Challenging Problem 6

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***Abstract*—This a simple document that explains Orthogonal vectors are Linearly independent.**

Download latex-tikz codes from

https://github.com/saranshbali/EE5609/blob/master/ChallengeProblem6

1 Problem

Show that the set of Orthogonal vectors $\mathbf{v_1}, \mathbf{v_2}, ..., \mathbf{v_n}$ is Linear independent.

2 Solution

2.1 n=2

Consider $\mathbf{v_1}$ and $\mathbf{v_2}$ be two orthogonal vectors. Now,

$$a_1\mathbf{v_1} + a_2\mathbf{v_2} = 0 \tag{2.1.1}$$

Taking dot product of 2.1.1 with $\mathbf{v_1}$, we get

$$a_1 \left\|\mathbf{v_1}\right\|^2 + a_2\mathbf{v_2}^T\mathbf{v_1} = 0 \tag{2.1.2}$$

$$a_1 \left\|\mathbf{v_1}\right\|^2 = 0 \quad \implies a_1 = 0 \tag{2.1.3}$$

$$\left\|\mathbf{v_1}\right\|^2 = 0 \iff \mathbf{v_1} = 0 \tag{2.1.4}$$

Similarly, taking dot product of 2.1.1 with $\mathbf{v_2}$, we get $a_2 = 0$.

Thus, $\mathbf{v_1}$ and $\mathbf{v_2}$ are linearly independent as well.

2.2 General Case

Consider, the expression

$$a_1\mathbf{v_1} + a_2\mathbf{v_2} + ... + a_n\mathbf{v_n} = 0 \tag{2.2.1}$$

Take the dot product of 2.2.1 with $\mathbf{v_1}$, we get

$$a_1 \left\|\mathbf{v_1}\right\|^2 + a_2\mathbf{v_2}^T\mathbf{v_1} + ... + a_n\mathbf{v_n}^T\mathbf{v_1} = 0 \tag{2.2.2}$$

$$a_1 \left\|\mathbf{v_1}\right\|^2 = 0 \quad (\mathbf{v_i}^T\mathbf{v_j} = 0 \quad \forall i \neq j) \tag{2.2.3}$$

$$\left\|\mathbf{v_1}\right\|^2 = 0 \quad \iff \mathbf{v_1} = 0 \tag{2.2.4}$$

Hence, $a_1 = 0$

Similarly, taking the dot product of 2.2.1 with $\mathbf{v_2}$, ...,$\mathbf{v_n}$, we find out $a_2 = 0, ..., a_n = 0$.

Thus, the set of Orthogonal vectors $\mathbf{v_1}, \mathbf{v_2}, ..., \mathbf{v_n}$ is Linear independent.