ELL793 REPORT

ASSIGNMENT 1(CAMERA CALIBRATION)

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CAMERA CALIBRATION

INTRODUCTION

To find intrinsic and extrinsic camera calibration parameters of our mobile phone's camera. Intrinsic calibration parameters include focal length, skew, radial distortion parameters, other distortion parameters, camera's optic centre. Extrinsic calibration parameters include Rotation, Translation (scale) computation for every photo taken by the mobile phone

DATASET CREATION

Calibrating using a checkerboard object. Print two (or three) copies of the checkerboard image and stick them to two (or three) orthogonal planes (wall corner). Click a picture of the checkerboard from your phone.

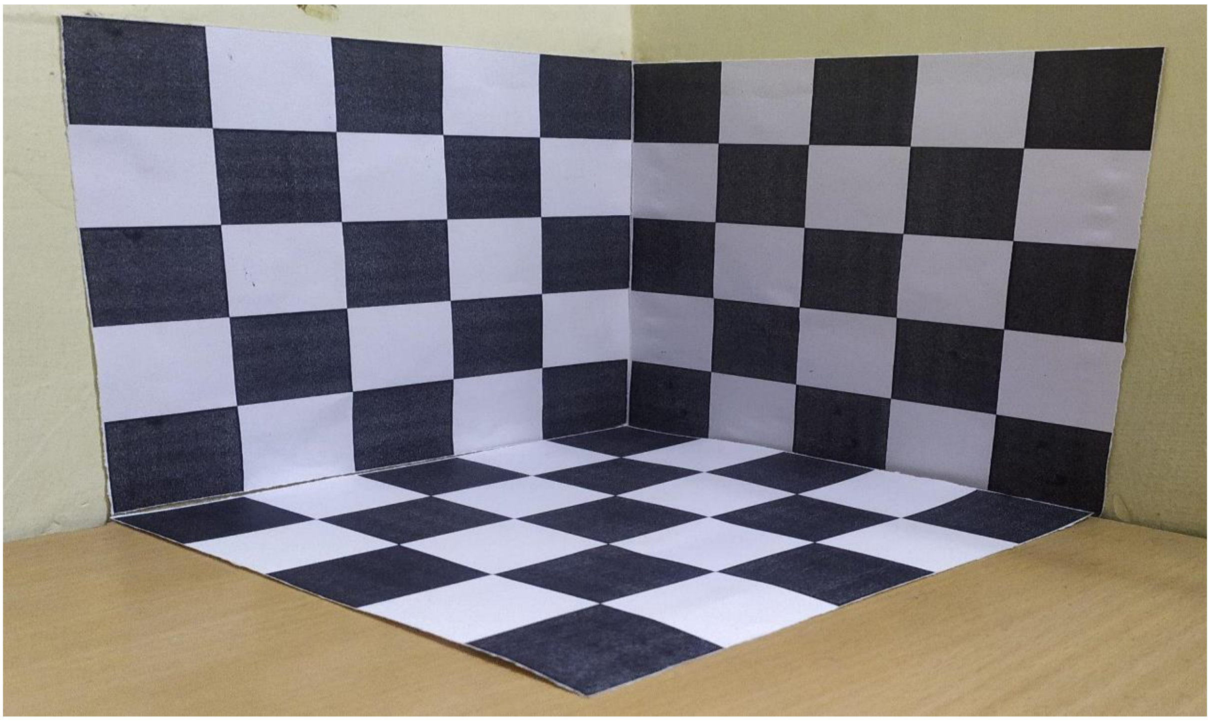

Figure 1: checkerboard image using phone camera

Camera calibration requires 2D and 3D correspondences. Create a dataset that contains XYZ coordinates of N points marked out on the wall checkerboard and also the XY coordinates of the corresponding points on the image.

The world and image coordinates of the 12 points were noted. The image coordinates in pixels were obtained by using the MATLAB image viewer tool.

X	Y	Z	x	y
1	0	0	577	681
3	0	0	376	734
0	1	0	665	557
0	3	0	665	334
0	0	1	742	678
0	0	3	930	735
3	3	0	362	343
3	0	3	641	849
0	3	3	954	362
4	4	0	225	197
4	0	4	632	943
0	4	4	1081	227

Figure 2: world and image coordinates

(X, Y, Z) are world coordinates of a point on image and (x ,y) are the corresponding image coordinates in pixels

Your code should follow the following steps:

STEP 1: NORMALIZATON OF DATASET

Normalize the data such that the centroid of 2D and 3D points are at origin and the average Euclidean distance of 2D and 3D points from the origin is $\sqrt{2}$ and $\sqrt{3}$, respectively. Find the transformation matrices T and U that achieve this for 2D and 3D respectively, i.e, $\hat{x} = Tx$ and $\hat{X} = UX$ where x and X are the unnormalized 2D and 3D points in homogeneous coordinates.

