Question 1. *Show that the unit direction vector inclined equally to the coordinate axes is* $\begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix}$.

Solution: Let $\mathbf{m}$ be the given unit vector such that $\mathbf{m} = \begin{pmatrix} m_x \\ m_y \\ m_z \end{pmatrix}$.Let $\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\mathbf{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $\mathbf{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ be the direction vectors of the coordinate axes. As $\mathbf{m}$ is a unit vector, so $\|\mathbf{m}\| = 1$ and also we are given is that $\mathbf{m}$ is inclined equally to the coordinate axis,

$$\mathbf{e}_1^T\mathbf{m} = \mathbf{e}_2^T\mathbf{m} = \mathbf{e}_3^T\mathbf{m} \tag{1}$$

Now, (1) implies

$$(\mathbf{e}_1 - \mathbf{e}_2)^T\mathbf{m} = 0 \tag{2}$$
$$(\mathbf{e}_2 - \mathbf{e}_3)^T\mathbf{m} = 0 \tag{3}$$
$$(\mathbf{e}_3 - \mathbf{e}_1)^T\mathbf{m} = 0 \tag{4}$$

Thus, converting above system of equations into matrix form, we get

$$\mathbf{Am} = 0 \tag{5}$$

To find the solution of (5), we find the echelon form of $\mathbf{A}$.

$$\begin{pmatrix} +1 & -1 & +0 \\ +0 & +1 & -1 \\ -1 & +0 & +1 \end{pmatrix} \xleftrightarrow{r_3 \leftarrow r_1 + r_3} \begin{pmatrix} +1 & -1 & +0 \\ +0 & +1 & -1 \\ +0 & -1 & +1 \end{pmatrix} \tag{6}$$
$$\begin{pmatrix} +1 & -1 & +0 \\ +0 & +1 & -1 \\ +0 & -1 & +1 \end{pmatrix} \xleftrightarrow{r_3 \leftarrow r_2 + r_3} \begin{pmatrix} +1 & -1 & +0 \\ +0 & +1 & -1 \\ +0 & +0 & +0 \end{pmatrix} \tag{7}$$
$$\begin{pmatrix} +1 & -1 & +0 \\ +0 & +1 & -1 \\ +0 & +0 & +0 \end{pmatrix} \xleftrightarrow{r_1 \leftarrow r_1 + r_2} \begin{pmatrix} +1 & +0 & -1 \\ +0 & +1 & -1 \\ +0 & +0 & +0 \end{pmatrix} \tag{8}$$

From (8), we find out that

$$m_x = m_y = m_z \tag{9}$$
$$\mathbf{m} = \begin{pmatrix} m_z \\ m_z \\ m_z \end{pmatrix} \implies \mathbf{m} = m_z \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{10}$$

Taking $m_z = 1$, then $\|\mathbf{m}\| = \frac{1}{\sqrt{3}}$ and for $\mathbf{m}$ to be a unit vector, we need to divide each element of $\mathbf{m}$ by $\|\mathbf{m}\|$. Hence

$$\mathbf{m} = \begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix} \tag{11}$$

Thus, we see that $\mathbf{m} = \begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix}$ is the unit direction vector inclined equally to the coordinate axes.

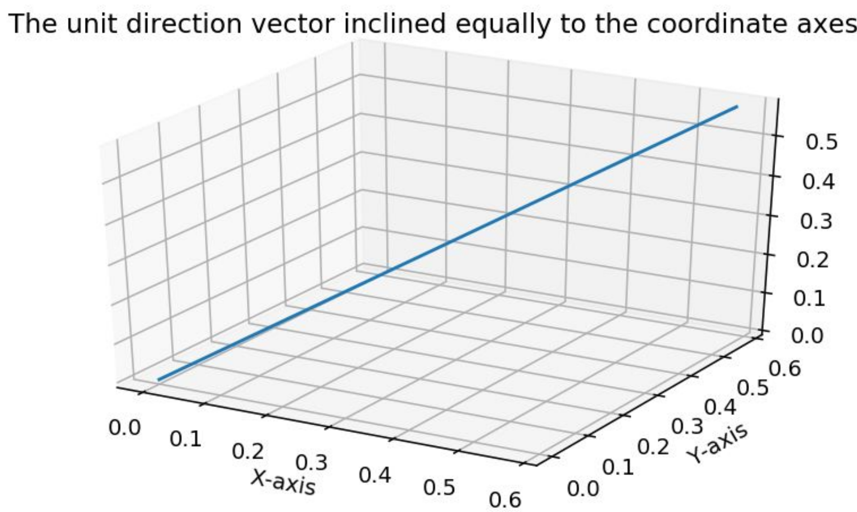

Figure 1