Assignment 1

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***Abstract*—This a simple document that explains how to find a unit vector inclined equally to the coordinate axes.**

Download all python codes from

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https://github.com/saranshbali/EE5609/blob/master/Assignment1/Code/assignment1_n.ipynb
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and latex-tikz codes from

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https://github.com/saranshbali/EE5609/tree/master/Assignment1/Latex
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1 Problem

Show that the unit direction vector inclined equally to the coordinate axes is $\begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix}$.

2 Solution

Let $\mathbf{m}$ be a unit vector such that $\mathbf{m} = \begin{pmatrix} m_x \\ m_y \\ m_z \end{pmatrix}$. Let $\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\mathbf{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $\mathbf{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ be the direction vectors of the coordinate axes.

As $\mathbf{m}$ is a unit vector, so $\|\mathbf{m}\| = 1$ and also we are given is that $\mathbf{m}$ is inclined equally to the coordinate axis,

$$\mathbf{e}_1^T\mathbf{m} = \mathbf{e}_2^T\mathbf{m} = \mathbf{e}_3^T\mathbf{m} \tag{2.0.1}$$

Now, 2.0.1 implies

$$(\mathbf{e}_1 - \mathbf{e}_2)^T\mathbf{m} = 0 \tag{2.0.2}$$
$$(\mathbf{e}_2 - \mathbf{e}_3)^T\mathbf{m} = 0 \tag{2.0.3}$$
$$(\mathbf{e}_3 - \mathbf{e}_1)^T\mathbf{m} = 0 \tag{2.0.4}$$

Thus, converting above system of equations into matrix form, we get

$$\mathbf{A}\mathbf{m} = 0 \tag{2.0.5}$$

To find the solution of 2.0.5, we find the echelon form of $\mathbf{A}$.

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{pmatrix} \xleftrightarrow{r_3 \leftarrow r_1 + r_3} \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} \tag{2.0.6}$$

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} \xleftrightarrow{r_3 \leftarrow r_2 + r_3} \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix} \tag{2.0.7}$$

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix} \xleftrightarrow{r_1 \leftarrow r_1 + r_2} \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix} \tag{2.0.8}$$

From 2.0.8, we find out that

$$m_x = m_y = m_z \tag{2.0.9}$$

$$\mathbf{m} = \begin{pmatrix} m_z \\ m_z \\ m_z \end{pmatrix} \implies \mathbf{m} = m_z \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{2.0.10}$$

Taking $m_z = 1$, then $\|\mathbf{m}\| = \frac{1}{\sqrt{3}}$ and for $\mathbf{m}$ to be a unit vector, we need to divide each element of $\mathbf{m}$ by $\|\mathbf{m}\|$.

Thus, we see that

$$\mathbf{m} = \begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix} \tag{2.0.11}$$

is the unit direction vector inclined equally to the coordinate axes.

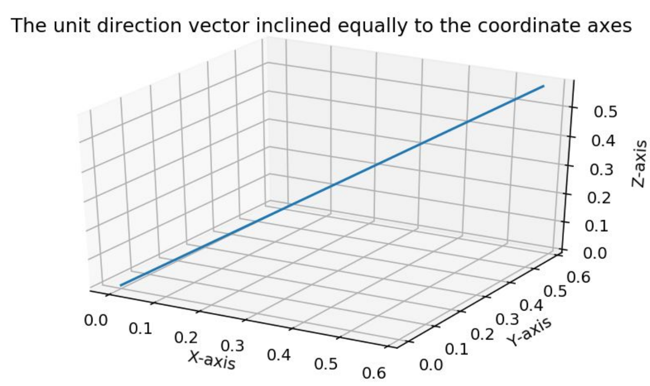

Fig. 1