Bit Manipulation - 2

Question 1

Given an integer array where every no. occurs 3 times except one. find the unique no. ?

A = [4 5 5 4 1 6 6 4 5 6]

Bruteforce : count frequency of every no. using Hashmap

TC = O(N)
SC = O(N)

	2	1	0
4	1	0	0
5	1	0	1
5	1	0	1
4	1	0	0
1	0	0	1
6	1	1	0
6	1	1	0
4	1	0	0
5	1	0	1
6	1	1	0

if # 1's is multiple of 3 then that bit is unset in unique no.

else bit is set

#1's 9 3 4

bit is unset

bit is set in unique no.

Code

consider no. is 32 bit integer

```
int ans = 0        //000.....0

for( i=0 to 31 ) {

    cnt=0
    for( j=0 to n-1 ) {
        if( (A[j] & (1<<i)) > 0 )
            cnt++
    }
    if( cnt%3 == 1 ) {
        // set ith bit in ans
        ans |= (1<<i)
    }
}

return ans
```

if a[i] ax long then use 63 here.

n is size of array

$ans = ans \mid (1<<i)$

total iterations $= 32 \times N$

$TC = O(N)$

$SC = O(1)$

Question Every element occurs K times except 1. find the unique no. ?

Use above sol^n with one change.

```
if ( cnt % K == 1 ) {
        ans |= (1<<i)
}
```

if K is even $\Rightarrow$ $\boxed{\text{ans} = \text{xor of all no.}}$

Dry run

	0	1	2	3	4	5	6	7	8	9
A = [	4	5	5	4	1	6	6	4	5	6]

ans = 0

i = 0	4 & (1<<0)	$\Rightarrow 0$	cnt = ~~0~~
	5 & (1<<0)	$\Rightarrow 2^0$	~~1~~
	5 & (1<<0)	$\Rightarrow 2^0$	~~2~~
	4 & (1<<0)	$= 0$	
	1 & (1<<0)	$= 2^0$	~~3~~
	6 & (1<<0)	$= 0$	
	6 & (1<<0)	$= 0$	

$4 \& (1<<0) = 0$

$5 \& (1<<0) = 2^0$

$6 \& (1<<0) = 0$

4 cnt%3 = 1

ans = ans | (1<<0)

= 1

cnt = ~~0~~

$i=1$

$4 \& (1<<1) = 0$

$5 \& (1<<1) = 0$

$5 \& (1<<1) = 0$

$4 \& (1<<1) = 0$

$1 \& (1<<1) = 0$

$6 \& (1<<1) = 2^1$

$6 \& (1<<1) = 2^1$

$4 \& (1<<1) = 0$

$5 \& (1<<1) = 0$

$6 \& (1<<1) = 2^1$

+

2

3 cnt%3 != 1

$i=2$

⋮

$i=3$

⋮

$i = 31$

Question 2

Given an integer array where every no. occurs 2 times except 2 numbers. find the 2 unique no. .

$A = [\ 4\ \ 5\ \ 5\ \ 4\ \ 1\ \ 6\ \ 6\ \ 2\ ]$ ans = 1, 2

$4 \wedge 5 \wedge 5 \wedge 4 \wedge 1 \wedge 6 \wedge 6 \wedge 2 = 1 \wedge 2 = 3$

let's assume two no. a and b. which are different

$a \wedge b = c \Rightarrow c > 0$

$\Rightarrow$ at least 1 bit in c is 1.

let's assume i^{th} bit is 1 in c.

$\Rightarrow$ i^{th} bit is set in only a OR b.

$\Rightarrow$ i^{th} bit is 1 in one no.
and 0 in another no.

Divide the array into 2 parts

1st part

i^{th} bit is 1

$\Downarrow$

xor all no. is part 1

give 1'st unique no.

2nd part

i^{th} bit is 0

$\Downarrow$

xor all no. is part 2

give 2nd unique no.

$A = [\ 4 \quad 5 \quad 5 \quad 4 \quad 1 \quad 6 \quad 6 \quad 2\ ]$

$xor = 4 \wedge 5 \wedge 5 \wedge 4 \wedge 1 \wedge 6 \wedge 6 \wedge 2 = 1 \wedge 2 = 3 \quad (11)_2$

Divide array based on 0th bit.

$A = [\ 4\ \ 5\ \ 5\ \ 4\ \ 1\ \ 6\ \ 6\ \ 2\ ]$

0th bit $= 1$ $\qquad\qquad$ 0th bit $= 0$

$\underbrace{5\ \ 5\ \ 1}$ $\qquad\qquad$ $\underbrace{4\ \ 4\ \ 6\ \ 6\ \ 2}$

$\downarrow$ xor $\qquad\qquad$ $\downarrow$ xor

$5 \wedge 5 \wedge 1 = 1$ $\qquad\qquad$ $4 \wedge 4 \wedge 6 \wedge 6 \wedge 2 = 2$

$A = [\ 4\ \ 5\ \ 5\ \ 4\ \ 8\ \ 6\ \ 6\ \ 2\ ]$

$\text{xor} = 4 \wedge 5 \wedge 5 \wedge 4 \wedge 8 \wedge 6 \wedge 6 \wedge 2 = 8 \wedge 2 = 10 \quad (\overset{3}{1}\overset{2}{0}\overset{1}{1}\overset{0}{0})_2$

Divide based on 1st bit

$A = [\ 4\ \ 5\ \ 5\ \ 4\ \ 8\ \ 6\ \ 6\ \ 2\ ]$

1th bit $= 1$ $\qquad\qquad$ 1th bit $= 0$

$\underbrace{6\ \ 6\ \ 2}$ $\qquad\qquad$ $\underbrace{4\ \ 5\ \ 5\ \ 4\ \ 8}$

$\downarrow$ xor $\qquad\qquad$ $\downarrow$ xor

2 $\qquad\qquad$ 8

$2 \wedge 3 = 1 \quad (01)_2$

$1 \wedge 7 = 6 \; (110)_2$

(001) (111)

Code

```
xor = 0
for (i=0 to n-1) {          → N times
    xor ^= a[i]
}
index = -1
for (i=0 to 31) {           → 32 times
    if((xor & (1<<i)) > 0) {
        index = i
        break
    }
}

ans1 = 0 , ans2 = 0

for (i=0 to n-1) {          → N times
    if((a[i] & (1<<index)) > 0) {   // bit is set (group1)
        ans1 ^= a[i]
    }
    else {   // bit is unset (group 2)
        ans2 ^= a[i]
```

```
    }
}
print( ans1 , ans2 )
```

iteration = N + 32 + N

TC = O(N)

SC = O(1)

Doubt

$a \wedge b = c$

$$
\begin{array}{rl}
a = & 1\ 0\ 1\ 0\ 1 \\
b = \wedge & 1\ 0\ 0\ 1\ 1 \\
\hline
 & 0\ 0\ 1\ 1\ 0
\end{array}
$$