(f) **[Bonus question]:** Can you identify which terms in $\vec{H}$ and $\vec{E}$ contributed to the $\vec{P}_{avg}$, and which terms did not? These field terms that contribute to the real radiated power are called the far-field terms, while the terms which do not contribute are called the near-field terms. [Bonus: upto +3 marks]

(g) **[Bonus question]:** Radiation resistance is defined as $R_{rad} = 2 * P_{total}/I_0^2$, where P_{total} is the total real time-averaged radiated power in Watts. What is the radiation resistance of this Herztian dipole? [Bonus: upto +3 marks]

[*Hint: Use spherical coordinates. There is no variation in quantities along* ϕ, *so* $\partial/\partial\phi = 0$. *Required formulas are given in the formula sheet.*]

Total for Question 6: 17

____________End of Questions ____________

Some useful formulas and equations are given on this and the next page.

Formula sheet

Vector calculus and identities

Gradient

Cartesian: $\nabla V = \frac{\partial V}{\partial x}\hat{x} + \frac{\partial V}{\partial y}\hat{y} + \frac{\partial V}{\partial z}\hat{z}$

Cylindrical: $\nabla V = \frac{\partial V}{\partial \varrho}\hat{\varrho} + \frac{1}{\varrho}\frac{\partial V}{\partial \phi}\hat{\phi} + \frac{\partial V}{\partial z}\hat{z}$

Spherical: $\nabla V = \frac{\partial V}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{\phi}$

Curl

Cartesian:

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$$

Cylindrical:

$$\nabla \times \vec{F} = \frac{1}{\varrho}\begin{vmatrix} \hat{\varrho} & \varrho\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial \varrho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ F_\varrho & \varrho F_\phi & F_z \end{vmatrix}$$

Divergence

Cartesian: $\nabla \cdot \vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$

Cylindrical: $\nabla \cdot \vec{F} = \frac{1}{\varrho}\frac{\partial(\varrho F_\varrho)}{\partial \varrho} + \frac{1}{\varrho}\frac{\partial F_\phi}{\partial \phi} + \frac{\partial F_z}{\partial z}$

Spherical: $\nabla \cdot \vec{F} = \frac{1}{r^2}\frac{\partial(r^2 F_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(F_\theta \sin\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial F_\phi}{\partial \phi}$

Spherical:

$$\nabla \times \vec{F} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ F_r & rF_\theta & r\sin\theta F_\phi \end{vmatrix}$$

$$\nabla \times \nabla \times \vec{F} = \nabla\left(\nabla \cdot \vec{F}\right) - \nabla^2\vec{F}$$

$$\nabla \cdot \left(\vec{A} \times \vec{B}\right) = \vec{B} \cdot \nabla \times \vec{A} - \vec{A} \cdot \nabla \times \vec{B}$$

Maxwell's Equations

Differential form

$\nabla \cdot \vec{D} = \rho_v$

$\nabla \cdot \vec{B} = 0$

$\nabla \times \vec{E} = -\mu\frac{\partial \vec{H}}{\partial t}$

$\nabla \times \vec{H} = \vec{J} + \epsilon\frac{\partial \vec{E}}{\partial t}$

Integral form

$\oint_{surface} \vec{D} \cdot d\vec{a} = Q_{enc}$

$\oint_{surface} \vec{B} \cdot d\vec{a} = 0$

$\oint \vec{E} \cdot d\vec{l} = -\iint \frac{\partial \vec{B}}{\partial t} \cdot d\vec{a}$

$\oint \vec{H} \cdot d\vec{l} = \iint \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) \cdot d\vec{a}$

$\vec{D} = \epsilon\vec{E}$; $\vec{B} = \mu\vec{H}$; Lorentz Force $= q\left(\vec{E} + \vec{v} \times \vec{B}\right)$; $\nabla \times \vec{A} = \vec{B}$; $\nabla \cdot \vec{A} = -\mu\epsilon\frac{\partial V}{\partial t}$ (Lorenz Gauge)

Constants values $\epsilon_0 = 8.85 \times 10^{-12} F/m$; $\mu_0 = 4\pi \times 10^{-7} H/m$

Polarization

$\gamma = \tan^{-1}(E_{y0}/E_{x0})$. Assuming propagation in $+z$ direction.

