}^{n} 1 = n \tag{2.0.5}$$

From Lemma 2.1,

$$M_{S_n}(t) = \frac{e^{\frac{nt}{1-2t}}}{(1-2t)^{\frac{n}{2}}} \tag{2.0.6}$$

and

$$Var(S_n) = E(S_n^2) - (E(S_n))^2 \tag{2.0.7}$$

From the properties of MGF,

$$E(S_n) = \frac{d}{dt}\left(M_{S_n}(t)\right)|_{t=0} \tag{2.0.8}$$

$$E(S_n) = \frac{2ne^{\frac{nt}{1-2t}}}{(1-2t)^{\frac{n}{2}+2}}(1-t)|_{t=0} \tag{2.0.9}$$

$$\implies E(S_n) = 2n \tag{2.0.10}$$

From the properties of MGF,

$$E(S_n^2) = \frac{d^2}{dt^2}\left(M_{S_n}(t)\right)|_{t=0} \tag{2.0.11}$$

$$E(S_n^2) = \frac{2ne^{\frac{nt}{1-2t}}}{(1-2t)^{\frac{n}{2}+4}}(n(1-t) - (1-2t)^2 + (1-t)(1-2t)(n+4))|_{t=0} \tag{2.0.12}$$

$$E(S_n^2) = \frac{2n}{1^{n/2+2}}\left(n - 1^2 + n + 4\right) \tag{2.0.13}$$

$$E(S_n^2) = 2n \times (2n+3) \tag{2.0.14}$$

$$\implies E(S_n^2) = 4n^2 + 6n \tag{2.0.15}$$

Substituting equations (2.0.10) and (2.0.15) in (2.0.7),we get

$$Var(S_n) = 4n^2 + 6n - (2n)^2 \tag{2.0.16}$$

$$\implies Var(S_n) = 6n \tag{2.0.17}$$

and

$$\lim_{n\to\infty} \frac{Var\,(S_n)}{n} = \lim_{n\to\infty} \frac{6n}{n} = \lim_{n\to\infty} 6 = 6$$

Hence,option(B) is correct.