Maths : Combinatorics Basics

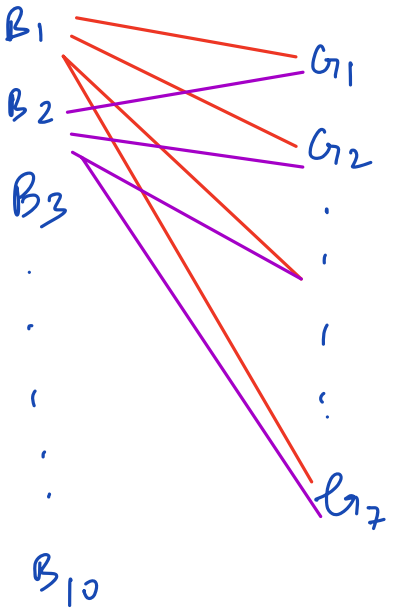
1. Addition & Multiplication Rule
2. Permutations Basics
3. Combination Basics & properties
4. Pascal Triangle
5. find N^{th} column title

Question

10 boys 7 girls

How many pairs can be created ?

pair : 1 boy + 1 girl

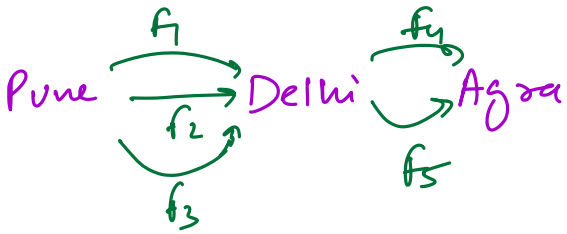


B_1 → 7 pairs (G_1 to G_7)

B_2 → 7 pairs

total pair = 7 × 10 = 70

Question



ways to travel from Pune to Agra via Delhi ?

Pune → Delhi **AND** Delhi → Agra

ways (Pune → Delhi) = 3

ways (Delhi → Agra) = 2

ways (Pune → Delhi) X ways (Delhi → Agra)

3 x 2 = 6

ways → $f_1 f_4$ $f_2 f_4$ $f_3 f_4$

$f_1 f_5$ $f_2 f_5$ $f_3 f_5$

Quiz

Pune → Agra ⇒ Pune → Delhi & Delhi → Agra

OR

Pune → Mumbai & Mumbai → Agra

ways (Pune → Delhi) : 3

ways (Delhi → Agra) : 4

ways (Pune → Delhi & Delhi → Agra) = 3×4 = 12

ways (Pune → Mumbai) = 2

ways (Mumbai → Agra) = 3

ways (Pune → Mumbai & Mumbai → Agra) = 2×3 = 6

total ways from Pune to Agra =

12 + 6 = 18

AND : used to count possibilities that occur together in sequence

OR : used to count possibilities that occur in separate ways

Question

Most variety of meal combos.

given A[n][3]

A[i][0] → no. of main courses

A[i][1] → no. of desserts

A[i][2] → no. of beverages

$$A = \begin{bmatrix} [3 & 2 & 2] \Rightarrow 3\times2\times2 = 12 \\ [4 & 3 & 3] \Rightarrow 4\times3\times3 = 36 \\ [1 & 1 & 1]\ ] \Rightarrow 1\times1\times1 = 1 \end{bmatrix}$$

Solution: Iterate the array & find the max. combo restaurant.