Let $(X_{cen}, Y_{cen}, Z_{cen}, 1)$ denote the centroids of world points in homogenous coordinate system and $(x_{cen}, y_{cen}, 1)$ denote the centroids of world points in homogenous coordinate system.

Let D_{3d} and D_{2d} denote the average distance of the points from the centroids of world and image points respectively.

Let $d_{3d} = D_{3d}$ / sqrt (3) and $d_{2d} = D_{2d}$ / sqrt (2)

Then the normalization matrix U and T will be

$$T = \begin{bmatrix} \frac{1}{d_{2d}} & 0 & \frac{-x_{centroid}}{d_{2d}} \\ 0 & \frac{1}{d_{2d}} & \frac{-y_{centroid}}{d_{2d}} \\ 0 & 0 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} \frac{1}{d_{3d}} & 0 & 0 & \frac{-X_{centroid}}{d_{3d}} \\ 0 & \frac{1}{d_{3d}} & 0 & \frac{-Y_{centroid}}{d_{3d}} \\ 0 & 0 & \frac{1}{d_{3d}} & \frac{-Z_{centroid}}{d_{3d}} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

STEP 2: ESTIMATION OF PROJECION MATRIX

Estimate the normalized projection matrix M using the DLT method. Denormalize the projection matrix M. ($M = T^{-1}MU$).

First calculate P matrix using the world and image coordinates.

$$\mathcal{P} = \begin{pmatrix} \mathbf{P}_1^T & 0^T & -x_1\mathbf{P}_1^T \\ 0^T & \mathbf{P}_1^T & -y_1\mathbf{P}_1^T \\ \dots & \dots & \dots \\ \mathbf{P}_n^T & 0^T & -x_n\mathbf{P}_n^T \\ 0^T & \mathbf{P}_n^T & -y_n\mathbf{P}_n^T \end{pmatrix}$$

Then Solve for M using PM = 0.

The solution to the equation i.e., M is given by the eigenvector corresponding to the smallest eigenvector of the matrix P^TP up to a constant of multiplication rho.

STEP 3: ESTIMATION OF PARAMETERS

Decompose the projection matrix M = K [R| −RXo] into intrinsic matrix K, rotation matrix R and the camera centre Xo. K and R can be estimated using RQ decomposition.

STEP 4: CALCULATION OF ERROR

Verify that the projection matrix is correctly estimated by computing the RMSE between the 2D points marked by you and the estimated 2D projections of the marked 3D points. Visualize the points on the image

STEP 5: ESTIMATION OF RADIAL DISTORTION COFFICIENTS

Let (u, v) be the ideal pixel image coordinates, and (˘u, v˘) the corresponding real observed image coordinates.

Let (x, y) be ideal normalized image coordinates and $(\breve{x}, \breve{y})$ be real normalized image coordinates.

We have

$$\breve{x} = x + x[k_1(x^2 + y^2) + k_2(x^2 + y^2)^2]$$
$$\breve{y} = y + y[k_1(x^2 + y^2) + k_2(x^2 + y^2)^2]$$

k_1 and k_2 are radial distortion cofficients. Let $\breve{u} = u_0 + \alpha\breve{x} + \gamma\breve{y}$ and $\breve{v} = v_0 + \beta\breve{y}$

We have

$$\breve{u} = u + (u - u_0)[k_1(x^2 + y^2) + k_2(x^2 + y^2)^2]$$
$$\breve{v} = v + (v - v_0)[k_1(x^2 + y^2) + k_2(x^2 + y^2)^2]$$

Now for estimating of k_1 and k_2 from above two equations in matrix form we get

$$\begin{bmatrix} (u-u_0)(x^2+y^2) & (u-u_0)(x^2+y^2)^2 \\ (v-v_0)(x^2+y^2) & (v-v_0)(x^2+y^2)^2 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} \breve{u}-u \\ \breve{v}-v \end{bmatrix}$$

Given m points in matrix form as $Dk = d$, where $k = [k_1, k_2]^T$. The linear least-squares solution is given by $k = (D^TD)^{-1}D^Td$. Once k1 and k2 are estimated, we refine the estimate using Levenberg-Marquardt Algorithm.

