

Challenging Problem 6

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Abstract—This a simple document that explains Orthogonal vectors are Linearly independent.

Download latex-tikz codes from

<https://github.com/saranshbali/EE5609/blob/master/ChallengeProblem6>

1 PROBLEM

Show that the set of Orthogonal vectors $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n$ is Linear independent.

2 SOLUTION

Consider, the expression

$$a_1 \mathbf{v}_1 + a_2 \mathbf{v}_2 + \dots + a_n \mathbf{v}_n = 0 \quad (2.0.1)$$

Take the dot product of 2.0.1 with \mathbf{v}_1 , we get

$$a_1 \|\mathbf{v}_1\|^2 + a_2 \mathbf{v}_2^T \mathbf{v}_1 + \dots + a_n \mathbf{v}_n^T \mathbf{v}_1 = 0 \quad (2.0.2)$$

$$a_1 \|\mathbf{v}_1\|^2 = 0 \quad (\mathbf{v}_i^T \mathbf{v}_j = 0 \quad \forall i \neq j) \quad (2.0.3)$$

$$\|\mathbf{v}_1\|^2 = 0 \quad \implies \mathbf{v}_1 = 0 \quad (2.0.4)$$

Hence, $a_1 = 0$

Similarly, taking the dot product of 2.0.1 with $\mathbf{v}_2, \dots, \mathbf{v}_n$, we find out $a_2 = 0, \dots, a_n = 0$.

Thus, the set of Orthogonal vectors $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n$ is Linear independent.