Math 411 - Fall 2025

Introduction to Abstract Algebra 1

Santiago Arango-Piñeros

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Contents

Coordinates

Instructor

Santiago Arango-Piñeros (he/him/his). My office is LGRT 1238, and you can contact me via email at:

- `sarangopiner@umass.edu`.

Lectures

Section	Days	Time	Place
01-LEC (64725)	Tu-Th	8:30 AM - 9:45 AM	LGRT 177
02-LEC (64726)	Tu-Th	10:00 AM - 11:15 AM	LGRT 206

Office hours

- Between 1:00 PM and 3:30 PM on Tuesdays.
- Between 1:00 PM and 2:15 PM on Thursdays.
- If the times above are not convenient, send me an email.

Course description

The focus of the course will be on studying *groups*. These are algebraic structures that capture the notion of symmetry. Groups are ubiquitous in

all areas of mathematics (and the world around us). If you commit to this class, you will master the essential concepts of group theory by the end of the course.

Prerequisites

MATH 235 and either **CMPSCI 250** or **MATH 300**. In other words, we will need some important concepts from linear algebra, and there will be an emphasis on proofs and development of careful mathematical reasoning and writing.

Textbook

- ***Algebra: Abstract and Concrete*** by Frederick M. Goodman. The book is freely available for download at the author's web-site.
- As a complement of the textbook, we will use some of Keith Conrad's blurbs on group theory.

Learning objectives

1. To learn the fundamental examples of groups: finitely generated abelian groups, dihedral groups, symmetric and alternating groups, and matrix groups.
2. To learn the axiomatic definition of a group and how to use it to prove basic properties.
3. To learn the concepts of subgroups, cosets, quotients, and how to combine these to derive Lagrange's theorem.
4. To learn the concepts of homomorphisms and isomorphisms and Noether's isomorphism theorems.
5. To understand the structure theorem of finitely generated abelian groups in terms of the Smith normal form of a matrix with integer coefficients.
6. To learn what it means for a group to act on a set as well as the natural actions of each of the fundamental examples.

Homework (300 points)

Homework assignments must be submitted through Gradescope by 11:59 PM on the due date. Each problem set contains 5 problems, and each problem is worth 10 points. This means that there is a 50 points bonus on the homework. **Late homework will not be graded.**

Collaboration with other students is highly encouraged! Nevertheless, every student must write down their own solutions.

The use of professional academic typing software, such as Typst or LaTeX (for example, via a free account at Overleaf), is highly recommended but not required. However, if the grader finds the writing difficult to read, they reserve the right not to grade that particular answer.

- PSET1: Some examples of groups (pdf)(tex) *<2025-09-11 Thu>*.
- PSET2: Basic properties of groups (pdf)(tex) *<2025-09-25 Thu>*.
- PSET3: Lagrange's theorem (pdf)(tex) *<2025-10-07 Tue>*.
- PSET4: The isomorphism theorems (pdf)(tex) *<2025-10-16 Thu>*.
- PSET5: Finitely generated abelian groups (pdf)(tex) *<2025-11-06 Thu>*.
- PSET6: Symmetries of regular polyhedra (pdf)(tex) *<2025-11-18 Tue>*.
- PSET7: Group actions (pdf)(tex) *<2025-12-09 Tue>*.

Exams (300 points)

There will be three midterm exams. Each exam will have 5 questions. Each question will be worth 20 points.

- EXAM1: Lectures 1-7. *<2025-09-25 Thu>*
- EXAM2: Lectures 8-14. *<2025-11-06 Thu>*
- EXAM3: Lectures 18-22. *<2025-12-09 Tue>*

Question one will ask you to define a concept. Question two will ask you to prove a result (of reasonable difficulty) from the assigned reading. Questions 3, 4, and 5 will be random problems related to the topics of the lectures.

