

Searching 2 : Binary Search Problems

3 steps of Binary Search

1. Define search space
2. Check if mid is the answer
3. Decide whether to go left or right

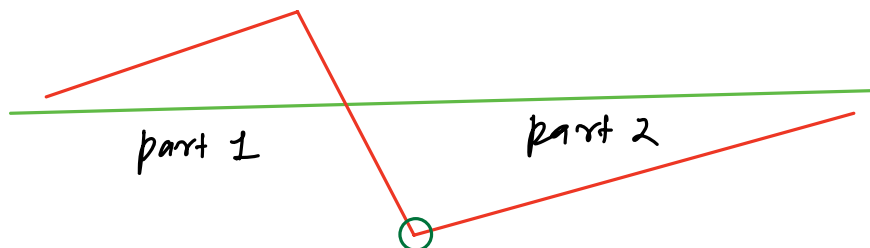
To find mid use,

$$l + \frac{(r-l)}{2} \quad \text{instead of} \quad \frac{(l+r)}{2}$$

Question 1

Given a sorted rotated array of unique elements.
find the index of given element K.

$A = [$ 6 7 8 1 2 3 4 5 $]$



find the smallest element ?

How to know if $A[mid]$ is in part 1 or part 2 ?

all elements in part 1 $>$ all elements in part 2

$A[0] >$ all elements in Part 2

$A[0] <$ all elements in Part 1

Corner case

check for not rotated ? $\longrightarrow A[0] < A[n-1]$

$TC = \underline{O(\log N)}$ $SC = O(1)$
total 3 binary searches

$l=0, r=n-1$

while ($l \leq r$) {

$mid = l + (r-l)/2$

 if ($mid \neq 0$ & $A[mid-1] > A[mid]$)

 return mid

 if ($A[mid] < A[0]$) { // part 2

$r = mid-1$

 }

\longleftarrow finding
smallest
element

```

    else {
        l = mid + 1
    }
}

```

After finding smallest element, do binary search on each part.

Now let's solve in 1 binary search

// Define search space

$l = 0$, $r = n - 1$

while ($l \leq r$) {

// check if mid is answer or not!

$mid = l + (r - l) / 2$

if ($A[mid] == K$) return mid

// decide whether to go left or right

if ($K < A[0]$) { // K is in Part 2

if ($A[mid] < A[0]$) { // A[mid] in Part 2

if ($A[mid] < K$) $l = mid + 1$

else $r = mid - 1$

```

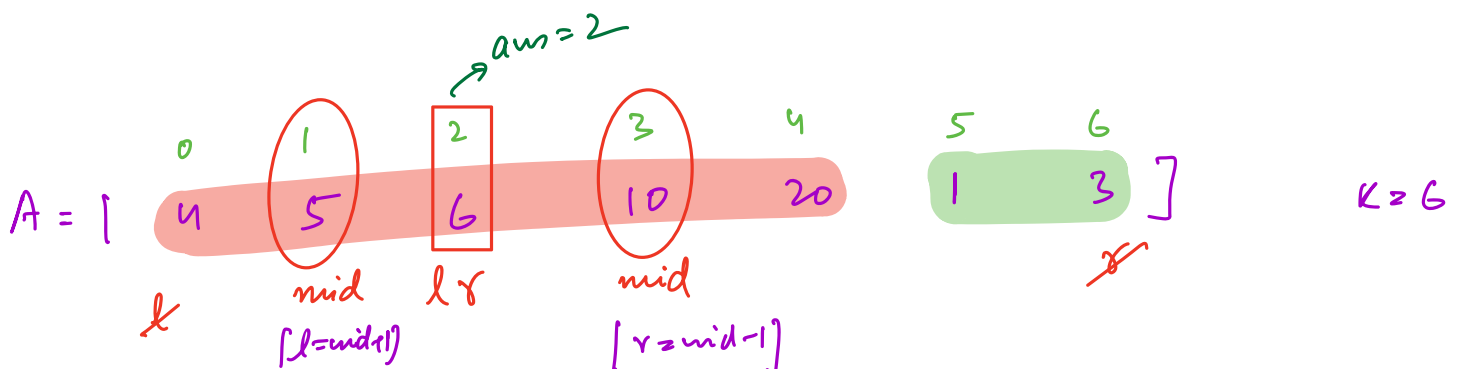
    }
    else { // A[mid] in Part 1
        l = mid + 1
    }
}
else { // K is in Part 1
    if (A[mid] > A[0]) { // A[mid] in Part 1
        if (A[mid] < K) l = mid + 1
        else r = mid - 1
    }
    else { // A[mid] in Part 2
        r = mid - 1
    }
}
}

return -1

```

Note: Take care of
case when array
is not rotated

TC = $O(\log N)$
 SL = $O(1)$



Question 2

find sqrt. of a given perfect sq. number.

$$N = 49$$

$$\text{ans} = 7$$

$$N = 25$$

$$\text{ans} = 5$$

$$N = 30 \leftarrow \text{invalid input}$$

Bruteforce

min. value of $\sqrt{N} = 1$
max value of $\sqrt{N} = N$ } range

```
for (i=1 to N) {  
    if (i*i == N)  
        return i  
}
```

$$TL = O(\cancel{N})$$
$$O(\sqrt{N})$$

$$SC = O(1)$$

$$N=25 \quad i=1, 2, 3, 4, 5$$

$$49 \quad i=1, 2, 3, 4, 5, 6, 7$$

$$81 \quad i=1, 2, 3, 4, 5, 6, 7, 8, 9$$

always stops after
 \sqrt{N} iterations

Use binary search

// Define search space

$l=1, r=N$

while ($l \leq r$) {

// check if mid is answer

$mid = l + (r-l)/2$

if ($mid * mid == N$) return mid

// decide whether to go left or right

if ($mid * mid < N$) $l = mid + 1$ // go right

else $r = mid - 1$

}

TC = $O(\log N)$

SL = $O(1)$

$N=36$

$l=1$ 5

$r=36$ 17 8

$mid = 18$ 9 4 6

$18^2 > 36$

$9^2 > 36$

$4^2 < 36$

$6^2 = 36$

ans

If input is not perfect sq. number,
find the nearest sq.

$$N = 30$$

$$am = 5$$

$$N = 37$$

$$am = 6$$

Doctr

$$A = 10^9 \Rightarrow \max 2^{32}$$

\Rightarrow convert to binary
110001010

$$N = 10^5$$

$$B(i) = 10^9 \quad \text{weight } 2^i$$