

Matrix theory - Challenge

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Abstract—This document illustrates proving properties of triangle using linear algebra

Download all latex-tikz from

https://github.com/shreeprasadbhat/matrix-theory/blob/master/challenging_problems/challenge3/

1 PROBLEM

Prove that, Sides opposite to equal angles of a triangle are equal.

2 SOLUTION

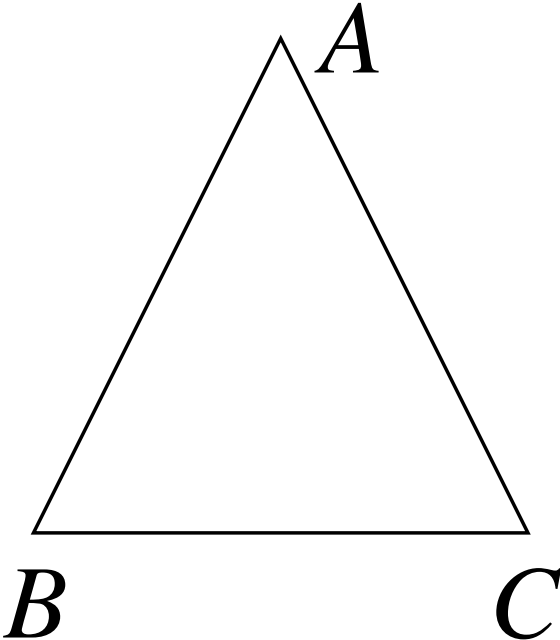


Fig. 0: $\triangle ABC$

Given that two angles of triangle are equal,

$$\angle ABC = \angle ACB \quad (2.0.1)$$

$$\cos \angle ABC = \cos \angle ACB \quad (2.0.2)$$

$$\frac{(\mathbf{B} - \mathbf{A})^T (\mathbf{B} - \mathbf{C})}{\|\mathbf{B} - \mathbf{A}\| \|\mathbf{B} - \mathbf{C}\|} = \frac{(\mathbf{C} - \mathbf{A})^T (\mathbf{C} - \mathbf{B})}{\|\mathbf{C} - \mathbf{A}\| \|\mathbf{C} - \mathbf{B}\|} \quad (2.0.3)$$

$$\frac{(\mathbf{B} - \mathbf{A})^T (\mathbf{B} - \mathbf{C})}{\|\mathbf{B} - \mathbf{A}\|} = \frac{(\mathbf{C} - \mathbf{A})^T (\mathbf{C} - \mathbf{B})}{\|\mathbf{C} - \mathbf{A}\|} \quad (2.0.4)$$

It can be showed that,

$$(\mathbf{B} - \mathbf{A})^T (\mathbf{B} - \mathbf{C}) = \|\mathbf{A} - \mathbf{B}\|^2 - (\mathbf{A} - \mathbf{C})^T (\mathbf{A} - \mathbf{B}) \quad (2.0.5)$$

$$(\mathbf{C} - \mathbf{A})^T (\mathbf{C} - \mathbf{B}) = \|\mathbf{A} - \mathbf{C}\|^2 - (\mathbf{A} - \mathbf{C})^T (\mathbf{A} - \mathbf{B}) \quad (2.0.6)$$

Substituting (2.0.5) and (2.0.6) in (2.0.4),

$$\frac{\|\mathbf{A} - \mathbf{B}\|^2 - (\mathbf{A} - \mathbf{C})^T (\mathbf{A} - \mathbf{B})}{\|\mathbf{B} - \mathbf{A}\|} = \frac{\|\mathbf{A} - \mathbf{C}\|^2 - (\mathbf{A} - \mathbf{C})^T (\mathbf{A} - \mathbf{B})}{\|\mathbf{C} - \mathbf{A}\|} \quad (2.0.7)$$

$$\|\mathbf{A} - \mathbf{B}\| - \frac{(\mathbf{A} - \mathbf{C})^T (\mathbf{A} - \mathbf{B})}{\|\mathbf{B} - \mathbf{A}\|} = \|\mathbf{A} - \mathbf{C}\| - \frac{(\mathbf{A} - \mathbf{C})^T (\mathbf{A} - \mathbf{B})}{\|\mathbf{C} - \mathbf{A}\|} \quad (2.0.8)$$

$$\|\mathbf{A} - \mathbf{B}\| - \|\mathbf{A} - \mathbf{C}\| \cos \angle BAC = \|\mathbf{A} - \mathbf{C}\| - \|\mathbf{A} - \mathbf{B}\| \cos \angle CAB \quad (2.0.9)$$

$$\|\mathbf{A} - \mathbf{B}\| + \|\mathbf{A} - \mathbf{B}\| \cos \angle CAB = \|\mathbf{A} - \mathbf{C}\| + \|\mathbf{A} - \mathbf{C}\| \cos \angle BAC \quad (2.0.10)$$

$$\|\mathbf{A} - \mathbf{B}\| (1 + \cos \angle CAB) = \|\mathbf{A} - \mathbf{C}\| (1 + \cos \angle BAC) \quad (2.0.11)$$

$$\Rightarrow \|\mathbf{A} - \mathbf{B}\| = \|\mathbf{A} - \mathbf{C}\| \quad (2.0.12)$$

Hence we proved that sides opposite to equal angles of triangle are equal.