"Mistakes were made" essay (`100 points`)

This is a **handwritten** essay, due on the day of the final exam. It must include at least:

- three mathematical mistakes you (the **student**) made during the course (either in a homework assignment, a previous test, or during self-study), and
- two mathematical mistakes your favorite AI made when prompted with some group theory question;

along with thorough explanations of the nature of the errors and their corrections. The essay will be graded on the mathematical accuracy of each explanation: an accurate description of each mistake, together with a complete correction, will be worth `20 points`.

Final exam (`300 points`)

The final exam will consist on six random problems related to the topics of the lectures. Each problem will be worth `50 points`. The emphasis will be on the topics of lectures 1-14 and 18-22.

Grades

The perfect final grade is `1000 points`. The alphabetical grade of the class will be calculated as follows:

- `Homework grade = min(300, PSET1 + ... + PSET7).`
- `Exams grade = EXAM1 + EXAM2 + EXAM3 + ESSAY + FINAL`
- `Final grade = Homework grade + Exams grade.`

Grade	A	A−	B+	B	B−
`points`	$[860, 1000]$	$[830, 860)$	$[780, 830)$	$[740, 780)$	$[690, 740)$

Grade	C+	C	C−	D+	D	F
`points`	$[650, 690)$	$[610, 650)$	$[560, 610)$	$[520, 560)$	$[480, 520)$	$[0, 480)$

Topics and schedule

It is the student's responsibility to read the material before the lecture. During the lectures, we will focus on reviewing the key concepts, answering questions, and working on examples.

Date	Lecture	Reading
<2025-09-02 Tue>	1. What is symmetry?	1.1 - 1.7, blurb
<2025-09-04 Thu>	2. Examples of groups	1.1 - 1.7
<2025-09-09 Tue>	3. Abstract groups: first results	1.10, 2.1
<2025-09-11 Thu>	4. Subgroups and cyclic groups	2.2
<2025-09-16 Tue>	5. Dihedral groups	2.3
<2025-09-18 Thu>	6. Homomorphisms and isomorphisms	2.4
<2025-09-23 Tue>	7. The sign of a permutation	Blurb
<2025-09-25 Thu>	**Exam 1**	
<2025-09-30 Tue>	8. Cosets	2.5
<2025-10-02 Thu>	9. Lagrange's theorem	2.5
<2025-10-07 Tue>	10. Noether's isomorphism theorems	2.7
<2025-10-09 Thu>	11. Direct products	3.1
<2025-10-14 Tue>	12. Semidirect products	3.2
<2025-10-16 Thu>	13. Linear algebra over the integers	3.5
<2025-10-21 Tue>	14. Finitely generated abelian groups	3.6
<2025-10-23 Thu>	15. Rotations of regular polyhedra	4.1
<2025-10-28 Tue>	16. The Dodecahedron and Icosahedron	4.2
<2025-10-30 Thu>	17. Reflections	4.3
<2025-11-04 Tue>	**No class** (election day)	
<2025-11-06 Thu>	**Exam 2**	
<2025-11-11 Tue>	**No class** (veterans day)	
<2025-11-13 Thu>	18. Group actions	5.1
<2025-11-18 Tue>	19. Counting orbits	5.2
<2025-11-20 Thu>	20. Symmetries of groups	5.3
<2025-11-25 Tue>	21. Group actions and group structure	5.4
<2025-11-27 Thu>	**No class** (thanksgiving)	
<2025-12-02 Tue>	22. The Sylow theorems	Blurb
<2025-12-04 Thu>	23. Questions?	
<2025-12-09 Tue>	**Exam 3**	

Philosophy

Adopt a growth mindset

Your effort and attitude determine your abilities. Embrace challenges and failure as an oportunity to grow. Find inspiration in other people's success.

Learning is the student's responsibility

Paraphrasing Galileo:

> "You cannot teach a person **anything**; you can only help them find it within themselves."

We are all here to understand. My job as a more experienced learner is to assist you on your journey. But you are responsible for investing the time and effort necessary to learn.

