

# Assignment 3

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**Abstract**—This a simple document that explains how to find results using congruency of triangles.

Download all latex-tikz codes from

<https://github.com/saranshbali/EE5609/tree/master/Assignment3>

## 3 SOLUTION

### 1 PROBLEM

ABCD is a quadrilateral in which  $AD = BC$  and  $\angle DAB = \angle CBA$ . Prove that

$$a) \quad \triangle ABD \cong \triangle BAC \quad (1.0.1)$$

$$b) \quad BD = AC \quad (1.0.2)$$

$$c) \quad \angle ABD = \angle BAC \quad (1.0.3)$$

### 2 SOME RESULTS USED

$$\|A - B\| = \|B - A\| \quad (2.0.1)$$

$$(B - A)^T (B - C) = \|A - B\|^2 - (A - C)^T (A - B) \quad (2.0.2)$$

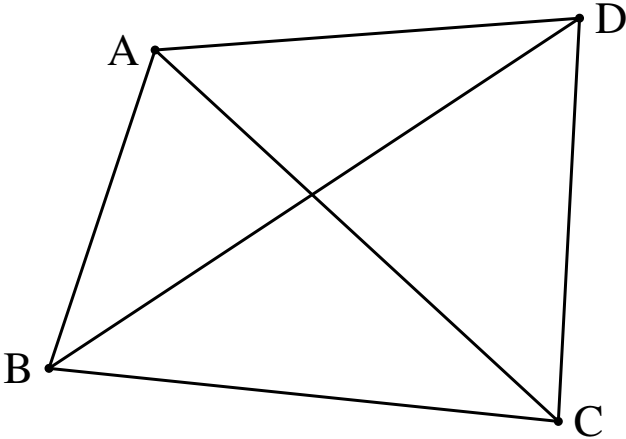


Fig. 0: Quadrilateral ABCD with  $AD = BC$  and  $\angle DAB = \angle CBA$

ABCD is a quadrilateral, where  $AD = BC$  and  $\angle DAB = \angle CBA$ .

To show  $\triangle ABD \cong \triangle BAC$ , we use

$$\angle DAB = \angle CBA \quad (\text{Given}) \quad (3.0.1)$$

$$AD = BC \quad (\text{Given}) \quad (3.0.2)$$

$$AB = BA \quad (\text{Common Side}) \quad (3.0.3)$$

Thus, by SAS Congruency Criteria,  $\triangle ABD \cong \triangle BAC$ .

Also, we are given that

$$\angle DAB = \angle CBA \quad (3.0.4)$$

$$\Rightarrow \cos \angle DAB = \cos \angle CBA \quad (3.0.5)$$

$$\frac{(A - B)^T (A - D)}{\|A - B\| \|A - D\|} = \frac{(B - A)^T (B - C)}{\|B - A\| \|B - C\|} \quad (3.0.6)$$

Since,

$$\|A - D\| = \|B - C\| \quad (3.0.7)$$

$$\Rightarrow \frac{(A - B)^T (A - D)}{\|A - B\|} = \frac{(B - A)^T (B - C)}{\|B - A\|} \quad (3.0.8)$$

$$\Rightarrow (A - B)^T (A - D) = (B - A)^T (B - C) \quad (3.0.9)$$

$$\Rightarrow \|A - B\|^2 - (B - A)^T (B - D) = \|A - B\|^2 - (A - B)^T (A - C) \quad (3.0.10)$$

$$(B - A)^T (B - D) = (A - B)^T (A - C) \quad (3.0.11)$$

$$\|B - A\| \|B - D\| \cos \angle ABD = \|A - B\| \|A - C\| \cos \angle BAC \quad (3.0.12)$$

$$\|B - D\| \cos \angle ABD = \|A - C\| \cos \angle BAC \quad (3.0.13)$$

1) To Prove  $\|B - D\| = \|A - C\|$ .

From (3.0.11),

$$(B - A)^T (B - D) = (A - B)^T (A - C) \quad (3.0.14)$$

$$\begin{aligned} \|\mathbf{B} - \mathbf{D}\|^2 - (\mathbf{D} - \mathbf{B})^T(\mathbf{D} - \mathbf{A}) = \\ \|\mathbf{A} - \mathbf{C}\|^2 - (\mathbf{C} - \mathbf{B})^T(\mathbf{C} - \mathbf{A}) \end{aligned} \quad (3.0.15)$$

$$\begin{aligned} \|\mathbf{B} - \mathbf{D}\|^2 - (\|\mathbf{A} - \mathbf{D}\|^2 - (\mathbf{A} - \mathbf{B})^T(\mathbf{A} - \mathbf{D})) = \\ \|\mathbf{A} - \mathbf{C}\|^2 - (\|\mathbf{B} - \mathbf{C}\|^2 - (\mathbf{B} - \mathbf{A})^T(\mathbf{B} - \mathbf{C})) \end{aligned} \quad (3.0.16)$$

We know that

$$\|\mathbf{A} - \mathbf{D}\| = \|\mathbf{B} - \mathbf{C}\| \quad (3.0.17)$$

$$\begin{aligned} \|\mathbf{B} - \mathbf{D}\|^2 + (\mathbf{A} - \mathbf{B})^T(\mathbf{A} - \mathbf{D}) = \\ \|\mathbf{A} - \mathbf{C}\|^2 + (\mathbf{B} - \mathbf{A})^T(\mathbf{B} - \mathbf{C}) \end{aligned} \quad (3.0.18)$$

$$\begin{aligned} \|\mathbf{B} - \mathbf{D}\|^2 + \|\mathbf{A} - \mathbf{B}\| \|\mathbf{A} - \mathbf{D}\| \cos \angle DAB = \\ \|\mathbf{A} - \mathbf{C}\|^2 + \|\mathbf{B} - \mathbf{A}\| \|\mathbf{B} - \mathbf{C}\| \cos \angle CBA \end{aligned} \quad (3.0.19)$$

Since, we are given that  $\angle DAB = \angle CBA$  and  $\|\mathbf{A} - \mathbf{D}\| = \|\mathbf{B} - \mathbf{C}\|$ . Then by (3.0.19)

$$\|\mathbf{B} - \mathbf{D}\|^2 = \|\mathbf{A} - \mathbf{C}\|^2 \quad (3.0.20)$$

$$\|\mathbf{B} - \mathbf{D}\| = \|\mathbf{A} - \mathbf{C}\| \quad (3.0.21)$$

Hence,  $BD = AC$ .

By, (3.0.13) and (3.0.21), we have

$$\cos \angle ABD = \cos \angle BAC \quad (3.0.22)$$

$$\angle ABD = \angle BAC \quad (3.0.23)$$

Hence, (c) establishes.