Permutations

Arrangement of objects,

order matters $\Rightarrow$ $(i,j) \neq (j,i)$

ways to arrange 3 distinct characters

a, b, c $\Rightarrow$ abc bac cab

acb bca cba

ans = 6

$3 \times 2 \times 1 = 6$ $(3!)$

4 distinct characters $\rightarrow$ $4 \times 3 \times 2 \times 1 = 24$ $(4!)$

ways to arrange N distinct characters =

$N \times (N-1) \times (N-2) \times \ldots \times 1 = N!$

Question

Find # ways to arrange R out of N characters.

d, a, t, e $\qquad \underline{4} \times \underline{3} = 12$

N = 4 , R = 2

1	2	3	. . . . .	R
# ways → N	N-1	N-2		N-(R-1) = N-R+1

$$N \times (N-1) \times (N-2) \times \cdots \times (N-R+1) \times (N-R) \times (N-R-1) \times \cdots \times 1$$

$$(N-R) \times (N-R-1) \times \cdots \times 1$$

$$= \frac{N!}{(N-R)!}$$

$$\boxed{{}^{N}P_{R} = \frac{N!}{(N-R)!}} \qquad R <= N$$

↑ # ways to arrange R elements out of total N elements

Combinations → selection of objects,

order doesn't matter → (i,j) = (j,i)

select 3 out of 4 distinct characters

d, a, t, e

d, a, t	d, a, e	a, t, e	d, t, e
d t a	d e a	a e t	d e t
a d t	a d e	t a e	t d e
a t d	a e d	t e a	t e d
t a d	e d a	e a t	e d t
t d a	e a d	e t a	e t d

selection AND arrangement = Permutation

$4 \times 3! = 24$

select R elements out of N elements

$$ {}^{N}C_{R} \times R! = {}^{N}P_{R} $$

$$ {}^{N}C_{R} = \frac{{}^{N}P_{R}}{R!} = \frac{N!}{(N-R)! \times R!} $$

Properties of ${}^{N}C_{R}$

1. # ways to not select anything $= {}^{N}C_{0} = \frac{N!}{(N-0)! \times 0!}$

$$ = \frac{N!}{N! \times 1} = 1 $$

2. # ways to select everything $= {}^{N}C_{N} = 1$

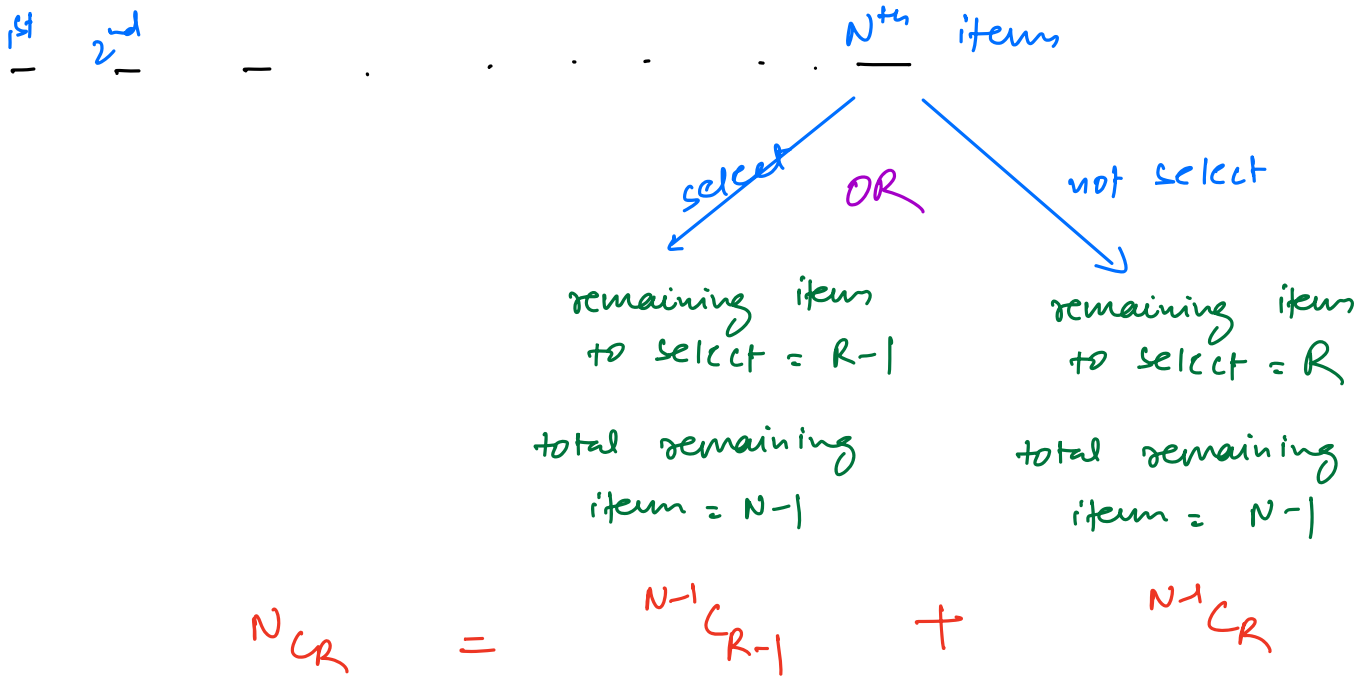
3. # ways to select $(N-R)$ items $= {}^{N}C_{N-R}$