Doing hard things

This is hard work, and it will be frustrating at times. In my opinion, the reward is well worth the investment, as it is often the case with challenging endeavors. In the words of JFK:

> "We choose to go to the Moon in this decade and do the other things, not because they are easy, but because they are hard; because that goal will serve to organize and measure the best of our energies and skills, because that challenge is one that we are willing to accept, one we are unwilling to postpone, and one we intend to win, and the others, too."

Everyone belongs in this classroom

We will subscribe to Federico's axioms.

- **Axiom 1.** Mathematical potential is equally present in different groups, irrespective of geographic, demographic, and economic boundaries.
- **Axiom 2.** Everyone can have joyful, meaningful, and empowering mathematical experiences.
- **Axiom 3.** Mathematics is a powerful, malleable tool that can be shaped and used differently by various communities to serve their needs.

- **Axiom 4.** Every student deserves to be treated with dignity and respect.

Administrative details

- Add/drop only through SPIRE.
- I do not keep a waiting list, and the mathematics department staff will not handle these matters.
- Final exams are kept by the mathematics department. Copies are available upon request.

Drops, withdrawals, and incompletes

- Last day to add or drop with no record: *<2025-09-08 Mon>*.
- Last day to drop with W: *<2025-10-28 Tue>*.
- See the academic calendar for other important dates.
- Incomplete grades are warranted only if a student is passing the course at the time of the request and if the course requirements can be completed by the end of the following semester. Read more here.

Make-up exam policy

You must take the regular exam unless you qualify for an official make-up exam approved by me, following the official make-up request procedure. Make sure you read and understand the make-up exam procedure.

- **Final exam conflict:** If you need a make-up exam due to a final exam schedule conflict, you must submit documentation from the Registrar's Office or other supporting documents at least two weeks before the scheduled exams. No exceptions will be made. No later than one week before the exam, you must submit a written request to me that includes: your name and UMass Amherst Student ID number, your section number, and the reason for requesting the make-up exam. You can request make-up exams through your SPIRE account: in SPIRE, go to Student Home > Final Exam Conflict.

- **Religious observance:** If you must miss an exam due to religious observance, you must contact me within two weeks of the beginning of the semester.
- **Medical reasons:** If you will be absent from an exam due to medical reasons, you must notify me at least one week in advance of the exam. If you have a medical emergency, you must notify me as soon as possible. In either case, you may need to provide documentation. You do not need to disclose personal details of your condition, but you must provide enough information to allow the absence to be excused.
- **Other circumstances:** It is impossible to anticipate all possible situations. In the event of an exceptional circumstance not covered above, you must contact me and explain the problem. You must be prepared to provide a written statement if necessary. I will evaluate the reasons you provide and make a decision.
- Note that there is **no re-taking of exams** in this course. If you are sick and take the exam anyway, you cannot re-take the exam later for a better grade. Regardless of the situation, if you do not feel you can take the exam on the scheduled date, you must inform me as soon as possible.
- Make-up exams will **not** be given to accommodate travel plans.
- I will ensure that taking a make-up exam does not represent any technical advantage. In particular, the questions will be completely different from those on the main exam.

Class attendance policy

By UMass policy, students are expected to attend all regularly scheduled classes at the University for which they are registered. When planning for the tests and homework, I will assume that you have been following my lectures. That being said, I will not enforce or grade for attendance. If you are not able to attend to one of the lectures, make sure you read the notes for that day and talk to other students to check if you missed anything important.

Class etiquette

- I expect you to be present and refrain from using your phone.

- Arrive on time. If you arrive late, try to minimize your disruption.
- Laptops and tablets are allowed during the lectures, provided that you do not disrupt your fellow classmates and the lectures.

Academic dishonesty

Academic dishonesty includes but is not limited to:

- **Cheating:** intentional use, and/or attempted use of trickery, artifice, deception, breach of confidence, fraud and/or misrepresentation of one's academic work.
- **Fabrication:** intentional and unauthorized falsification and/or invention of any information or citation in any academic exercise.
- **Plagiarism:** knowingly representing the words or ideas of another as one's own work in any academic exercise. This includes submitting without citation, in whole or in part, prewritten term papers of another or the research of another, including but not limited to commercial vendors who sell or distribute such materials.
- **Facilitating dishonesty:** knowingly helping or attempting to help another commit an act of academic dishonesty, including substituting for another in an examination, or allowing others to represent as their own one's papers, reports, or academic works.

