Evaluation of asset pricing models on NASDAQ100

QF603 - Quantitative Analysis of Financial Markets

Group member

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- **Question** : To what extent can the variance of NASDAQ 100 be explained by one of the four models? Which model can explain the variance the best?

- **Overview** :

1.

Data Collection

2.

Portfolio Construction

- CAPM
- Fama-French 3 Factor model
- Cahart 4 Factor model
- Fama-French 5 Factor model

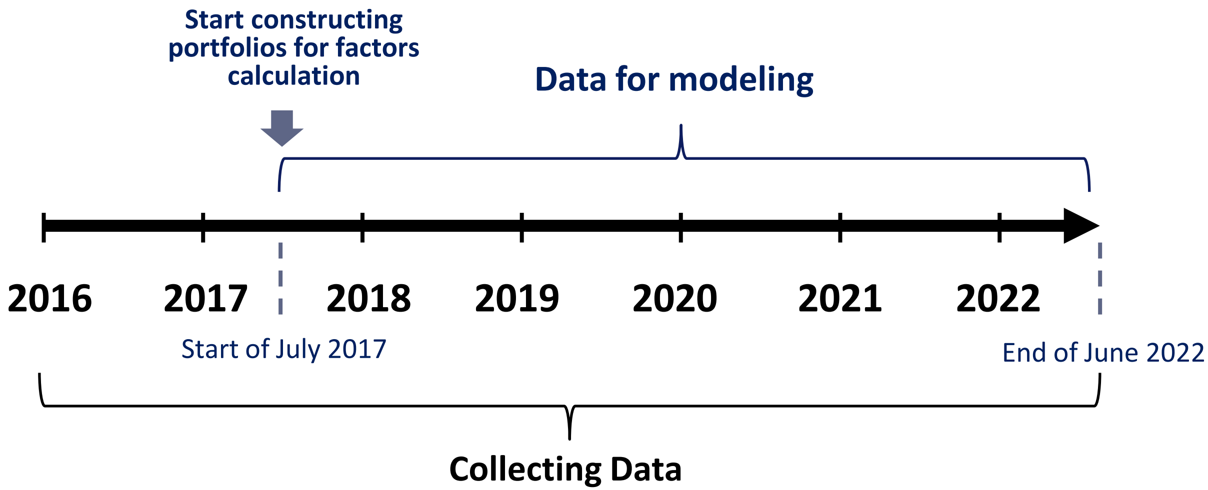
3.

Model Validation and Performance Measure

- Variance Inflation Factor (VIF)
- Studentized Breusch-Pegan
- Breusch-Godfrey test
- ACF – Autocorrelation Function

- Bayesian Information Criterion (BIC)
- Adjusted R^2
- F-Statistics

1. Data Collection | **2. Portfolio Construction** | **3. Model Validation and Performance measure**

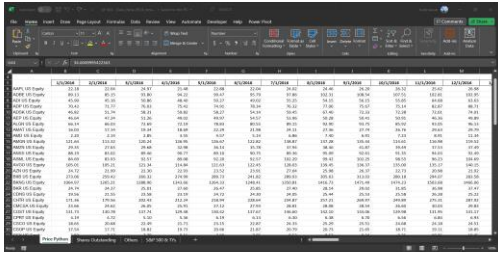


For each of stock in Nasdaq 100:

- Monthly Closing Price
- Quarterly Shares Outstanding
- Annual Operating Income
- Annual Interest Expense
- Annual Total Assets
- Annual Total Equity

Other data:

- US treasury yield (Monthly)
- S&P 500 monthly index



Our main source of financial data are Bloomberg and alpha vantage library in Python.

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1. Data Collection | 2. Portfolio Construction | 3. Model Validation and Performance measure

▶ Models

1.) Capital market model (CAPM) : $R_i - R_f = \alpha_i + \beta(\boldsymbol{R_m - R_f}) + \epsilon_{i_t}$

2.) Fama-French 3 factor model (FF3) : $R_i - R_f = \alpha_i + \beta_1(\boldsymbol{R_m - R_f}) + \beta_2(\boldsymbol{SMB_t}) + \beta_3(\boldsymbol{HML}) + \epsilon_t$

3.) Carhart 4 factor model (CH4) : $R_i - R_f = \alpha_i + \beta_1(\boldsymbol{R_m - R_f}) + \beta_2(\boldsymbol{SMB_t}) + \beta_3(\boldsymbol{HML}) + \beta_4(\boldsymbol{MOM}) + \epsilon_t$

4.) Fama-French 5 factor model (FF5) : $R_i - R_f = \alpha_i + \beta_1(\boldsymbol{R_m - R_f}) + \beta_2(\boldsymbol{SMB_{ff5}}) + \beta_3(\boldsymbol{HML}) + \beta_4(\boldsymbol{RMW}) + \beta_5(\boldsymbol{CMA}) + \epsilon_t$

