Assignment7

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***Abstract*—This a simple document that explains how to transform a matrix into identity matrix using product of elementary matrices.**

Download all latex-tikz codes from

https://github.com/saranshbali/EE5609/blob/master/Assignment7

1 Problem

For the matrix $\mathbf{A} = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 0 & 1 \\ 3 & 0 & 1 \end{pmatrix}$, find elementary matrices $\mathbf{E_1}$,$\mathbf{E_2}$,...,$\mathbf{E_k}$ such that

$$\mathbf{E_k}...\mathbf{E_2}\mathbf{E_1}\mathbf{A} = \mathbf{I} \tag{1.0.1}$$

2 Solution

Given,

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 0 & 1 \\ 3 & 0 & 1 \end{pmatrix} \tag{2.0.1}$$

Take,

$$\mathbf{E_1} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.0.2}$$

Now,

$$\begin{aligned} \mathbf{E_1}\mathbf{A} &= \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & -1 & 1 \\ 2 & 0 & 1 \\ 3 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 1 & -1 & 1 \\ 0 & 2 & -1 \\ 3 & 0 & 1 \end{pmatrix} \end{aligned} \tag{2.0.3}$$

Take

$$\mathbf{E_2} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{pmatrix} \tag{2.0.4}$$

$$\begin{aligned} \mathbf{E_2}(\mathbf{E_1}\mathbf{A}) &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & -1 & 1 \\ 0 & 2 & -1 \\ 3 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 1 & -1 & 1 \\ 0 & 2 & -1 \\ 0 & 3 & 2 \end{pmatrix} \end{aligned} \tag{2.0.5}$$

Take

$$\mathbf{E_3} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.0.6}$$

$$\begin{aligned} \mathbf{E_3}(\mathbf{E_2}\mathbf{E_1}\mathbf{A}) &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & -1 & 1 \\ 0 & 2 & -1 \\ 0 & 3 & 2 \end{pmatrix} \\ &= \begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & \frac{-1}{2} \\ 0 & 3 & 2 \end{pmatrix} \end{aligned} \tag{2.0.7}$$

Take

$$\mathbf{E_4} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.0.8}$$

$$\begin{aligned} \mathbf{E_4}(\mathbf{E_3}\mathbf{E_2}\mathbf{E_1}\mathbf{A}) &= \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & \frac{-1}{2} \\ 0 & 3 & 2 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{-1}{2} \\ 0 & 3 & 2 \end{pmatrix} \end{aligned} \tag{2.0.9}$$

Take

$$\mathbf{E_5} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{pmatrix} \tag{2.0.10}$$

$$\mathbf{E_5}(\mathbf{E_4E_3E_2E_1A}) = \begin{pmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & -3 & 1\end{pmatrix}\begin{pmatrix}1 & 0 & \frac{1}{2}\\0 & 1 & \frac{-1}{2}\\0 & 3 & 2\end{pmatrix}$$

$$= \begin{pmatrix}1 & 0 & \frac{1}{2}\\0 & 1 & \frac{-1}{2}\\0 & 0 & \frac{7}{2}\end{pmatrix} \quad (2.0.11)$$

Take

$$\mathbf{E_6} = \begin{pmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & \frac{2}{7}\end{pmatrix} \quad (2.0.12)$$

$$\mathbf{E_6}(\mathbf{E_5E_4E_3E_2E_1A}) = \begin{pmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & \frac{2}{7}\end{pmatrix}\begin{pmatrix}1 & 0 & \frac{1}{2}\\0 & 1 & \frac{-1}{2}\\0 & 0 & \frac{7}{2}\end{pmatrix}$$

$$= \begin{pmatrix}1 & 0 & \frac{1}{2}\\0 & 1 & \frac{-1}{2}\\0 & 0 & 1\end{pmatrix} \quad (2.0.13)$$

Take

$$\mathbf{E_7} = \begin{pmatrix}1 & 0 & \frac{-1}{2}\\0 & 1 & 0\\0 & 0 & 1\end{pmatrix} \quad (2.0.14)$$

$$\mathbf{E_7}(\mathbf{E_6E_5E_4E_3E_2E_1A}) = \begin{pmatrix}1 & 0 & \frac{-1}{2}\\0 & 1 & 0\\0 & 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0 & \frac{1}{2}\\0 & 1 & \frac{-1}{2}\\0 & 0 & 1\end{pmatrix}$$

$$= \begin{pmatrix}1 & 0 & 0\\0 & 1 & \frac{-1}{2}\\0 & 0 & 1\end{pmatrix} \quad (2.0.15)$$

Take

$$\mathbf{E_8} = \begin{pmatrix}1 & 0 & 0\\0 & 1 & \frac{1}{2}\\0 & 0 & 1\end{pmatrix} \quad (2.0.16)$$

$$\mathbf{E_8}(\mathbf{E_7E_6E_5E_4E_3E_2E_1A}) = \begin{pmatrix}1 & 0 & 0\\0 & 1 & \frac{1}{2}\\0 & 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0 & 0\\0 & 1 & \frac{-1}{2}\\0 & 0 & 1\end{pmatrix}$$

$$= \begin{pmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{pmatrix} \quad (2.0.17)$$

Now, to verify the above result, we calculate

$$\mathbf{E_8E_7E_6E_5E_4E_3E_2E_1} = \begin{pmatrix}0 & -1 & 1\\-1 & 2 & -1\\0 & 3 & -2\end{pmatrix} \quad (2.0.18)$$

Hence,

$$(\mathbf{E_8E_7E_6E_5E_4E_3E_2E_1})\,\mathbf{A} = \begin{pmatrix}0 & -1 & 1\\-1 & 2 & -1\\0 & 3 & -2\end{pmatrix}\begin{pmatrix}1 & -1 & 1\\2 & 0 & 1\\3 & 0 & 1\end{pmatrix}$$

$$\begin{pmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{pmatrix} \quad (2.0.19)$$