

Challenging Problem EigenValue Decomposition

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Abstract—This a simple document that explains that not every matrix cannot have Spectral (EigenValue) decomposition.

Download latex-tikz codes from

<https://github.com/saranshbali/EE5609/blob/master/ChallengeProblemEigenValueDecomposition/challenge.pdf>

1 PROBLEM

Does Spectral (EigenValue) decomposition always exist for a matrix ?

2 SOLUTION

Consider a 2×2 matrix $\mathbf{A} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ over \mathbb{R} field.

Now, calculating eigen values of \mathbf{A}

$$|\mathbf{A} - \lambda \mathbf{I}| = 0 \quad (2.0.1)$$

$$\begin{vmatrix} -\lambda & 1 \\ -1 & -\lambda \end{vmatrix} = 0 \quad (2.0.2)$$

$$\lambda^2 + 1 = 0 \quad (2.0.3)$$

Thus, by (2.0.3) we found out there is no real value of λ that satisfies the equation (2.0.3).

Thus, matrix \mathbf{A} over \mathbb{R} has no real eigen value and hence cannot have a spectral (EigenValue) Decomposition over \mathbb{R} .