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Assignment7

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Abstract—This a simple document that explains how to transform a matrix into identity matrix using product of elementary matrices.

Download all latex-tikz codes from

https://github.com/saranshbali/EE5609/blob/master/ Assignment7

1 Problem

For the matrix $\mathbf{A} = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 0 & 1 \\ 3 & 0 & 1 \end{pmatrix}$, find elementary matrices $\mathbf{E_1}, \mathbf{E_2}, \dots, \mathbf{E_k}$ such that

$$\mathbf{E_k}...\mathbf{E_2}\mathbf{E_1}\mathbf{A} = \mathbf{I}$$
 (1.0.1)

2 Solution

Given,

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 0 & 1 \\ 3 & 0 & 1 \end{pmatrix} \tag{2.0.1}$$

Take,

$$\mathbf{E_1} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.0.2}$$

$$\mathbf{E_2} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{pmatrix} \tag{2.0.3}$$

$$\mathbf{E_3} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.0.4}$$

$$\mathbf{E_4} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.0.5}$$

$$\mathbf{E_5} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{pmatrix} \tag{2.0.6}$$

$$\mathbf{E_6} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{2}{7} \end{pmatrix} \tag{2.0.7}$$

$$\mathbf{E_7} = \begin{pmatrix} 1 & 0 & \frac{-1}{2} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.0.8}$$

$$\mathbf{E_8} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 1 \end{pmatrix} \tag{2.0.9}$$

No, we calculate

$$\mathbf{E_8}\mathbf{E_7}\mathbf{E_6}\mathbf{E_5}\mathbf{E_4}\mathbf{E_3}\mathbf{E_2}\mathbf{E_1} = \begin{pmatrix} 0 & -1 & 1 \\ -1 & 2 & -1 \\ 0 & 3 & -2 \end{pmatrix}$$
 (2.0.10)

Hence,

$$(\mathbf{E_8}\mathbf{E_7}\mathbf{E_6}\mathbf{E_5}\mathbf{E_4}\mathbf{E_3}\mathbf{E_2}\mathbf{E_1}) \mathbf{A} = \begin{pmatrix} 0 & -1 & 1 \\ -1 & 2 & -1 \\ 0 & 3 & -2 \end{pmatrix} \begin{pmatrix} 1 & -1 & 1 \\ 2 & 0 & 1 \\ 3 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
 (2.0.11)