X

```
X = ( -3.   5.   0.
      -3.   4.  -1.
      -4.   0.  -1.
      -1.  -3.  -3. )
```

$$X = \begin{pmatrix} -3. & 5. & 0. \\ -3. & 4. & -1. \\ -4. & 0. & -1. \\ -1. & -3. & -3. \end{pmatrix}$$

```
{{-3., 5., 0.}, {-3., 4., -1.}, {-4., 0., -1.}, {-1., -3., -3.}}
```

Z

```
μ = Mean[X]
```

```
{-2.75, 1.5, -1.25}
```

```
(Z = Map[Function[x, x - μ], X]) // TableForm
```

-0.25	3.5	1.25
-0.25	2.5	0.25
-1.25	-1.5	0.25
1.75	-4.5	-1.75

C

```
(c = Covariance[Z]) // TableForm
```

1.58333	-2.5	-1.25
-2.5	13.6667	4.16667
-1.25	4.16667	1.58333

V

```
(V = Eigenvectors[c]) // TableForm
```

0.194466	-0.934828	-0.29712
0.889962	0.295524	-0.347322
0.412492	-0.196883	0.889431

Is each row of V an eigenvector? Or is it each column? Let us check ...

```
(c.V[[1]]) / V[[1]]
```

```
{15.511, 15.511, 15.511}
```

The fact that identical entries appear in the above result suggests that λ=15.511 is a solution of c.V[[1]]=λ V[[1]]. So the rows of V must be eigenvectors.

```
(c.V[[All, 1]]) / V[[All, 1]]
```

```
{-12.5092, 15.0516, 9.98371}
```

The fact that differing entries appear in the above result suggests that no λ can satisfy c.V[[All,1]]=λ V[[All,1]]. So the columns of V cannot be eigenvectors.

P

```
(P = Z.Transpose[V]) // TableForm
```

-3.69191	0.377691	0.319575
-2.45997	0.429489	-0.372973
1.08488	-1.64257	0.00206666
5.067	0.835389	0.0513319

Note that the PCA process decorrelates the features (columns) in the original data i.e. Covariance[P] must be a diagonal matrix (or very nearly so)

```
Covariance[P] // Chop // TableForm
```

15.511	0	0
0	1.24101	0
0	0	0.081292

Note that the features contained in P are arranged in decreasing order of importance

```
Eigenvalues[c]
```

{15.511, 1.24101, 0.081292}

R

```
(R = P.V) // TableForm
```

-0.25	3.5	1.25
-0.25	2.5	0.25
-1.25	-1.5	0.25
1.75	-4.5	-1.75

```
(Xrecovered = Map[Function[x, x + μ], R]) // Chop // TableForm
```

-3.	5.	0
-3.	4.	-1.
-4.	0	-1.
-1.	-3.	-3.