Bit Manipulation Basics

Decimal Number System → {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

base 10

$342 = 300 + 40 + 2 = 3\times10^2 + 4\times10^1 + 2\times10^0$

$2536 = 2\times10^3 + 5\times10^2 + 3\times10^1 + 6\times10^0$

3 2 1 0

Binary Number System → {0, 1}

base 2

$110 = 1\times2^2 + 1\times2^1 + 0\times2^0 = 4+2 = 6$ ← decimal no. equivalent of $(110)_2$

2 1 0

$1011 = 1\times2^3 + 0\times2^2 + 1\times2^1 + 1\times2^0$

$= 8 + 2 + 1 = 11$

$(1011)_2 = (11)_{10}$

Binary to Decimal conversion

$$\begin{array}{ccccc} 1 & 0 & 1 & 0 & 1 \\ 4 & 3 & 2 & 1 & 0 \end{array} = 1\times 2^4 + 0\times 2^3 + 1\times 2^2 + 0\times 2^1 + 1\times 2^0$$

$$= 16 + 0 + 4 + 0 + 1 = 21$$

$$\begin{array}{ccccccc} 6 & 5 & 4 & 3 & 2 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \end{array}$$

$0\times 2^0 = 0$

$1\times 2^1 = 2$

$0\times 2^2 = 0$

$1\times 2^3 = 8$

$1\times 2^4 = 16$

$0\times 2^5 = 0$

$1\times 2^6 = 64$

$\overline{\quad 90 \quad}$

$$\begin{array}{ccccccc} 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ \downarrow & & \downarrow & \downarrow & & \downarrow & \\ 2^6 & & 2^4 & 2^3 & & 2^1 & \end{array}$$

$64 + 16 + 8 + 2$

$= 90$

$$(1011010)_2 = (90)_{10}$$

Decimal to Binary

$(20)_{10}$

		remainder
2	20	0
2	10	0
2	5	1
2	2	0
2	1	1
	0	

$\Rightarrow (\overset{4}{1}\overset{3}{0}\overset{2}{1}\overset{1}{0}\overset{0}{0})_2$

$2^4 + 2^2 = 16+4 = 20$

Binary representation of $(45)_{10}$

2	45	1
2	22	0
2	11	1
2	5	1
2	2	0
2	1	1
	0	

$\Rightarrow (101101)_2$

$2^5 \quad 2^3 \quad 2^2 \quad 2^0$

$32+8+4+1 = 45$

Addition of Decimal numbers

$$\begin{array}{rccc} & 1 & 1 & \\ & 3 & 6 & 8 \\ + & 4 & 3 & 5 \\ \hline & 8 & 10 & 13 \end{array}$$

$= 803$

$9+9 = 18$

$1+9+9 = 19$

Addition of Binary no.

$$
\begin{array}{rccccc}
 & 1 & 0 & 1 & 0 & 1 \\
+ & & 1 & 1 & 0 & 1 \\
\hline
10 & 10 & 10 & 1 & 10 &
\end{array}
$$

$\Rightarrow 100010$

$$
\begin{array}{rccccc}
 & 1 & 0 & 1 & 1 & 0 \\
+ & 0 & 0 & 1 & 1 & 1 \\
\hline
1 & 1 & 1 & 1 & 10 & 1
\end{array}
$$

$(3)_{10} = (11)_2$

$(2)_{10} = (10)_2$

$\Rightarrow 11101$

Bitwise Operators $\longrightarrow$ AND , OR , XOR , NOT

& | ^ ~ / !

AND : $x \,\&\, y = 1$ iff both x and y are 1.

$= 0$ if any is 0

OR : $x \mid y = 1$ if any one is 1

$= 0$ iff both are 0

XOR: $x \wedge y$ = 1 if x and y are different

0 if they are same

NOT $\sim x$ = 1 if x is 0

= 0 if x is 1

$\sim 0 = 1$

$\sim 1 = 0$

A	B	A&B	A|B	A^B
0	0	0	0	0
0	1	0	1	1
1	0	0	1	1
1	1	1	1	0

5 & 6 $\Rightarrow$

$$\begin{array}{rccc} 5 = & 1 & 0 & 1 \\ 6 = & 1 & 1 & 0 \\ \hline & 1 & 0 & 0 \end{array} = (4)_{10}$$

