

Question 1. Show that the unit direction vector inclined equally to the coordinate axes is $\begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix}$.

Solution: Let \mathbf{m} be the given unit vector such that $\mathbf{m} = \begin{pmatrix} m_x \\ m_y \\ m_z \end{pmatrix}$. Let $\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$,

$\mathbf{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $\mathbf{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ be the direction vectors of the coordinate axes. As \mathbf{m} is a unit vector, so $\|\mathbf{m}\| = 1$ and also we are given is that \mathbf{m} is inclined equally to the coordinate axis,

$$\mathbf{e}_1^T \mathbf{m} = \mathbf{e}_2^T \mathbf{m} = \mathbf{e}_3^T \mathbf{m} \quad (1)$$

Now, (1) implies

$$(\mathbf{e}_1 - \mathbf{e}_2)^T \mathbf{m} = 0 \quad (2)$$

$$(\mathbf{e}_2 - \mathbf{e}_3)^T \mathbf{m} = 0 \quad (3)$$

$$(\mathbf{e}_3 - \mathbf{e}_1)^T \mathbf{m} = 0 \quad (4)$$

Thus, converting above system of equations into matrix form, we get

$$\mathbf{A}\mathbf{m} = 0 \quad (5)$$

To find the solution of (5), we find the echelon form of \mathbf{A} .

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{pmatrix} \xrightarrow{r_3 \leftarrow r_1 + r_3} \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} \quad (6)$$

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} \xrightarrow{r_3 \leftarrow r_2 + r_3} \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix} \quad (7)$$

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix} \xrightarrow{r_1 \leftarrow r_1 + r_2} \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix} \quad (8)$$

From (8), we find out that

$$m_x = m_y = m_z \quad (9)$$

$$\mathbf{m} = \begin{pmatrix} m_z \\ m_z \\ m_z \end{pmatrix} \implies \mathbf{m} = m_z \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad (10)$$

Taking $m_z = 1$, then $\|\mathbf{m}\| = \frac{1}{\sqrt{3}}$ and for \mathbf{m} to be a unit vector, we need to divide each element of \mathbf{m} by $\|\mathbf{m}\|$. Hence

$$\mathbf{m} = \begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix} \quad (11)$$

Thus, we see that

$$\mathbf{m} = \begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix} \quad (12)$$

is the unit direction vector inclined equally to the coordinate axes.

The unit direction vector inclined equally to the coordinate axes

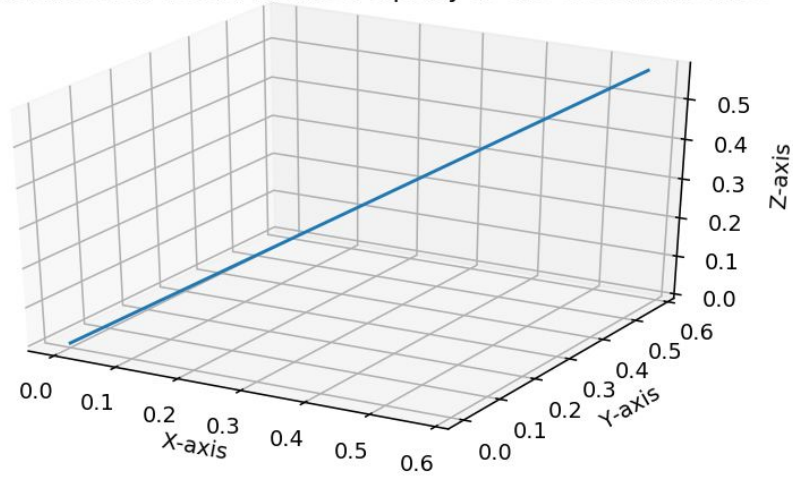


Figure 1