Orthogonality

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***Abstract*—This a simple document that explains how to prove the eigen values of an Orthogonal matrix have magnitude one.**

Download all latex-tikz codes from

https://github.com/saranshbali/EE5609/blob/master/Orthogonality

1 Problem

Show that the eigen values λ of an Orthogonal matrix $\mathbf{Q}$ are such that $|\lambda| = 1$.

2 Solution

Here $\mathbf{Q}$ is given to be an orthogonal matrix, then

$$\mathbf{Q}\mathbf{Q}^T = \mathbf{Q}^T\mathbf{Q} = \mathbf{I} \tag{2.0.1}$$

Let λ be an eigen value of $\mathbf{Q}$ and $\mathbf{v}$ be corresponding eigen vector. Then

$$\mathbf{Q}\mathbf{v} = \lambda\mathbf{v} \tag{2.0.2}$$
$$\|\mathbf{Q}\mathbf{v}\|^2 = \|\lambda\mathbf{v}\|^2 \tag{2.0.3}$$
$$(\mathbf{Q}\mathbf{v})^T\,\mathbf{Q}\mathbf{v} = |\lambda|^2\,\|\mathbf{v}\|^2 \tag{2.0.4}$$
$$\left(\mathbf{v}^T\mathbf{Q}^T\right)\mathbf{Q}\mathbf{v} = |\lambda|^2\,\|\mathbf{v}\|^2 \tag{2.0.5}$$
$$\tag{2.0.6}$$

By, (2.0.1), we have

$$\mathbf{v}^T\mathbf{v} = |\lambda|^2\,\|\mathbf{v}\|^2 \tag{2.0.7}$$
$$\|\mathbf{v}\|^2 = |\lambda|^2\,\|\mathbf{v}\|^2 \tag{2.0.8}$$

Thus, we have

$$|\lambda|^2 = 1 \tag{2.0.9}$$
$$|\lambda| = 1 \tag{2.0.10}$$

Thus, magnitude of eigen values of an orthogonal matrix is 1.