Assignment - 2

1. (a) (**5 points**) Use the Gaussian elimination method to find two matrices L, U such that $A = LU$ where L is lower triangular and U is upper triangular.

    (b) (**5 points**) Then solve the linear system $Ax = b$ using the above matrices L and U.

    The code should work for any non-singular square matrix A. Use a proper A and b to verify the results.

2. (**5 points**) Find the conditional number of the following matrix for $n = 3, 4, 3, 5, 6$ using the row-norm, column-norm and Euclidean-norm. Do not use the inbuilt norm function from the Matlab library. What do you observe in the results?

$$H_n = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \cdots & \frac{1}{n} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \cdots & \frac{1}{n+1} \\ \vdots & & & & \vdots \\ \frac{1}{n} & \frac{1}{n+1} & \cdots & \cdots & \frac{1}{2n-1} \end{bmatrix}.$$

3. (**5 points**) A tridiagonal system with n unknowns is given by,

$$a_i x_{i-1} + b_i x_i + c_i x_{i+1} = d_i; \ i = 1, 2, \ldots, n$$

where $a_1 = c_n = 0$. Solve the above system using the following method which is a special case of Gaussian elimination method:

$$c_i' = \begin{cases} \dfrac{c_i}{b_i} & ; \ i = 1 \\ \dfrac{c_i}{b_i - a_i c_{i-1}'} & ; \ i = 2, 3, \ldots, n-1 \end{cases} \qquad d_i' = \begin{cases} \dfrac{d_i}{b_i} & ; \ i = 1 \\ \dfrac{d_i - a_i d_{i-1}'}{b_i - a_i c_{i-1}'} & ; \ i = 2, 3, \ldots, n. \end{cases}$$

$$x_n = d_n'$$
$$x_i = d_i' - c_i' x_{i+1} \qquad ; \ i = n-1, n-2, \ldots, 1.$$

Instructions:

- Any descriptive answer should be written at the top of the code. Use '%' to comment inside the code.

- Make Matlab script for each of the above problems and submit only the '.m' file in gradescope.

- The final code should run without any error. **Sample inputs required for the code should be specified by yourself.**

- Code will be checked manually. Checker will only hit run, and he/she will not provide any input during checking. Everything should be specified in each code.

- **Please do not take the risk of copying or sharing your code with classmates. Code similarity will be checked.**

- For any clarification feel free to comments on team under this assignment posted.

- No partial marks will be given unless you use the correct method for each question. For example, use of general Gaussian elimination method in question 3 will be entitled a zero mark.