1

Matrix theory - Challenge

Shreeprasad Bhat AI20MTECH14011

Abstract—This document illustrates proving properties of traingle using linear algebra

Download all latex-tikz from

https://github.com/shreeprasadbhat/matrix-theory/blob/master//challenging problems/challenge3/

1 Problem

Prove that, Sides opposite to equal angles of a triangle are equal.

2 Solution

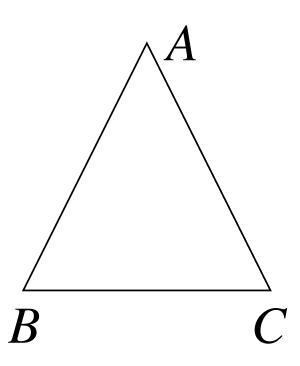


Fig. 0: $\triangle ABC$

Given that two angles of traingle are equal,

$$\angle ABC = \angle ACB \tag{2.0.1}$$

$$\cos \angle ABC = \cos \angle ACB \qquad (2.0.2)$$

$$\frac{(\mathbf{B} - \mathbf{A})^{T}(\mathbf{B} - \mathbf{C})}{\|\mathbf{B} - \mathbf{A}\| \|\mathbf{B} - \mathbf{C}\|} = \frac{(\mathbf{C} - \mathbf{A})^{T}(\mathbf{C} - \mathbf{B})}{\|\mathbf{C} - \mathbf{A}\| \|\mathbf{C} - \mathbf{B}\|}$$
(2.0.3)

$$\frac{(\mathbf{B} - \mathbf{A})^T (\mathbf{B} - \mathbf{C})}{\|\mathbf{B} - \mathbf{A}\|} = \frac{(\mathbf{C} - \mathbf{A})^T (\mathbf{C} - \mathbf{B})}{\|\mathbf{C} - \mathbf{A}\|}$$
(2.0.4)

It can be showed that,

$$(\mathbf{B} - \mathbf{A})^{T}(\mathbf{B} - \mathbf{C}) = \|\mathbf{A} - \mathbf{B}\|^{2} - (\mathbf{A} - \mathbf{C})^{T}(\mathbf{A} - \mathbf{B})$$
(2.0.5)

$$(\mathbf{C} - \mathbf{A})^{T}(\mathbf{C} - \mathbf{B}) = \|\mathbf{A} - \mathbf{C}\|^{2} - (\mathbf{A} - \mathbf{C})^{T}(\mathbf{A} - \mathbf{B})$$
(2.0.6)

Substituting (2.0.5) and (2.0.6) in (2.0.4),

$$\frac{\|\mathbf{A} - \mathbf{B}\|^2 - (\mathbf{A} - \mathbf{C})^T (\mathbf{A} - \mathbf{B})}{\|\mathbf{B} - \mathbf{A}\|} = \frac{\|\mathbf{A} - \mathbf{C}\|^2 - (\mathbf{A} - \mathbf{C})^T (\mathbf{A} - \mathbf{B})}{\|\mathbf{C} - \mathbf{A}\|}$$
(2.0.7)

$$\|\mathbf{A} - \mathbf{B}\| - \frac{(\mathbf{A} - \mathbf{C})^T (\mathbf{A} - \mathbf{B})}{\|\mathbf{B} - \mathbf{A}\|} = \|\mathbf{A} - \mathbf{C}\| - \frac{(\mathbf{A} - \mathbf{C})^T (\mathbf{A} - \mathbf{B})}{\|\mathbf{C} - \mathbf{A}\|}$$
(2.0.8)

$$\|\mathbf{A} - \mathbf{B}\| - \|\mathbf{A} - \mathbf{C}\| \cos \angle BAC = \|\mathbf{A} - \mathbf{C}\| - \|\mathbf{A} - \mathbf{B}\| \cos \angle CAB$$
(2.0.9)

$$\|\mathbf{A} - \mathbf{B}\| + \|\mathbf{A} - \mathbf{B}\| \cos \angle CAB = \|\mathbf{A} - \mathbf{C}\| + \|\mathbf{A} - \mathbf{C}\| \cos \angle BAC$$
(2.0.10)

$$\|\mathbf{A} - \mathbf{B}\| (1 + \cos \angle CAB) = \|\mathbf{A} - \mathbf{C}\| (1 + \cos \angle BAC)$$
(2.0.11)

$$\implies ||\mathbf{A} - \mathbf{B}|| = ||\mathbf{A} - \mathbf{C}|| \qquad (2.0.12)$$

Hence we proved that sides opposite to equal angles of triangle are equal.