▶ Factors

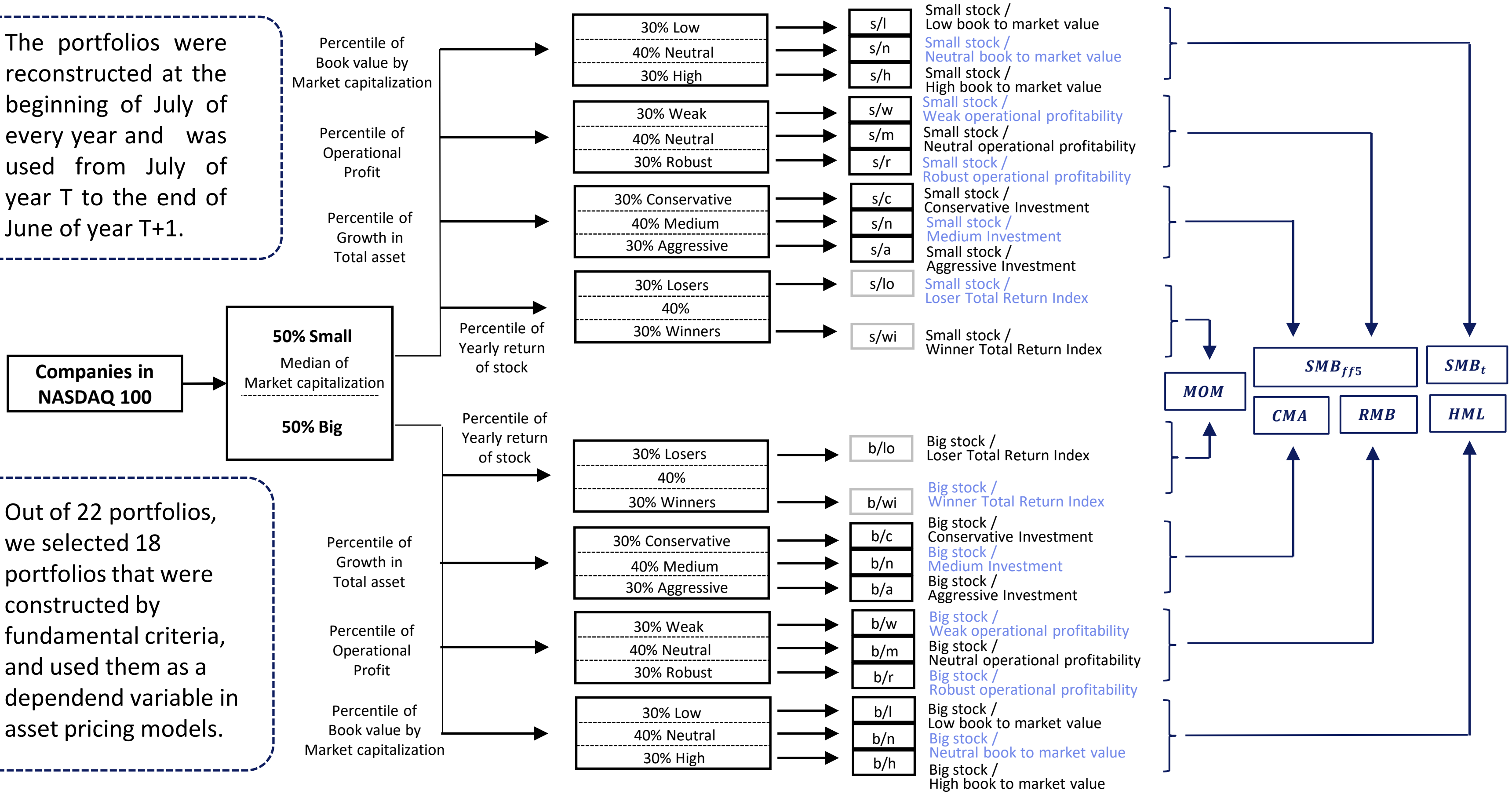
- $\boldsymbol{SMB_t} = \frac{s\backslash l + s\backslash m + s\backslash h}{3} - \frac{b\backslash l + b\backslash m + b\backslash h}{3}$
- $\boldsymbol{HML} = \frac{s\backslash \mathrm{h} + b\backslash h}{2} - \frac{s\backslash l + b\backslash \mathrm{l}}{2}$
- $\boldsymbol{SMB_{INV}} = \frac{s\backslash \mathrm{a} + s\backslash \mathrm{n} + s\backslash \mathrm{c}}{3} - \frac{b\backslash \mathrm{a} + b\backslash \mathrm{n} + b\backslash \mathrm{c}}{3}$
- $\boldsymbol{CMA} = \frac{s\backslash \mathrm{c} + b\backslash \mathrm{c}}{2} - \frac{s\backslash \mathrm{a} + b\backslash \mathrm{a}}{2}$
- $\boldsymbol{RMW} = \frac{s\backslash \mathrm{r} + b\backslash \mathrm{r}}{2} - \frac{s\backslash \mathrm{w} + b\backslash \mathrm{w}}{2}$
- $\boldsymbol{MOM} = \frac{s\backslash \mathrm{w} + b\backslash \mathrm{w}}{2} - \frac{s\backslash lo + b\backslash \mathrm{lo}}{2}$
- $\boldsymbol{SMB_{FF5}} = \frac{SMB_{B/M} + SMB_{OP} + SMB_{INV}}{3}$
- $\boldsymbol{SMB_{op}} = \frac{s\backslash \mathrm{w} + s\backslash \mathrm{n} + s\backslash \mathrm{r}}{3} - \frac{b\backslash \mathrm{w} + b\backslash \mathrm{n} + b\backslash \mathrm{r}}{3}$