Formal definitions of academic dishonesty, examples of various forms of dishonesty, and the procedures which faculty must follow to penalize dishonesty are detailed on the Academic Honesty website. Appeals must be filed within ten days of notification by the Academic Honesty Office that a formal charge has been filed by an instructor that suspects dishonesty. Contact the Academic Honesty Office for more information on the process. The Ombuds Office is also available to support individuals engaging with the Academic Honesty process. The Provost's Office is where appeals are processed and filed.

Required statements

Academic honesty statement

Since the integrity of the academic enterprise of any institution of higher education requires honesty in scholarship and research, academic honesty is re-

quired of all students at the University of Massachusetts Amherst. Academic dishonesty is prohibited in all programs of the University. Academic dishonesty includes but is not limited to: cheating, fabrication, plagiarism, and facilitating dishonesty. Appropriate sanctions may be imposed on any student who has committed an act of academic dishonesty. Instructors should take reasonable steps to address academic misconduct. Any person who has reason to believe that a student has committed academic dishonesty should bring such information to the attention of the appropriate course instructor as soon as possible. Instances of academic dishonesty not related to a specific course should be brought to the attention of the appropriate department Head or Chair. Since students are expected to be familiar with this policy and the commonly accepted standards of academic integrity, ignorance of such standards is not normally sufficient evidence of lack of intent (`http://www.umass.edu/dean_students/codeofconduct/acadhonesty/`).

Academic integrity statement

UMass Amherst is strongly committed to academic integrity, which is defined as completing all academic work without cheating, lying, stealing, or receiving unauthorized assistance from any other person, or using any source of information not appropriately authorized or attributed. As a community, we hold each other accountable and support each other's knowledge and understanding of academic integrity. Academic dishonesty is prohibited in all programs of the University and includes but is not limited to: Cheating, fabrication, plagiarism, lying, and facilitating dishonesty, via analogue and digital means. Sanctions may be imposed on any student who has committed or participated in an academic integrity infraction. Any person who has reason to believe that a student has committed an academic integrity infraction should bring such information to the attention of the appropriate course instructor as soon as possible. All students at the University of Massachusetts Amherst have read and acknowledged the Commitment to Academic Integrity and are knowingly responsible for completing all work with integrity and in accordance with the policy: (`https://www.umass.edu/senate/book/academic-regulations-academic-integrity-policy`).

Accommodation statement

The University of Massachusetts Amherst is committed to providing an equal educational opportunity for all students. If you have a documented physical, psychological, or learning disability on file with Disability Services (DS),

you may be eligible for reasonable academic accommodations to help you succeed in this course. If you have a documented disability that requires an accommodation, please notify me within the first two weeks of the semester so that we may make appropriate arrangements. For further information, please visit Disability Services (`https://www.umass.edu/disability/`).

Title IX statement

In accordance with Title IX of the Education Amendments of 1972 that prohibits gender-based discrimination in educational settings that receive federal funds, the University of Massachusetts Amherst is committed to providing a safe learning environment for all students, free from all forms of discrimination, including sexual assault, sexual harassment, domestic violence, dating violence, stalking, and retaliation. This includes interactions in person or online through digital platforms and social media. Title IX also protects against discrimination on the basis of pregnancy, childbirth, false pregnancy, miscarriage, abortion, or related conditions, including recovery. There are resources here on campus to support you. A summary of the available Title IX resources (confidential and non-confidential) can be found at the following link: `https://www.umass.edu/titleix/resources`. You do not need to make a formal report to access them. If you need immediate support, you are not alone. Free and confidential support is available 24 hours a day / 7 days a week / 365 days a year at the SASA Hotline 413-545-0800.