Arrays 2 : Two Dimensional

Question!

Given a <mark>row-wise</mark> & <mark>column-wise</mark> sorted matrix, find whether element K is present or not?

	0	1	2	3
0	-5	-2	1	13
1	-4	0	3	14
2	-3	2	6	18

A =

K = 13 ✓

K = 0 ✓

K = -1 ✗

Bruteforce → $f_{i,j}$ check $a[i][j]$ TC = $O(N*M)$

	0	1	2	3
0	-5	-2	1	13
1	-4	0	3	14
2	-3	2	6	18

K = 0

-5 < 0

⟹ go to larger value

-5 ⟶ increasing
↓
increasing

$13 > 0$

$\Rightarrow$ go to smaller value

decreasing $\longleftarrow$ 13
                              $\downarrow$
                         increasing

$K = 0$

	0	1	2	3
0	-5	-2	1	13
1	-4	0	3	14
2	-3	2	6	18

start

$13 > 0$

$\Rightarrow$ go left

	0	1	2
0	-5	-2	1
1	-4	0	3
2	-3	2	6

$1 > 0$

$\Rightarrow$ go left

	0	1
0	-5	-2
1	-4	0
2	-3	2

$-2 < 0$

$\Rightarrow$ go down

	0	1
0		
	-4	0
1	-3	2

FOUND?!

return true

Code

```
i=0, j= M-1        // top right

while ( i<n  && j>=0) {

    if ( a[i][j] == K)     return true

    else if ( a[i][j] > K)      j--   // go left

    else      i++       // go down

}

return false
```

$i \rightarrow 0$ to $n-1$
       N times

$j \rightarrow m-1$ to 0
       m times

$$TC : O(N+M)$$

$$SC = O(1)$$

Question 2

Given the square matrix, print boundary element

clockwise

N cols

0,0   0,N-1

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

N rows

N-1,0   N-1,N-1

N=5

output: 1 2 3 4 5 10 15 20 25

24 23 22 21 16 11 6

1	2	3
4	5	6
7	8	9

output: 1 2 3 6 9 8 7 4

Code

```
i=0, j=0

for(k=0 to n-2) {          → N-1 times
    print(a[i][j])
    j++    // left → right
}
```

i=0, j=0

} first row

```
for(k=0 to n-2) {          → N-1 times
    print(a[i][j])
    i++    // top → down
}
```

i=0, j=N-1

} last

```
for(k=0 to n-2) {          → N-1 times
    print(a[i][j])
    j++    // left → right
}
```

```
for(K=0  to  n-2)  {          → N-1 times                    i=N-1, j=N-1
    print ( a(i)(j))                              }  last
    j--   // right → left                            row
}
for(K=0  to  n-2)  {          → N-1 times                    i=N-1, j=0
    print ( a(i)(j))                              }  first
    i--   // down → top                              column
}
```
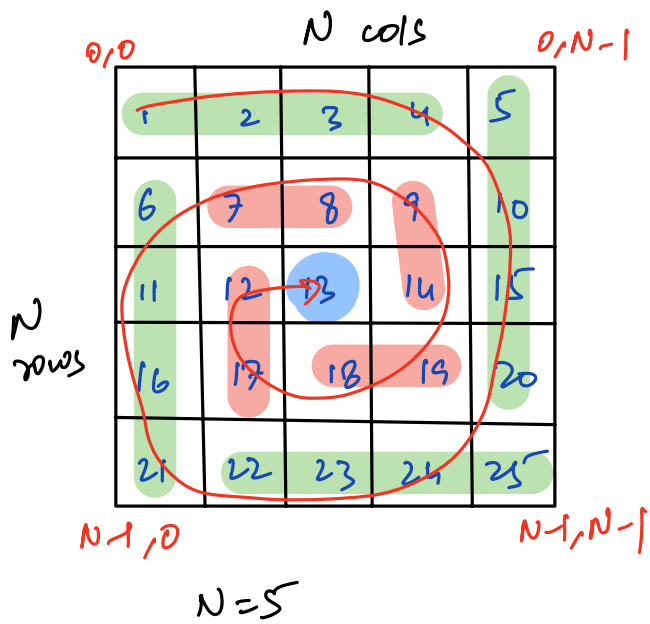
$$TC = O(N-1 + N-1 + N-1 + N-1)$$

$$= O(N)$$

$$SC = O(1)$$

Question 3

Print elements in spiral order in clockwise direction. (square matrix).

