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CUMULATIVE EXAM-I (MATHEMATICS)

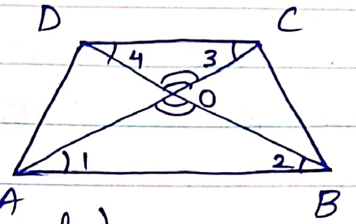
CLASS-X ANSWER KEY DATE: 02/05/20

SECTION-A

1. (b) - 6 years
2. (b) - Isosceles triangle
3. (b) - 5 units
4. (a) - $(x, 0)$
5. (c) - $(-4, -15)$
6. (b) - one solution
7. (b) - 4
8. (a) - 1
9. (c) - 6
10. (a) - rationals
11. (b) - 0
12. (a) - similar
13. (d) - 6 cm
14. (b) - $90°$
15. (b) - $\frac{4}{3}$
16. (c) - 3, -1
17. (a) - Three
18. (a) - 36 minutes
19. (a) - 24
20. (a) - irrational.

SECTION – B (5×2 = 10)

(21)



In ΔOCD and ΔOAB,

$AB \parallel DC$ (given)

$\therefore \angle 1 = \angle 3$

$\angle 2 = \angle 4$ } (Alternate interior angles)

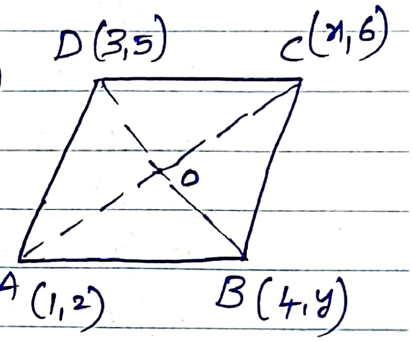
Also, $\angle DOC = \angle BOA$ [vertically opposite angles]

$\therefore \Delta OCD \sim \Delta OAB$ [by AAA similarity criterion]

$\Rightarrow \dfrac{OC}{OA} = \dfrac{OD}{OB}$ [Since, ratios of the corresponding sides of similar triangles are equal]

$\boxed{\dfrac{OA}{OC} = \dfrac{OB}{OD}}$ [Reciprocal]

(22)



Let $A(1,2)$ $B(4,y)$ $C(x,6)$ and $D(3,5)$ are the vertices of a parallelogram.

$\therefore$ Diagonals AC and BD will bisect each other, so, the mid point of AC and BD will be same.

Thus, mid-point of AC = Mid-Point of BD.

$$\Rightarrow \left(\frac{1+x}{2}, \frac{2+6}{2}\right) = \left(\frac{4+3}{2}, \frac{y+5}{2}\right)$$

[$\because$ mid point $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$]

$\dfrac{1+x}{\cancel{2}} = \dfrac{7}{\cancel{2}}$ | $\dfrac{8}{\cancel{2}} = \dfrac{y+5}{\cancel{2}}$

$1+x = 7$ | $8 = y+5$

$x = 7-1$ | $8-5 = y$

$\boxed{x = 6}$ | $\boxed{y = 3}$

(23) Given equation $2x^2 + Kx + 3 = 0$

$a = 2, \ b = K, \ c = 3$

Acc. to given it has equal roots

$\therefore b^2 - 4ac = 0$

$b^2 = 4ac$

$(K)^2 = 4(2)(3)$

(3)

$$K^2 = 24$$
$$K = \sqrt{6 \times 4}$$
$$K = \pm 2\sqrt{6}$$

(24) Given pair of linear equations is

$99x + 101y = 499$ → (1) & $101x + 99y = 501$ → (2)

on adding eqⁿ (1) & (2)

$$\begin{array}{rl} & 99x + 101y = 499 \\ (+) & 101x + 99y = 501 \\ \hline & \frac{200x}{200} + \frac{200y}{200} = \frac{1000}{200} \end{array}$$

$$x + y = 5 \quad \to (3)$$

on subtracting eq. (1) from eqⁿ (2).

