Question 1. *Show that the vectors* $\begin{pmatrix} +2 \\ -1 \\ +1 \end{pmatrix}$, $\begin{pmatrix} +1 \\ -3 \\ -5 \end{pmatrix}$ *and* $\begin{pmatrix} +3 \\ -4 \\ -4 \end{pmatrix}$ *form the vertices of a right angled trianle.*

Solution. Let A, B and C be given vectors such that $A = \begin{pmatrix} +2 \\ -1 \\ +1 \end{pmatrix}$,

$B = \begin{pmatrix} +1 \\ -3 \\ -5 \end{pmatrix}$ and $C = \begin{pmatrix} +3 \\ -4 \\ -4 \end{pmatrix}$.

To show that A, B and C form the vertices of a right angled triangle. First we need to show that A, B and C are indeed vertices of a triangle.
For this we need to see if the vertices satisfy triangle inequality. Let a, b and c denote the length of vectors $\overrightarrow{A-B}$, $\overrightarrow{B-C}$ and $\overrightarrow{C-A}$. Now,
$a = \sqrt{41}$, $b = \sqrt{6}$ and $c = \sqrt{35}$. We can see that $a + b > c$, $a + c > b$ and $b + c > a$. Thus, the given vectors A, B and C form the vertices of a triangle.

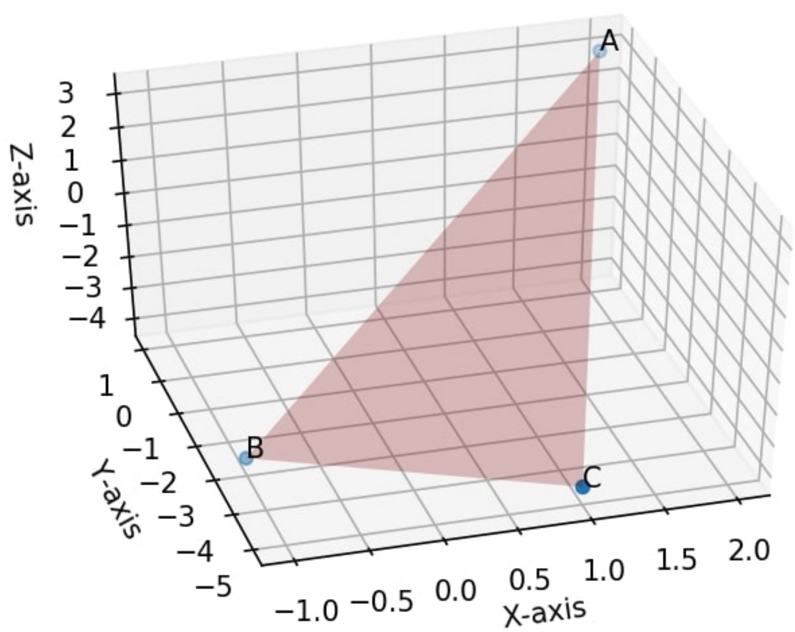

Figure 1

To prove, the $\triangle ABC$ is a right triangle, we need to calculate the inner product of all the below vectors and check if any one of them is 0.:

1. $\langle \overrightarrow{A-C}, \overrightarrow{B-C} \rangle = \overrightarrow{(A-C)}^T \ \overrightarrow{(B-C)} = (-1 \ \ 3 \ \ 5) \begin{pmatrix} -2 \\ +1 \\ -1 \end{pmatrix} = 0$

2. $\langle\overrightarrow{A-B},\overrightarrow{C-B}\rangle = \overrightarrow{(A-B)}^T\ \overrightarrow{(C-B)} = (1\ \ 2\ \ 6)\begin{pmatrix}+2\\-1\\+1\end{pmatrix} = 6$

3. $\langle\overrightarrow{B-A},\overrightarrow{C-A}\rangle = \overrightarrow{(B-A)}^T\ \overrightarrow{(C-A)} = (-1\ \ -2\ \ -6)\begin{pmatrix}+1\\-3\\-5\end{pmatrix} = 35$

Clearly, from (1). we can see that $\triangle$ABC is right angled at C.