

Question 1. Show that the vectors $\mathbf{A} = \begin{pmatrix} -2 \\ +1 \\ -1 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} +2 \\ -1 \\ +1 \end{pmatrix}$ and $\begin{pmatrix} +1 \\ -3 \\ -5 \end{pmatrix}$ form the vertices of a right angled triangle.

Solution. Let \mathbf{A}, \mathbf{B} and \mathbf{C} be given vectors such that $\mathbf{A} = \begin{pmatrix} -2 \\ +1 \\ -1 \end{pmatrix}$
 $\mathbf{B} = \begin{pmatrix} +2 \\ -1 \\ +1 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} +1 \\ -3 \\ -5 \end{pmatrix}$ Given below is the figure formed by \mathbf{A}, \mathbf{B} and \mathbf{C} .

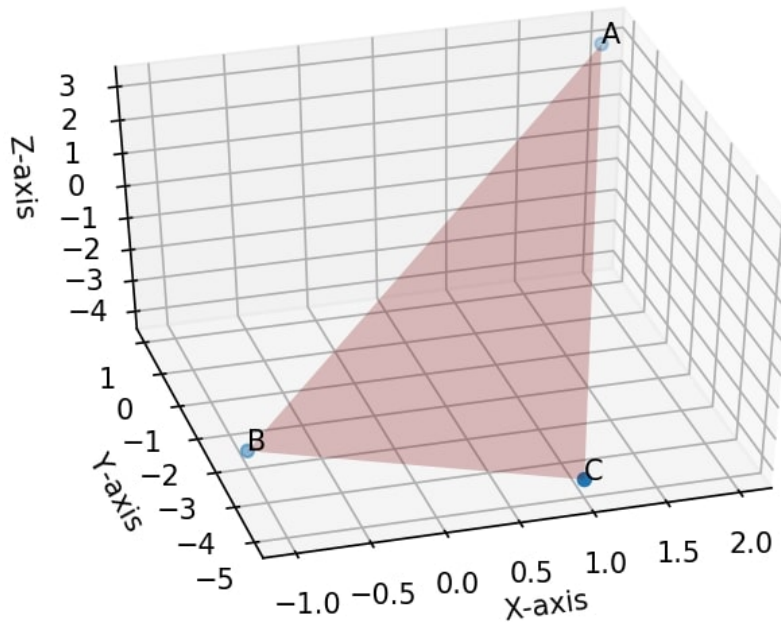


Figure 1

Clearly, \mathbf{ABC} is a triangle. To prove, the $\triangle ABC$ is a right triangle, we need to calculate the inner product of all the below vectors and check if any one of

them is 0.:

$$(\mathbf{A} - \mathbf{C})^T(\mathbf{B} - \mathbf{C}) = \begin{pmatrix} -1 & +3 & +5 \end{pmatrix} \begin{pmatrix} -2 \\ +1 \\ -1 \end{pmatrix} = 00 \quad (1)$$

$$(\mathbf{A} - \mathbf{B})^T(\mathbf{C} - \mathbf{B}) = \begin{pmatrix} +1 & +2 & +6 \end{pmatrix} \begin{pmatrix} +2 \\ -1 \\ +1 \end{pmatrix} = 06 \quad (2)$$

$$(\mathbf{B} - \mathbf{A})^T(\mathbf{C} - \mathbf{A}) = \begin{pmatrix} -1 & -2 & -6 \end{pmatrix} \begin{pmatrix} +1 \\ -3 \\ -5 \end{pmatrix} = 35 \quad (3)$$

Clearly, from (1) we can see that $\triangle ABC$ is right angled at C.