Matrix theory - Challenge

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***Abstract*—This document illustrates proving properties of traingle using linear algebra**

Download all latex-tikz from

https://github.com/shreeprasadbhat/matrix−theory/blob/master//challenging_problems/challenge3/

1 Problem

Prove that, Sides opposite to equal angles of a triangle are equal.

2 Solution

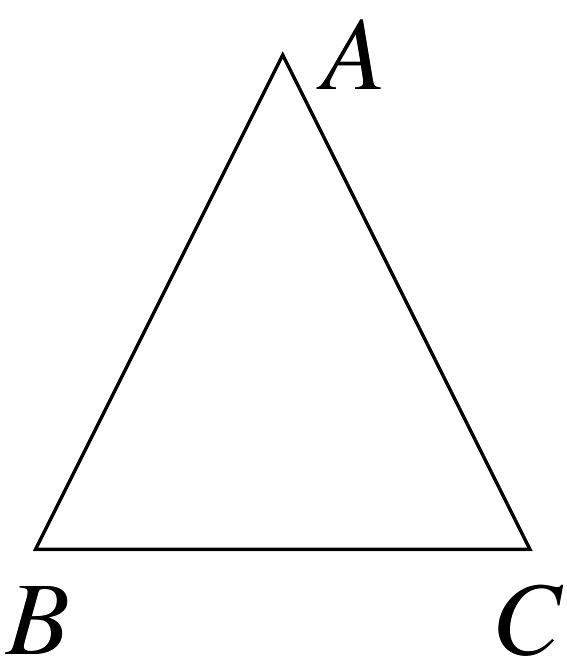

Fig. 0: $\triangle ABC$

Given that two angles of traingle are equal,

$$\angle ABC = \angle ACB \tag{2.0.1}$$

$$\cos \angle ABC = \cos \angle ACB \tag{2.0.2}$$

$$\frac{(\mathbf{B}-\mathbf{A})^T(\mathbf{B}-\mathbf{C})}{\|\mathbf{B}-\mathbf{A}\|\,\|\mathbf{B}-\mathbf{C}\|} = \frac{(\mathbf{C}-\mathbf{A})^T(\mathbf{C}-\mathbf{B})}{\|\mathbf{C}-\mathbf{A}\|\,\|\mathbf{C}-\mathbf{B}\|} \tag{2.0.3}$$

$$\frac{(\mathbf{B}-\mathbf{A})^T(\mathbf{B}-\mathbf{C})}{\|\mathbf{B}-\mathbf{A}\|} = \frac{(\mathbf{C}-\mathbf{A})^T(\mathbf{C}-\mathbf{B})}{\|\mathbf{C}-\mathbf{A}\|} \tag{2.0.4}$$

It can be showed that,

$$(\mathbf{B}-\mathbf{A})^T(\mathbf{B}-\mathbf{C}) = \|\mathbf{A}-\mathbf{B}\|^2 - (\mathbf{A}-\mathbf{C})^T(\mathbf{A}-\mathbf{B}) \tag{2.0.5}$$

$$(\mathbf{C}-\mathbf{A})^T(\mathbf{C}-\mathbf{B}) = \|\mathbf{A}-\mathbf{C}\|^2 - (\mathbf{A}-\mathbf{C})^T(\mathbf{A}-\mathbf{B}) \tag{2.0.6}$$

Substituting (2.0.5) and (2.0.6) in (2.0.4),

$$\frac{\|\mathbf{A}-\mathbf{B}\|^2 - (\mathbf{A}-\mathbf{C})^T(\mathbf{A}-\mathbf{B})}{\|\mathbf{B}-\mathbf{A}\|} = \frac{\|\mathbf{A}-\mathbf{C}\|^2 - (\mathbf{A}-\mathbf{C})^T(\mathbf{A}-\mathbf{B})}{\|\mathbf{C}-\mathbf{A}\|} \tag{2.0.7}$$

$$\|\mathbf{A}-\mathbf{B}\| - \frac{(\mathbf{A}-\mathbf{C})^T(\mathbf{A}-\mathbf{B})}{\|\mathbf{B}-\mathbf{A}\|} = \|\mathbf{A}-\mathbf{C}\| - \frac{(\mathbf{A}-\mathbf{C})^T(\mathbf{A}-\mathbf{B})}{\|\mathbf{C}-\mathbf{A}\|} \tag{2.0.8}$$

$$\|\mathbf{A}-\mathbf{B}\| - \|\mathbf{A}-\mathbf{C}\| \cos \angle BAC = \|\mathbf{A}-\mathbf{C}\| - \|\mathbf{A}-\mathbf{B}\| \cos \angle CAB \tag{2.0.9}$$

$$\|\mathbf{A}-\mathbf{B}\| + \|\mathbf{A}-\mathbf{B}\| \cos \angle CAB = \|\mathbf{A}-\mathbf{C}\| + \|\mathbf{A}-\mathbf{C}\| \cos \angle BAC \tag{2.0.10}$$

$$\|\mathbf{A}-\mathbf{B}\|\,(1 + \cos \angle CAB) = \|\mathbf{A}-\mathbf{C}\|\,(1 + \cos \angle BAC) \tag{2.0.11}$$

$$\implies \|\mathbf{A}-\mathbf{B}\| = \|\mathbf{A}-\mathbf{C}\| \tag{2.0.12}$$

Hence we proved that sides opposite to equal angles of triangle are equal.