$$ = \frac{N!}{(N-(N-R))! \times (N-R)!} $$

$$ = \frac{N!}{R!\,(N-R)!} = {}^{N}C_{R} $$

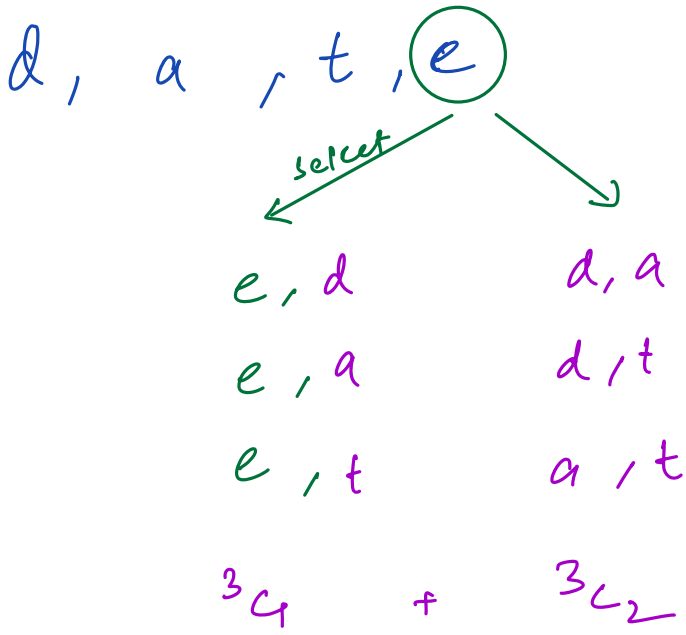
$$ {}^{N}C_{R} = {}^{N}C_{N-R} $$

4. # ways to select R items from N items.



$$N_{C_R} = {}^{N-1}C_{R-1} + {}^{N-1}C_R$$

$$\boxed{N_{C_R} = {}^{N-1}C_{R-1} + {}^{N-1}C_R}$$



$n = 4$, $r = 2$

$${}^4C_2 = \frac{4 \times 3}{2} = 6$$

$${}^3C_1 + {}^3C_2$$

Question

Generate Pascal Triangle for given input N.

