Assignment 4

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Abstract—**This a simple document that explains how to check whether a second degree equation represents pair of straight lines .**

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1 PROBLEM

Find the value of k so that the following equation may represent the pair of staright lines:

$$2x^2 + xy - y^2 + kx + 6y - 9 = 0 \qquad (1.0.1)$$

2 SOLUTION

Here we are given

$$2x^2 + xy - y^2 + kx + 6y - 9 = 0 \qquad (2.0.1)$$

We need to find the value of k for which (2.0.1) represents a pair of straight lines.

Converting (2.0.1) into vector form, we get

$$\mathbf{x}^T \begin{pmatrix} 2 & 1/2 \\ 1/2 & -1 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} k/2 \\ 3 \end{pmatrix} \mathbf{x} - 9 = 0 \qquad (2.0.2)$$

Here, we have

$$\mathbf{V} = \mathbf{V}^T = \begin{pmatrix} 2 & 1/2 \\ 1/2 & -1 \end{pmatrix} \qquad (2.0.3)$$

$$\mathbf{u} = \begin{pmatrix} k/2 \\ 3 \end{pmatrix} \qquad (2.0.4)$$

$$f = -9 \qquad (2.0.5)$$

The above represents a pair of straight lines if

$$\begin{vmatrix} \mathbf{V} & \mathbf{u} \\ \mathbf{u}^T & f \end{vmatrix} = 0 \qquad (2.0.6)$$

Since (2.0.1) represents a pair of straight lines, then by (2.0.6), we have

$$\begin{vmatrix} 2 & 1/2 & k/2 \\ 1/2 & -1 & 3 \\ k/2 & 3 & -9 \end{vmatrix} = 0 \qquad (2.0.7)$$

By solving, above determinant we get

$$2(9-9) + \frac{-1}{2}(\frac{-9}{2} + \frac{-3k}{2}) + \frac{k}{2}(\frac{3}{2} + \frac{k}{2}) = 0 \quad (2.0.8)$$

$$\frac{(9 + 3k)}{4} + \frac{k(3 + k)}{4} = 0 \qquad (2.0.9)$$

$$k^2 + 6k + 9 = 0 \qquad (2.0.10)$$

$$(k + 3)^2 = 0 \qquad (2.0.11)$$

$$k = -3 \qquad (2.0.12)$$

Hence by (2.0.12), we have

$$2x^2 + xy - y^2 - 3x + 6y - 9 = 0 \qquad (2.0.13)$$

represents family of straight lines for $k = -3$.

To find the staright lines, we write each of thrm in their vector form as

$$\mathbf{n_1}^T \mathbf{x} = c_1 \qquad (2.0.14)$$

$$\mathbf{n_2}^T \mathbf{x} = c_2 \qquad (2.0.15)$$

Equating the product of above with (2.0.2), we have

$$\left(\mathbf{n_1}^T \mathbf{x} - c_1\right)\left(\mathbf{n_2}^T \mathbf{x} - c_2\right) =$$
$$\mathbf{x}^T \begin{pmatrix} 2 & 1/2 \\ 1/2 & -1 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} k/2 \\ 3 \end{pmatrix} \mathbf{x} - 9 \quad (2.0.16)$$

$$\implies \mathbf{n_1} * \mathbf{n_2} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \qquad (2.0.17)$$

$$c_2\mathbf{n_1} + c_1\mathbf{n_1} = -2\begin{pmatrix} -3/2 \\ 3 \end{pmatrix} \qquad (2.0.18)$$

$$c_1c_2 = -9 \qquad (2.0.19)$$

Here, the slope of these lines are given by the roots

of the polynomial

$$-m^2 + m + 2 = 0 \qquad (2.0.20)$$

$$m^2 - m - 2 = 0 \qquad (2.0.21)$$

$$m = \frac{1 \pm \sqrt{1 + 8}}{2} \qquad (2.0.22)$$

$$m_1 = \frac{1 + 3}{2} = 2 \qquad (2.0.23)$$

$$m_2 = \frac{1 - 3}{2} = -1 \qquad (2.0.24)$$

$$n_1 = k_1 \begin{pmatrix} -2 \\ 1 \end{pmatrix} \qquad (2.0.25)$$

$$n_2 = k_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad (2.0.26)$$

Substituing (2.0.25) and (2.0.26) in (2.0.17), we get

$$k_1 k_2 = -1 \qquad (2.0.27)$$

Taking $k_1 = -1$ and $k_2 = 1$, we get

$$n_1 = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \qquad (2.0.28)$$

$$n_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad (2.0.29)$$

Substituting in (2.0.18) for above values of n_1 and n_2

$$\left(n_1 n_2 \right) \begin{pmatrix} c_2 \\ c_1 \end{pmatrix} = \begin{pmatrix} 3 \\ -6 \end{pmatrix} \qquad (2.0.30)$$

$$\begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} c_2 \\ c_1 \end{pmatrix} = \begin{pmatrix} 3 \\ -6 \end{pmatrix} \qquad (2.0.31)$$

Solving (2.0.31), we get

$$c_1 = -3 \qquad (2.0.32)$$

$$c_2 = 3 \qquad (2.0.33)$$

Thus, pair of staright lines are

$$\begin{pmatrix} 2 & -1 \end{pmatrix} \mathbf{x} = -3 \qquad (2.0.34)$$

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = 3 \qquad (2.0.35)$$

where

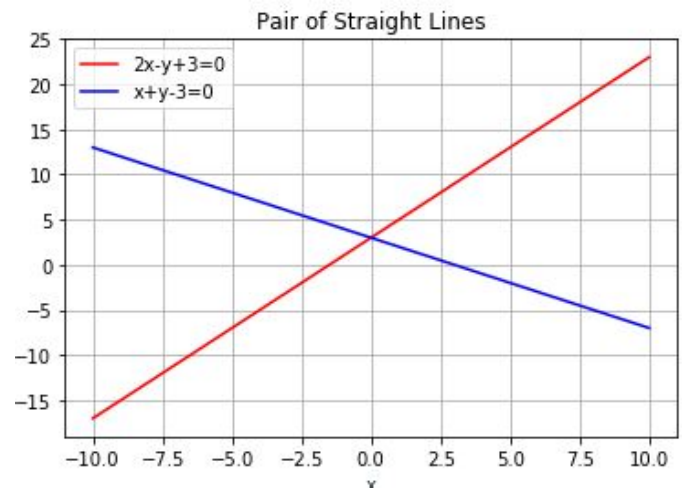$$\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix} \qquad (2.0.36)$$

The plot of above is shown below

Fig. 0: Pair of Straight Lines