Assignment 4

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***Abstract*—This a simple document that explains how to check whether a second degree equation represents pair of straight lines .**

Download all python codes from

https://github.com/saranshbali/EE5609/tree/master/Asssignment4/Python%20Code

and all latex-tikz codes from

https://github.com/saranshbali/EE5609/tree/master/Asssignment4/Latex

1 Problem

Find the value of k so that the following equation may represent the pair of staright lines:

$$2x^2 + xy - y^2 + kx + 6y - 9 = 0 \tag{1.0.1}$$

2 Explaination about Pair of Straight Lines

The general equation of second degree is given by

$$ax^2 + 2bxy + cy^2 + 2dx + 2ey + f = 0 \tag{2.0.1}$$

and the (2.0.1) can be expressed as

$$\mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{2.0.2}$$

where

$$\mathbf{V} = \mathbf{V}^T = \begin{pmatrix} a & b \\ b & c \end{pmatrix} \tag{2.0.3}$$

$$\mathbf{u} = \begin{pmatrix} d \\ e \end{pmatrix} \tag{2.0.4}$$

(2.0.1) refers a pair of staright lines if

$$\begin{vmatrix} \mathbf{V} & \mathbf{u} \\ \mathbf{u}^T & f \end{vmatrix} = 0 \tag{2.0.5}$$

Otherwise (2.0.1) represents a conic.

3 Solution

Here we are given

$$2x^2 + xy - y^2 + kx + 6y - 9 = 0 \tag{3.0.1}$$

We need to find the value of k for which (3.0.1) represents a pair of straight lines.

From (2.0.1) and (2.0.2), we can convert (2.0.5) as

$$\mathbf{x}^T \begin{pmatrix} 2 & 1/2 \\ 1/2 & -1 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} k/2 \\ 3 \end{pmatrix} \mathbf{x} - 9 = 0 \tag{3.0.2}$$

Here, we have

$$\mathbf{V} = \mathbf{V}^T = \begin{pmatrix} 2 & 1/2 \\ 1/2 & -1 \end{pmatrix} \tag{3.0.3}$$

$$\mathbf{u} = \begin{pmatrix} k/2 \\ 3 \end{pmatrix} \tag{3.0.4}$$

$$f = -9 \tag{3.0.5}$$

Since (3.0.1) represents a pair of straight lines, then by (2.0.5), we have

$$\begin{vmatrix} 2 & 1/2 & k/2 \\ 1/2 & -1 & 3 \\ k/2 & 3 & -9 \end{vmatrix} = 0 \tag{3.0.6}$$

By solving, above determinant we get

$$2(9-9) + \frac{-1}{2}(\frac{-9}{2} + \frac{-3k}{2}) + \frac{k}{2}(\frac{3}{2} + \frac{k}{2}) = 0 \tag{3.0.7}$$

$$\frac{(9+3k)}{4} + \frac{k(3+k)}{4} = 0 \tag{3.0.8}$$

$$k^2 + 6k + 9 = 0 \tag{3.0.9}$$

$$(k+3)^2 = 0 \tag{3.0.10}$$

$$k = -3 \tag{3.0.11}$$

Hence by (3.0.11), we have

$$2x^2 + xy - y^2 - 3x + 6y - 9 = 0 \tag{3.0.12}$$

represents family of straight lines for $k = -3$.

The plot of above is shown below

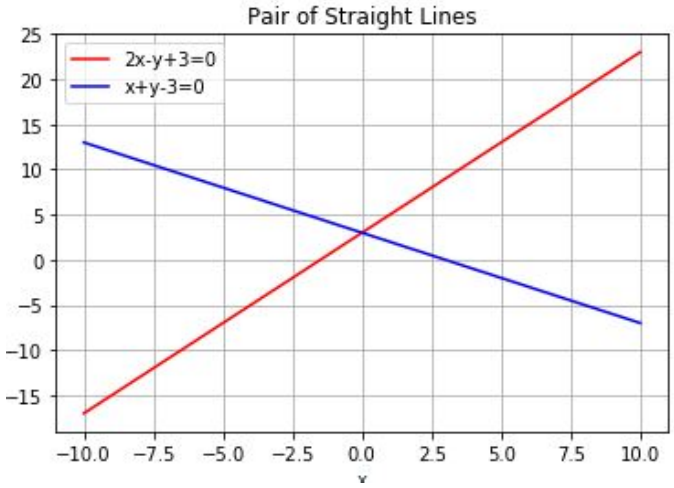

Fig. 0: Pair of Straight Lines