$$\begin{array}{rl} & 101x + 99y = 501 \\ (-) & \underset{(-)}{99x} \underset{(-)}{+} 101y \underset{(-)}{=} 499 \\ \hline & \frac{2x}{2} - \frac{2y}{2} = \frac{2}{2} \end{array}$$

$$x - y = 1 \quad \to (4)$$

Add eqⁿ (3) & (4)

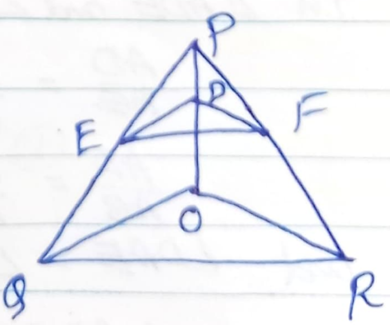
$$\begin{array}{r} x + y = 5 \\ x - y = 1 \\ \hline 2x = 6 \end{array} \Rightarrow x = \frac{6}{2}; \boxed{x = 3}$$

substitute x in (3) $\quad 3 + y = 5$

$$\boxed{y = 2}$$

(25) In $\triangle POQ$, $DE \parallel OQ$

$\therefore \frac{PE}{EQ} = \frac{PD}{DO}$ ($\because$ from BPT) → (1)



In $\triangle POR$, $DF \parallel OR$.

$\frac{PF}{FR} = \frac{PD}{DO}$ ($\because$ from BPT) → (2)

From Eq^n 1 & 2 $\frac{PE}{EQ} = \frac{PF}{FR}$

In $\triangle PQR$, we have $\frac{PE}{EQ} = \frac{PF}{FR}$

$\boxed{EF \parallel QR}$ ($\because$ from converse of BPT).

SECTION – C (6×3=18)

(26) Given, Point $P(x, y)$ is equidistant from $A(3,6)$ $B(-3,4)$

$\therefore AP = BP$ (Distances)

$A(3,6)$ $P(x,y)$ with $x_1\ y_1$, $x_2\ y_2$ $\qquad$ $B(-3,4)$ $P(x,y)$ with $x_1\ y_1$, $x_2\ y_2$

$\sqrt{(x-3)^2+(y-6)^2} = \sqrt{(x+3)^2+(y-4)^2}$ $\left[\because \text{Distance } \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}\right]$

S.o.b.s

$\left(\sqrt{x^2-6x+9+y^2-12y+36}\right)^2 = \left(\sqrt{x^2+6x+9+y^2-8y+16}\right)^2$

$\cancel{x^2}-6x+\cancel{9}+\cancel{y^2}-12y+36 = \cancel{x^2}+6x+\cancel{9}+\cancel{y^2}-8y+16$

$-6x-6x-12y+8y+36-16=0$

$\frac{-12x}{-4} - \frac{4y}{-4} + \frac{20}{-4} = 0$

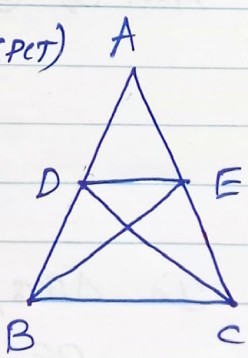
$\boxed{3x+y-5=0}$

(27) Given, $\triangle ABE \cong \triangle ACD$

$\Rightarrow AB = AC$ and $AE = AD$ (by CPCT)

$\Rightarrow \frac{AB}{AC} = 1$ and

$\frac{AD}{AE} = 1 \Rightarrow \frac{AB}{AC} = \frac{AD}{AE}$ → (1)



In $\triangle ADE$ and $\triangle ABC$, we have

$\frac{AD}{AE} = \frac{AB}{AC}$ ($\because$ from (1))

$\frac{AD}{AB} = \frac{AE}{AC}$

and $\angle DAE = \angle BAC$ (common angle)

$\therefore \triangle ADE \sim \triangle ABC$ (by SAS criterion)

