

Orthogonality

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Abstract—This a simple document that explains how to prove the eigen values of an Orthogonal matrix have magnitude one.

Download all latex-tikz codes from

<https://github.com/saranshbali/EE5609/tree/master/Assignment1/Latex>

1 PROBLEM

Show that the eigen values λ of an Orthogonal matrix \mathbf{Q} are such that $|\lambda| = 1$.

2 SOLUTION

Here \mathbf{Q} is given to be an orthogonal matrix, then

$$\mathbf{Q}\mathbf{Q}^T = \mathbf{Q}^T\mathbf{Q} = \mathbf{I} \quad (2.0.1)$$

Let λ be an eigen value of \mathbf{Q} and \mathbf{v} be corresponding eigen vector. Then

$$\mathbf{Q}\mathbf{v} = \lambda\mathbf{v} \quad (2.0.2)$$

$$\|\mathbf{Q}\mathbf{v}\|^2 = \|\lambda\mathbf{v}\|^2 \quad (2.0.3)$$

$$(\mathbf{Q}\mathbf{v})^T \mathbf{Q}\mathbf{v} = |\lambda|^2 \|\mathbf{v}\|^2 \quad (2.0.4)$$

$$(\mathbf{v}^T \mathbf{Q}^T) \mathbf{Q}\mathbf{v} = |\lambda|^2 \|\mathbf{v}\|^2 \quad (2.0.5)$$

$$(2.0.6)$$

By, (2.0.1), we have

$$\mathbf{v}^T \mathbf{v} = |\lambda|^2 \|\mathbf{v}\|^2 \quad (2.0.7)$$

$$\|\mathbf{v}\|^2 = |\lambda|^2 \|\mathbf{v}\|^2 \quad (2.0.8)$$

Thus, we have

$$|\lambda|^2 = 1 \quad (2.0.9)$$

$$|\lambda| = 1 \quad (2.0.10)$$

Thus, magnitude of eigen values of an orthogonal matrix is 1.