1. Data Collection

2. Portfolio Construction

3. Model Validation and Performance measure

The portfolios were reconstructed at the beginning of July of every year and was used from July of year T to the end of June of year T+1.



Out of 22 portfolios, we selected 18 portfolios that were constructed by fundamental criteria, and used them as a dependend variable in asset pricing models.

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1. Data Collection | **2. Portfolio Construction** | **3. Model Validation and Performance measure**

3.1) Variance Inflation Factor (VIF)

$$\boxed{VIF = \frac{1}{1 - R_i^2}}$$

- Fama French 3 Factor model :

$$R_m - R_f \;\; / \;\; SMB \;\; / \;\; HML$$

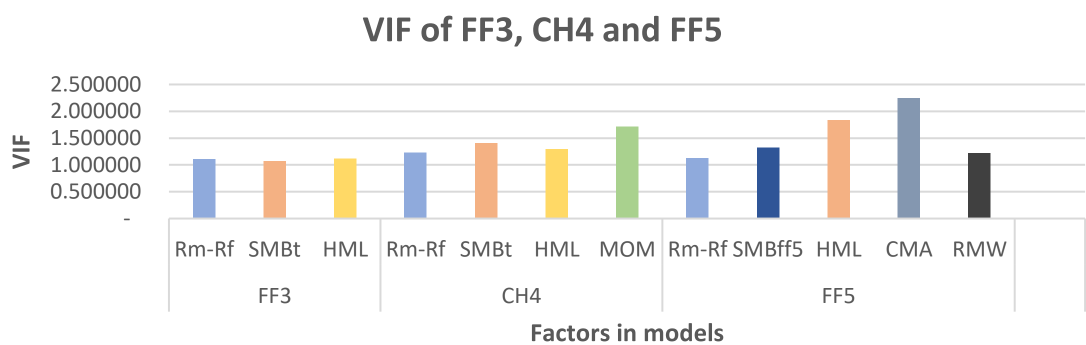
- Carhart 4 Factor model :

$$R_m - R_f \; / \, SMB \; / \; HML \, / \, MOM$$

- Fama French 5 Factor model :

$$R_m - R_f \,/\, SMB_{ff5} \,/\, HML \,/\, CMA \,/\, RMW$$

Result of VIF:



- One of the methods we used to validate the model is the VIF.
- The VIF measures the severity of multicollinearity in the ordinary least square regression analysis.
- Generally, a VIF above 4 indicates that multicollinearity might exist and further investigation is required.
- Based on the results we have achieved, the VIF is generally less than 2.5, which indicates moderate multicollinearity.
- Hence, it suggests that multicollinearity is not a substantial concern among the variables.

