Matrix theory - Assignment3

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Abstract—This document illustrates finding determinant of matrix using properties of determinant

Download all python codes from

https://github.com/shreeprasadbhat/matrix-theory/ tree/master/assignment3/codes

and latex-tikz codes from

https://github.com/shreeprasadbhat/matrix-theory/ blob/master/assignment3/

1 Problem

Show that

$$\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix} = 4a^2b^2c^2$$
 (1.0.1)

2 Solution

$$\begin{vmatrix} -a^{2} & ab & ac \\ ba & -b^{2} & bc \\ ca & cb & -c^{2} \end{vmatrix} = abc \begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix}$$
 (2.0.1)
$$= abc \begin{vmatrix} -a & a & a \\ b & -b & b \\ c & c & -c \end{vmatrix}$$
 (2.0.2)
$$= a^{2}b^{2}c^{2} \begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$
 (2.0.3)

$$= abc \begin{vmatrix} -a & a & a \\ b & -b & b \\ c & c & -c \end{vmatrix}$$
 (2.0.2)

$$= a^2 b^2 c^2 \begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$
 (2.0.3)

$$\xrightarrow[R_2 \leftrightarrow R_3]{R_1 \leftrightarrow R_2} (-1)(-1)a^2b^2c^2 \begin{vmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{vmatrix}$$
 (2.0.4)

$$\xrightarrow[R_3 \leftarrow R_3 - R_1]{R_2 \leftarrow R_2 - R_1} a^2 b^2 c^2 \begin{vmatrix} 1 & -1 & 1 \\ 0 & 2 & -2 \\ 0 & 0 & 2 \end{vmatrix}$$
 (2.0.5)

$$= a^2 b^2 c^2 \times (1 \times 2 \times 2) \qquad (2.0.6)$$

$$=4a^2b^2c^2$$
 (2.0.7)