

# Assignment 12

Saransh Bali

*Abstract*—This a simple document that explains about annihilators of a set  $S$ .

Download latex-tikz from

<https://github.com/saranshbali/EE5609/blob/master/Assignment12>

## 1 PROBLEM

Let  $\alpha_1 = (1, 0, -1, 2)$  and  $\alpha_2 = (2, 3, 1, 1)$  and let  $\mathbf{W}$  be the subspace of  $\mathbb{R}^4$  spanned by  $\alpha_1$  and  $\alpha_2$ . Which linear functionals  $\mathbf{f}$  :

$$f(x_1, x_2, x_3, x_4) = c_1x_1 + c_2x_2 + c_3x_3 + c_4x_4 \quad (1.0.1)$$

are in the annihilator of  $\mathbf{W}$ ?

## 2 DEFINITIONS

Linear Functional	If $\mathbf{V}$ is a vector space over the field $\mathbf{F}$ , a linear transformation $\mathbf{f}$ from $\mathbf{V}$ into the scalar field $\mathbf{F}$ is also called a linear functional on $\mathbf{V}$ .
Dual Space of $\mathbf{V}$	If $\mathbf{V}$ is a vector space, the collection of all linear functionals on $\mathbf{V}$ forms a vector space. It is the space $\mathbf{L}(\mathbf{V}, \mathbf{F})$ which we denote by $\mathbf{V}^*$ and is called as dual space of $\mathbf{V}$ .
Annihilator of $S$	$\mathbf{V}$ is a vector space over the field $\mathbf{F}$ and $S$ is a subset of $\mathbf{V}$ , the annihilator of $S$ is the set $S^\circ$ of linear functionals $\mathbf{f}$ on $\mathbf{V}$ such that $\mathbf{f}(\alpha) = 0$ for every $\alpha$ in $S$ .

## 3 SOLUTION

Given two vectors

$$\alpha_1 = (1, 0, -1, 2) \quad \text{and} \quad \alpha_2 = (2, 3, 1, 1) \quad (3.0.1)$$

Since,  $\alpha_1$  is not a scalar multiple of  $\alpha_2$ . Thus,  $\alpha_1$  and  $\alpha_2$  are linearly independent. Also, given  $\alpha_1$  and  $\alpha_2$  span  $\mathbf{W}$ , thus  $\{\alpha_1, \alpha_2\}$  form basis for  $\mathbf{W}$ . Hence,  $\mathbf{W}$  has dimension 2.

Now, a functional  $\mathbf{f}$  is in the annihilator of  $\mathbf{W}$  if and only if  $\mathbf{f}(\alpha_1) = \mathbf{f}(\alpha_2) = 0$ . We find such  $\mathbf{f}$  by solving the system

$$\mathbf{f}(\alpha_1) = 0 \quad (3.0.2)$$

$$\mathbf{f}(\alpha_2) = 0 \quad (3.0.3)$$

or equivalently

$$c_1 - c_3 + 2c_4 = 0 \quad (3.0.4)$$

$$2c_1 + 3c_2 + c_3 + c_4 = 0 \quad (3.0.5)$$

Converting, above into system of equations, we have

$$\begin{pmatrix} 1 & 0 & -1 & 2 \\ 2 & 3 & 1 & 1 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix} = 0 \quad (3.0.6)$$

Converting (3.0.6) into row reduced echelon form

$$\begin{pmatrix} 1 & 0 & -1 & 2 \\ 2 & 3 & 1 & 1 \end{pmatrix} \xrightarrow{r_2 = r_2 - 2r_1} \begin{pmatrix} 1 & 0 & -1 & 2 \\ 0 & 3 & 3 & -3 \end{pmatrix} \quad (3.0.7)$$

$$\begin{pmatrix} 1 & 0 & -1 & 2 \\ 0 & 3 & 3 & -3 \end{pmatrix} \xrightarrow{r_2 = \frac{r_2}{3}} \begin{pmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & 1 & -1 \end{pmatrix} \quad (3.0.8)$$

from (3.0.8), we have

$$c_1 = c_3 - 2c_4 \quad (3.0.9)$$

$$c_2 = -c_3 + c_4 \quad (3.0.10)$$

The general element of  $\mathbf{W}^o$  is therefore

$$\begin{aligned} \mathbf{f}(x_1, x_2, x_3, x_4) = \\ (c_3 - 2c_4)x_1 + (c_3 + c_4)x_2 + c_3x_3 + c_4x_4 \end{aligned} \quad (3.0.11)$$

for arbitrary constants  $c_3$  and  $c_4$ .

Also,  $\mathbf{W}^o$  has dimension 2.