N cols

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

0,0   0,N-1

N rows

N-1,0   N-1,N-1

N = 5

output: 1 2 3 4 5 10 15

20 25 24 23 22 21

16 11 6 7 8 9 14

19 18 17 12 13

r	c	N
$+1$ 0	0 $)+1$	5 $)-2$
1	1	3
$+1$ 2	2 $)+1$	1 $)-2$

$r = 0, \ c = 0$

while (N > 1) {

    $i = r \ , \ j = c$

     for (K = 0   to   n-2) {

       print (a[i][j])

       j++

    }

   for (K = 0   to   n-2) {

```
        print (a[i][j))
        i++
    }
    for ( K=0    to    n-2 ) {
        print (a[i][j))
        j--
    }
    for ( K=0    to    n-2 ) {
        print (a[i][j))
        i--
    }
    r++, c++  , N-=2
}

if ( N==1 )
    print ( a[r][c] )        } → executed if N is odd


TC =  O(N²)

SC = O(1)
```
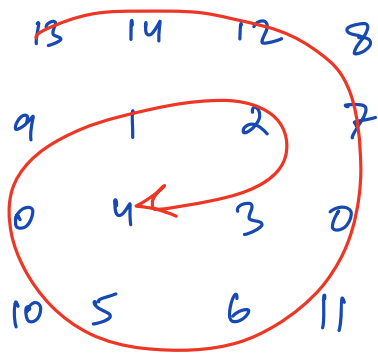

N=4

N=2

N-

13 14 12 8

9 1 2 7

0 4 3 0

10 5 6 11
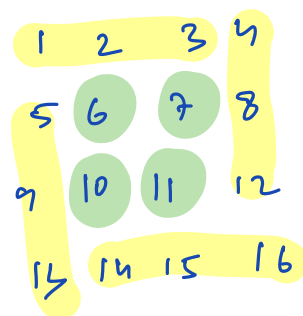
13 14 12 8 7 0 11 6 5 10

0 9 1 2 3 4

Sub matrix

subarray : continous part of array

sub-matrix : continous part of matrix

	0	1	N_2 cols 3		4
0	1	2	3	4	5
1	6	7	8	9	10
2	11	12	13	14	15
3	16	17	18	19	20
4	21	22	23	24	25

N rows

→ submatrix

(1,2) (3,4)

top left

bottom right

N=5
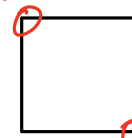
Question

find the # submatrices in which a[i][j] is present for given i,j.

	0	1	2
0	1	2	3
1	4	5	6
2	7	8	9

$(i,j) = (1,2)$ $N = 3$

ans = 12

$(1+1) \times (2+1) \times (3-1) \times (3-2)$

$2 \times 3 \times 2 \times 1 = 12$

```
1 2 3        2 3        3           4 5 6        5 6        6
4 5 6        5 6        6
```

```
4 5 6        5 6        6           1 2 3        2 3        3
7 8 9        8 9        9           4 5 6        5 6        6
                                    7 8 9        8 9        9
```

j=3 → possible top left cells = $(i+1) \times (j+1)$

	0	1	2	3	4
0	1	2	3	4	5
1	6	7	8	9	10
2	11	12	13	14	15
3	16	17	18	19	20
4	21	22	23	24	25

i=2

possible bottom right cells = $(N-i) \times (N-j)$

N=5

Sub matrices containing $a(i)(j)$ =

$$(i+1) \times (j+1) \times (n-i)(n-j)$$

Quiz $N = 4$, $M = 5$ $(i+1) \times (j+1) \times (N-i) \times (M-j)$

$(1,2) = ?$

$$(1+1) \times (2+1) \times (4-1) \times (5-2)$$

$$2 \times 3 \times 3 \times 3 = 54$$

Question

find the sum of all submatrices sum.

$sum = 0$

for $(i=0$ to $N-1)$ {

    for $(j=0$ to $M-1)$ {

        $sum += (i+1) \times (j+1) \times (N-i) \times (M-j) \times a(i)(j)$

    }

}

$TC = O(N \times M)$

$SC = O(1)$