Question 1. *Show that the unit direction vector inclined equally to the coordinate axes is* $\begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix}$.

Solution: Let $\mathbf{m}$ be the given unit vector such that $\mathbf{m} = \begin{pmatrix} m_x \\ m_y \\ m_z \end{pmatrix}$.Let $\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\mathbf{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $\mathbf{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ be the direction vectors of the coordinate axes. As $\mathbf{m}$ is a unit vector, so $||\mathbf{m}|| = 1$ and also we are given is that $\mathbf{m}$ is inclined equally to the coordinate axis,

$$\mathbf{e}_1^T\mathbf{m} = \mathbf{e}_2^T\mathbf{m} = \mathbf{e}_2^T\mathbf{m} \tag{1}$$

Now, (1) implies

$$\mathbf{m}_x = \mathbf{m}_y = \mathbf{m}_z$$

Thus,

$$\mathbf{m} = \begin{pmatrix} m_x \\ m_x \\ m_x \end{pmatrix} \tag{2}$$

Also, we know that $||\mathbf{m}|| = 1$ and by (2), we have

$$\sqrt{\mathbf{m_x}^2 + \mathbf{m_x}^2 + \mathbf{m_x}^2} = 1 \qquad \implies \mathbf{m_x} = \frac{+}{-}\frac{1}{\sqrt{3}}$$

Taking, the positive sign, we see that $\mathbf{m} = \begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix}$ is the unit direction vector inclined equally to the coordinate axes.

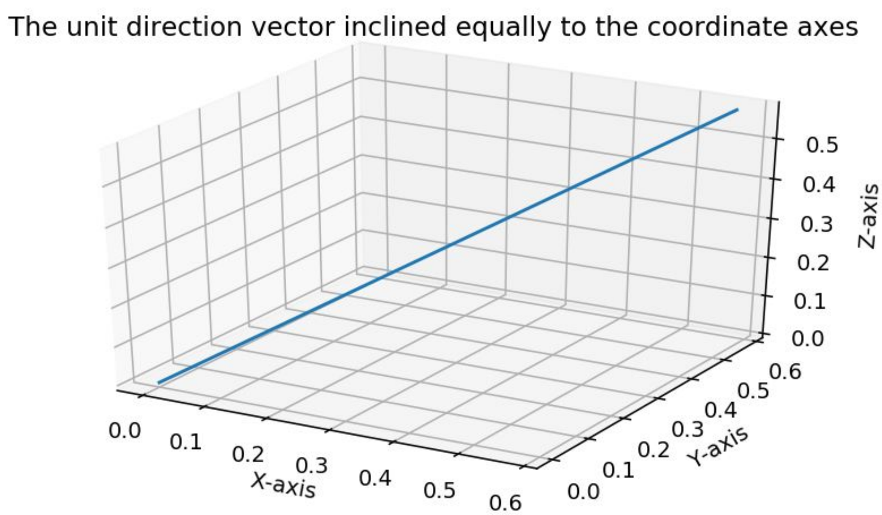
The unit direction vector inclined equally to the coordinate axes
0.0
0.1
0.2
0.3
0.4
0.5
0.6
X-axis
0.0
0.1
0.2
0.3
0.4
0.5
0.6
Y-axis
0.0
0.1
0.2
0.3
0.4
0.5
Z-axis

Figure 1