1. Data Collection

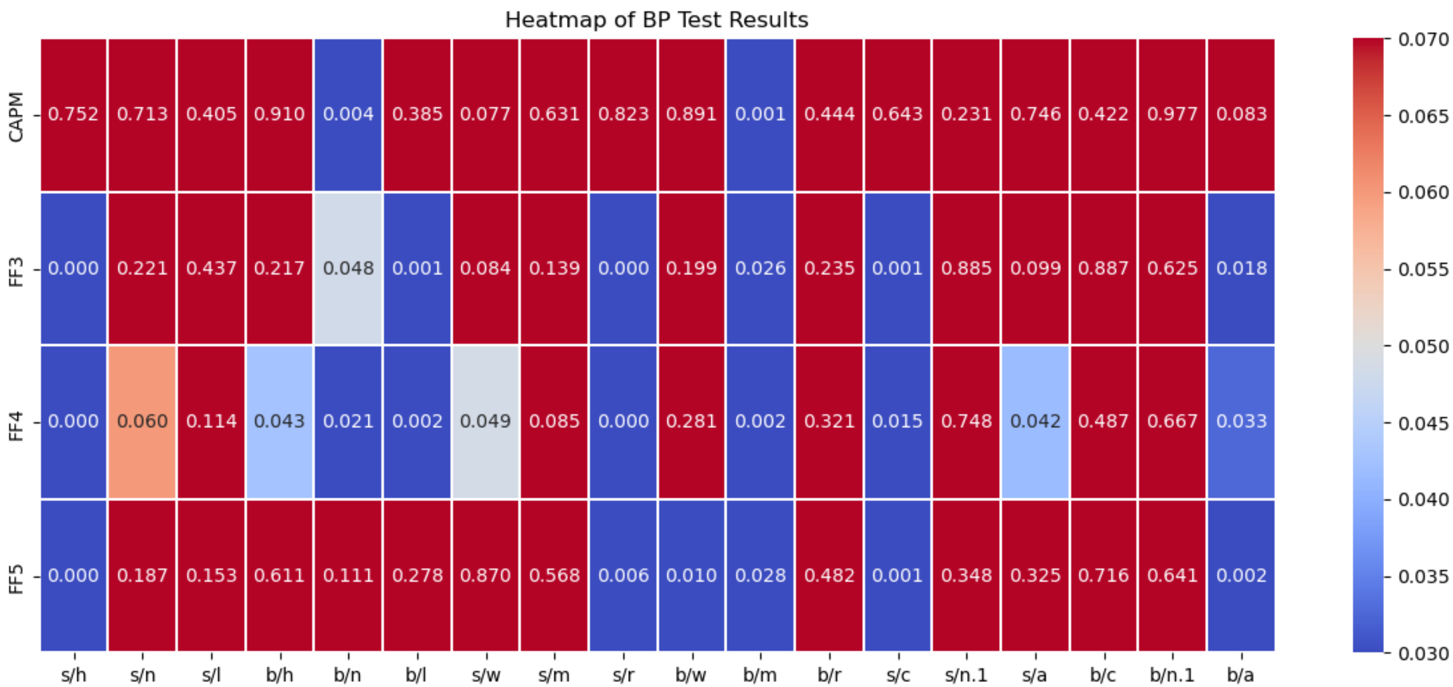
2. Portfolio Construction

3. Model Validation and Performance measure

3.2) Breusch–Pagan test

$$H_0 = \sigma^2_{\epsilon 1} = \sigma^2_{\epsilon 2} = \sigma^2_{\epsilon 3} = \;\ldots = \sigma^2_{\epsilon}$$

Result of BP test



1. Data Collection

2. Portfolio Construction

3. Model Validation and Performance measure

3.2) Breusch–Pagan test

$$H_0 = \sigma^2_{\epsilon 1} = \sigma^2_{\epsilon 2} = \sigma^2_{\epsilon 3} = \ \dots . = \sigma^2_{\epsilon}$$

Explanation for BP test and result

- One of the assumptions of the classical linear regression model is the constant variance in the error terms. Breaking this assumption means estimators are no longer the best Linear Unbiased Estimators because their variances are not the lowest. If we assume there is no heteroskedasticity but in reality there exists, we will get unbiased but inefficient estimates and biased standard errors.

- And BP test is used to test the heteroskedasticity in a linear regression model. The test is derived from Lagrange multiplier, and approximately follows the χ 2 distribution with the null hypothesis of a constant variance. So, if the p-value is below than 0.05 then we reject the null hypothesis.

- From our result, the variable b/m in all 4 models is less than 0.05, which means "b/m" has a significant impact on the heteroskedasticity . We may consider transforming this variable. Like non-linear transformations.
And s/h, s/r, s/c and b/a show the similar situation in other models except CAPM. We could consider some correlation methods like weighted least squares model or heteroskedasticity -robust method to address the effect.

- Overall, CAPM is more tolerant of the heteroskedasticity , while other models could be more sensitive. And for our dataset, the Fama-French 5-factor model is also an acceptable model that provides relatively stable results.

