

**Question 1.** Show that the unit direction vector inclined equally to the coordinate axes is  $\begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix}$ .

**Solution:** Let  $\mathbf{m}$  be the given unit vector such that  $\mathbf{m} = \begin{pmatrix} m_x \\ m_y \\ m_z \end{pmatrix}$ . Let  $\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ ,

$\mathbf{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$  and  $\mathbf{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$  be the direction vectors of the coordinate axes. As  $\mathbf{m}$  is a unit vector, so  $\|\mathbf{m}\| = 1$  and also we are given is that  $\mathbf{m}$  is inclined equally to the coordinate axis,

$$\mathbf{e}_1^T \mathbf{m} = \mathbf{e}_2^T \mathbf{m} = \mathbf{e}_3^T \mathbf{m} \quad (1)$$

Now, (1) implies

$$(\mathbf{e}_1 - \mathbf{e}_2)^T \mathbf{m} = 0$$

$$(\mathbf{e}_2 - \mathbf{e}_3)^T \mathbf{m} = 0$$

$$(\mathbf{e}_3 - \mathbf{e}_1)^T \mathbf{m} = 0$$

Thus, converting above system of equations into matrix form, we get

$$\begin{pmatrix} +1 & -1 & +0 \\ +0 & +1 & -1 \\ -1 & +0 & +1 \end{pmatrix} \begin{pmatrix} m_x \\ m_y \\ m_z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \xrightarrow[R_1+R_3]{R_3=} \quad (2)$$

$$\begin{pmatrix} +1 & -1 & +0 \\ +0 & +1 & -1 \\ +0 & -1 & +1 \end{pmatrix} \begin{pmatrix} m_x \\ m_y \\ m_z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \xrightarrow[R_2+R_3]{R_3=} \quad (3)$$

$$\begin{pmatrix} +1 & -1 & +0 \\ +0 & +1 & -1 \\ +0 & +0 & +0 \end{pmatrix} \begin{pmatrix} m_x \\ m_y \\ m_z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \xrightarrow[R_1+R_2]{R_1=} \quad (4)$$

$$\begin{pmatrix} +1 & +0 & +1 \\ +0 & +1 & -1 \\ +0 & +0 & +0 \end{pmatrix} \begin{pmatrix} m_x \\ m_y \\ m_z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad (5)$$

From(5) we find out that

$$m_x = m_y = m_z$$

Taking,  $m_x = m_y = m_z = 1$ , we get

$$\mathbf{m} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

Also, we know that  $\|\mathbf{m}\| = 1$  as  $\mathbf{m}$  is a unit vector, thus

$$\mathbf{m} = \begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix}$$

Thus, we see that  $\mathbf{m} = \begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix}$  is the unit direction vector inclined equally to the coordinate axes.

The unit direction vector inclined equally to the coordinate axes

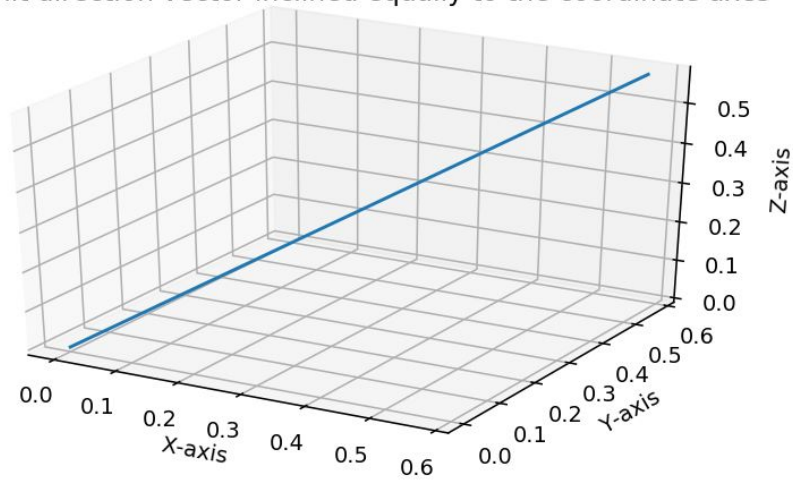


Figure 1