Assignment 2

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***Abstract*—This a simple document that explains how to find multipliers that balances a chemical reaction.**

Download all latex-tikz codes from

https://github.com/saranshbali/EE5609/tree/master/Assignment2/latex

1 Problem

Balance the following chemical equation.

$$NaOH + H_2SO_4 \rightarrow Na_2SO_4 + H_2O$$

2 Solution

Let $\mathbf{m}$ be a vector consisting of $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$ such that

$$x_1NaOH + x_2H_2SO_4 \rightarrow x_3Na_2SO_4 + x_4H_2O \quad (2.0.1)$$

For balancing the two equations, we need to calculate number of occurences of each element on left hand side and right hand side of 2.0.1 and equate the two.

Number of elements of Na in Left side of 2.0.1 is x_1 and number of elements of Na in Left side of 2.0.1 is $2x_3$. Thus,

$$x_1 = 2x_3 \quad (2.0.2)$$

Number of elements of O in Left side of 2.0.1 is $x_1 + 4x_2$ and number of elements of O in Left side of 2.0.1 is $4x_3 + x_4$. Thus,

$$x_1 + 4x_2 = 4x_3 + x_4 \quad (2.0.3)$$

Number of elements of H in Left side of 2.0.1 is $x_1 + 2x_2$ and number of elements of O in Left side of 2.0.1 is $2x_4$. Thus,

$$x_1 + 2x_2 = 2x_4 \quad (2.0.4)$$

Number of elements of S in Left side of 2.0.1 is x_2 and number of elements of O in Left side of 2.0.1 is x_3. Thus,

$$x_2 = x_3 \quad (2.0.5)$$

By 2.0.2, 2.0.3, 2.0.4 and 2.0.5 we find out that

$$x_1 + 2x_2 + 0x_3 - 2x_4 = 0 \quad (2.0.6)$$
$$x_1 + 0x_2 - 2x_3 + 0x_4 = 0 \quad (2.0.7)$$
$$0x_1 + x_2 - x_3 + 0x_4 = 0 \quad (2.0.8)$$
$$x_1 + 4x_2 - 4x_3 - x_4 = 0 \quad (2.0.9)$$

Converting, 2.0.6, 2.0.7, 2.0.8 and 2.0.9 into matrix form we get,

$$\mathbf{Am} = 0 \quad (2.0.10)$$

The matrix $\mathbf{A}$ in above is given as:

$$\begin{pmatrix} 1 & 2 & 0 & -2 \\ 1 & 0 & -2 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 4 & -4 & -1 \end{pmatrix} \quad (2.0.11)$$

To find the solution of 2.0.10, we reduce $\mathbf{A}$ into its Echelon form and solve consequently. The Echolen form of $\mathbf{A}$ can be found as

$$\begin{pmatrix} 1 & 2 & 0 & -2 \\ 1 & 0 & -2 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 4 & -4 & -1 \end{pmatrix} \xleftrightarrow{r_2 \leftarrow r_2 - r_1} \begin{pmatrix} 1 & 2 & 0 & -2 \\ 0 & -2 & -2 & 2 \\ 0 & 1 & -1 & 0 \\ 1 & 4 & -4 & -1 \end{pmatrix} \quad (2.0.12)$$

$$\begin{pmatrix} 1 & 2 & 0 & -2 \\ 0 & -2 & -2 & 2 \\ 0 & 1 & -1 & 0 \\ 1 & 4 & -4 & -1 \end{pmatrix} \xleftrightarrow{r_4 \leftarrow r_4 - r_1} \begin{pmatrix} 1 & 2 & 0 & -2 \\ 0 & -2 & -2 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 2 & -4 & 1 \end{pmatrix} \quad (2.0.13)$$

$$\begin{pmatrix} 1 & 2 & 0 & -2 \\ 0 & -2 & -2 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 2 & -4 & 1 \end{pmatrix} \xleftrightarrow{r_2 \leftarrow r_1 + r_2} \begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & -2 & -2 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 2 & -4 & 1 \end{pmatrix} \quad (2.0.14)$$

$$\begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & -2 & -2 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 2 & -4 & 1 \end{pmatrix} \xleftrightarrow{r_2 \leftarrow -r_2/2} \begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 2 & -4 & 1 \end{pmatrix} \quad (2.0.15)$$

$$\begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 2 & -4 & 1 \end{pmatrix} \xleftrightarrow{r_3 \leftarrow r_3 - r_2} \begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -2 & 1 \\ 0 & 2 & -4 & 1 \end{pmatrix} \tag{2.0.16}$$

$$\begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -2 & 1 \\ 0 & 2 & -4 & 1 \end{pmatrix} \xleftrightarrow{r_4 \leftarrow r_4 - 2r_2} \begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & -6 & 3 \end{pmatrix} \tag{2.0.17}$$

$$\begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -2 & 1 \\ 0 & 2 & -4 & 1 \end{pmatrix} \xleftrightarrow{r_3 \leftarrow -r_3/2} \begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & -1/2 \\ 0 & 0 & -6 & 3 \end{pmatrix} \tag{2.0.18}$$

$$\begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -2 & 1 \\ 0 & 2 & -4 & 1 \end{pmatrix} \xleftrightarrow{r_2 \leftarrow r_2 - r_3} \begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 0 & -1/2 \\ 0 & 0 & 1 & -1/2 \\ 0 & 0 & -6 & 3 \end{pmatrix} \tag{2.0.19}$$

$$\begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -2 & 1 \\ 0 & 2 & -4 & 1 \end{pmatrix} \xleftrightarrow{r_1 \leftarrow r_1 + 2r_3} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1/2 \\ 0 & 0 & 1 & -1/2 \\ 0 & 0 & -6 & 3 \end{pmatrix} \tag{2.0.20}$$

$$\begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -2 & 1 \\ 0 & 2 & -4 & 1 \end{pmatrix} \xleftrightarrow{r_4 \leftarrow r_4 + 6r_3} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1/2 \\ 0 & 0 & 1 & -1/2 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{2.0.21}$$

2.0.21 is the Echelon form of matrix **A** and solving for **m**, we get

$$x_1 = x_4, \quad x_2 = \frac{x_4}{2} \quad and \quad x_3 = \frac{x_4}{2} \tag{2.0.22}$$

Hence, we find out that

$$\mathbf{m} = \begin{pmatrix} x_4 \\ x_4/2 \\ x_4/2 \\ x_4 \end{pmatrix} \implies \mathbf{m} = x_4 \begin{pmatrix} 1 \\ 1/2 \\ 1/2 \\ 1 \end{pmatrix} \tag{2.0.23}$$

Taking $x_4 = 2$ in 2.0.23, we find out that

$$\mathbf{m} = \begin{pmatrix} 2 \\ 1 \\ 1 \\ 2 \end{pmatrix} \tag{2.0.24}$$

Thus, by 2.0.24 we find out one set of multipliers which balance the given chemical equation and the balanced chemical equation is :

$$2NaOH + H_2SO_4 \rightarrow Na_2SO_4 + 2H_2O$$