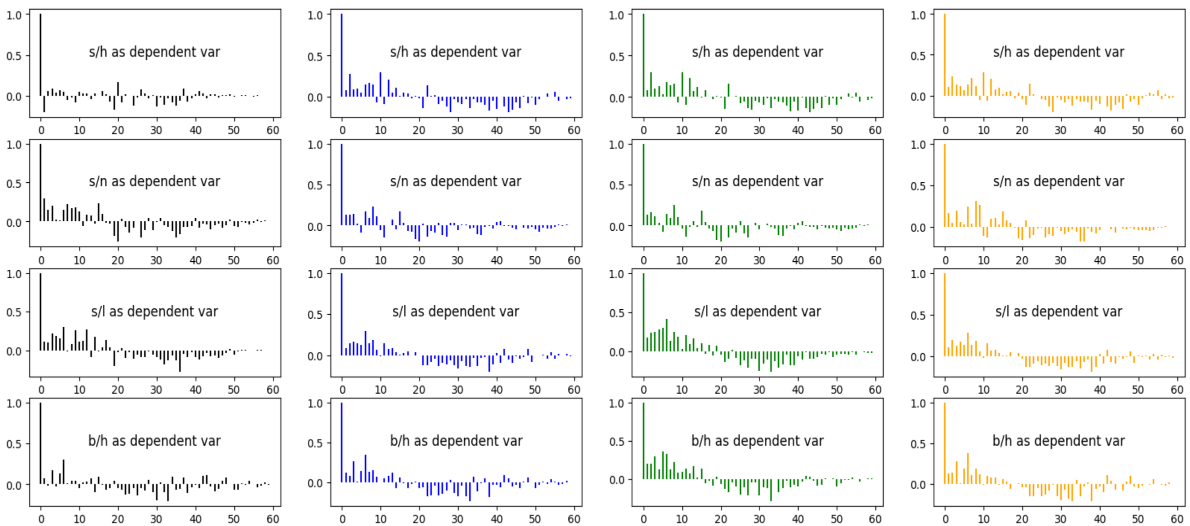
1. Data Collection | **2. Portfolio Construction** | **3. Model Validation and Performance measure**

3.3) Autocorrelation Function

$$r_k = \frac{\sum_{t=k+1}^{T}(y_t - \bar{y})(y_{t-k} - \bar{y})}{\sum_{t=1}^{T}(y_t - \bar{y})^2}$$

18 plots per model

ACF of an error term:



- We use Autocorrelation Function (ACF) with the error to check if there's any hidden pattern or correlation in the residuals

- Overall, for all of the models, there is no significant autocorrelation between the error term and the lag value of itself.