N = 4

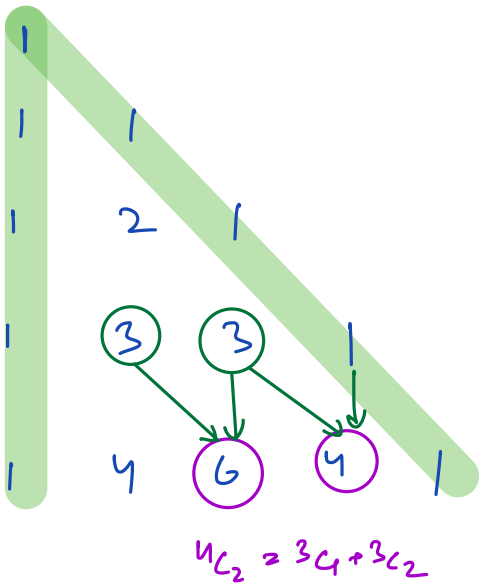
$^0C_0$

$^1C_0$ $^1C_1$

$^2C_0$ $^2C_1$ $^2C_2$

$^3C_0$ $^3C_1$ $^3C_2$ $^3C_3$

$^4C_0$ $^4C_1$ $^4C_2$ $^4C_3$ $^4C_4$



$^4C_2 = {}^3C_1 + {}^3C_2$

Bruteforce

if ^iC_j => calculate it using $\frac{i!}{j!\,(i-j)!}$

TC to calculate N! = O(N)

total TC = $O(N^2 \times N) = O(N^3)$

$^{N}C_0 = 1$ $^{N}C_N = 1$ $^{N}C_R = {}^{N-1}C_{R-1} + {}^{N-1}C_R$

c[i][j] = c[i-1][j-1] + c[i-1][j]

```
def pascalTriangle(n) {
    nCr[n+1][n+1] = {0}
    for(i=0 to n) {
        nCr[i][0] = 1
        nCr[i][i] = 1
        for(j=1 to i-1) {
            nCr[i][j] = (nCr[i-1][j-1] + nCr[i-1][j]) %M
        }
    }
    return nCr;
}
```

TC = O(N^2)

SC = O(N^2)

Question

Given a +ive integer N, find N^{th} column title.

A B C . . . . Z AA AB . . . . AZ BA BB . . . . . ZZ AAA . . . .

1 2 3 26 27 28 52 53

N = 4 am = "D"

N = 28 am = "AB"

N = 50 am = "AX"

observation → base 26 number system

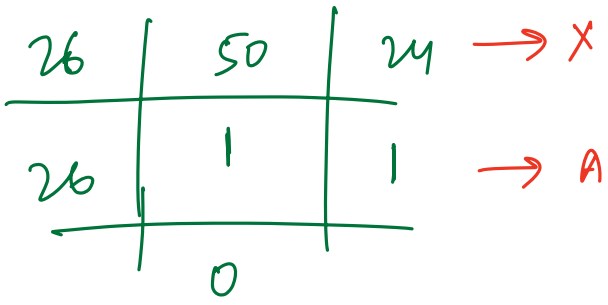
0 1 25

A B . . . . Z

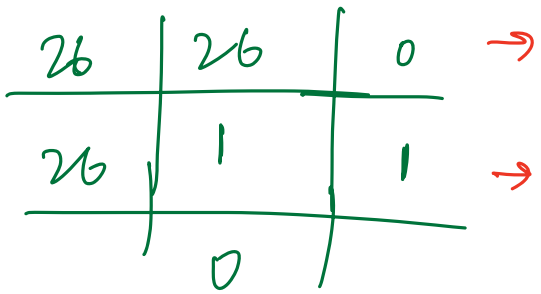
N = 50

26	(50 - 1)	23	→ X
26	(1 - 1)	0	→ A
	0		

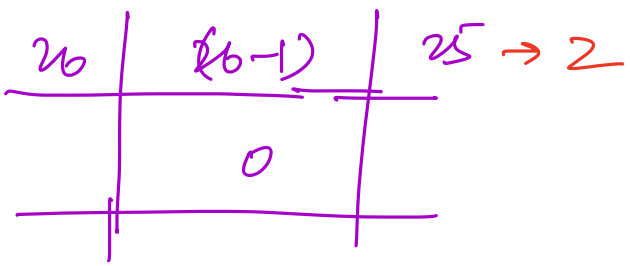
AX



N = 26



N = 26



Code

```
def columnTitle(n) {
    ans = ""
    while (n > 0) {
        n = n - 1
        ans = (char)('A' + (n % 26)) + ans;
        n = n / 26
    }
```

optimize if get TLE

return ans

}

$TC = O(\log_{26} N)$

$SC = O(1)$