5

(28) Given Equation $(1+m^2)x^2+2mcx+(c^2-a^2)=0$

$a=(1+m^2)$, $b=2mc$, $c=(c^2-a^2)$

Acc- to given it has equal roots

$$b^2-4ac=0$$

$$b^2=4ac$$

$$(2mc)^2=4[(1+m^2)(c^2-a^2)]$$

$$4m^2c^2=4[c^2-a^2+m^2c^2-m^2a^2]$$

$$\cancel{4m^2c^2}=4c^2-4a^2+\cancel{4m^2c^2}-4m^2a^2$$

$$4a^2+4m^2a^2=4c^2$$

$$\cancel{4}a^2(1+m^2)=\cancel{4}c^2$$

$$\boxed{a^2(1+m^2)=c^2}$$

Hence Proved

(29) Let the digit at units place be x and the digit at ten's place be y. Then.

$$\text{Number}=10y+x$$

Acc. to the given conditions, we have

$$10y+x=8(x+y)+1 \Rightarrow 7x-2y+1=0 \quad \text{①}$$

and, $$10y+x=13(y-x)+2 \Rightarrow 14x-3y-2=0 \quad \text{②}$$

multiply Eq^n ① by '3' and Eq^n ② by '2'. we get

$$21x-6y+3=0$$

$$\underset{(-)}{28x}\ \underset{(+)}{-6y}\ \underset{(+)}{-4}=0 \quad (-)$$

$$-7x+7=0$$

$$-7x=-7$$

$$\boxed{x=1}$$

substitute x in Eq^n ①

$$7(1)-2y+1=0$$

$$-2y=-8$$

$$\boxed{y=4}$$

Hence, the number $=10y+x=10\times4+1=\boxed{41}$

30. It is given that α and β are the zeros of the polynomial $f(x) = 2x^2 + 5x + k$

$\therefore \alpha + \beta = -\frac{5}{2}$ and, $\alpha\beta = \frac{k}{2}$

we have,

$$\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$$

$$(\alpha^2 + \beta^2 + 2\alpha\beta) - \alpha\beta = \frac{21}{4}$$

$$(\alpha + \beta)^2 - \alpha\beta = \frac{21}{4}$$

$$\left(-\frac{5}{2}\right)^2 - \left(\frac{k}{2}\right) = \frac{21}{4}$$

$$\frac{25}{4} - \frac{k}{2} = \frac{21}{4}$$

$$-\frac{k}{2} = \frac{21}{4} - \frac{25}{4}$$

$$-\frac{k}{2} = -1$$

$$\boxed{k = 2}$$

31. Clearly, the required number is the HCF of the numbers

$398 - 7 = 391$, $436 - 11 = 425$, & $542 - 15 = 527$

$$17 \,\underline{|\, 391}\quad 23 \qquad 5 \,\underline{|\, 425}\quad 5 \,\underline{|\, 85}\quad 17 \qquad 17 \,\underline{|\, 527}\quad 31$$

$\therefore 391 = 17 \times 23$

$425 = 5 \times 5 \times 17$

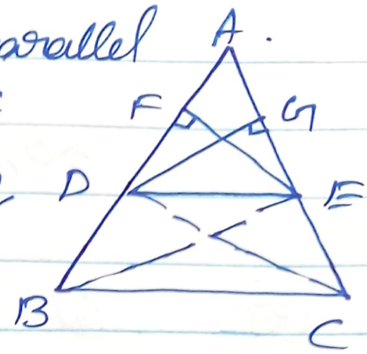
$527 = 17 \times 31$

$\therefore$ The HCF of $391, 425$ & 527 is $\boxed{17}$

Hence, the required number is 17.

SECTION – D (4×5 = 20)

(32) Statement: If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.



Given:- $DE \parallel BC$

TO Prove:- $\frac{AD}{DB} = \frac{AE}{EC}$

Construction:- Join BE and CD. Draw $EF \perp AB$ & $DG \perp AC$.

Proof:- In $\triangle ADE$ & $\triangle BED$

$$\frac{ar(\triangle ADE)}{ar(\triangle BDE)} = \frac{\frac{1}{2} \times AD \times EF}{\frac{1}{2} \times DB \times EF}$$

$$= \frac{AD}{DB} \rightarrow (1)$$

In $\triangle ADE$ & $\triangle DEC$

$$\frac{ar(\triangle ADE)}{ar(\triangle BDE)} = \frac{\frac{1}{2} \times AE \times DG}{\frac{1}{2} \times EC \times DG}$$

$$= \frac{AE}{EC} \rightarrow (2)$$

Since, $\triangle BDE$ and $\triangle DEC$ stand on the same base DE and between parallel lines DE and BC.