1. Data Collection

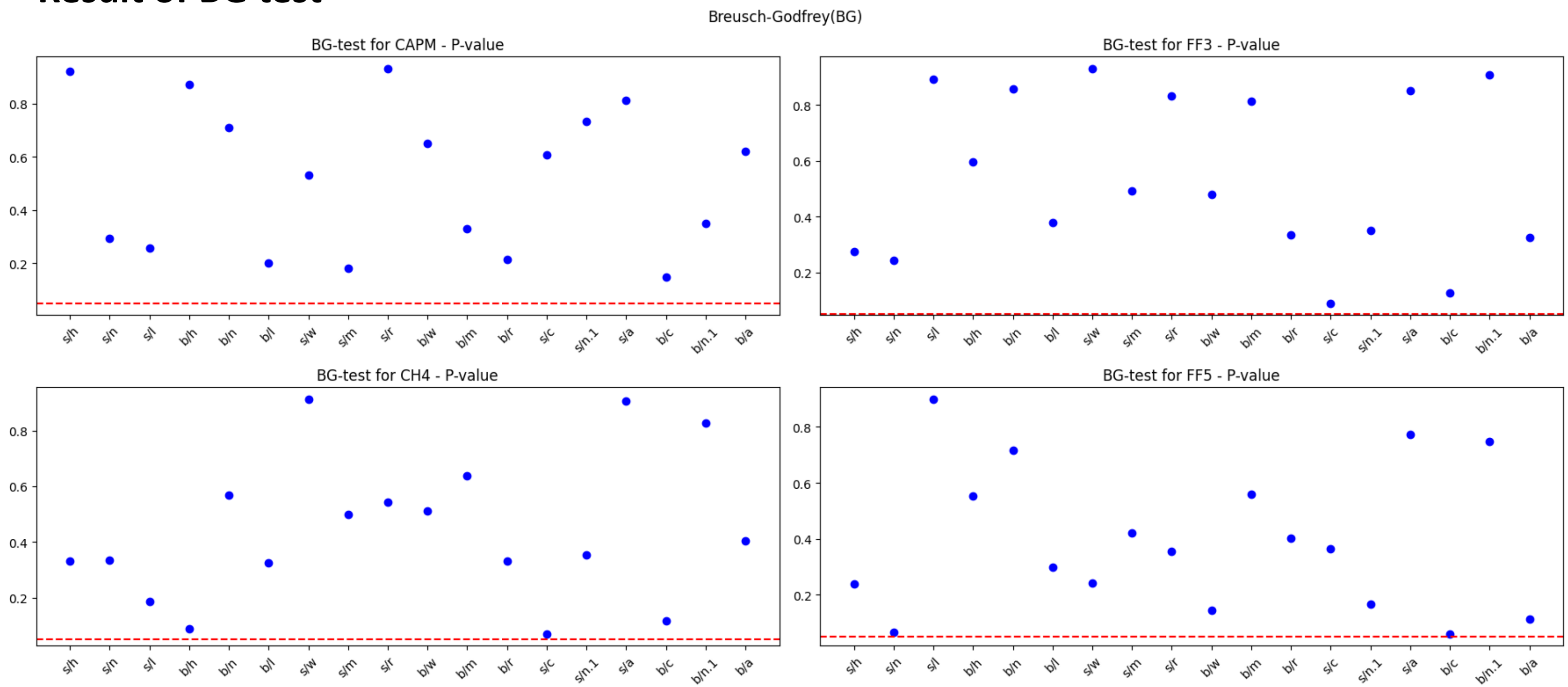
2. Portfolio Construction

3. Model Validation and Performance measure

3.4) Breusch-Godfrey test

$$H_0 = \rho_1 = \rho_2 = \rho_3 = \ldots = \rho_p = 0$$

Result of BG test



1. Data Collection

2. Portfolio Construction

3. Model Validation and Performance measure

3.4) Breusch-Godfrey test

$$H_0 = \rho_1 = \rho_2 = \rho_3 = \ldots = \rho_p = 0$$

Explanation for BG test and result

The Breusch-Godfrey test, often abbreviated as BG test, is a diagnostic tool used to detect the presence of autocorrelation in the residuals of a regression model. If present, autocorrelation can invalidate the results of our regression model, as it violates the assumption that the residuals are not autocorrelated.

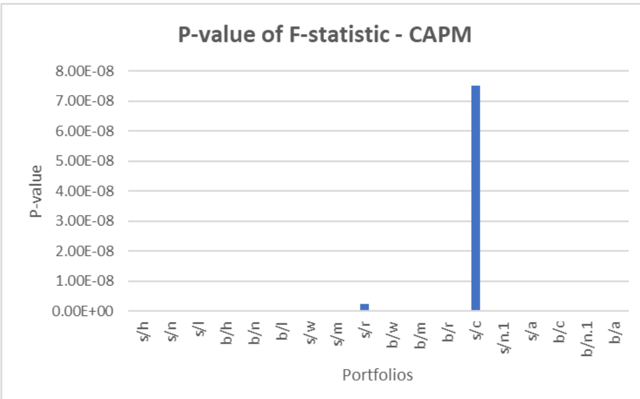
We first estimate our regression model and obtain the residuals. Next, we regress these residuals on the original independent variable and the lagged values of the residuals. The test statistic is then computed, which follows a chi-squared distribution. We compare the test statistic to the critical value from the chi-squared distribution to determine the presence of autocorrelation. The null hypothesis, Ho, states that there is no autocorrelation up to the p-th order. If the p-value is below a 0.05 significance level, we reject the null hypothesis, indicating the presence of autocorrelation.

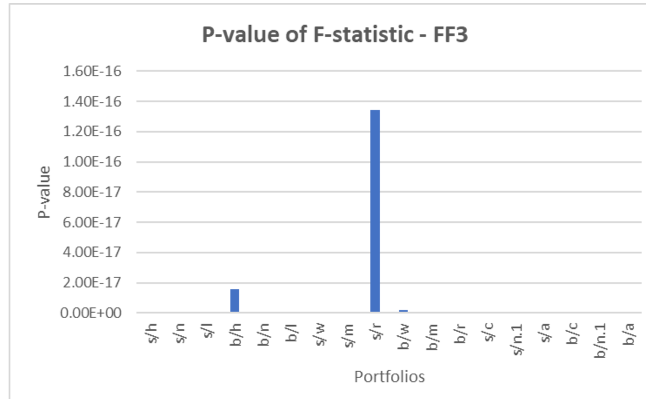
we've applied the BG test for each of four models: CAPM, FF3, CH4 and FF5. The dotted red line represents the significance level at 0.05. Looking at the p-values, any value below this line suggests the presence of autocorrelation in the residuals of that model.

