Question 1. *Show that the vectors* $\mathbf{A} = \begin{pmatrix} -2 \\ +1 \\ -1 \end{pmatrix}$,$\mathbf{B} = \begin{pmatrix} +2 \\ -1 \\ +1 \end{pmatrix}$ *and* $\begin{pmatrix} +1 \\ -3 \\ -5 \end{pmatrix}$ *form the vertices of a right angled triangle.*

Solution. Let $\mathbf{A}$,$\mathbf{B}$ and $\mathbf{C}$ be given vectors such that $\mathbf{A} = \begin{pmatrix} -2 \\ +1 \\ -1 \end{pmatrix}$ $\mathbf{B} = \begin{pmatrix} +2 \\ -1 \\ +1 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} +1 \\ -3 \\ -5 \end{pmatrix}$ Given below is the figure formed by $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$.

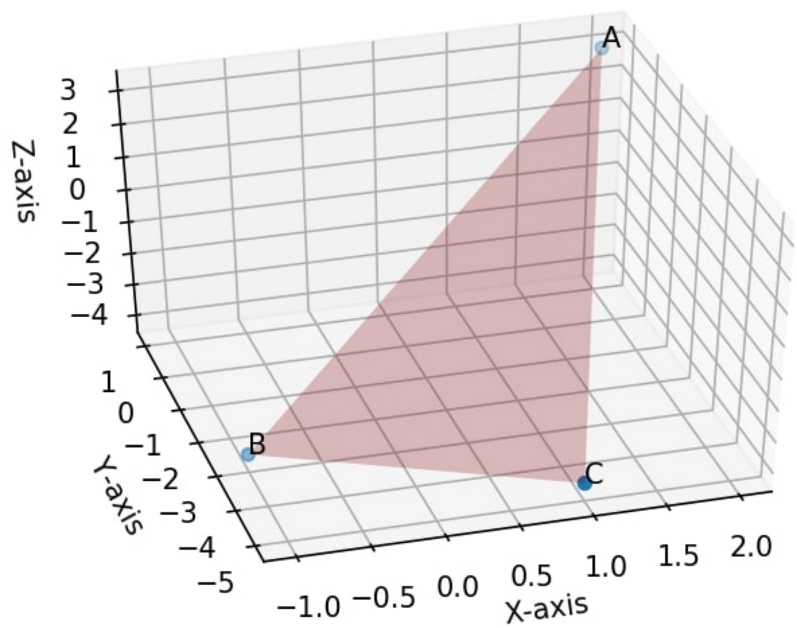

Figure 1

Clearly, **ABC** is a triangle. To prove, the $\triangle ABC$ is a right triangle, we need to calculate the inner product of all the below vectors and check if any one of them is 0.:

$$\langle \mathbf{A}-\mathbf{C}, \mathbf{B}-\mathbf{C}\rangle = (\mathbf{A}-\mathbf{C})^T(\mathbf{B}-\mathbf{C}) = \begin{pmatrix}-1 & +3 & +5\end{pmatrix}\begin{pmatrix}-2\\+1\\-1\end{pmatrix} = 00 \tag{1}$$

$$\langle \mathbf{A}-\mathbf{B}, \mathbf{C}-\mathbf{B}\rangle = (\mathbf{A}-\mathbf{B})^T(\mathbf{C}-\mathbf{B}) = \begin{pmatrix}+1 & +2 & +6\end{pmatrix}\begin{pmatrix}+2\\-1\\+1\end{pmatrix} = 06 \tag{2}$$

$$\langle \mathbf{B}-\mathbf{A}, \mathbf{C}-\mathbf{A}\rangle = (\mathbf{B}-\mathbf{A})^T(\mathbf{C}-\mathbf{A}) = \begin{pmatrix}-1 & -2 & -6\end{pmatrix}\begin{pmatrix}+1\\-3\\-5\end{pmatrix} = 35 \tag{3}$$

Clearly, from (1) we can see that $\triangle$ABC is right angled at C.