Electrical to Ellipse

$$\begin{aligned}\tan 2\tau &= \tan 2\gamma \cos\phi \\ \sin 2\alpha &= \sin 2\gamma \sin\phi\end{aligned}$$

Ellipse to electrical

$$\begin{aligned}\cos 2\gamma &= \cos 2\alpha \cos 2\tau \\ \tan\phi &= \frac{\tan 2\alpha}{\sin 2\tau}\end{aligned}$$

Stokes parameter

$$\begin{aligned}S_0 &= E_x^2 + E_y^2 \\ S_1 &= E_x^2 - E_y^2\end{aligned}$$

$$\begin{aligned}S_2 &= 2E_xE_y\cos\phi \\ S_3 &= 2E_xE_y\sin\phi\end{aligned}$$

Transformation of vector

Cart→Cyl

$$\begin{bmatrix}F_\varrho \\ F_\phi \\ F_z\end{bmatrix} = \begin{bmatrix}\cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}F_x \\ F_y \\ F_z\end{bmatrix}$$

Cart→Sph

$$\begin{bmatrix}F_r \\ F_\theta \\ F_\phi\end{bmatrix} = \begin{bmatrix}\sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0\end{bmatrix}\begin{bmatrix}F_x \\ F_y \\ F_z\end{bmatrix}$$

Elemental length

$$\begin{aligned}\text{Cart: } d\vec{l} &= dx\ \hat{x} + dy\ \hat{y} + dz\ \hat{z} \\ \text{Cyl: } d\vec{l} &= d\varrho\ \hat{\varrho} + \varrho d\phi\ \hat{y} + dz\ \hat{z} \\ \text{Sph: } d\vec{l} &= dr\ \hat{r} + rd\theta\ \hat{\theta} + r\sin\theta\ \hat{\phi}\end{aligned}$$

Elemental volume

$$\begin{aligned}\text{Cart: } d\vec{v} &= dx\ dy\ dz \\ \text{Cyl: } d\vec{v} &= \varrho d\varrho\ d\phi\ dz \\ \text{Sph: } d\vec{v} &= r^2\sin\theta drd\theta d\phi\end{aligned}$$

Elemental surface

Cart:

$$d\vec{a} = \begin{cases}dy\ dz\hat{x} \\ dx\ dz\hat{y} \\ dy\ dz\hat{z}\end{cases}$$

Cyl:

$$d\vec{a} = \begin{cases}\varrho\ d\phi\ dz\hat{\varrho} \\ d\varrho\ dz\hat{\phi} \\ \varrho d\varrho\ d\phi\hat{z}\end{cases}$$

Sph:

$$d\vec{a} = \begin{cases}r^2\sin\theta\ d\theta d\phi\ \hat{r} \\ r\sin\theta drd\phi\hat{\theta} \\ rdrd\theta\hat{\phi}\end{cases}$$

Boundary conditions

Dielectric-Dielectric

$$\begin{aligned}(\vec{D}_1 - \vec{D}_2)\cdot\hat{n} &= 0 \\ (\vec{B}_1 - \vec{B}_2)\cdot\hat{n} &= 0 \\ (\vec{E}_1 - \vec{E}_2)\times\hat{n} &= 0 \\ (\vec{H}_1 - \vec{H}_2)\times\hat{n} &= 0\end{aligned}$$

Dielectric-metal

$$\begin{aligned}\vec{D}_1\cdot\hat{n} &= \rho_s \\ \vec{B}_1\cdot\hat{n} &= 0 \\ \vec{E}_1\times\hat{n} &= 0 \\ \vec{H}_1\times\hat{n} &= \vec{J}_s\end{aligned}$$

Media interface

TE

$$\begin{aligned}R_\perp &= \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} \\ T_\perp &= \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}\end{aligned}$$

$\eta = \sqrt{\mu/\epsilon}$

TM

$$\begin{aligned}R_\parallel &= \frac{\eta_1\cos\theta_i - \eta_2\cos\theta_t}{\eta_1\cos\theta_i + \eta_2\cos\theta_t} \\ T_\parallel &= \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}\end{aligned}$$