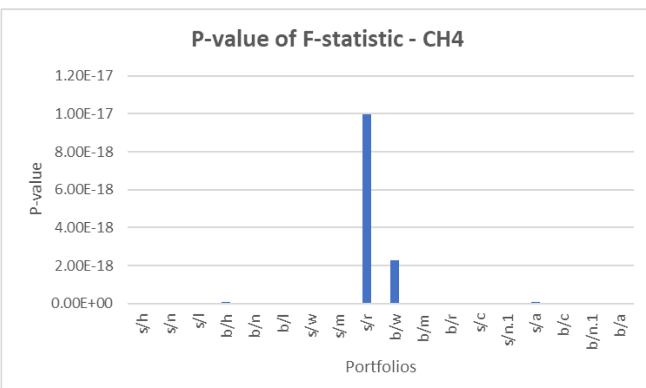
From our BG test results, it show that all model consistently shows p-values above the 0.05 significance line, indicating no significant autocorrelation in its residuals across all the time frames considered. However, the absence of autocorrelation is just one aspect of model reliability. Other factors should also be considered when determining the overall best model.

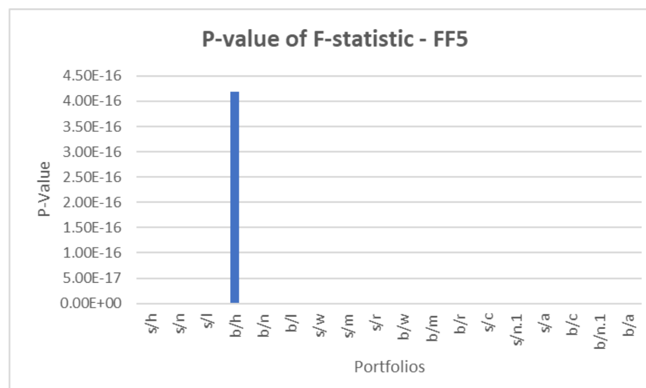
1. Data Collection | **2. Portfolio Construction** | **3. Model Validation and Performance measure**

3.5) Performance measurement – F-Statistic









Model	Average p-val
CAPM	4.32e-09
FF3	8.43e-18
CH4	6.93e-19
FF5	2.33e-17

- F-statistic is used to measure the performance of the models. If the p-value is lower that 0.05, it means that the overall of our regression model is significant

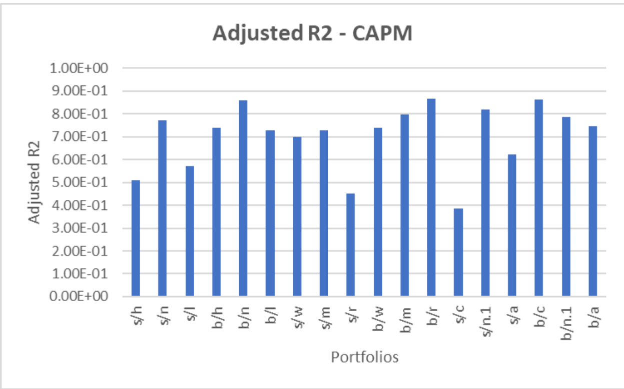
- Based on these results, all the models are statistically significant since their p-values are very close to zero.

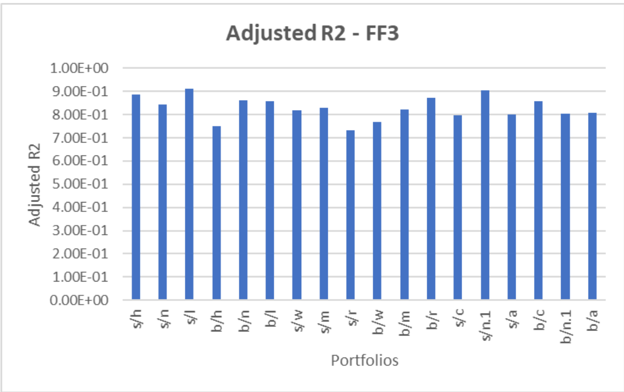
1. Data Collection

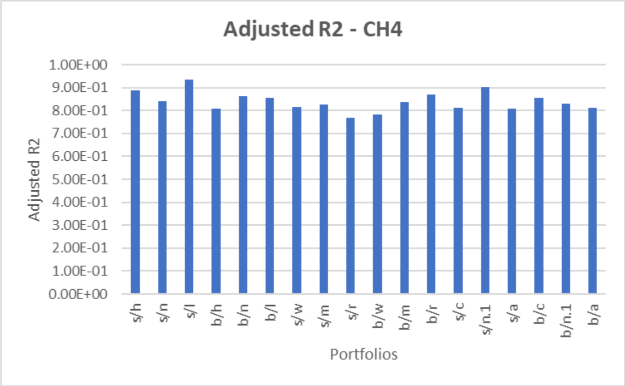
2. Portfolio Construction

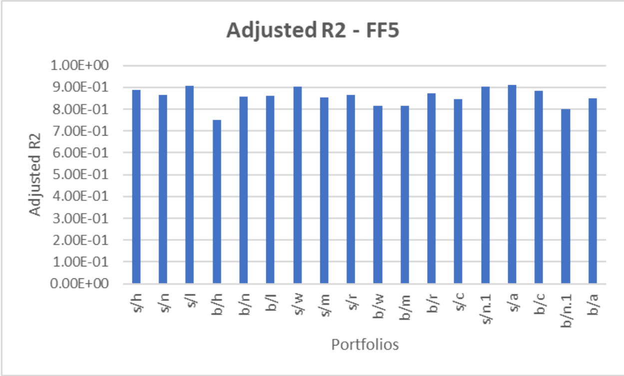
3. Model Validation and Performance measure

