Question 1. *Show that the unit direction vector inclined equally to the coordinate axes is* $\begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix}$.

Solution: Let $\mathbf{a}$ be the given unit vector such that $\mathbf{a} = (\mathbf{a}_x, \mathbf{a}_y, \mathbf{a}_z)$. The direction cosines of $\mathbf{a}$ are given as:

$$\cos\alpha = \frac{\mathbf{a}_x}{|a|}, \qquad \cos\beta = \frac{\mathbf{a}_y}{|a|} \qquad and \qquad \cos\gamma = \frac{\mathbf{a}_z}{|a|} \tag{1}$$

As $\mathbf{a}$ is a unit vector, so $|a| = 1$ and also we are given is that $\mathbf{a}$ is inclined equally to the coordinate axis, thus we have by (1)

$$\cos\alpha = \cos\beta = \cos\gamma = \mathbf{a}_x = \mathbf{a}_y = \mathbf{a}_z \tag{2}$$

Using, (2) and

$$\begin{aligned} \cos^2\alpha + \cos^2\beta + \cos^2\gamma &= 1 \\ 3\cos^2\alpha &= 1 \\ \cos\alpha &= \frac{\pm 1}{\sqrt{3}} \end{aligned}$$

Thus, taking positive sign in above, we get

$$\mathbf{a}_x = \frac{1}{\sqrt{3}}, \quad \mathbf{a}_y = \frac{1}{\sqrt{3}} \quad and \quad \mathbf{a}_z = \frac{1}{\sqrt{3}} \tag{3}$$

Thus, by (3) we have proved that the unit direction vector inclined equally to the coordinate axes is $\begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix}$.

The below figure shows the unit vector $\mathbf{a}$ inclined equally to coordinate axes.

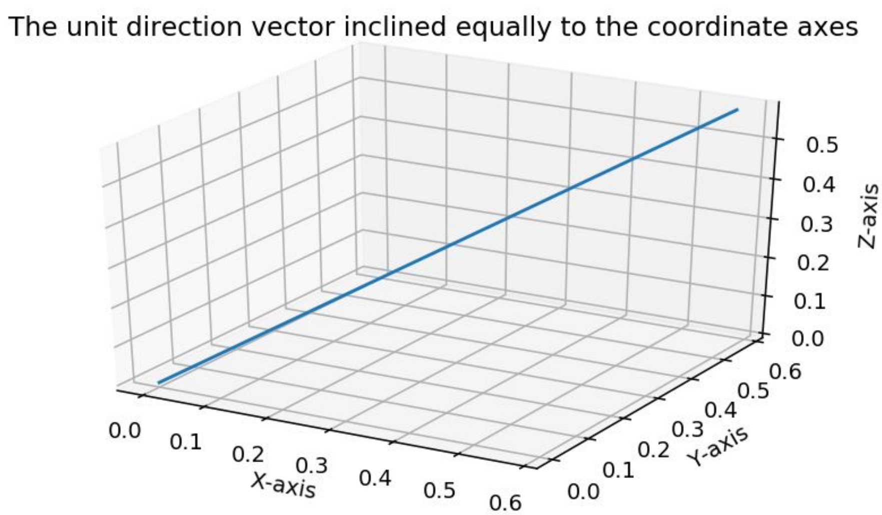
The unit direction vector inclined equally to the coordinate axes
0.0
0.1
0.2
0.3
0.4
0.5
0.6
X-axis
Y-axis
Z-axis

Figure 1