RESULTS OF PHONE CAMERA CALIBRATION

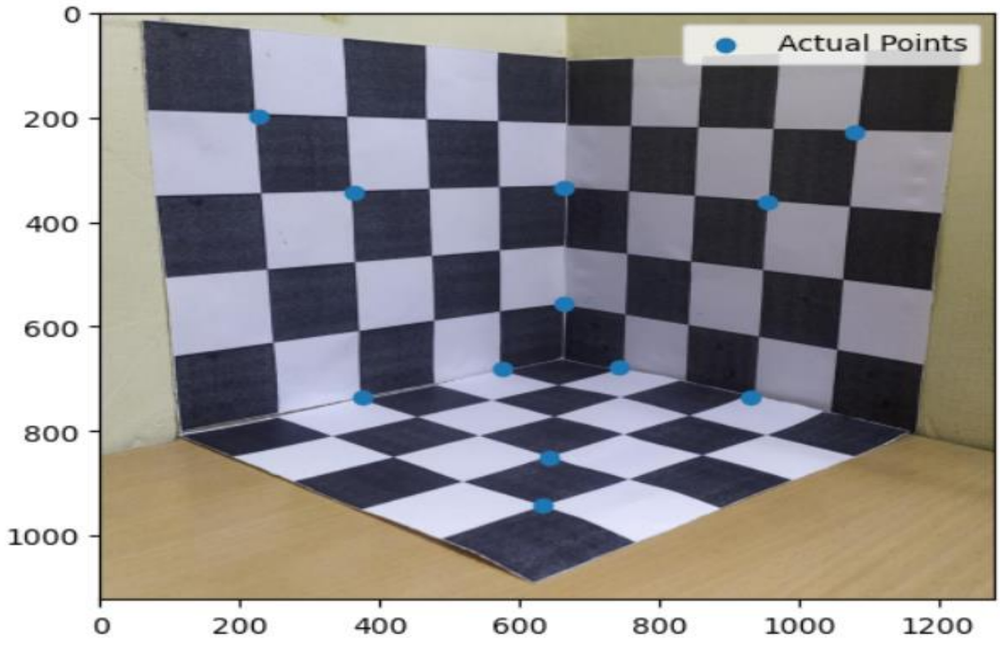

Figure 3: plotting world and image coordinates

Transformation Matrixes (T AND U)

```
Transformation matrix U :
[[ 0.61769751  0.          0.         -0.92654627]
 [ 0.          0.61769751  0.         -0.92654627]
 [ 0.          0.          0.61769751 -0.92654627]
 [ 0.          0.          0.          1.        ]]
```

```
Transformation matrix T :
[[ 0.00459691  0.         -3.00714836]
 [ 0.          0.00459691 -2.54362613]
 [ 0.          0.          1.        ]]
```

Projection Matrixes (M)

```
PROJECTION MATRIX :
[[ 9.75360010e+02  7.78609291e+01 -3.77257269e+02 -5.54932591e+03]
 [ 1.28220439e+02  9.54547222e+02  7.21297894e+01 -5.52019868e+03]
 [ 4.54810768e-01  1.41185113e-01  3.93400741e-01 -8.37778479e+00]]
```

Intrinsic Parameters

Intrinsic calibration parameters include focal length, skew, radial distortion parameters, other distortion parameters, camera's optic centre.

Camera Centre:

$\mathbf{X_0}$ = 802.47

$\mathbf{Y_0}$ = 580.42

Alpha and Beta:

Alpha = 1496.04

Beta = 1451.82

Angle (Theta):

Angle = 89.616°

Skewness(s): s is equal to -alpha*cot(theta)

s = -10.007

Radial distortion coefficients

$\mathbf{K_1}$ = -0.02832031

$\mathbf{K_2}$ = 0.296875

Intrinsic parameter matrix

```
INTRINSIC MATRIX :
[[ 1.49604785e+03 -1.00079903e+01  8.02473014e+02]
 [ 0.00000000e+00  1.45185540e+03  5.80421130e+02]
 [ 0.00000000e+00  0.00000000e+00  1.00000000e+00]]
```

Extrinsic Parameters

Extrinsic calibration parameters include Rotation, Translation (scale) computation

Rotation Matrix:

```
ROTATION MATRIX :
[[ 0.65950383 -0.03183758 -0.75102668]
 [-0.15138293  0.97300751 -0.17418266]
 [ 0.73630015  0.22856675  0.63688251]]
```

Translation Matrix:

```
TRANSLATION MATRIX :
[  1.26510254  -0.73321798 -13.56292465]
```

Original Points

x	y
577	681
376	734
665	557
665	334
742	678
930	735
362	343
641	849
954	362
225	197
632	943
1081	227