3.6) Performance measurement – Adjusted R^2





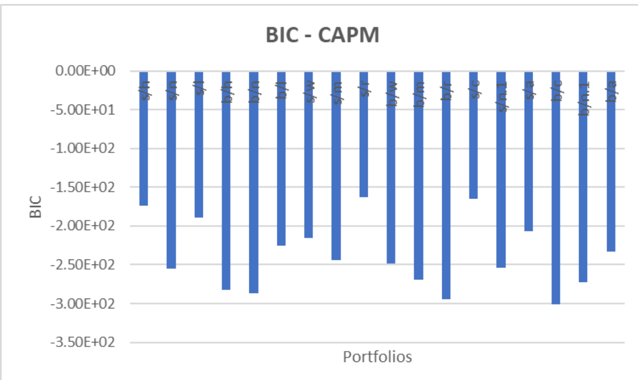


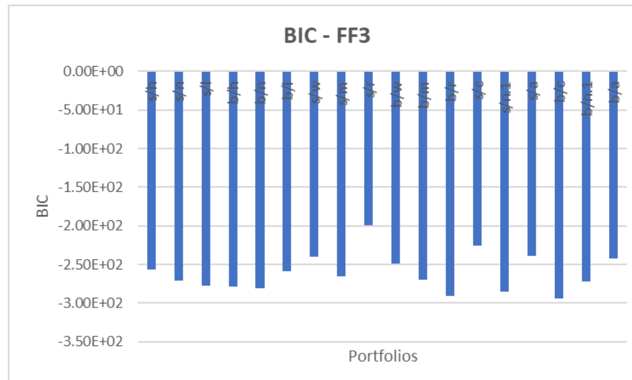


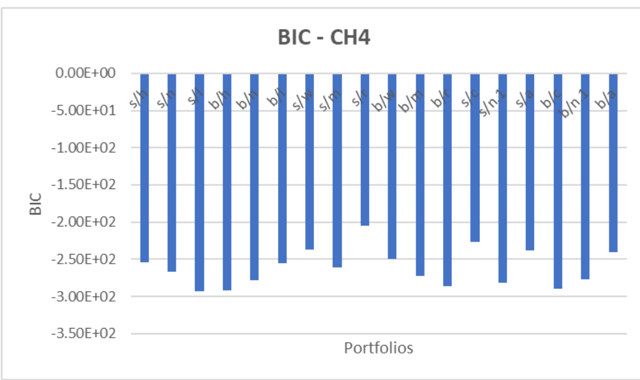
Model	Average Adj. R2
CAPM	0.705
FF3	0.829
CH4	0.840
FF5	0.859

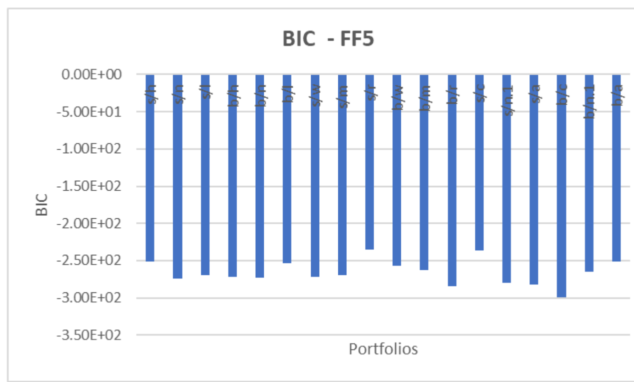
- We used adjusted R2 to measure the performance of our models. It refers to how much of total variation is explained, with a penalizing factor for more parameters.
- Based on the result on the adjusted R2, FF5 seems to have the best explanatory power among all the models, followed closely by CH4, then FF3, and lastly, CAPM.

3.7) Performance measurement – Bayesian Information Criterion (BIC)









Model	Average BIC
CAPM	-238
FF3	-261
CH4	-261
FF5	-266

- We also used BIC to measure the performance of models. a lower BIC value indicates a better model fit.
- The result of BIC aligns in the same way as adjusted R2, that FF5 is the best fit model followed by CH4, FF3 and CAPM

Q & A