

# Assignment 2

Saransh Bali

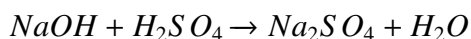
**Abstract**—This a simple document that explains how to find multipliers that balances a chemical reaction.

Download all latex-tikz codes from

<https://github.com/saranshbali/EE5609/tree/master/Assignment2/latex>

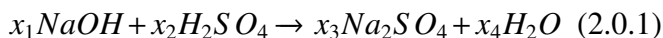
## 1 PROBLEM

Balance the following chemical equation.



## 2 SOLUTION

Let  $\mathbf{m}$  be a vector consisting of  $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$  such that



For balancing the two equations, we need to calculate number of occurrences of each element on left hand side and right hand side of 2.0.1 and equate the two.

Number of elements of Na in Left side of 2.0.1 is  $x_1$  and number of elements of Na in Left side of 2.0.1 is  $2x_3$ . Thus,

$$x_1 = 2x_3 \quad (2.0.2)$$

Number of elements of O in Left side of 2.0.1 is  $x_1 + 4x_2$  and number of elements of O in Left side of 2.0.1 is  $4x_3 + x_4$ . Thus,

$$x_1 + 4x_2 = 4x_3 + x_4 \quad (2.0.3)$$

Number of elements of H in Left side of 2.0.1 is  $x_1 + 2x_2$  and number of elements of H in Left side of 2.0.1 is  $2x_4$ . Thus,

$$x_1 + 2x_2 = 2x_4 \quad (2.0.4)$$

Number of elements of S in Left side of 2.0.1 is  $x_2$  and number of elements of S in Left side of 2.0.1 is  $x_3$ . Thus,

$$x_2 = x_3 \quad (2.0.5)$$

By 2.0.2, 2.0.3, 2.0.4 and 2.0.5 we find out that

$$x_1 + 2x_2 + 0x_3 - 2x_4 = 0 \quad (2.0.6)$$

$$x_1 + 0x_2 - 2x_3 + 0x_4 = 0 \quad (2.0.7)$$

$$0x_1 + x_2 - x_3 + 0x_4 = 0 \quad (2.0.8)$$

$$x_1 + 4x_2 - 4x_3 - x_4 = 0 \quad (2.0.9)$$

Converting, 2.0.6, 2.0.7, 2.0.8 and 2.0.9 into matrix form we get,

$$\mathbf{A}\mathbf{m} = \mathbf{0} \quad (2.0.10)$$

The matrix  $\mathbf{A}$  in above is given as:

$$\begin{pmatrix} 1 & 2 & 0 & -2 \\ 1 & 0 & -2 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 4 & -4 & -1 \end{pmatrix} \quad (2.0.11)$$

To find the solution of 2.0.10, we reduce  $\mathbf{A}$  into its Echelon form and solve consequently. The Echelon form of  $\mathbf{A}$  can be found as

$$\begin{pmatrix} 1 & 2 & 0 & -2 \\ 1 & 0 & -2 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 4 & -4 & -1 \end{pmatrix} \xrightarrow{r_2 \leftarrow r_2 - r_1} \begin{pmatrix} 1 & 2 & 0 & -2 \\ 0 & -2 & -2 & 2 \\ 0 & 1 & -1 & 0 \\ 1 & 4 & -4 & -1 \end{pmatrix} \quad (2.0.12)$$

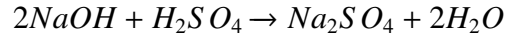
$$\begin{pmatrix} 1 & 2 & 0 & -2 \\ 0 & -2 & -2 & 2 \\ 0 & 1 & -1 & 0 \\ 1 & 4 & -4 & -1 \end{pmatrix} \xrightarrow{r_4 \leftarrow r_4 - r_1} \begin{pmatrix} 1 & 2 & 0 & -2 \\ 0 & -2 & -2 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 2 & -4 & 1 \end{pmatrix} \quad (2.0.13)$$

$$\begin{pmatrix} 1 & 2 & 0 & -2 \\ 0 & -2 & -2 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 2 & -4 & 1 \end{pmatrix} \xrightarrow{r_2 \leftarrow r_2 + r_3} \begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & -2 & -2 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 2 & -4 & 1 \end{pmatrix} \quad (2.0.14)$$

$$\begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & -2 & -2 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 2 & -4 & 1 \end{pmatrix} \xrightarrow{r_2 \leftarrow -r_2/2} \begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 2 & -4 & 1 \end{pmatrix} \quad (2.0.15)$$

$$\begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 2 & -4 & 1 \end{pmatrix} \xleftrightarrow{r_3 \leftarrow r_3 - r_2} \begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -2 & 1 \\ 0 & 2 & -4 & 1 \end{pmatrix} \quad (2.0.16)$$

balanced chemical equation is :



$$\begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -2 & 1 \\ 0 & 2 & -4 & 1 \end{pmatrix} \xleftrightarrow{r_4 \leftarrow r_4 - 2r_2} \begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & -6 & 3 \end{pmatrix} \quad (2.0.17)$$

$$\begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -2 & 1 \\ 0 & 2 & -4 & 1 \end{pmatrix} \xleftrightarrow{r_3 \leftarrow -r_3/2} \begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & -1/2 \\ 0 & 2 & -4 & 1 \end{pmatrix} \quad (2.0.18)$$

$$\begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -2 & 1 \\ 0 & 2 & -4 & 1 \end{pmatrix} \xleftrightarrow{r_2 \leftarrow r_2 - r_3} \begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 0 & -1/2 \\ 0 & 0 & 1 & -1/2 \\ 0 & 2 & -4 & 1 \end{pmatrix} \quad (2.0.19)$$

$$\begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -2 & 1 \\ 0 & 2 & -4 & 1 \end{pmatrix} \xleftrightarrow{r_1 \leftarrow r_1 + 2r_3} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1/2 \\ 0 & 0 & 1 & -1/2 \\ 0 & 2 & -4 & 1 \end{pmatrix} \quad (2.0.20)$$

$$\begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -2 & 1 \\ 0 & 2 & -4 & 1 \end{pmatrix} \xleftrightarrow{r_4 \leftarrow r_4 + 6r_3} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1/2 \\ 0 & 0 & 1 & -1/2 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (2.0.21)$$

2.0.21 is the Echelon form of matrix **A** and solving for **m**, we get

$$x_1 = x_4, \quad x_2 = \frac{x_4}{2} \quad \text{and} \quad x_3 = \frac{x_4}{2} \quad (2.0.22)$$

Hence, we find out that

$$\mathbf{m} = \begin{pmatrix} x_4 \\ x_4/2 \\ x_4/2 \\ x_4 \end{pmatrix} \Rightarrow \mathbf{m} = x_4 \begin{pmatrix} 1 \\ 1/2 \\ 1/2 \\ 1 \end{pmatrix} \quad (2.0.23)$$

Taking  $x_4 = 2$  in 2.0.23, we find out that

$$\mathbf{m} = \begin{pmatrix} 2 \\ 1 \\ 1 \\ 2 \end{pmatrix} \quad (2.0.24)$$

Thus, by 2.0.24 we find out one set of multipliers which balance the given chemical equation and the