$\therefore ar(\triangle BDE) = ar(DEC) \rightarrow (3)$

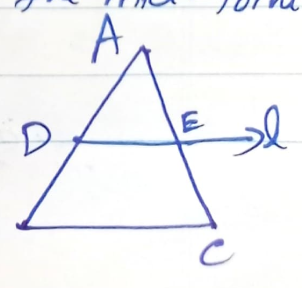
from (1), (2) & (3)

$$\boxed{\frac{AD}{DB} = \frac{AE}{EC}}$$

Hence Proved.

Consider $\triangle ABC$, in which D is the mid-point of AB.

Then, $\frac{AD}{DB} = 1 \rightarrow (i)$



(8)

line l is drawn through D such that
$l \parallel BC$.

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{AE}{EC} = 1$$

$$AE = EC$$

so, E is the mid-point of AC.

(33)

Let the speed of the train be x km/hr.
Distance travelled by the train = 480 km.
∴ time taken for travelling 480 km.

$$= \frac{480}{x} \text{ h} \quad \left(T = \frac{D}{S}\right)$$

If the speed had been 8 km/hr less,
i.e. $(x-8)$ km/hr.

Time taken for travelling 480 km

$$= \frac{480}{(x-8)} \text{ h}$$

Acc. to the question.

$$\frac{480}{(x-8)} = 3 + \frac{480}{x} \Rightarrow \frac{480}{(x-8)} - \frac{480}{x} = 3$$

$$\Rightarrow \frac{480x - 480(x-8)}{x(x-8)} = 3$$

$$\Rightarrow \cancel{480x} - \cancel{480x} + 3840 = 3x(x-8)$$

$$3840 = 3x^2 - 24x$$

$$\frac{3x^2}{3} - \frac{24x}{3} - \frac{3840}{3} = 0$$

$$x^2 - 8x - 1280 = 0$$

$$x^2 - 40x + 32x - 1280 = 0$$

$$x(x-40) + 32(x-40) = 0$$

$$(x-40)(x+32) = 0$$

$$x = 40, -32$$

∴ Speed of the train is $\boxed{40 \text{ km/hr}}$

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(34) Let us assume $3+2\sqrt{7}$ is rational

$3+2\sqrt{7} = \frac{p}{q}$ $(p,q \in I)$, $q \neq 0$

$3+2\sqrt{7} = \frac{a}{b}$ (a, b are co-primes)

$2\sqrt{7} = \frac{a}{b} - \frac{3}{1}$

$\sqrt{7} = \frac{a-3b}{2b}$

Since, a is an integer and 2b is also integer $(2b \neq 0)$,

so, $\frac{a-3b}{2b}$ is a rational number.

Let us assume $\sqrt{7}$ is rational

$\sqrt{7} = \frac{p}{q}$ $(p,q \in I)$, $q \neq 0$

$\sqrt{7} = \frac{a}{b}$ (a, b are co-primes)

$\sqrt{7}b = a$

S.on.b.s

$(\sqrt{7}b)^2 = a^2$

$7b^2 = a^2$ → (1)

7 is a factor of a^2

7 is a factor of a.

$a = 7k$ (k is constant)

substitute a in (1)

$7b^2 = (7k)^2$

$7b^2 = 49k^2$

$b^2 = 7k^2$

∴ 7 is a factor of b^2

7 is a factor of b

Thus, 7 is a common factor of a and b. But this contradicts the fact that a and b have no common factor other than 1. The contradiction arises by assuming that $\sqrt{7}$ is rational.

