Question 1. *Show that the vectors* $\begin{pmatrix}-2\\+1\\-1\end{pmatrix}, \begin{pmatrix}+2\\-1\\+1\end{pmatrix}$ *and* $\begin{pmatrix}+1\\-3\\-5\end{pmatrix}$ *form the vertices of a right angled triangle.*

Solution. Let **A**,**B** and **C** be given vectors such that:

$$\mathbf{A} = \begin{pmatrix}-2\\+1\\-1\end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix}+2\\-1\\+1\end{pmatrix} \quad \textit{and} \quad \mathbf{C} = \begin{pmatrix}+1\\-3\\-5\end{pmatrix} \tag{1}$$

Given below is the figure formed by **A**, **B** and **C**.

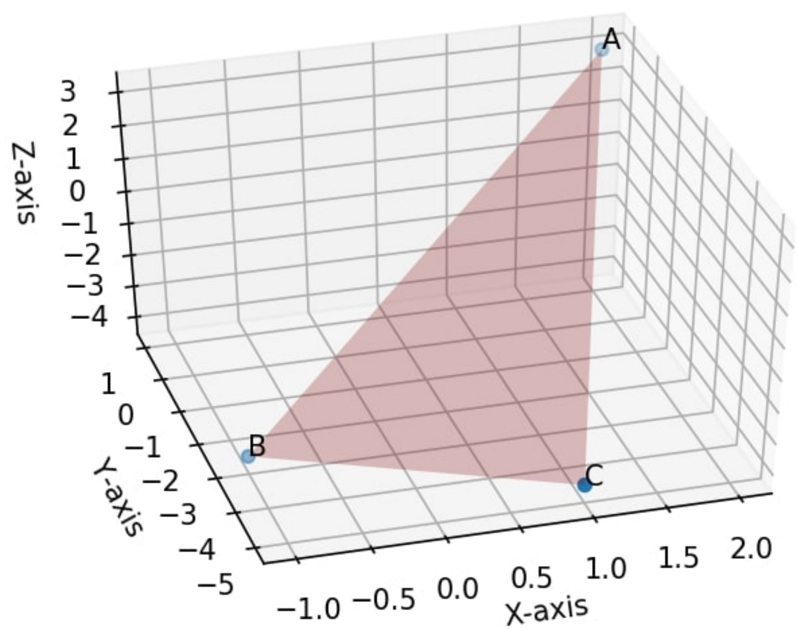

Figure 1

Clearly, **ABC** is a triangle. To prove, the $\triangle ABC$ is a right triangle, we need to calculate the inner product of all the below vectors and check if any one of them is 0:

$$(\mathbf{A}-\mathbf{C})^T(\mathbf{B}-\mathbf{C}) = \begin{pmatrix}-1 & +3 & +5\end{pmatrix}\begin{pmatrix}-2\\+1\\-1\end{pmatrix} = 00 \tag{2}$$

$$(\mathbf{A}-\mathbf{B})^T(\mathbf{C}-\mathbf{B}) = \begin{pmatrix}+1 & +2 & +6\end{pmatrix}\begin{pmatrix}+2\\-1\\+1\end{pmatrix} = 06 \tag{3}$$

$$(\mathbf{B}-\mathbf{A})^T(\mathbf{C}-\mathbf{A}) = \begin{pmatrix}-1 & -2 & -6\end{pmatrix}\begin{pmatrix}+1\\-3\\-5\end{pmatrix} = 35 \tag{4}$$

Clearly, from (2) we can see that $\triangle$ABC is right angled at C.