Question 1. *Show that the vectors* $\begin{pmatrix} +2 \\ -1 \\ +1 \end{pmatrix}$, $\begin{pmatrix} +1 \\ -3 \\ -5 \end{pmatrix}$ *and* $\begin{pmatrix} +3 \\ -4 \\ -4 \end{pmatrix}$ *form the vertices*
of a right angled triangle.

Solution. Let **A**,**B** and **C** be given vectors such that $\mathbf{A} = \begin{pmatrix} +2 \\ -1 \\ +1 \end{pmatrix}$,

$\mathbf{B} = \begin{pmatrix} +1 \\ -3 \\ -5 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} +3 \\ -4 \\ -4 \end{pmatrix}$.

To show that **A**, **B** and **C** form the vertices of a right angled triangle. First we
need to show that **A**, **B** and **C** are indeed vertices of a triangle.
For this we need to see if the vertices satisfy triangle inequality. Let a, b and c
denote the length of vectors $(\mathbf{A} - \mathbf{B})$, $(\mathbf{B} - \mathbf{C})$ and $(\mathbf{C} - \mathbf{A})$. Now,
$a = \sqrt{41}$, $b = \sqrt{6}$ and $c = \sqrt{35}$. We can see that $a + b > c$, $a + c > b$ and
$b + c > a$. Thus, the given vectors **A**, **B** and **C** form the vertices of a triangle.
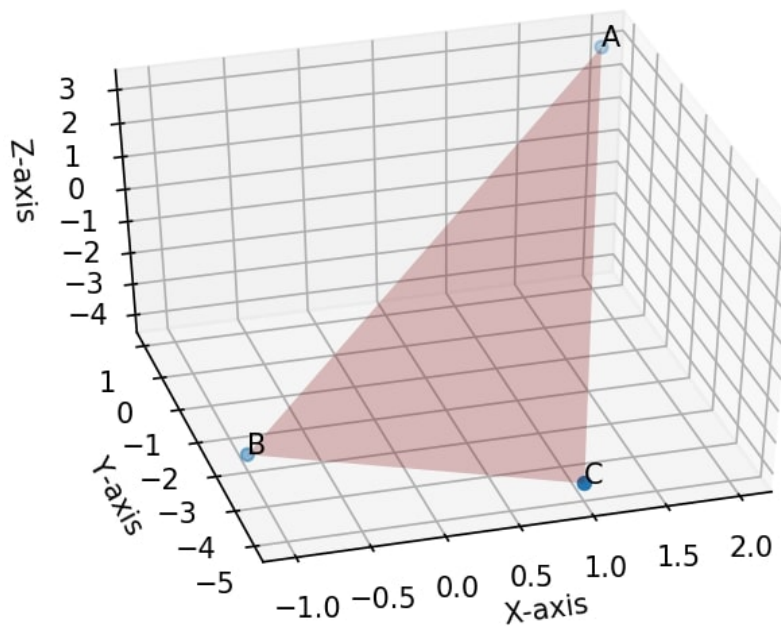

Figure 1

To prove, the $\triangle ABC$ is a right triangle, we need to calculate the inner

product of all the below vectors and check if any one of them is 0.:

$$\langle \mathbf{A} - \mathbf{C}, \mathbf{B} - \mathbf{C} \rangle = (\mathbf{A} - \mathbf{C})^T (\mathbf{B} - \mathbf{C}) = \begin{pmatrix} -1 & +3 & +5 \end{pmatrix} \begin{pmatrix} -2 \\ +1 \\ -1 \end{pmatrix} = 00 \quad (1)$$

$$\langle \mathbf{A} - \mathbf{B}, \mathbf{C} - \mathbf{B} \rangle = (\mathbf{A} - \mathbf{B})^T (\mathbf{C} - \mathbf{B}) = \begin{pmatrix} +1 & +2 & +6 \end{pmatrix} \begin{pmatrix} +2 \\ -1 \\ +1 \end{pmatrix} = 06 \quad (2)$$

$$\langle \mathbf{B} - \mathbf{A}, \mathbf{C} - \mathbf{A} \rangle = (\mathbf{B} - \mathbf{A})^T (\mathbf{C} - \mathbf{A}) = \begin{pmatrix} -1 & -2 & -6 \end{pmatrix} \begin{pmatrix} +1 \\ -3 \\ -5 \end{pmatrix} = 35 \quad (3)$$

Clearly, from (1) we can see that $\triangle ABC$ is right angled at C.