Hence, $\sqrt{7}$ is irrational

so, $\frac{a-3b}{2b}$ is a rational

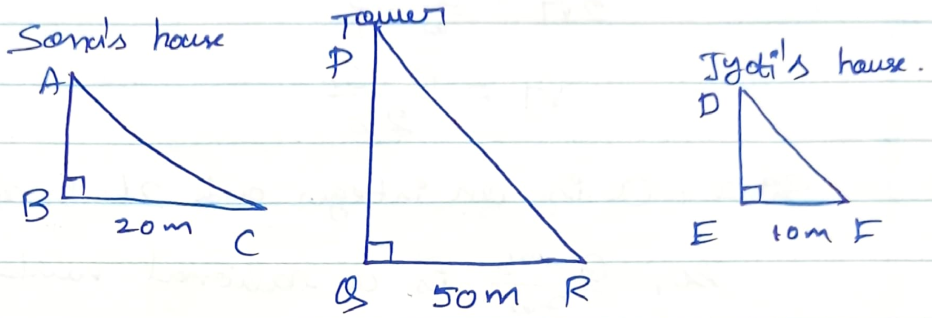
Therefore, our assumption is wrong

Hence $3+2\sqrt{7}$ is irrational number.

(35) Graph.

SECTION- E

(36)



(i) Since the triangles are similar
Their sides are proportional

In $\triangle PQR$ and $\triangle DEF$

(i) $$\frac{\text{Height of the tower } (PQ)}{DE} = \frac{QR}{EF}$$

$$\frac{PQ}{20} = \frac{50}{10}$$

$$\boxed{PQ = 100m}$$

(ii) In $\triangle PQR$ and $\triangle DEF$

$$\frac{PQ}{DE} = \frac{\text{Length of shadow of tower } (QR)}{EF}$$

$$\frac{100}{20} = \frac{QR}{15}$$

$$5 \times 15 = QR$$

$$\boxed{QR = 75\ m}$$

In $\triangle ABC$ and $\triangle PQR$

(11) (iii) $\frac{\text{Height of sonu's house (AB)}}{PQ} = \frac{20}{50} = \frac{BC}{QR}$

$$\frac{AB}{100} = \frac{2}{5}$$

$$AB = \frac{2}{\cancel{5}} \times \cancel{100}^{20}$$

$$\boxed{AB = 40m}$$

In $\triangle ABC$ and $\triangle PQR$ (or)

(iv) $\frac{AB}{PQ} = \frac{\text{length of shadow of sonu's house (BC)}}{QR}$

$$\frac{\cancel{40}^{2}}{\cancel{100}_{5}} = \frac{BC}{40}$$

$$40 \times \frac{2}{5} = BC$$

$$\boxed{BC = 16\ m}$$

(37) (i) If A is taken as origin
The coordinates of triangle PQR in
P(4,6) Q(3,2) R(6,5)

(ii) If C is taken as origin
The coordinates of point P(-6,-4)

(iii) If B is taken as origin
the coordinates of point P(4,-4)
(or)
If origin is A, Distance b/w PQ

$P(\underset{x_1}{4},\underset{y_1}{6})\ Q(\underset{x_2}{3},\underset{y_2}{2})$ $\quad PQ = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$$= \sqrt{(3-4)^2+(2-6)^2}$$

$$= \sqrt{1+16}$$

(i) Since, the graph of the polynomial intersect the x-axis at $x = 1, 3$

$\therefore$ required zeroes are $1, 3$

(ii) If α, β are zeroes

$\alpha = 1, \beta = 3$

$K[x^2 - (\alpha+\beta)x + \alpha\beta]$

$K[x^2 - (1+3)x + (1)(3)]$

$K[x^2 - 4x + 3]$

(iii) Let $f(x) = x^2 - 4x + 3$

Then $f(4) = (4)^2 - 4(4) + 3$

$= 16 - 16 + 3$

$= 3$

(iv) Given one zero is '7'

Other zero $= -\frac{35}{7} = -5$

Thus, the zeroes are 7 and -5

$K[x^2 - (\alpha+\beta)x + \alpha\beta]$

$K[x^2 - (7-5)x + 7(-5)]$

$K[x^2 - 2x - 35]$

(or)

Let $P(x) = -x^2 + 5x - 6$

$-x^2 + 2x + 3x - 6$

$-x(x-2) + 3(x-2)$

$(x-2)(-x+3)$

For zeroes, we consider

$x - 2 = 0$; $-x + 3 = 0$

$x = 2$, $x = 3$

— x —