$\boxed{5 \& 6 = 4}$

20 & 45

$$
\begin{array}{rcccccccl}
20 = & 0 & 1 & 0 & 1 & 0 & 0 & & \\
\& & 1 & 0 & 1 & 1 & 0 & 1 & & \\
\hline
& (0 & 0 & 0 & 1 & 0 & 0)_2 & = & (4)_{10}
\end{array}
$$

92 | 154

$$
\begin{array}{rccccccccl}
92 = & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & \\
\text{OR} & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & \\
\hline
& 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & = (222)_{10}
\end{array}
$$

20 ^ 45

$$
\begin{array}{rccccccl}
20 = & 0 & 1 & 0 & 1 & 0 & 0 & \\
45 = & 1 & 0 & 1 & 1 & 0 & 1 & \\
\hline
\text{XOR} & 1 & 1 & 1 & 0 & 0 & 1 & = (57)_{10}
\end{array}
$$

Negative numbers

$(-45)_{10} = (\ ?\ )_2$

2's complement

→ assume all no. are 8 bits

5 → 0 0 0 0 0 1 0 1

1's complement of 5 → 1 1 1 1 1 0 1 0
(invert all bits)

+ 1

2's complement of 5: 1 1 1 1 1 0 1 1

$2^7\ 2^6\ 2^5\ 2^4\ 2^3\ \ 2^1\ 2^0$

-128

$= 128 + 64 + 32 + 16 + 8 + 2 + 1$

$= 251$ $= -5$

7	6	5	4	3	2	1	0

Most significant bit (MSB) ⇒ signed bit (-ive bit)

$(-3)_{10} = (?)_2$

$(3)_{10}$ = 0 0 0 0 0 0 1 1

1's complement = 1 1 1 1 1 1 0 0
(~3)

+ 1

2's complement 1 1 1 1 1 1 0 1 = $(-3)_{10}$

$(-10)_{10}$

$(10)_{10}$ = 0 0 0 0 1 0 1 0

1's complement = 1 1 1 1 0 1 0 1

+ 1

1 1 1 1 0 1 1 0

Range of 8 bit no. : -128 to 127

10000000 01111111

-2^7 to 2^7-1

Unsigned 8 bit no: 0 to 255

0 to 2^8-1

for n bit no. range = -2^{n-1} to $2^{n-1}-1$

32 bit integer

$\Rightarrow -2^{31}$ to $2^{31}-1$

$\Rightarrow -2\times10^{9}$ to 2×10^{9}

$= -2147483648$ to

2147483647

$2^{31} = 2^{30}\times 2$

$= (2^{10})^{3}\times 2$

$\approx (10^{3})^{3}\times 2$

$\approx 2\times10^{9}$

64 bit no.

$= -2^{63}$ to $2^{63}-1$

$= -8\times10^{18}$ to 8×10^{18}

$2^{10} \approx 10^{3}$

1024 1000

$2^{63} = 2^{60}\times 2^{3}$

$= 8\times 2^{60}$

$= 8\times(2^{10})^{6}$

$= 8\times(10^{3})^{6}$

$= 8\times10^{18}$

Importance of constraints

int $a = 10^5$

int $b = 10^6$

int $c = a \times b$ ✗ $[a \times b = 10^5 \times 10^6 = 10^{11}]$

↳ overflow, wrong answer

long $c = a \times b$ ✗

↳ overflow during multiplication

MUL a, b, temp — int

CPY temp, c

long c = long$(a \times b)$ ✗

↳ overflow during multiplication

long c = (long)a $\times$ (long)b ✓

will work

MUL a, b, temp — long

CPY temp, c

long c = (long)a $\times$ b ✓

will work

Ques: Given an array a[N], calculate sum of all elements.

$1 <= N <= 10^5$

$1 <= a[i] <= 10^6$

```
~~int~~ long ans = 0

for (i = 0 to n-1) {
    ans += a[i]
}            ↳ ans = ans + a[i]
               long   long   int
print(ans)
```

if all $a[i] = 10^6$

$n = 10^5$

$sum = 10^6 \times 10^5 = \underline{10^{11}}$

overflow in int.

Doubts

[0 0 0 -1 -3]

0 0 0 3 4

exists = 1