Assignment 2

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Abstract—**This a simple document that explains how to find multipliers that balances a chemical reaction.**

Download all python codes from

https://github.com/saranshbali/EE5609/tree/master/Assignment2/Code

and all latex-tikz codes from

https://github.com/saranshbali/EE5609/tree/master/Assignment2/latex

1 Problem

Balance the following chemical equation.

$$NaOH + H_2SO_4 \rightarrow Na_2SO_4 + H_2O$$

2 Solution

Let $\mathbf{m}$ be a vector consisting of $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$ such that

$$x_1NaOH + x_2H_2SO_4 \rightarrow x_3Na_2SO_4 + x_4H_2O \tag{2.0.1}$$

For balancing the two equations, we need to calculate number of occurences of each element on left hand side and right hand side of 2.0.1 and equate the two. Thus,

$$(x_1 - 2x_3)Na = 0 \tag{2.0.2}$$
$$(x_1 + 4x_2 - 4x_3 + x_4)O = 0 \tag{2.0.3}$$
$$(x_1 - 2x_4)H = 0 \tag{2.0.4}$$
$$(x_2 - x_3)S = 0 \tag{2.0.5}$$

From 2.0.2,2.0.3,2.0.4 and 2.0.5, we get

$$x_1 + 2x_2 + 0x_3 - 2x_4 = 0 \tag{2.0.6}$$
$$x_1 + 0x_2 - 2x_3 + 0x_4 = 0 \tag{2.0.7}$$
$$0x_1 + x_2 - x_3 + 0x_4 = 0 \tag{2.0.8}$$
$$x_1 + 4x_2 - 4x_3 - x_4 = 0 \tag{2.0.9}$$

Converting, 2.0.6, 2.0.7, 2.0.8 and 2.0.9 into matrix form we get,

$$\mathbf{A}\mathbf{m} = 0 \tag{2.0.10}$$

The matrix $\mathbf{A}$ in above is given as:

$$\begin{pmatrix} 1 & 2 & 0 & -2 \\ 1 & 0 & -2 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 4 & -4 & -1 \end{pmatrix} \tag{2.0.11}$$

To find the solution of 2.0.10, we reduce $\mathbf{A}$ into its Echelon form and solve consequently. The Echolen form of $\mathbf{A}$ can be found as

$$\begin{pmatrix} 1 & 2 & 0 & -2 \\ 1 & 0 & -2 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 4 & -4 & -1 \end{pmatrix} \xleftrightarrow{r_2 \leftarrow r_2 - r_1} \begin{pmatrix} 1 & 2 & 0 & -2 \\ 0 & -2 & -2 & 2 \\ 0 & 1 & -1 & 0 \\ 1 & 4 & -4 & -1 \end{pmatrix} \tag{2.0.12}$$

$$\begin{pmatrix} 1 & 2 & 0 & -2 \\ 0 & -2 & -2 & 2 \\ 0 & 1 & -1 & 0 \\ 1 & 4 & -4 & -1 \end{pmatrix} \xleftrightarrow{r_4 \leftarrow r_4 - r_1} \begin{pmatrix} 1 & 2 & 0 & -2 \\ 0 & -2 & -2 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 2 & -4 & 1 \end{pmatrix} \tag{2.0.13}$$

$$\begin{pmatrix} 1 & 2 & 0 & -2 \\ 0 & -2 & -2 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 2 & -4 & 1 \end{pmatrix} \xleftrightarrow{r_2 \leftarrow r_1 + r_2} \begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & -2 & -2 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 2 & -4 & 1 \end{pmatrix} \tag{2.0.14}$$

$$\begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & -2 & -2 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 2 & -4 & 1 \end{pmatrix} \xleftrightarrow{r_2 \leftarrow -r_2/2} \begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 2 & -4 & 1 \end{pmatrix} \tag{2.0.15}$$

$$\begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 2 & -4 & 1 \end{pmatrix} \xleftrightarrow{r_3 \leftarrow r_3 - r_2} \begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -2 & 1 \\ 0 & 2 & -4 & 1 \end{pmatrix} \tag{2.0.16}$$

$$\begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -2 & 1 \\ 0 & 2 & -4 & 1 \end{pmatrix} \xleftrightarrow{r_4 \leftarrow r_4 - 2r_2} \begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & -6 & 3 \end{pmatrix} \tag{2.0.17}$$

$$\begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -2 & 1 \\ 0 & 2 & -4 & 1 \end{pmatrix} \xleftrightarrow{r_3 \leftarrow -r_3/2} \begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & -1/2 \\ 0 & 0 & -6 & 3 \end{pmatrix} \tag{2.0.18}$$

$$\begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -2 & 1 \\ 0 & 2 & -4 & 1 \end{pmatrix} \xleftrightarrow{r_2 \leftarrow r_2 - r_3} \begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 0 & -1/2 \\ 0 & 0 & 1 & -1/2 \\ 0 & 0 & -6 & 3 \end{pmatrix} \tag{2.0.19}$$

$$\begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -2 & 1 \\ 0 & 2 & -4 & 1 \end{pmatrix} \xleftrightarrow{r_1 \leftarrow r_1 + 2r_3} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1/2 \\ 0 & 0 & 1 & -1/2 \\ 0 & 0 & -6 & 3 \end{pmatrix} \tag{2.0.20}$$

$$\begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -2 & 1 \\ 0 & 2 & -4 & 1 \end{pmatrix} \xleftrightarrow{r_4 \leftarrow r_4 + 6r_3} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1/2 \\ 0 & 0 & 1 & -1/2 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{2.0.21}$$

2.0.21 is the Echelon form of matrix **A** and solving for **m**, we get

$$x_1 = x_4, \quad x_2 = \frac{x_4}{2} \quad and \quad x_3 = \frac{x_4}{2} \tag{2.0.22}$$

Hence, we find out that

$$\mathbf{m} = \begin{pmatrix} x_4 \\ x_4/2 \\ x_4/2 \\ x_4 \end{pmatrix} \implies \mathbf{m} = x_4 \begin{pmatrix} 1 \\ 1/2 \\ 1/2 \\ 1 \end{pmatrix} \tag{2.0.23}$$

Taking $x_4 = 2$ in 2.0.23, we find out that

$$\mathbf{m} = \begin{pmatrix} 2 \\ 1 \\ 1 \\ 2 \end{pmatrix} \tag{2.0.24}$$

Thus, by 2.0.24 we find out one set of multipliers which balance the given chemical equation and the balanced chemical equation is :

$$2NaOH + H_2SO_4 \rightarrow Na_2SO_4 + 2H_2O$$