Matrix theory - Assignment3

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***Abstract*—This document illustrates finding determinant of matrix using properties of determinant**

Download all python codes from

```
https://github.com/shreeprasadbhat/matrix−theory/
    tree/master/assignment3/codes
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and latex-tikz codes from

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https://github.com/shreeprasadbhat/matrix−theory/
    blob/master/assignment3/
```

1 Problem

Show that

$$\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix} = 4a^2b^2c^2 \tag{1.0.1}$$

2 Solution

We will show (1.0.1) using only properties of determinants.

By property of determinant we can take multiple of a row out of determinant. Using this property we can take a out of $row1$ and b out of $row2$ and c out of $row3$, we get

$$\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix} = abc \begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix} \tag{2.0.1}$$

By property of determinant that $\left|A^T\right| = \left|A\right|$, we can rewrite (2.0.1) as

$$= abc \begin{vmatrix} -a & a & a \\ b & -b & b \\ c & c & -c \end{vmatrix} \tag{2.0.2}$$

Again using property we can take a out of $row1$ and b out of $row2$ and c out of $row3$, we get

$$= a^2b^2c^2 \begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} \tag{2.0.3}$$

Property of determinant says that interchange rows changes sign of determinant. Using this property interchange $row1$ with $row3$, then interchange $row2$ with $row3$, we get

$$= a^2b^2c^2 \begin{vmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{vmatrix} \tag{2.0.4}$$

By property of determinant that elementary row operations does not change value of the determinant, we get

$$= a^2b^2c^2 \begin{vmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{vmatrix} \xrightarrow[R_3 \leftarrow R_3 - R_1]{R_2 \leftarrow R_2 - R_1} a^2b^2c^2 \begin{vmatrix} 1 & -1 & 1 \\ 0 & 2 & -2 \\ 0 & 0 & 2 \end{vmatrix} \tag{2.0.5}$$

Determinant of a triangular matrix is equal to product of it's diagonals, so we get

$$= a^2b^2c^2 \times (1 \times 2 \times 2) \tag{2.0.6}$$

$$= 4a^2b^2c^2 \tag{2.0.7}$$

Hence we showed that

$$\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix} = 4a^2b^2c^2 \tag{2.0.8}$$