Reconstructed Points

x	y
577	680
374	732
664	554
668	334
742	682
928	737
363	345
644	843
952	360
223	198
633	947
1081	226

Root Mean Square Error (RMSE)

The root means square error (RMSE) between the original points and reconstructed points in pixel units comes around

RMSE = 2.33

Visualizing Actual Points and Reconstructed Points

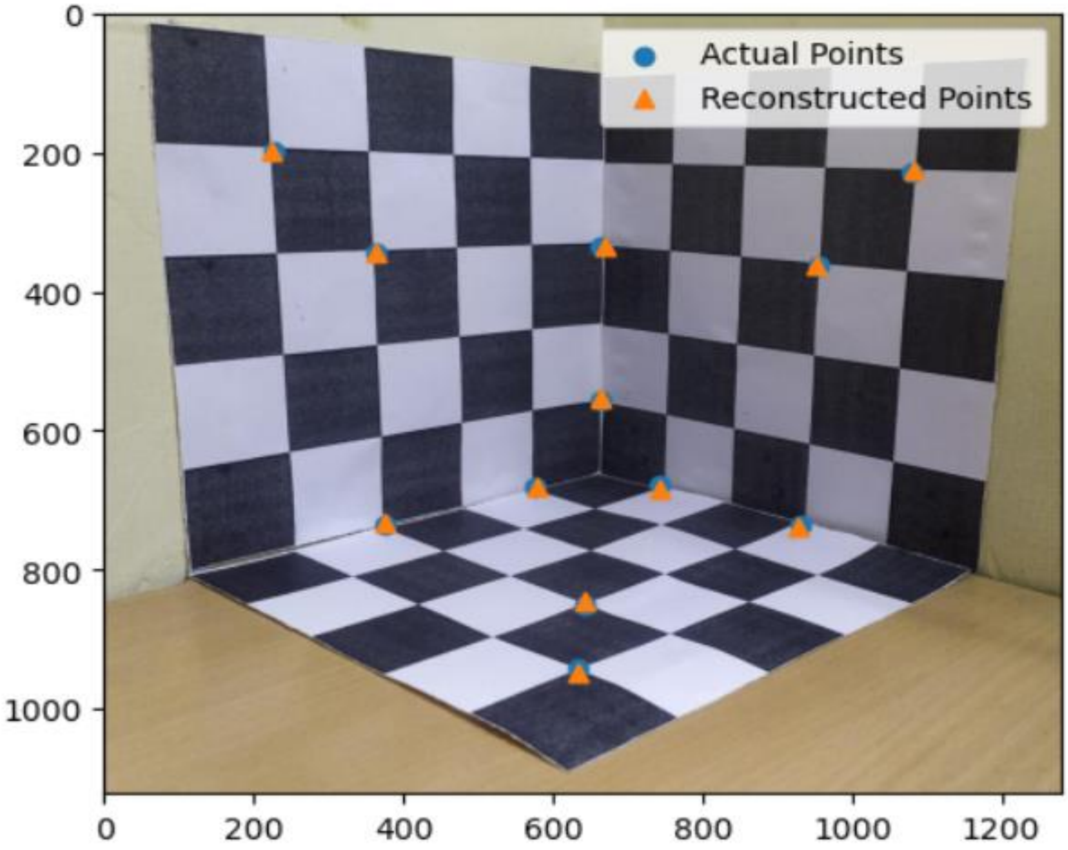

Figure 4: plotting actual and reconstructed points

Also mention why it is a good idea to normalize the points before performing DLT.

X	Y	Z	x	y
1	0	0	577	681
3	0	0	376	734
0	1	0	665	557
0	3	0	665	334
0	0	1	742	678
0	0	3	930	735
3	3	0	362	343
3	0	3	641	849
0	3	3	954	362
4	4	0	225	197
4	0	4	632	943
0	4	4	1081	227

Figure 5: world and image coordinates

From the above table we can observe that world coordinates (X, Y, Z) are much smaller while the image coordinates (x, y) measured in pixels are much bigger. The units of measurement of world coordinates and image coordinates are much different which leads to variances. Also Due to which the matrix P may become ill-conditioned or even singular. Normalization decreases the variance and also decrease the root mean square error (RMSE) between the actual points and reconstructed points

CONCLUSION

We have calibrated Intrinsic calibration parameters which include focal length, skew, radial distortion parameters, other distortion parameters, camera's optic centre and Extrinsic calibration parameters which include Rotation, Translation (scale) computation. We have also calculated the Root mean square error between the actual points and reconstructed points which comes about to be 2.33 which implies that estimated projection matrix is very close to true projection matrix.