

Assignment7

Saransh Bali

Abstract—This a simple document that explains how to transform a matrix into identity matrix using product of elementary matrices.

Download all latex-tikz codes from

<https://github.com/saranshbali/EE5609/blob/master/Assignment7>

$$\mathbf{E}_5 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{pmatrix} \quad (2.0.6)$$

$$\mathbf{E}_6 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{2}{7} \end{pmatrix} \quad (2.0.7)$$

$$\mathbf{E}_7 = \begin{pmatrix} 1 & 0 & \frac{-1}{2} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (2.0.8)$$

$$\mathbf{E}_8 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 1 \end{pmatrix} \quad (2.0.9)$$

1 PROBLEM

For the matrix $\mathbf{A} = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 0 & 1 \\ 3 & 0 & 1 \end{pmatrix}$, find elementary matrices $\mathbf{E}_1, \mathbf{E}_2, \dots, \mathbf{E}_k$ such that

$$\mathbf{E}_k \dots \mathbf{E}_2 \mathbf{E}_1 \mathbf{A} = \mathbf{I} \quad (1.0.1)$$

Now, we calculate

$$\mathbf{E}_8 \mathbf{E}_7 \mathbf{E}_6 \mathbf{E}_5 \mathbf{E}_4 \mathbf{E}_3 \mathbf{E}_2 \mathbf{E}_1 = \begin{pmatrix} 0 & -1 & 1 \\ -1 & 2 & -1 \\ 0 & 3 & -2 \end{pmatrix} \quad (2.0.10)$$

Hence,

$$(\mathbf{E}_8 \mathbf{E}_7 \mathbf{E}_6 \mathbf{E}_5 \mathbf{E}_4 \mathbf{E}_3 \mathbf{E}_2 \mathbf{E}_1) \mathbf{A} = \begin{pmatrix} 0 & -1 & 1 \\ -1 & 2 & -1 \\ 0 & 3 & -2 \end{pmatrix} \begin{pmatrix} 1 & -1 & 1 \\ 2 & 0 & 1 \\ 3 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (2.0.11)$$

2 SOLUTION

Given,

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 0 & 1 \\ 3 & 0 & 1 \end{pmatrix} \quad (2.0.1)$$

Take,

$$\mathbf{E}_1 = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (2.0.2)$$

$$\mathbf{E}_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{pmatrix} \quad (2.0.3)$$

$$\mathbf{E}_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (2.0.4)$$

$$\mathbf{E}_4 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (2.0.5)$$