

Assignment 2

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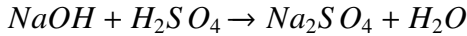
Abstract—This a simple document that explains how to find multipliers that balances a chemical reaction.

%beginlstlisting Download all latex-tikz codes from

<https://github.com/saranshbali/EE5609/tree/master/Assignment1/Latex>

1 PROBLEM

Balance the following chemical equation.



2 SOLUTION

Let \mathbf{m} be a vector consisting of $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$ such that

$$x_1 NaOH + x_2 H_2SO_4 \rightarrow x_3 Na_2SO_4 + x_4 H_2O \quad (2.0.1)$$

For balancing the two equations, we need to calculate number of occurrences of each element on left hand side and right hand side of 2.0.1 and equate the two. Thus, doing so we found out that

$$x_1 + 2x_2 = 2x_4 \implies x_1 + 2x_2 + 0x_3 - 2x_4 = 0 \quad (2.0.2)$$

$$x_1 = 2x_3 \implies x_1 + 0x_2 - 2x_3 + 0x_4 = 0 \quad (2.0.3)$$

$$x_2 = x_3 \implies 0x_1 + x_2 - x_3 + 0x_4 = 0 \quad (2.0.4)$$

$$x_1 + 4x_2 = 4x_3 + x_4 \implies x_1 + 4x_2 - 4x_3 - x_4 = 0 \quad (2.0.5)$$

Converting, 2.0.2, 2.0.3, 2.0.4 and 2.0.5 into matrix form we get,

$$\mathbf{A}\mathbf{m} = 0 \quad (2.0.6)$$

The matrix \mathbf{A} in above is given as:

$$\begin{pmatrix} 1 & 2 & 0 & -2 \\ 1 & 0 & -2 & 0 \\ 0 & 1 & -1 & -0 \\ 1 & 4 & -4 & -1 \end{pmatrix} \quad (2.0.7)$$

To find the solution of 2.0.6, we reduce \mathbf{A} into its Echelon form and solve consequently. The Echelon form of \mathbf{A} can be found as

$$\begin{pmatrix} 1 & 2 & 0 & -2 \\ 1 & 0 & -2 & 0 \\ 0 & 1 & -1 & -0 \\ 1 & 4 & -4 & -1 \end{pmatrix} \xleftrightarrow{r_2 \leftarrow r_2 - r_1} \begin{pmatrix} 1 & 2 & 0 & -2 \\ 0 & -2 & -2 & 2 \\ 0 & 1 & -1 & 0 \\ 1 & 4 & -4 & -1 \end{pmatrix} \quad (2.0.8)$$

$$\begin{pmatrix} 1 & 2 & 0 & -2 \\ 0 & -2 & -2 & 2 \\ 0 & 1 & -1 & 0 \\ 1 & 4 & -4 & -1 \end{pmatrix} \xleftrightarrow{r_4 \leftarrow r_4 - r_1} \begin{pmatrix} 1 & 2 & 0 & -2 \\ 0 & -2 & -2 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 2 & -4 & 1 \end{pmatrix} \quad (2.0.9)$$

$$\begin{pmatrix} 1 & 2 & 0 & -2 \\ 0 & -2 & -2 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 2 & -4 & 1 \end{pmatrix} \xleftrightarrow{r_2 \leftarrow r_1 + r_2} \begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & -2 & -2 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 2 & -4 & 1 \end{pmatrix} \quad (2.0.10)$$

$$\begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & -2 & -2 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 2 & -4 & 1 \end{pmatrix} \xleftrightarrow{r_2 \leftarrow -r_2/2} \begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 2 & -4 & 1 \end{pmatrix} \quad (2.0.11)$$

$$\begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 2 & -4 & 1 \end{pmatrix} \xleftrightarrow{r_3 \leftarrow r_3 - r_2} \begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -2 & 1 \\ 0 & 2 & -4 & 1 \end{pmatrix} \quad (2.0.12)$$

$$\begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -2 & 1 \\ 0 & 2 & -4 & 1 \end{pmatrix} \xleftrightarrow{r_4 \leftarrow r_4 - 2r_2} \begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & -6 & 3 \end{pmatrix} \quad (2.0.13)$$

$$\begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -2 & 1 \\ 0 & 2 & -4 & 1 \end{pmatrix} \xleftrightarrow{r_3 \leftarrow -r_3/2} \begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & -1/2 \\ 0 & 0 & -6 & 3 \end{pmatrix} \quad (2.0.14)$$

$$\begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -2 & 1 \\ 0 & 2 & -4 & 1 \end{pmatrix} \xleftrightarrow{r_2 \leftarrow r_2 - r_3} \begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 0 & -1/2 \\ 0 & 0 & 1 & -1/2 \\ 0 & 0 & -6 & 3 \end{pmatrix} \quad (2.0.15)$$

$$\begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -2 & 1 \\ 0 & 2 & -4 & 1 \end{pmatrix} \xleftrightarrow{r_1 \leftarrow r_1 + 2r_3} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1/2 \\ 0 & 0 & 1 & -1/2 \\ 0 & 0 & -6 & 3 \end{pmatrix} \quad (2.0.16)$$

$$\begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -2 & 1 \\ 0 & 2 & -4 & 1 \end{pmatrix} \xleftrightarrow{r_4 \leftarrow r_4 + 6r_3} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1/2 \\ 0 & 0 & 1 & -1/2 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (2.0.17)$$

2.0.17 is the Echelon form of matrix **A** and solving for **m**, we get

$$x_1 = x_4, \quad x_2 = \frac{x_4}{2} \quad \text{and} \quad x_3 = \frac{x_4}{2} \quad (2.0.18)$$

Hence, we find out that

$$\mathbf{m} = \begin{pmatrix} x_4 \\ x_4/2 \\ x_4/2 \\ x_4 \end{pmatrix} \implies \mathbf{m} = x_4 \begin{pmatrix} 1 \\ 1/2 \\ 1/2 \\ 1 \end{pmatrix} \quad (2.0.19)$$

Taking $x_4 = 2$ in 2.0.19, we find out that

$$\mathbf{m} = \begin{pmatrix} 2 \\ 1 \\ 1 \\ 2 \end{pmatrix} \quad (2.0.20)$$

Thus, by 2.0.20 we find out one set of multipliers which balance the given chemical equation.