Assignment 3

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***Abstract*—This a simple document that explains how to find results using congruency of triangles.**

Download all latex-tikz codes from

https://github.com/saranshbali/EE5609/tree/master/Assignment3

1 Problem

ABCD is a quadrilateral in which AD = BC and $\angle DAB = \angle CBA$. Prove that

$$a) \quad \triangle ABD \cong \triangle BAC \tag{1.0.1}$$
$$b) \quad BD = AC \tag{1.0.2}$$
$$c) \quad \angle ABD = \angle BAC \tag{1.0.3}$$

2 Some Results used

$$\|\mathbf{A}-\mathbf{B}\| = \|\mathbf{B}-\mathbf{A}\| \tag{2.0.1}$$
$$(\mathbf{B}-\mathbf{A})^T(\mathbf{B}-\mathbf{C}) = \|\mathbf{A}-\mathbf{B}\|^2 - (\mathbf{A}-\mathbf{C})^T(\mathbf{A}-\mathbf{B}) \tag{2.0.2}$$

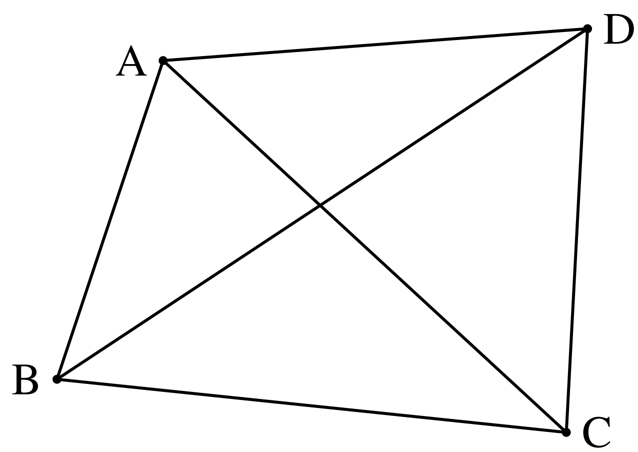

Fig. 0: Quadrilateral ABCD with AD = BC and ∠ DAB = ∠CBA

3 Solution

ABCD is a quadrilateral, where AD=BC and $\angle DAB = \angle CBA$.

1) To show $\triangle ABD \cong \triangle BAC$, we use

$$\angle DAB = \angle CBA \quad (Given) \tag{3.0.1}$$
$$AD = BC \quad (Given) \tag{3.0.2}$$
$$AB = BA \quad (Common\ Side) \tag{3.0.3}$$

Thus, by SAS Congruency Criteria, $\triangle ABD \cong \triangle BAC$.

Also, we are given that

$$\angle DAB = \angle CBA \tag{3.0.4}$$
$$\implies \cos\angle DAB = \cos\angle CBA \tag{3.0.5}$$
$$\frac{(\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{D})}{\|\mathbf{A}-\mathbf{B}\|\,\|\mathbf{A}-\mathbf{D}\|} = \frac{(\mathbf{B}-\mathbf{A})^T(\mathbf{B}-\mathbf{C})}{\|\mathbf{B}-\mathbf{A}\|\,\|\mathbf{B}-\mathbf{C}\|} \tag{3.0.6}$$

Since,

$$\|\mathbf{A}-\mathbf{D}\| = \|\mathbf{B}-\mathbf{C}\| \tag{3.0.7}$$
$$\implies \frac{(\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{D})}{\|\mathbf{A}-\mathbf{B}\|} = \frac{(\mathbf{B}-\mathbf{A})^T(\mathbf{B}-\mathbf{C})}{\|\mathbf{B}-\mathbf{A}\|} \tag{3.0.8}$$
$$\implies (\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{D}) = (\mathbf{B}-\mathbf{A})^T(\mathbf{B}-\mathbf{C}) \tag{3.0.9}$$

$$\implies \|\mathbf{A}-\mathbf{B}\|^2 - (\mathbf{B}-\mathbf{A})^T(\mathbf{B}-\mathbf{D}) = \|\mathbf{A}-\mathbf{B}\|^2 - (\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{C}) \tag{3.0.10}$$

$$(\mathbf{B}-\mathbf{A})^T(\mathbf{B}-\mathbf{D}) = (\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{C}) \tag{3.0.11}$$

$$\|\mathbf{B}-\mathbf{A}\|\,\|\mathbf{B}-\mathbf{D}\|\cos\angle ABD = \|\mathbf{A}-\mathbf{B}\|\,\|\mathbf{A}-\mathbf{C}\|\cos\angle BAC \tag{3.0.12}$$

$$\|\mathbf{B}-\mathbf{D}\|\cos\angle ABD = \|\mathbf{A}-\mathbf{C}\|\cos\angle BAC \tag{3.0.13}$$

1) To Prove $\|\mathbf{B}-\mathbf{D}\| = \|\mathbf{A}-\mathbf{C}\|$.
From (3.0.11),

$$(\mathbf{B}-\mathbf{A})^T(\mathbf{B}-\mathbf{D}) = (\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{C}) \tag{3.0.14}$$

$$\|\mathbf{B}-\mathbf{D}\|^2 - (\mathbf{D}-\mathbf{B})^T(\mathbf{D}-\mathbf{A}) = \|\mathbf{A}-\mathbf{C}\|^2 - (\mathbf{C}-\mathbf{B})^T(\mathbf{C}-\mathbf{A}) \quad (3.0.15)$$

$$\|\mathbf{B}-\mathbf{D}\|^2 - (\|\mathbf{A}-\mathbf{D}\|^2 - (\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{D})) = \|\mathbf{A}-\mathbf{C}\|^2 - (\|\mathbf{B}-\mathbf{C}\|^2 - (\mathbf{B}-\mathbf{A})^T(\mathbf{B}-\mathbf{C})) \quad (3.0.16)$$

We know that

$$\|\mathbf{A}-\mathbf{D}\| = \|\mathbf{B}-\mathbf{C}\| \quad (3.0.17)$$

$$\|\mathbf{B}-\mathbf{D}\|^2 + (\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{D}) = \|\mathbf{A}-\mathbf{C}\|^2 + (\mathbf{B}-\mathbf{A})^T(\mathbf{B}-\mathbf{C}) \quad (3.0.18)$$

$$\|\mathbf{B}-\mathbf{D}\|^2 + \|\mathbf{A}-\mathbf{B}\| \, \|\mathbf{A}-\mathbf{D}\| \cos\angle DAB = \|\mathbf{A}-\mathbf{C}\|^2 + \|\mathbf{B}-\mathbf{A}\| \, \|\mathbf{B}-\mathbf{C}\| \cos\angle CBA \quad (3.0.19)$$

Since, we are given that $\angle DAB = \angle CBA$ and $\|\mathbf{A}-\mathbf{D}\| = \|\mathbf{B}-\mathbf{C}\|$. Then by (3.0.19)

$$\|\mathbf{B}-\mathbf{D}\|^2 = \|\mathbf{A}-\mathbf{C}\|^2 \quad (3.0.20)$$

$$\|\mathbf{B}-\mathbf{D}\| = \|\mathbf{A}-\mathbf{C}\| \quad (3.0.21)$$

Hence, BD = AC.

By, (3.0.13) and (3.0.21), we have

$$\cos\angle ABD = \cos\angle BAC \quad (3.0.22)$$

$$\angle ABD = \angle BAC \quad (3.0.23)$$

Hence, (c) establishes.